Heat Transfer in Oscillating Flows
With Sudden Change in Cross Section

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ABSTRACT

Oscillating fluid flow (zero mean) with heat transfer, between two parallel plates with a sudden change in cross section, was examined computationally. The flow was assumed to be laminar and incompressible with inflow velocity uniform over the channel cross section but varying sinusoidally with time. Over 30 different cases were examined; these cases cover wide ranges of \( \text{Re}_{\text{max}} \) (187.5 to 30 000), \( \text{Va} \) (1 to 350), expansion ratio (1:2, 1:4, 1:8, and 1:12) and \( A_i \) (0.68 to 4). Three different geometric cases were considered (asymmetric expansion/contraction, symmetric expansion/contraction, and symmetric blunt body). The heat transfer cases were based on constant wall temperature at higher (heating) or lower (cooling) value than the inflow fluid temperature. As a result of the oscillating flow, the fluid undergoes sudden expansion in one-half of the cycle and sudden contraction in the other half. In this paper, one heating case is examined in detail, and conclusions are drawn from all the cases (documented in detail elsewhere). Instantaneous friction factors and heat transfer coefficients, for some ranges of \( \text{Re}_{\text{max}} \) and \( \text{Va} \), deviated substantially from those predicted with steady-state correlations.

NOMENCLATURE

\begin{align*}
A_i & \quad \text{relative fluid displacement (2 } X_{\text{avg}}/L) \\
D_h & \quad \text{hydraulic diameter of the smaller size channel} \\
f & \quad \text{instantaneous friction factor (2 } \frac{f}{\rho U^2}) \\
L & \quad \text{length of channel} \\
\text{Re} & \quad \text{instantaneous Reynolds number} \\
\text{Re}_{\text{max}} & \quad \text{maximum Reynolds number (} U_{\text{max}} D_h/\nu) \\
S & \quad \text{step size (see Fig. 1)} \\
\text{St} & \quad \text{Strouhal number (4 } \text{Va}/\text{Re}_{\text{max}}) \\
U & \quad \text{X-component of velocity} \\
V & \quad \text{Y-component of velocity} \\
\text{Va} & \quad \text{Valensi number (} \text{to} D_h^2/4\nu) \\
X & \quad \text{distance along channel axis} \\
X_{\text{avg}} & \quad \text{amplitude of fluid displacement} \\
Y & \quad \text{distance normal to channel axis} \\
\rho & \quad \text{density of fluid} \\
\mu & \quad \text{dynamic viscosity of fluid} \\
\omega & \quad \text{frequency of oscillation} \\
\text{Subscripts:} \\
in & \quad \text{inlet condition} \\
\text{ss} & \quad \text{steady state} \\
w & \quad \text{at the wall}
\end{align*}

INTRODUCTION

Several engineering applications involve unsteady flow as well as sudden changes in cross-sectional area. In free-piston Stirling engines, the flow oscillates around a zero mean while sudden changes in cross section take place at component interfaces. For example, in the Space Power Research Engine (SPRE), the flow goes through sudden changes in cross section at the expansion space/heater, heater/regenerator, regenerator/cooler, and cooler/compression space interfaces. Currently, steady-state correlations for fluid friction and heat transfer (which also account for steady-state \( U/D_h \) or end effects) are used in the design of Stirling engines.

For oscillating, zero-mean flow through a sudden change in cross-sectional area, the fluid undergoes sudden expansion in one-half of the cycle and sudden contraction in the other half. In addition to the imposed (sinusoidal) flow reversal, the sudden expansion portion of the cycle results in a time varying recirculation zone downstream of the step.

Experiments have been conducted to investigate the hydrodynamics of oscillating flow in a circular pipe [1,2]. These measurements show velocity profiles differing significantly from steady flow; they also indicate a friction factor significantly larger than predicted by steady-flow correlations, during the laminar flow part of the cycle.

Several numerical investigations have been conducted to examine laminar oscillating flows with heat transfer in uniform circular pipes and parallel plate channels [3,4]. Velocity and temperature profiles were found to differ significantly from the steady-flow case. Also, friction factor and heat transfer coefficients were considerably different than indicated by steady-flow correlations.

A literature survey showed that several investigations have been conducted for flows with a sudden change in cross-sectional area. Examples involve steady unidirectional flow over a backward facing step [5-8] and flow through a sudden contraction in a channel [9-11]. These results showed higher friction factor as compared to a uniform geometry under similar flow conditions.

In this paper, results from a computational study of oscillating fluid flow with heat transfer, between two parallel plates with a sudden change in cross section, are presented. The flow parameters were selected to emulate the SPRE. A more complete documentation of the computational results of the study, including cases for different geometries and expansion ratios, is given by Hashim [12].
ANALYSIS

ASSUMPTIONS - Figure 1(a) shows a parallel plate channel with a sudden change in cross section; it also shows the Cartesian coordinate system used for this analysis. The analysis concentrated on the asymmetric expansion/contraction of Fig. 1(a), after it was determined that the symmetric expansion/contraction of Fig. 1(b) and the symmetric blunt body of Fig. 1(c) did not yield any unique computational results of interest for this study. The following assumptions were made: (1) the flow is laminar incompressible with constant thermophysical properties; (2) the inlet velocity is uniform, spatially, but varies sinusoidally with time; (3) the location of the step is far away from either end. Under these assumptions the Navier-Stokes equations were simplified [12].

\[ \begin{align*}
0 &< \theta < 180 \\
0 &< \theta < 360 \\
\end{align*} \]

(a) Asymmetric expansion/contraction.

(b) Symmetric expansion/contraction.

(c) Symmetric blunt body.

Figure 1.—Different geometries examined.

BOUNDARY CONDITIONS - The momentum and energy equations are parabolic in time and elliptic in space coordinates. Therefore, the boundary conditions are required at all spatial boundaries of the solution domain: (1) solid walls: no slip and no jump conditions, (2) axis of symmetry: zero velocity and enthalpy gradients normal to the axis, (3) inlet plane: uniform velocity and temperature profiles, but the velocity varies sinusoidally with time, and (4) outlet plane: zero velocity and enthalpy gradients in the axis direction; this is consistent with the assumption of fully-developed flow and thermal fields at the outlet. It should be noted that for oscillating flows the inlet and outlet planes are switched at the appropriate time step so that a flow reversal is implemented numerically.

NUMERICAL METHOD - The analysis utilizes a modified version of the computer code, CAST, developed by Perle and Scheuerer [13]. CAST solves two-dimensional (2-D) Navier-Stokes equations for laminar or turbulent flows utilizing a collocated grid. It has been modified to handle time dependent boundary conditions for oscillating flows. A special velocity-pressure coupling [13] is used, based on the staggered grid concept to prevent an oscillatory pressure solution [14]. The solution procedure employed is Patankar’s well known SIMPLE algorithm [14].

For all cases investigated, the flow cycle was divided into 60 time steps of 6° intervals. At least three cycles were run for each case to achieve a converged solution (0.2 percent convergence criteria). CPU time required on Cray X-MP/Y-MP’s ranged from 3600 to 8000 sec (for three cycles), depending upon the mesh size.

CODE VALIDATIONS - Several computational experiments were conducted to validate the CAST code: (1) Steady-flow predictions for reattachment length and the minimum and maximum velocities at various locations along the channel axis, for Re = 50 and 150 and expansion ratios of 1:1.2 and 1:1.5, were compared with similar results by Morgan et al. [8]; the comparisons showed good agreement. (2) Comparisons were made with numerical computations of Chiu [6] for steady flow over a backward facing step, asymmetric channel with 1:1.5 expansion ratio and Re = 916. Predictions for friction factor were within 5 percent of Chiu’s results. (3) Predicted size of the recirculation zone before the step, for a forward facing step with Re = 200, was compared with the numerical computations of Mei, and Plotkin [11]. The agreement was within 2 percent. (4) Also, computations were made for impulsively started flow over a backward facing step with Re = 400 and 1:2 expansion ratio. The solution was marched in time. Friction factor and reattachment length were compared with steady flow results for the same case. The agreement was within 1 percent. Similar agreement was found upon examining the case of a forward facing step. (5) Finally, unsteady-flow calculations were made for oscillating flow in a straight channel. The results were in excellent agreement with Kurzweg’s analytical solutions for fully developed channel flow [4].

RESULTS AND DISCUSSION

Tables I and II list some of the cases computed by Hashim [12] for the geometry of Fig. 1(a). These cases cover wide ranges of

Table I.-Summary of test cases corresponding to Stirling engine heater conditions.

<table>
<thead>
<tr>
<th>Test case</th>
<th>Expansion ratio</th>
<th>Re_{max}</th>
<th>Va</th>
<th>Ar</th>
<th>Tw/Tin</th>
<th>MESH</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>1:2</td>
<td>16 500</td>
<td>88.0</td>
<td>2.0</td>
<td>1.05</td>
<td>84x22</td>
</tr>
<tr>
<td>H2</td>
<td>1:2</td>
<td>16 500</td>
<td>88.0</td>
<td>1.32</td>
<td>1.05</td>
<td>94x22</td>
</tr>
<tr>
<td>H3</td>
<td>1:2</td>
<td>8 250</td>
<td>44.0</td>
<td>1.56</td>
<td>1.05</td>
<td>92x22</td>
</tr>
<tr>
<td>H4</td>
<td>1:4</td>
<td>16 500</td>
<td>88.0</td>
<td>1.32</td>
<td>1.05</td>
<td>2x4x22</td>
</tr>
<tr>
<td>H5</td>
<td>1:8</td>
<td>16 500</td>
<td>88.0</td>
<td>1.32</td>
<td>1.05</td>
<td>92x42</td>
</tr>
<tr>
<td>H6</td>
<td>1:12</td>
<td>16 500</td>
<td>88.0</td>
<td>1.32</td>
<td>1.05</td>
<td>92x42</td>
</tr>
</tbody>
</table>

Table II.-Summary of test cases corresponding to Stirling engine cooler.

<table>
<thead>
<tr>
<th>Test case</th>
<th>Expansion ratio</th>
<th>Re_{max}</th>
<th>Va</th>
<th>Ar</th>
<th>Tw/Tin</th>
<th>MESH</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1:2</td>
<td>30 000</td>
<td>350.0</td>
<td>2.0</td>
<td>0.955</td>
<td>84x22</td>
</tr>
<tr>
<td>C2</td>
<td>1:2</td>
<td>30 000</td>
<td>350.0</td>
<td>0.955</td>
<td>955</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>1:2</td>
<td>30 000</td>
<td>350.0</td>
<td>0.955</td>
<td>955</td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>1:4</td>
<td>30 000</td>
<td>350.0</td>
<td>0.955</td>
<td>955</td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>1:8</td>
<td>30 000</td>
<td>350.0</td>
<td>0.955</td>
<td>955</td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>1:12</td>
<td>30 000</td>
<td>350.0</td>
<td>0.955</td>
<td>955</td>
<td></td>
</tr>
</tbody>
</table>
Re\textsubscript{max} (187.5 to 30 000), Va (1 to 350), expansion ratio (1:2, 1:4, 1:8, and 1:12), and \( A_e \) (0.68 to 4). For heating, inflow temperature was 300 K and wall temperature was 315 K; for cooling, inflow temperature was 315 K and wall temperature was 300 K. All Table I cases had the same Strouhal number, St, as the design SPRE heater (0.02133). The Table II cases correspond to the design SPRE cooler (\( St = 0.04667 \)). The inlet velocity, at either end of the channel, varies sinusoidally with time. One of the cases calculated, test case H2 from Table I, will be discussed in detail and conclusions will be drawn for all cases (see [12] for other case results).

Figure 2 shows the oscillating flow streamlines at different velocity phase angles for case H2 (Re\textsubscript{max} = 16 500, Va = 88, expansion ratio of 1:2). During the fluid acceleration with sudden expansion (0° to 90°), the size of the recirculation zone increases in magnitude. During the flow deceleration (90° to 180°), the recirculation zone continues to grow slightly and then remains approximately the same size. This behavior demonstrates the unsteady nature of the flow. It contrasts with quasi-steady oscillating flow (which takes place at lower Re\textsubscript{max} and Va and is not shown in this paper) where the size of the recirculation zone increases during flow acceleration but then decreases during flow deceleration; the unsteadiness increases with Re\textsubscript{max} or Va. During flow reversal (Fig. 2, 180° to 360°), the recirculation zone disappears completely (sudden contraction).

Figure 3 shows temperature contours at different velocity phase angles for the above case (H2). During fluid acceleration with sudden expansion (0° to 90°), the thermal front advances from the inlet at the left; at the same time the thermal front remaining from the previous half cycle (near outlet, at right), retreats to the right. The front appears relatively stationary during flow acceleration (90° to 180°). During flow deceleration (90° to 180°), a similar phenomenon occurs in the reverse direction.

For another case, C2 of Table II (plots not shown here), the oscillating-flow streamlines show similar trends to those in Fig. 2. However, the size of the recirculation zone is smaller than for case H2 at the same cycle time. This is attributed to Case C2's higher Va. Also, for case C2, temperature contour trends are similar to those discussed in Fig. 3, except that the movement of the thermal front is less than for case H2. This can be explained by the difference in St for the two cases, as follows: From the definition of St (nomenclature), and Tables I and II, it is found that St for case C2 is twice that of case H2. Since St is the inverse of "the number of hydraulic diameters the fluid travels during the amplitude of its oscillation," the relative (to D\textsubscript{h}) amplitude of oscillation for case C2 is half that of case H2.

**FRICITION FACTOR** – Figure 4 shows the instantaneous friction factor (as calculated from the previously discussed 2-D computations) normalized by "the corresponding quasi-steady-flow friction factor" (i.e., calculated using the correlation for steady fully developed laminar flat-plate flow, but here based on the instantaneous value of the cross-sectional average Re); it is plotted as a function of dimensionless axial distance at different velocity phase angles, for case H2. Therefore, all plots would collapse on the horizontal line defined by \( \text{ff}_{\text{qst}} = 1 \), if the 2-D computations had agreed precisely with the quasi-steady-flow friction factor for all spatial locations and times during the cycle. It is seen that the instantaneous friction factor can be an order of magnitude larger than the corresponding quasi-steady value. The differences between the instantaneous friction factor and the corresponding quasi-steady one are larger to the right of the step (which occurs at \( x/S = 0 \)); the differences also increased with either Re\textsubscript{max} or Va.

**HEAT TRANSFER COEFFICIENT** – Figure 5 shows the instantaneous Nusselt number (averaged over both walls) versus dimensionless axial distance at different velocity phase angles, for case H2. It can be seen that the instantaneous Nusselt number can be an order of magnitude larger than the corresponding quasi-steady value (which is a constant = 7.541). Again, as noticed in the friction-factor results, the difference between the instantaneous Nusselt number and the corresponding quasi-steady one is larger to the right of the step; the differences also increased with either Re\textsubscript{max} or Va. Also, it can be seen that "area-change" effects are noticeable at the
Figure 3.—Temperature contours for oscillating flow with asymmetric 1:2 change in cross-section. Re max = 16500.0, Va = 88.00, A = 312 k, B = 303 k.

Figure 4.—Normalized friction factor versus dimensionless channel length at different velocity phase angles for asymmetric 1:2 change in cross-section. Re max = 16500.0, Va = 88.00.
two ends of the channel and at the step, while fully developed conditions are observed throughout most of the smaller channel.

CONCLUDING REMARKS

The following conclusions are drawn from all of the computational results documented by Hashim [12]:

1. For all cases examined, a recirculation zone appeared downstream of the sudden expansion and grew in size as the flow accelerated. \( \text{Re}_{\text{max}} \) affects the size of the recirculation zone for a given channel ratio or step height and, also, affects the fluid momentum during flow acceleration and, therefore, the strength of the recirculation zone. For low \( \text{Re}_{\text{max}} \) of 187.5, the recirculation zone decreased in size during deceleration (quasi-steady behavior). For higher \( \text{Re}_{\text{max}} \) > or = 1000, the recirculation zone continued growing during deceleration (unsteady behavior). The degree of "unsteadiness" increased with either \( \text{Re}_{\text{max}} \) or \( \text{Va} \). For values of \( \text{Re}_{\text{max}} \) beyond 5000, the recirculation zone formed during sudden expansion experienced a drift away from the step; this drift increased with \( \text{Re}_{\text{max}} \).

2. \( \text{Va} \) affects the shape of velocity and temperature profiles which in turn affect the values of friction factor and heat transfer coefficient. For small \( \text{Re}_{\text{max}} \), \( \text{Va} \) is the dominant parameter affecting recirculation zone size. Higher \( \text{Va} \) general restricts recirculation zone growth. For Strouhal number, \( \text{St} \), from 0.02 to 0.04, \( \text{Re}_{\text{max}} \) seems to have the dominant effect on recirculation zone size. However, for \( \text{St} \) from 0.1 to 0.2, the effect of \( \text{Va} \) overrides the effect of \( \text{Re}_{\text{max}} \).

3. The expansion/contraction ratio, for given \( \text{Re}_{\text{max}} \) and \( \text{Va} \), affects recirculation zone size. Larger ratios result in larger recirculation zones; however, a decrease in velocity magnitude within the recirculation zone accompanies an increase in size.

4. Three geometries were examined. It was concluded that an asymmetric geometry is representative of symmetric and blunt body geometries as well, for purposes of the study.

5. Relative fluid displacement, \( A_p \), accounts for the effect of \( L/D_h \) on heat transfer for given \( \text{St} \) and expansion/contraction ratio. As \( A_p \) is increased (\( L/D_h \) decreased), the heating or cooling surface area decreases, affecting the temperature of the fluid in the channel.

6. Axial pressure drop is affected by \( \text{Re}_{\text{max}} \), \( \text{Va} \) and the expansion/contraction ratio. Increasing any of these parameters, independently or together, increases the axial pressure drop.

7. Axial temperature variation is time dependent and is affected by the axial temperature distribution from the previous half cycle (a "history" effect). Temperature lags behind velocity. For a heater, the inlet low-temperature fluid produces a temperature distribution which gradually increases from the inlet, by the end of the first half cycle. At flow reversal, the fluid near the new inlet is at a higher temperature than the incoming fluid; in this region a relatively large axial temperature gradient occurs. As the flow accelerates, the fluid temperature gradient is gradually reduced. Finally, towards the end of the second half cycle, a gradually increasing axial temperature distribution is established from inlet to exit.

8. The instantaneous friction factor and heat transfer coefficient deviate substantially from quasi-steady state values for the same flow parameters. These coefficients can be a factor of 10 larger than the quasi-steady-flow values near sudden changes in cross-sectional area.

9. For thermally-expandable flows similar observations, as for incompressible flows, were made. However, the fluid acceleration during the sinusoidal velocity variation is complemented by the acceleration due to density change resulting from fluid heating. The values of friction factor and wall heat flux are therefore higher for thermally-expandable flows than for incompressible flows.

A few final remarks are in order regarding the value of the results for engine design. The results discussed in this paper and documented in full by Hashim [12]: (1) permit visualization of oscillating-flow and temperature fields in the presence of sudden expansion/contractions, (2) show the sensitivity of these fields to the

Figure 5.—Nusselt number versus dimensionless channel length at different velocity phase angles for asymmetric 1:2 change in cross-section. \( \text{Re}_{\text{max}} = 16500.0 \), \( \text{Va} = 88.00 \).
hydrodynamic oscillating-flow dimensionless parameters, and (3) convey a qualitative understanding of how a sudden expansion/contraction affects oscillating-flow friction factors and heat transfer. Thus a study of these results is likely to provide the Stirling designer with insights which enable him to modify hardware in order to produce performance improvements.

When several ongoing tasks are completed, oscillating-flow sudden expansion/contraction effects should be adequately characterized for direct use in Stirling one-dimensional (1-D) design codes. These tasks are: (1) oscillating-flow friction-factor and heat-transfer coefficient (or Nusselt number) correlations must be developed for uniform (no area change) channels; (2) oscillating-flow sudden expansion/contraction computations must be analyzed to characterize how friction factor is affected by cross-sectional area changes, relative to oscillating-flow friction factors for a uniform channel. Resulting "sudden expansion/contraction pressure-loss coefficients" can then be compared with existing steady-flow expansion/contraction pressure-loss coefficients, which are currently used in Stirling 1-D design codes; and (3) oscillating-flow sudden expansion/contraction computations must be analyzed to characterize how heat transfer is affected by cross-sectional area changes, relative to oscillating flow with heat transfer in uniform channels.

The comments in the preceding two paragraphs are made with respect to the laminar, incompressible flow computations discussed in this paper. Two-dimensional code improvements are in process that will further impact the derivation of friction-factor and heat transfer correlations for 1-D codes. These are: (1) a transition model is needed to predict laminar/turbulent transition during the course of the cycle. Transition typically occurs from laminar to turbulent and back, twice per engine cycle. A new empirical transition model for oscillating-flow is under development in cooperation with the University of Minnesota; (2) oscillating-flow and oscillating-pressure level calculations will be made with full compressibility accounted for in the basic equations. In Stirling engines, significant changes in density occur with spatial location due to large temperature gradients and with time due to cyclic oscillations in the pressure level (even though the Mach number typically remains low, <0.2). In is anticipated that oscillating-flow sudden cross-sectional area-change effects will need to account for transition and compressibility.

At the same time as these computational efforts are proceeding, experimental flow and heat transfer measurements are underway in the presence of oscillating flow and oscillating pressure level. It is anticipated that interaction between the experimental and computational efforts will result in improved friction-factor and heat transfer correlations for use in Stirling machine design.

REFERENCES

Oscillating fluid flow (zero mean) with heat transfer, between two parallel plates with a sudden change in cross section, was examined computationally. The flow was assumed to be laminar and incompressible with inflow velocity uniform over the channel cross section but varying sinusoidally with time. Over 30 different cases were examined; these cases cover wide ranges of $Re_{max}$ (187.5 to 30 000), $Va$ (1 to 350), expansion ratio (1:2, 1:4, 1:8 and 1:12) and $Ar$ (0.68 to 4). Three different geometric cases were considered (asymmetric expansion/contraction, symmetric expansion/contraction, and symmetric blunt body). The heat transfer cases were based on constant wall temperature at higher (heating) or lower (cooling) value than the inflow fluid temperature. As a result of the oscillating flow, the fluid undergoes sudden expansion in one-half of the cycle and sudden contraction in the other half. In this paper, one heating case is examined in detail, and conclusions are drawn from all the cases (documented in detail elsewhere). Instantaneous friction factors and heat transfer coefficients, for some ranges of $Re_{max}$ and $Va$, deviated substantially from those predicted with steady-state correlations.