Annual Status Report

For the Period: April 1, 1991 - March 31, 1992

Under NASA Grant No. NAG 9-525, Basic

PHYSICS OF HEAT PIPE REWETTING

by

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Prepared for

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NAG9-525.RTP/CHAN
This is a brief, annual status report which includes the theoretical achievement and the experimental setup during the report period of April 1, 1991 to March 31, 1992.

The theoretical achievement is based on our paper entitled "Rewetting Theory and the Dryout Heat Flux of Smooth and Grooved Plates with a Uniform Heating" (see Appendix A) which has been accepted in the "Heat Pipes and Thermosyphons" session at the 1992 ASME Winter Annual Meeting. Although several investigations have been made to determine the rewetting characteristics of liquid films on heated rods, tubes and flat plates, no solutions are yet available to describe the rewetting process of a hot plate subjected to a uniform heating. In our paper, a model is presented to analyze the rewetting process of such plates with and without grooves. Approximate analytical solutions are presented for the prediction of the rewetting velocity and the transient temperature profiles of the plates. It is shown that the present rewetting velocity solution reduces correctly to the existing solution for the rewetting of an initially hot isothermal plate without heating from beneath the plate. Numerical solutions have also been obtained to validate the analytical solutions. In Appendix A, the successful prediction of the rewetting curve by the approximate analytical solution for the grooved (or smooth) plate with a uniform heating is illustrated. It shows that the approximate closed form solution is in reasonably good agreement with the numerical solution. Furthermore, a simple method is presented in Appendix A to predict the dryout heat flux of a liquid film flowing over a heated smooth or grooved plate. The results of the prediction are found to be in reasonable agreement with the existing experimental data.

During this first annual period, an experimental system has been set up, design of which has been reported in our semiannual status report on October 18, 1991, which includes (1)
microfin surface geometry of the grooved plate; (2) the selection of the test fluid and plate material; (3) surface temperature measurement mechanism; (4) liquid flow rate control and measurement design; (5) heat source of constant temperature/variable heat flux system. The complete experimental system including the data acquisition system was being built and assembled during the second half of the year. Some shake-up experiments to measure rewetting velocity and the surface temperature distribution of the plate will be conducted. Figure 1 shows the overall experimental system while Figure 2 shows the test section. Some experimental results will be reported in the next semiannual status report.

Fig. 1. Heat Pipe Rewetting Experimental System
Fig. 2. Test Section
APPENDIX

REWETTING THEORY AND THE DRYOUT HEAT FLUX OF SMOOTH AND GROOVED PLATES WITH A UNIFORM HEATING

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ABSTRACT

The evaporation and condensation of thin liquid films are of significant importance in a wide variety of problems ranging from specific applications in the heat pipe field to more general ones in chemical, nuclear and petrochemical industries. Although several investigations have been conducted to determine the rewetting characteristics of liquid films on heated rods, tubes and flat plates, no solutions are yet available to describe the rewetting process of a hot plate subjected to a uniform heating. A model is presented to analyze the rewetting process of such plates with and without grooves. Approximate analytical solutions are presented for the prediction of the rewetting velocity and the transient temperature profiles of the plates. It is shown that the present rewetting velocity solution reduces correctly to the existing solution for the rewetting of an initially hot isothermal plate without heating from beneath the plate. Numerical solutions have also been obtained to validate the analytical solutions. Finally, a simple method is presented to predict the dryout heat flux of a liquid film flowing over a heated smooth or grooved plate. The results of the prediction are found to be in reasonable agreement with the existing experimental data.

NOMENCLATURE

\begin{align*}
K & \text{ thermal conductivity (W/m·°C)} \\
L & \text{ length of the plate (m)} \\
L_1 & \text{ length of wet region (m)} \\
L_2 & \text{ length of dry region (m)} \\
N & \text{ grooved geometric coefficient} \\
n & \text{ exponent defined in eq. (65)} \\
P & \text{ dimensionless rewetting velocity} \\
qu & \text{ uniform heat flux (W/m²)} \\
qu_{\text{CHF}} & \text{ critical heat flux (CHF) (W/m²)} \\
qu_{\text{ib}} & \text{ incipient boiling heat flux (W/m²)} \\
qu_{\text{max}} & \text{ maximum heat flux (W/m²)} \\
s & \text{ plate thickness for smooth (s₁) or grooved (s₁ - ℓ₁) plates (m)} \\
s₁ & \text{ plate thickness (m)} \\
s₂ & \text{ liquid film thickness (m)} \\
T & \text{ temperature (°C)} \\
T_e & \text{ environmental temperature (°C)} \\
T_f & \text{ inlet liquid temperature (°C)} \\
T_i & \text{ incipient boiling temperature (°C)} \\
T_o & \text{ Leidenfrost temperature (°C)} \\
T_s & \text{ saturation temperature (°C)} \\
T₁ & \text{ initial hot surface temperature (°C)} \\
t & \text{ time (sec)} \\
U_r & \text{ rewetting front velocity (m/s)} \\
x & \text{ length of liquid film in moving front coordinate (m)} \\
x' & \text{ length of liquid film in stationary system (m)}
\end{align*}
**Greek symbols**

\( \alpha_1 \) constant defined in eq. (23)

\( \alpha_2 \) constant defined in eq. (45)

\( \beta_1 \) constant defined in eq. (23)

\( \beta_2 \) constant defined in eq. (45)

\( \eta \) dimensionless length coordinate with respect to \( x \)

\( \eta' \) dimensionless length coordinate with respect to \( x' \)

\( \theta \) dimensionless temperature

\( \theta_b \) dimensionless temperature of the transient part

\( \theta_s \) dimensionless temperature of the steady-state part

\( \tau \) dimensionless time

\( \rho \) density (Kg/m\(^3\))

**Subscripts**

\( d \) dry region

\( g \) grooved plate

\( L \) total length of the plate

\( L_1 \) length of wet region

\( L_2 \) length of dry region

\( s \) smooth plate

\( w \) wet region

**INTRODUCTION**

The rewetting process is a conjugated heat transfer problem involving interactions between a solid wall and flowing fluids. The process for rewetting of a grooved plate with a uniform heating is complicated, as the rewetting velocity varies with time, physical geometry of the grooves, plate properties, fluid properties and the applied heat flux. Although several investigations (Yamanouchi, 1968; Thompson, 1972; Duffey, 1973; Sun, 1974; Alario, 1983; Grimley, 1988; Stroes, 1990; Peng, 1991; Peng and Peterson, 1991) have been made to determine the rewetting characteristics of liquid films on heated rods, tubes and flat plates, none has yet presented the solution for the rewetting process of a heated plate with a smooth or a grooved surface subjected to a uniform heat flux. It is noted that the surface with small grooves has received increasing attention as it has many practical applications. For instance, the microgrooved surface is employed most often to enhance heat transfer (Grimley, 1988). Microgrooves are also useful for replacing the wicking material in heat pipes. In fact, it is used in the innovative monogroove heat pipes. In fact, it is used in the innovative monogroove heat pipes.

The above governing equation is different from the equation in all prior rewetting models (Yamanouchi, 1968; Duffey, 1973; Sun, 1974; Peng, 1991 et al.) in that an extra term, namely, the transient term on the left-hand side of the equation, is added. The additional of this transient term is essential in analyzing the rewetting process of the heated plate subjected to a heat flux conduction as shown in Fig. 1. This is because the temperature profile of the plate in the Lagrangian coordinate (x, y) is no longer invariant with time as in the case of rewetting an initially hot isothermal plate without heating from beneath the plate. Without

**REWETTING MODEL AND SOLUTIONS**

Consider the rewetting process of a hot plate initially at a uniform temperature \( T_1 \) with no liquid on the plate as shown in Fig. 1, the plate is heated below by a uniform heat flux and is quenched by a liquid advancing along the direction of the grooves on the top surface of the plate. In order to simplify the complexity of the physical phenomena of the rewetting process, we consider first a heated smooth surface plate with no parallel grooves.

1. Smooth Surface Plate

The rewetting process of a hot dry plate is sketched in Fig. 1. A liquid film from a liquid reservoir, driven by its surface tension, is to advance along the hot plate. Similar to prior studies by Yamanouchi (1968), Duffey and Porthouse (1973), and Sun et al. (1974), the initial temperature of the plate is assumed to be higher than Leidenfrost temperature \( T_0 \) such that the rewetting process is assumed to be conduction-controlled. Also, an averaged heat transfer coefficient is assumed in the wet region to remove the heat from the thin plate to the liquid film, no heat loss to the environment is assumed in the dry region, the plate at the rewet front is assumed to remain at a constant Leidenfrost temperature, and the plate is thin enough that the one-dimensional rewetting model can be invoked. It is therefore proposed to solve the following governing equation (see Appendix A) on a Lagrangian coordinate moving with the rewetting front,

\[
\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + P \frac{\partial \theta}{\partial \eta} - B \theta + A
\]

where

\[
P = \frac{U\gamma_s \rho C_p}{K} \quad B = \frac{h_s I}{K} \quad A = \frac{q s_1}{K (T_0 - T_s)} \quad \theta (\eta, \tau) = \frac{T - T_s}{T_0 - T_s} \quad \eta = \frac{x}{s_1} \quad \tau = \frac{t}{(s_1 \rho C_p \Delta K)}
\]

where, \( B \) is the Biot number; \( A, P, \theta(\eta, \tau), \eta \) and \( \tau \) are the dimensionless heat source, rewetting velocity, temperature, length and time respectively. In the wet region \((-L_1 \leq x \leq 0)\), \( h = \text{constant} \neq 0 \) while in the dry region \((0 \leq x \leq L_2)\), \( h = B = 0 \).

The above governing equation is different from the equation in all prior rewetting models (Yamanouchi, 1968; Duffey, 1973; Sun, 1974; Peng, 1991 et al.) in that an extra term, namely, the transient term on the left-hand side of the equation, is added. The addition of this transient term is essential in analyzing the rewetting process of the heated plate subjected to a heat flux conduction as shown in Fig. 1. This is because the temperature profile of the plate in the Lagrangian coordinate (x, y) is no longer invariant with time as in the case of rewetting an initially hot isothermal plate without heating from beneath the plate. Without
it, the incompatibility difficulty as noted above will arise and the final solution can not be expected to satisfy the transient boundary condition at $x = \infty$. The addition of the transient term, however, renders some mathematical complications in the solution of the rewetting velocity.

The initial condition is

$$\theta (\xi, 0) = \frac{T_1 - T_s}{T_0 - T_s} = \theta_1$$  \hspace{1cm} (3)

while the boundary conditions are,

$$\theta (-\eta_{L1}, r) = 0$$  \hspace{1cm} (4)

$$\theta (0, r) = 1$$  \hspace{1cm} (5)

$$\theta (\eta_{L2}, r) = \frac{T_L - T_s}{T_0 - T_s} = \theta_1 + A\tau$$  \hspace{1cm} (6)

where $\eta_{L1}$ and $\eta_{L2}$ are defined as

$$\eta_{L1} = \frac{L_1}{s_1}, \eta_{L2} = \frac{L_2}{s_1}$$  \hspace{1cm} (7)

and $T_L$ is the dry plate temperature at $\eta_{L2}$. The condition given in eq. (6) implies that the plate is long enough such that, within the time period of interest, the plate temperature at $\eta_{L2}$ is not thermally affected by the rewetting process. Consequently, it can be expressed as

$$T_L = T_1 + \frac{q_l}{\rho C_p s_1}$$  \hspace{1cm} (8)

which is used in eq. (6).

In the wet region ($\eta_{L1} \leq \eta \leq 0$), the mathematical model of this problem is given as

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + P \frac{\partial \theta}{\partial \eta} - B \theta + A$$  \hspace{1cm} (9)

$$\theta (\eta, 0) = \theta_1$$  \hspace{1cm} (10)

$$\theta (-\eta_{L1}, \tau) = 0$$  \hspace{1cm} (11)

$$\theta (0, \tau) = 1$$  \hspace{1cm} (12)

To solve for the exact, analytical solution of the above appears to be difficult, if not impossible. Therefore an approximate, analytical solution is sought. This is made possible by treating the Peclet number, $P$, as a constant value in the mathematical deliberation in order to achieve a closed form solution. This approximation appears to be reasonable as the rewetting velocity tends to reach a quasi-steady state after an initial period when the liquid film is brought in contact with the hot plate. The numerically exact solution will also be presented later to check the accuracy of the closed form solution. Accordingly, the solution is split into two parts,

$$\theta (\eta, \tau) = \theta_{s,w} (\eta) + \theta_{h,w} (\eta, \tau)$$  \hspace{1cm} (13)

where $\theta_{s,w}$ is the solution for the steady-state part of the problem,

$$\frac{\partial^2 \theta_{s,w}}{\partial \eta^2} + P \frac{\partial \theta_{s,w}}{\partial \eta} - B \theta_{s,w} + A = 0$$  \hspace{1cm} (14)

$$\theta_{s,w} (0) = 1$$  \hspace{1cm} (15)

$$\theta_{s,w} (-\eta_{L1}) = 0$$  \hspace{1cm} (16)

while $\theta_{h,w}$ is the solution to the transient part of the problem

$$\frac{\partial \theta_{h,w}}{\partial \tau} = \frac{\partial^2 \theta_{h,w}}{\partial \eta^2} + P \frac{\partial \theta_{h,w}}{\partial \eta} - B \theta_{h,w}$$  \hspace{1cm} (17)

$$\theta_{h,w} (\eta, 0) = \theta_1 - \theta_{s,w} (\eta)$$  \hspace{1cm} (18)

$$\theta_{h,w} (0, \tau) = 0$$  \hspace{1cm} (19)

$$\theta_{h,w} (-\eta_{L1}, \tau) = 0$$  \hspace{1cm} (20)

The solution of the steady-state problem is given as

$$\theta_{s,w} (\eta) = \left[ 1 - \frac{A}{B} \right] e^{-\frac{\eta}{s_1}} - \left[ \frac{(1 - \frac{A}{B}) e^{-\frac{\eta_{L1}}{s_1}} + \frac{A}{B}}{e^{-\frac{\eta_{L1}}{s_1}} - e^{-\frac{\eta_{L2}}{s_1}}} \right] e^{rac{\eta}{s_1}}$$  \hspace{1cm} (21)

$$+ \frac{1}{e^{-\frac{\eta_{L1}}{s_1}} - e^{-\frac{\eta_{L2}}{s_1}}} \left[ \frac{1 + \frac{A}{B} e^{-\frac{\eta_{L1}}{s_1}} + \frac{A}{B}}{e^{-\frac{\eta_{L1}}{s_1}} - e^{-\frac{\eta_{L2}}{s_1}}} \right] e^{rac{\eta}{s_1}} + \frac{A}{B}$$

$$r_1 = -P + \sqrt{P^2 + 4B}$$

$$r_2 = -P - \sqrt{P^2 + 4B}$$

To solve for the transient solution, the following transformation is introduced,

$$\theta_{h,w} (\eta, \tau) = e^{r_1 \eta + r_2 \tau} \nu(\eta, \tau)$$  \hspace{1cm} (22)
where
\[ \alpha_1 = -\frac{P}{2}, \beta_1 = -B - P^2/4 \]  

(23)

Then eqs. (17) to (20) become
\[ \frac{\partial \nu}{\partial \tau} = \frac{\partial^2 \nu}{\partial \eta^2} \]  

(24)

\[ \nu (\eta,0) = e^{-\alpha_1 \theta} (\theta_1 - \theta_{s,w}(\eta)) = f_1(\eta) \]  

(25)

\[ \nu (0,\tau) = 0 \]  

(26)

\[ \nu (-\eta_{L1},\tau) = 0 \]  

(27)

The solution of the above can be readily obtained from Carslaw (1959) as follows:
\[ \nu (\eta,\tau) = \sum_{n=1}^{\infty} a_{n1} \sin \left( \frac{n \pi \eta}{\eta_{L1}} \right) e^{-n^2 \frac{\pi^2 \tau}{\eta_{L1}^2}} \]  

(28)

where
\[ a_{n1} = -\frac{2}{\eta_{L1}} \int_{-\eta_{L1}}^{0} f_1(V) \sin \left( \frac{n \pi V}{\eta_{L1}} \right) dV \]  

(29)

\[ f_1(V) = e^{-\alpha_1 V} \{ \theta_1 - \left[ 1 - \frac{A}{B} \right] e^{-r_1 \eta_{L1}} + \frac{A}{B} \} e^{r_1 V} \]  

(30)

Therefore, the combined solution is
\[ \theta (\eta,\tau) = \theta_{s,w}(\eta) + \theta_{h,w}(\eta,\tau) \]

\[ = \left[ 1 - \frac{A}{B} \right] - \frac{(1 - \frac{A}{B}) e^{-r_1 \eta_{L1}} + \frac{A}{B}}{e^{-r_1 \eta_{L1}} - e^{-r_2 \eta_{L1}}} e^{r_1 \eta} \]  

(31)

\[ + \left[ 1 + \frac{A}{B} \right] e^{-r_2 \eta_{L1}} + \frac{A}{B} \frac{A}{B} \]  

\[ e^{r_2 \eta} - e^{-r_2 \eta_{L1}} \]

\[ + \frac{A}{B} e^{a_1 \eta + \beta_1 \tau} \sum_{n=1}^{\infty} a_{n1} \sin \left( \frac{n \pi \eta}{\eta_{L1}} \right) e^{-n^2 \frac{\pi^2 \tau}{\eta_{L1}^2}} \]

where \( a_{n1} \) and \( f_1(V) \) are given by eqs. (29) and (30) respectively, \( \alpha_1 \) and \( \beta_1 \) are given by eq. (23).

Before proceeding to the solution for the dry region, it is noted that the above steady-state solution is identical to the solution presented by previous workers (Sun et al., 1974) when \( A = 0 \) (or \( q = 0 \)).

Similarly, the problem in the dry region (\( 0 \leq \eta \leq \eta_{L2} \)) is described by
\[ \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + P \frac{\partial \theta}{\partial \eta} + A \]  

(32)

\[ \theta (\eta,0) = \theta_1 \]  

(33)

\[ \theta (0,\tau) = 1 \]  

(34)

\[ \theta (\eta_{L2},\tau) = \theta_1 + A \tau \]  

(35)

which is likewise split up into the steady-state and transient solution,
\[ \frac{d^2 \theta_{s,d}}{d\eta^2} + P \frac{d\theta_{s,d}}{d\eta} + A = 0 \]  

(36)

\[ \theta_{s,d}(0) = 1 \]  

(37)

\[ \theta_{s,d}(\eta_{L2}) = \theta_1 \]  

(38)
\[
\frac{\partial \theta_{h,d}}{\partial \tau} = \frac{\partial^2 \theta_{h,d}}{\partial \eta^2} + P \frac{\partial \theta_{h,d}}{\partial \eta} 
\]

\[
\theta_{h,d}(\eta,0) = \theta_{i} - \theta_{s,d}(\eta) 
\]

\[
\theta_{h,d}(0,\tau) = 0 
\]

\[
\theta_{h,d}(\eta_{L_2},\tau) = A \tau 
\]

The solution of the steady-state problem is

\[
\theta_{s,d}(\eta) = \left[ 1 - \frac{\theta_1 + \frac{A}{P} \eta_{L_2} - 1}{e^{-\eta_{L_2}} - 1} \right] - \frac{A}{P} \eta 
\]

\[
+ \left[ \frac{\theta_1 + \frac{A}{P} \eta_{L_2} - 1}{e^{-\eta_{L_2}} - 1} \right] e^{-\eta_{L_2}} 
\]

A similar transformation is introduced

\[
\theta_{h,d}(\eta, \tau) = e^{\alpha_2 \eta} \beta_2^2 u(\eta, \tau) 
\]

where: \( \alpha_2 = -p/2, \beta_2 = -p^2/4 \)

To reduce eqs. (39) to (42) to the form

\[
\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \eta^2} 
\]

\[
u(\eta, 0) = (\theta_1 - \theta_{s,d}(\eta)) e^{-\alpha_2 \eta} = f_2(\eta) 
\]

\[
u(0, \tau) = 0 
\]

\[
u(\eta_{L_2}, \tau) = A \tau \cdot e^{-(\alpha_2 \eta_{L_2} + \beta_2 \tau)} = \phi_2(\tau) 
\]

which can be solved by the method of linear superposition. Thus, the transient temperature profile in the dry region can be readily found as

\[
\theta(\eta, \tau) = e^{\alpha_2 \eta} \beta_2^2 \left[ \sum_{n=1}^{\infty} a_{n_2} \sin \left( \frac{n \pi \eta}{\eta_{L_2}} \right) e^{-n^2 \pi^2 \eta_{L_2}} \right] 
\]

\[
+ \frac{\eta_{L_2}}{\eta_{L_2}} (A \tau) e^{-\alpha_2 \eta_{L_2} + \beta_2 \tau} + \frac{2}{\eta_{L_2}} \sum_{n=1}^{\infty} (-1)^n \sin \frac{\beta_n \eta}{\beta_n} 
\]

\[
\left[ e^{-\rho_2^2 - \beta_2^2} (\rho_2^2 - \beta_2^2) \right] + 1 - \frac{\theta_1 + \frac{A}{P} \eta_{L_2} - 1}{e^{-\rho_2^2 \eta_{L_2}} - 1} 
\]

\[
+ \left[ \frac{\theta_1 + \frac{A}{P} \eta_{L_2} - 1}{e^{-\rho_2^2 \eta_{L_2}} - 1} \right] e^{-\rho_2^2 \eta_{L_2}} - \frac{A}{P} \eta 
\]

where \( \beta_n = \pi \eta_{L_2} \) and

\[
a_{n_2} = \frac{2}{\eta_{L_2}} \int_{0}^{\eta_{L_2}} f_2(w) \sin \frac{n \pi \eta}{\eta_{L_2}} dw 
\]

At the rewetting front, the conductive heat flux is continuous, i.e.,

\[
\frac{\partial \theta}{\partial \eta} \bigg|_{\eta=0^+} = \frac{\partial \theta}{\partial \eta} \bigg|_{\eta=0^-} 
\]

Upon the substitution of eqs. (31) and (50) into eq. (51), the dimensionless rewetting velocity \( P \) can be determined by the expression

\[
- \frac{P}{2} \left[ 1 + \frac{A}{B} \right] + \frac{\sqrt{P^2 + 4B}}{2} \left[ 1 - \frac{A}{B} \right] - \sqrt{P^2 + 4B} 
\]

\[
\left[ \frac{(1 - \frac{A}{B}) e^{-\rho_1 \eta_{L_1}} + \frac{A}{B} e^{-\rho_1 \eta_{L_1}} - e^{-\rho_1 \eta_{L_1}} - e^{-\rho_2 \eta_{L_1}}}{e^{-\rho_1 \eta_{L_1}} - e^{-\rho_2 \eta_{L_1}}} \right] + e^{\beta_1^2} \sum_{n=1}^{\infty} \frac{\pi \eta}{\eta_{L_1}} a_{n_1} e^{-n^2 \pi^2 \eta_{L_1}} 
\]

\[
e^{\beta_2^2} \sum_{n=1}^{\infty} a_{n_2} \left( \frac{\pi \eta}{\eta_{L_2}} \right) e^{-n^2 \pi^2 \eta_{L_2}} + \frac{1}{\eta_{L_2}} (A \tau) e^{-\alpha_2 \eta_{L_2}} 
\]

\[
+ 2 \frac{e^{\beta_2^2}}{\eta_{L_2}} \sum_{n=1}^{\infty} (-1)^n \left( e^{-\alpha_2 \eta_{L_2} - \beta_2^2} \right) \left( e^{(\alpha_2^2 + \beta_2^2) \tau - \gamma_2^2} \right) \left( e^{(\alpha_2^2 + \beta_2^2) \tau - \gamma_2^2} \right) 
\]

\[
+ \frac{\beta_2}{(\beta_2^2 - \beta_2)} \left( e^{(\alpha_2^2 + \beta_2) \tau - \gamma_2^2} \right) \right) - P \left[ \frac{\theta_1 + \frac{A}{P} \eta_{L_2} - 1}{e^{-\rho_2^2 \eta_{L_2}} - 1} \right] - \frac{A}{\eta_{L_2}} 
\]

(52)
It is of interest to examine the limiting solution of the above for the case that has been investigated by Yamanouchi (1968), namely, the rewetting of an infinitely long, hot and isothermal plate without any heating. In this case, \( A=0 \) (i.e., \( q=0 \)) and then by setting \( \eta_{L1}=\eta_{L2} \rightarrow \infty, \tau \rightarrow \infty \), the above solution reduces to

\[
U_r = \left\{ \frac{\rho C_p}{2} \right\} \left[ \frac{s_1}{kh} \left[ \frac{2(T_1-T_0)}{T_0-T_s} + 1 \right] - 1 \} \right]^{-1} \tag{53}
\]

which is exactly the same as the well known Yamanouchi's solution (1968).

Due to the absence of data on rewetting velocity on a hot plate heated by a uniform heat flux beneath the plate, the solution given by eq. (52) cannot be compared with experimental data. However, experimental data are available on a hot plate without heating from below. Yamanouchi (1968), for example, has confirmed reasonable agreement between the limiting solution given by eq. (53) and his data. The general solution presented above for a smooth plate will be used in the following section to yield the solution for the plate with axial grooves.

2. Grooved Plate

The rewetting model of the grooved plate is based on that of the smooth surface plate. The coolant is driven by the wicking (surface tension) effect of microfins and is assumed, without loss of generality, to fill up the grooves as shown in Fig. 2.

At the level of \( y = 0 \),

\[
-K \frac{\partial T_1}{\partial y} \bigg|_{y=0} \cdot (2t_2 + t_3) = h(T - T_s)(2t_1 + t_2)
\]

\( + h_s(T - T_a) \cdot 2t_2 \)

(54)

where, \( h \) is the convective coefficient of a smooth surface, \( T_s \) is the environmental temperature, \( h_s \) is the convective coefficient between the plate surface and the environmental gas above (\( h_s = 0 \)). At the level of \( y = s_1 - t_1 \),

\[
\frac{\partial T_1}{\partial y} \bigg|_{y=(s_1-t_1)} = \frac{q}{K}
\]

(55)

Since the plate is thin,

\[
\frac{\partial^2 T}{\partial y^2} = \lim_{\delta y \to 0} \left[ \frac{\partial T_1}{\partial y} \bigg|_{y=0} - \frac{\partial T_1}{\partial y} \bigg|_{y=(s_1-t_1)} \right] \frac{s_1 - t_1}{s_1 - \ell_1}
\]

Combining Eqs. (54), (55) and (56) yields

\[
\frac{\partial^2 T}{\partial y^2} = - \left[ \frac{Nh}{K} (T - T_s) - \frac{q}{K} \right] / (s_1 - t_1)
\]

(57)

where the grooved geometric coefficient is defined as

\[
N = (2t_1 + t_3)/(2t_2 + t_3)
\]

The result given by eq. (57) suggests two useful simplifications. First, the factor \((Nh)\) is the equivalent convective heat transfer coefficient of the grooved plate and can be approximated by that of a thin liquid film on a smooth plate multiplied by a factor of \(N\), namely,

\[
\left( \text{heat transfer coefficient of a grooved plate} \right) = N \left( \text{heat transfer coefficient of a smooth plate} \right)
\]

(58)

where \(N\) is more generally defined as

\[
N = \frac{\text{the wetted perimeter}}{\text{width of the cross section}}
\]

(59)

Second, the governing equation for the grooved plate with a uniform heating remains unchanged provided \( h \) is replaced by \((Nh)\) and the dimensionless variables are properly scaled as follows:

\[
P = U_r \rho C_p (s_1 - t_1) / K ; \quad B = Nh (s_1 - t_1) / K
\]

\[
A = q (s_1 - t_1) / \left[ K(T_s - T_a) \right] ; \quad \eta = x / (s_1 - t_1)
\]

\[
\eta_{L1} = L_1 / (s_1 - t_1) ; \quad \eta_{L2} = L_2 / (s_1 - t_1)
\]

\[
\tau = t / [(s_1 - t_1)^2 \rho C_p / K]
\]

(59)

where \( h \) is the convective coefficient of the smooth plate with a uniform heating. Therefore, the solution of the grooved plate with a uniform heating in the wet region is the same as eq. (31) with the above modification and the solution for the dry region is identical to eq. (50).

Numerical solutions have also been obtained by solving for the original governing equation (see Appendix A, eq. (A7)) fixed to the nonmoving coordinates \((x', t)\),
\[ \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} - B \theta + A \]  

subject to

\[ \theta (\eta', 0) = \theta_1 \]  

\[ \theta (0, \tau) = 0 \]  

\[ \theta (\eta'_L, \tau) = \theta_1 + A \tau \]  

where

\[ \eta' = x'/s, \quad \eta'_L = L/s \]

and \( s = s_1 \) and \( (s_1 - \ell_1) \) for the smooth and grooved plates respectively. The rewetting front location, \( \eta'_L,1 \), is determined from \( \theta(\eta'_L,1, \tau) = 1 \). The differential equation is discretized by a standard finite-difference approach. Figure 3 illustrates the computed wall temperature of the grooved (or smooth) plate versus the plate length, the so-called rewetting temperature curve, at various times when the liquid film is FC-72, which is Case 1 of Fig. 4. These numerical values are selected from the experimental condition of Fig. 5 of Grimley et al. (1988), namely, for FC-72 fluid on the smooth surface, \( T_i - T_s = 11^\circ C, T_0 - T_s = 17.5^\circ C, T_s = 56^\circ C, T_1 = 100^\circ C \) from which \( \theta_1 \) is calculated. Other dimensionless parameters \( A, B \) and \( \eta_1 \) are estimated from the following values. The needed heat transfer coefficient \( h \) is calculated from their boiling curve by eq. (65). For the grooved copper plate, \( C_p = 383.1 \text{ J/kg}^\circ C, \rho = 8954 \text{ kg/m}^3, K = 386 \text{ W/m}^\circ C. \) When the liquid fills up the grooves as shown in Fig. 2, the grooved geometric coefficient is \( N = 1.75 \), which is based on their geometric dimension of \( \ell_1 = 0.5 \text{ mm}, \ell_2 = 0.2 \text{ mm}, \ell_3 = 0.4 \text{ mm}, s_1 = 6 \text{ mm} \), and \( q = 24000 \) and \( 100000 \text{ W/m}^2 \) for case 1 and 2 respectively. They also reported dryout heat flux data which will be used in the later comparison. From Fig. 3, it is clearly shown that the rewetting temperature profiles are transient in nature, even in the coordinate frame moving with the rewetting front.

The successful prediction of the rewetting curve by the approximate analytical solution, eq. (52), for the grooved (or smooth) plate with a uniform heating is illustrated in Fig. 4. It shows that the approximate closed form solution is in reasonably good agreement with the numerical solution. This may appear to be somewhat of a surprise in view of the seemingly inconsistent approximation of treating \( P \) (or \( U_r \)) as constant. As mentioned above, such an approximation was necessarily made in the mathematical manipulation to achieve an approximate closed form solution. It is equivalent to neglecting higher order terms attributed to the transient components of \( P \). Figure 4 shows that such an approximation does yield good results as is expected because all rewetting velocities tend to level off quickly. In fact, beyond \( \tau = 5.208 \) (or \( t = 1.4 \text{ sec} \)), the difference between the closed form and the numerical solutions is also indiscernible.

In a recent study, Grimley et al. (1988) conducted experiments to investigate the enhancement of convective boiling heat transfer by grooves on a plate heated from beneath. Unfortunately no rewetting velocity data were reported which could otherwise be useful to check the validity of the solutions discussed above. However, they did report interesting data on the maximum heat flux (the critical heat flux, CHF) that the heater could supply to the plate without causing the dryout of the flowing liquid film. Since the dryout and the subsequent rewet of a heated surface are an integrated problem in heat pipe applications, it is desirable to be able to explain or predict the heat flux condition that leads to dryout of the plate. To achieve this objective, a simple method is presented next and comparisons will be made with the reported data.

**PREDICTION OF THE MAXIMUM DRYOUT HEAT FLUX**

Under consideration is a smooth or a grooved plate initially covered by a thin flowing liquid film at a temperature \( T_s \). The plate is subjected to a uniform heating. It is of interest to predict the maximum heat flux that triggers the dryout of the film.

A simple method based on the above rewetting concept is now extended to provide a means of estimating this maximum dryout heat flux. For a flowing liquid film over a plate heated by a heat flux which exceeds the maximum heat removal capability by convection and boiling, the liquid will cease to advance and begin to recede. Thus dryout will occur. On this physical premise, the maximum rate of the heat removal is given by

\[ q_{\text{max},s} = \bar{h} (T_{0,s} - T_s) \]

where, following the work of Howard et al. (1975), the average convective boiling heat transfer coefficient of the liquid film is estimated from

\[ \bar{h} = \frac{1}{T_{0,s} - T_i} \int_{T_{0,s}}^{T_f} \frac{Q_b}{T - T_f} dT \]

in which \( T_i \) is the plate temperature at the onset of boiling, and \( T_{0,s} \) is the smooth plate Leidenfrost temperature of the rewetting front and is approximated by the plate temperature at the CHF location. \( Q_b \) is the boiling curve of the liquid film over the plate. As an approximation, a form \( Q_b = a (T - T_s)^\beta \), which fits the boiling curve, can be used. In the event that the boiling curve of the flowing liquid film is unavailable, the pool boiling curve could be used as the first order approximation (Howard et al. 1975).

In the case of the grooved plate, the same analogy developed above is adopted here. The maximum dryout heat flux is estimated from

\[ q_{\text{max},g} = N \cdot \bar{h} \cdot (T_{0,g} - T_s) \]

where \( T_{0,g} \) is the Leidenfrost temperature of the grooved plate, also approximated by its CHF temperature. Based on the experimental data of Grimley et al. (1988) for a fluorocarbon (FC-72) liquid film falling over heated smooth and grooved plates, \( T_{0,g} \) was found to be slightly lower than \( T_{0,s} \). Thus, if \( T_{0,g} \) of the grooved plate is unavailable due to the lack of the boiling curve for the grooved plate, one may attempt to approximate \( T_{0,g} \) from the smooth surface data \( T_{0,s} \). Then the predicted maximum heat flux may be slightly overestimated, namely,

\[ q_{\text{max},g} < N \cdot \bar{h} \cdot (T_{0,s} - T_s) \]

As an application to show the feasibility of the above method, the experimental conditions and geometries of Grimley et
al. (1988) are used. In their experiments, the liquid film completely covered the smooth and grooved plates, \( t_1 = 0.5 \, \text{mm}, \ t_2 = 0.2 \, \text{mm} \), and \( t_3 = 0.4 \, \text{mm} \), such that \( N = 2.25 \) (from eq. (58)). \( (T_1 - T_0) \) and \( (T_0 - T_2) \) are taken from their boiling curves. The average boiling heat transfer coefficient is calculated from their smooth surface boiling curve and eq. (65). The two correlation constants \( a_1 \) and \( n \) are determined by arbitrarily collocating the boiling curve of the smooth surface plate at two locations, \( T_2 \) and \( T_0 \), where the boiling heat fluxes are designated by \( q_b \) and \( q_{CHF} \), respectively. Table 1 shows the comparison between the predicted maximum dryout heat flux, \( q_{max} \), using eqs. (64) and (66), and the reported dryout data \( q_{CHF} \) for both types of plates. The agreement is satisfactory particularly in view of the simplicity of the method proposed for the grooved plate. The same agreement is shown in Table 2 when the mean inlet velocity of the falling film is increased from 0.5 m/s to 1.0 m/s.

CONCLUSIONS

The rewetting process of a smooth surface plate subjected to a uniform heating has been investigated. A proper governing transient heat conduction with a convective boiling condition has been solved to yield an approximate closed form solution for the plate temperature profiles in the wet and dry regions of the plate. From the temperature profiles, an approximate closed form solution for the rewetting velocity over the heated plate has been obtained. Numerical solutions have also been presented to check the validity of the closed form solution. The closed form rewetting velocity was found to be in good agreement with that of the numerical solution. It is shown that a limiting condition the present rewetting velocity solution reduces correctly to the existing solution for the rewetting of a hot, isothermal plate without heating. However, contrary to the case without heating, the rewetting process on the plate with a uniform heating is found to be transient (time variant) even on the coordinate frame moving with the rewetting front. The rewetting velocity is found to be much faster initially and then levels off later.

A method to address the rewetting process of the grooved plate based on the smooth plate rewetting model has been developed. It has been shown that, by properly defined scalings, the solution for the smooth plate can be made to be applicable for the grooved plate.

Finally, the dryout of a liquid film over a heated plate has been investigated. A simple method has been proposed to predict the dryout critical heat flux of the smooth and grooved plates. The results of the prediction were compared and found in reasonable agreement with the existing experimental data.

ACKNOWLEDGMENT

This work is supported by NASA L. B. Johnson Space Center, Contract No. NAG9 - 525, under the technical management of Eugene K. Unger.

REFERENCES

eq. (A1) can be written as

\[
\frac{\partial T}{\partial t} \frac{\partial T}{\partial x} + \frac{\partial T}{\partial x} \frac{\partial T}{\partial x} = 0.
\]

For a thin plate subjected to heating from below, it further reduces to

\[
\frac{\partial T}{\partial t} \left( \frac{\partial T}{\partial x} \right) = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).
\]

or in dimensionless form

\[
\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} + P \frac{\partial \theta}{\partial \eta} - B \theta + A
\]

where the dimensionless variables and parameters are given by eq. (2).

For the purposes of comparison and numerical computation, the above is written in untransformed coordinates \((x', t')\) as

\[
\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} - B \theta + A
\]

where

\[
\eta' = x'/s_1.
\]
Table 1. Comparison Between Predicted and Measured (Grimley, 1988, Fig. 8,9)
Maximum Dryout Heat Flux

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Mean Inlet Velocity [m/s]</th>
<th>$T_r-T_a$ [°C]</th>
<th>$T_e-T_s$ [°C]</th>
<th>$\tau_1$</th>
<th>n</th>
<th>$\bar{h}$ [W/m²°C]</th>
<th>$q_e$ [W/m²]</th>
<th>$q_{exp}$ [W/m²] (data)</th>
<th>$q_{pred}$ [W/m²] (pred.)</th>
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<tr>
<td>Smooth Surface</td>
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<td></td>
<td>15.39256</td>
<td>2286.43</td>
<td>18000</td>
<td>57000</td>
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<tr>
<td>Grooved Surface</td>
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<td>15</td>
<td></td>
<td></td>
<td></td>
<td>N $\cdot$ $\bar{h}$</td>
<td>5144.46</td>
<td>80000</td>
<td>77167.01</td>
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</table>

Table 2. Comparison Between Predicted and Measured (Grimley, 1988, Fig. 5)
Maximum Dryout Heat Flux

<table>
<thead>
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<th>Geometry</th>
<th>Mean Inlet Velocity [m/s]</th>
<th>$T_r-T_a$ [°C]</th>
<th>$T_e-T_s$ [°C]</th>
<th>$\tau_1$</th>
<th>n</th>
<th>$\bar{h}$ [W/m²°C]</th>
<th>$q_e$ [W/m²]</th>
<th>$q_{exp}$ [W/m²] (data)</th>
<th>$q_{pred}$ [W/m²] (pred.)</th>
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<tr>
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<td></td>
<td>230.5495</td>
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<td>59000</td>
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<tr>
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<td>N $\cdot$ $\bar{h}$</td>
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