SUMMARY

Equations for the force produced by an air core electromagnet on a permanent magnet core as a function of coil height, coil inner and outer radii, and core position are developed. The magnetization vector of the permanent magnet core is assumed to be aligned with the central axis of the electromagnet and the forces which are produced lie along the same axis. Variations in force due to changes in electromagnet parameters and core position are investigated and parametric plots which should be useful for coil design are presented.

INTRODUCTION

The force produced by a single magnetic suspension actuator element is typically an attractive force and is directly proportional to the square of the flux in the magnetic gap. The flux, under ideal assumptions, is directly proportional to the current in the actuator element coil and inversely proportional to the magnetic gap. In order to produce a bidirectional force, previous designs have utilized two or more actuator element pairs acting together along a single axis. Many approaches for controlling the force produced by this type of actuator have been investigated (ref. 1). A proposed approach which requires only one electromagnet element to produce a bidirectional force along a given axis is presented in reference 2. This paper develops the force equations for this type of actuator as a function of electromagnet coil height, coil inner and outer radii, and permanent magnet core position. These equations are then used to produce parametric plots which should be useful for coil design.

FORCE EQUATIONS

Figure 1 is a schematic representation of the single element magnetic suspension actuator and presents all the component parts for a complete actuator for bidirectional control along a single axis. Shown is an electromagnet, a suspended element, a permanent magnet element (or core) mounted on the suspended element to provide a magnetic field for the electromagnet to interact with, and a sensor to measure the position of the element being controlled. The center of the permanent magnet core is located a distance \( z \) from the face of the electromagnet.

In order to develop the force equations for the single element actuator, the simplified schematic of figure 2 is introduced. Shown is a cross section of the electromagnet with inner radius \( r_1 \), outer radius \( r_2 \), and height, \( h \). The center of the permanent magnet core is located a distance \( z \) from the face of the electromagnet. The flux produced by the electromagnet along the axis through the center at the distance \( z \) is obtained by expanding the equation for the on-axis field, distributed current case in reference 3. Making the appropriate substitutions results in

\[
B_z = \frac{\mu_0 J}{2} \left\{ \ln \frac{(z^2 + r_1^2)^{1/2} + r_2}{z} - \ln \frac{(z^2 + r_2^2)^{1/2} + r_1}{z} \right\}
+ \left\{ z \ln \frac{(z^2 + r_1^2)^{1/2} + r_1}{z} - z \ln \frac{(z^2 + r_2^2)^{1/2} + r_2}{z} \right\}
\] (1)
where $B_z$ is flux density, $\mu_0$ is the permeability of free space, $J$ is current density, and $\zeta$ is defined as

$$\zeta = z + h$$  \hfill (2)

Combining terms results in

$$B_z = \frac{\mu_0 J}{2} \left\{ \zeta \ln \left[ \frac{(\zeta^2 + r_2^2)^{1/2} + r_2}{(\zeta^2 + r_1^2)^{1/2} + r_1} \right] + \zeta \ln \left[ \frac{(\zeta^2 + r_2^2)^{1/2} + r_2}{(\zeta^2 + r_2^2)^{1/2} + r_2} \right] \right\}$$  \hfill (3)

From reference 4, the force on the permanent magnet core, along the axis through the center of the electromagnet, can be approximated by

$$F_z = (\text{Vol} \times M_z) \frac{\partial B_z}{\partial z}$$  \hfill (4)

where Vol is the volume of the core and $M_z$ is magnetization of the core along the $z$ axis. Taking $\partial B_z/\partial z$ results in

$$\frac{\partial B_z}{\partial z} = \frac{\mu_0 J}{2} \left\{ \begin{array}{c} \frac{\zeta^2}{[\zeta^2 + r_2^2 + r_1 (\zeta^2 + r_1^2)^{1/2}]^2} - \frac{\zeta^2}{[\zeta^2 + r_1^2 + r_2 (\zeta^2 + r_1^2)^{1/2}]^2} \\ + \begin{array}{c} \frac{z^2}{[z^2 + r_2^2 + r_1 (z^2 + r_1^2)^{1/2}]^2} - \frac{z^2}{[z^2 + r_1^2 + r_2 (z^2 + r_1^2)^{1/2}]^2} \\ \end{array} \\ + \ln \left[ \frac{(\zeta^2 + r_2^2)^{1/2} + r_2}{(\zeta^2 + r_2^2)^{1/2} + r_2} \right] + \ln \left[ \frac{(\zeta^2 + r_1^2)^{1/2} + r_1}{(\zeta^2 + r_1^2)^{1/2} + r_1} \right] \end{array} \right\}$$  \hfill (5)

The coil parameters can be made dimensionless by factoring out $\zeta^2$ and $z^2$ which results in

$$\frac{\partial B_z}{\partial z} = \frac{\mu_0 J}{2} \left\{ \begin{array}{c} \frac{1}{\left[ 1 + r_2^2 + \frac{r_1^2}{z^2} \left( 1 + \frac{r_2^2}{z^2} \right)^{1/2} \right]^2} - \frac{1}{\left[ 1 + r_1^2 + \frac{r_2^2}{z^2} \left( 1 + \frac{r_1^2}{z^2} \right)^{1/2} \right]^2} \\ + \begin{array}{c} \frac{1}{\left[ 1 + r_2^2 + \frac{r_1^2}{z^2} \left( 1 + \frac{r_2^2}{z^2} \right)^{1/2} \right]^2} - \frac{1}{\left[ 1 + r_1^2 + \frac{r_2^2}{z^2} \left( 1 + \frac{r_1^2}{z^2} \right)^{1/2} \right]^2} \\ \end{array} \\ + \ln \left[ \frac{\left( 1 + \frac{r_2^2}{z^2} \right)^{1/2} + \frac{r_2}{z^2}}{\left( 1 + \frac{r_2^2}{z^2} \right)^{1/2} + \frac{r_2}{z^2}} \right] + \ln \left[ \frac{\left( 1 + \frac{r_1^2}{z^2} \right)^{1/2} + \frac{r_1}{z^2}}{\left( 1 + \frac{r_1^2}{z^2} \right)^{1/2} + \frac{r_1}{z^2}} \right] \end{array} \right\}$$  \hfill (6)
To further simplify (6) define

\[ \frac{r_1}{z} = W_1 \]  
\[ \frac{r_2}{z} = W_2 \]  
\[ \frac{r_1}{\bar{r}} = \bar{W}_1 \]  
and

\[ \frac{r_2}{\bar{r}} = \bar{W}_2 \]  

By making the appropriate substitutions, the equation for force becomes

\[
F_z = (\text{Vol})(M_z)\left(\frac{\mu_0 J}{2}\right)\left\{\frac{1}{(1 + W_2^2) + W_2 (1 + W_2^2)^{1/2}} - \frac{1}{(1 + W_1^2) + W_1 (1 + W_1^2)^{1/2}} \right. \\
+ \frac{1}{(1 + W_2^2) + W_1 (1 + W_2^2)^{1/2}} - \frac{1}{(1 + W_2^2) + W_2 (1 + W_2^2)^{1/2}} \\
+ \ln \left[\frac{(1 + W_2^2)^{1/2} + W_2}{(1 + W_2^2)^{1/2} + W_1}\right] + \ln \left[\frac{(1 + W_1^2)^{1/2} + W_1}{(1 + W_2^2)^{1/2} + W_2}\right]\right\}
\]

\[ \text{(11)} \]

**ANALYSIS APPROACH**

For purposes of determining the effects of different coil parameters on force, the first part of equation (11), \((\text{Vol})(M_z)(\mu_0 J/2)\), is assumed to be constant and is defined as

\[ (\text{Vol})(M_z)(\mu_0 J/2) = K \]  

(12)

To put the results in more useful form, the dimensionless parameters \(K_r\) and \(K_h\) are introduced where

\[ K_r = \frac{r_2}{r_1} \]  

(13)

and

\[ K_h = \frac{h}{z} \]  

(14)

These two parameters, together with \(W_1\) (see eq. (7)), can be used to define a coil size for a given gap (z). The other parameters in the second part of (11) can be defined in terms of these parameters as follows:

\[ W_2 = K_r W_1 \]  

(15)

\[ W_1 = W_1/(1 + K_h) \]  

(16)

\[ W_2 = K_r W_1 \]  

(17)

The approach was to set \(K_h\) and \(W_1\) to arbitrary values and calculate \(F_z/K\) as a function of \(K_r\). The results of these calculations were used to determine the value of \(K_r\), within reasonable physical constraints, which produced the maximum value of \(F_z/K\). \(K_r\) was then set at this value and \(K_h\) varied. Finally, \(K_h\) and \(K_r\) were fixed and \(W_1\) varied.
RESULTS AND DISCUSSION

Using the approach described above, $K_h$ and $W_1$ were fixed at a value of 1.0 and $F_z/K$ was calculated as $K_r$ was varied over a wide range. Figure 3 is a plot of the results. As shown in the figure, $F_z/K$ appears essentially constant for values of $K_r$ above 10.0 and changes very little for values of $K_r$ between 5.0 and 10.0. Figure 4 is a plot of $F_z/K$ as $K_r$ is varied from 1.0 to 5.0. As can be seen using this expanded scale, very little is gained beyond a value of 3.0 for $K_r$. Also, above this value practical coil size becomes a factor. For example, a value of 3.0 for $K_r$ would require a coil with an inner radius of 1.0 inch to have an outer radius of 3.0 inches. Using a value of 3.0 for $K_r$ and 1.0 for $W_1$, $F_z/K$ was next calculated as a function of $K_h$. A plot of the results is presented in figure 5. This figure indicates a small change in $F_z/K$ above a value of 2.0 for $K_h$. Finally, using values of 3.0 and 2.0 for $K_r$ and $K_h$ respectively, $F_z/K$ was calculated as a function of $W_1$. A plot of the results of these calculations is presented in figure 6. This figure indicates a maximum value of $F_z/K$ at a value of approximately 0.65 for $W_1$. Although $W_1$ is useful for determining coil parameters, the trend for $F_z/K$ that is indicated is one that results from $r_1$ being varied around a fixed $z$ or gap. A more useful trend results from plotting $F_z/K$ as a function of $1/W_1$. This indicates the variation of $F_z/K$ as $z$ is varied with respect to a fixed $r_1$. This plot is presented in figure 7. This figure indicates that the operating point for a suspended element should be above the maximum force point since the change in force with respect to $z$ is much smaller in this region than it is below the maximum force point.

CONCLUDING REMARKS

Equations for the force produced by an air core electromagnet on a permanent magnet core as a function of coil height, coil inner and outer radii, and core position have been developed. The magnetization vector of the permanent magnet core was assumed to be aligned with the central axis of the electromagnet and the forces produced were assumed to lie along the same axis. Variations in force due to changes in electromagnet parameters and core position were investigated and the results presented in the form of plots. The plots indicated that a value of approximately 0.65 for the ratio of the coil inner radius to core position produced a maximum force value. They also indicated that very small gains in force could be obtained for values greater than 3.0 for the ratio of outer to inner coil radius and 2.0 for the ratio of coil height to core position. A given application may require that smaller ratios be selected because of size constraints. In this case the plots should be useful for conducting trade-offs on the different parameters so that the best design within the constraints can be obtained.

REFERENCES

Figure 1.-Schematic representation of a single element bidirectional magnetic suspension actuator.
Figure 2: Electromagnet cross section
Figure 3.- Variation of $F_z/K$ with $K_r$
Figure 4.- Variation of $F_z/K$ with $K_r$ (expanded scale)

$K_h = 1$
$W_1 = 1$
Figure 5. Variation of $Fz/K$ with $Kh$

$K_r = 3$
$W_1 = 1$
Figure 6.- Variation of $Fz/K$ with $W1$

$K_r = 3$
$K_h = 2$
Figure 7.- Variation of $Fz/K$ with $1/W1$
## Design Considerations for an Air Core Magnetic Actuator

### Abstract (Maximum 200 words)

Equations for the force produced by an air core electromagnet on a permanent magnet core as a function of coil height, coil inner and outer radii, and core displacement are developed. The magnetization vector of the permanent magnet core is assumed to be aligned with the central axis of the electromagnet and the forces which are produced lie along the same axis. Variations in force due to changes in electromagnet parameters and core displacement are investigated and parameter plots which should be useful for coil design are presented.

### Subject Terms

- Magnetic Suspension
- Magnetic Actuator
- Air Core Magnetic Actuator