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EXPERIMENTAL MEASUREMENT AND THEORETICAL MODELING OF MICROWAVE SCATTERING AND THE STRUCTURE OF THE SEA SURFACE INFLUENCING RADAR OBSERVATIONS FROM SPACE

INTRODUCTION

The electromagnetic bias is an error present in radar altimetry of the ocean due to the non-uniform reflection from wave troughs and crests. The electromagnetic bias was first measured by Yaplee et al. [1970] from an ocean platform using a one nanosecond pulse X-band radar. Examples of the normalized radar cross section versus wave displacement for a calm and wind driven sea were reported. It was demonstrated that the reflectivity in these two cases was not uniform but increased toward the trough. The mean reflecting surface was 5% of the significant wave height (SWH) lower than the mean sea surface for both examples. The difference between the mean reflecting surface and the mean sea surface has been named the electromagnetic bias.

A study of the electromagnetic bias became necessary to permit error reduction in mean sea level measurements of satellite radar altimeters. If not corrected the electromagnetic bias could introduce errors in mean sea level measurements as large as 50 cm [Slinn, 1990]. During 1980 three airborne electromagnetic bias experiments were performed. Walsh et al. [1984] measured the electromagnetic bias at 36 GHz as 1.1% of the SWH. Choy et al. [1984] measured the electromagnetic bias at 10 GHz as 3.5% of the SWH. During 1988 Melville et al. [1991] measured the electromagnetic bias at 14 GHz as 3.3% of the SWH from an ocean platform. At optical frequencies the electromagnetic bias was measured by Hoge et al. [1984] as biased toward the crests by 2% of the SWH for a low wind speed case and biased toward the trough by 0.75% of the SWH for a high wind case. Walsh et al. [1989] report additional measurements of the electromagnetic bias at optical frequencies for high wind conditions. They found the electromagnetic bias at optical frequencies to be unbiased or biased toward the crests as much as 0.5% of the SWH.
Satellite radar altimeter measurements have been used to find upper and lower bounds for the electromagnetic bias. Studies of the GEOS-3, SEASAT and GEOSAT altimeter data [Lipa and Barrick, 1981; Born et al., 1982; Hayne and Hancock, 1982; Douglas and Agreen, 1983; Nerem and Shum, 1990; Fu and Glazman, 1991] lead to an electromagnetic bias that is about 2-4% of the SWH.

Jackson [1979] was the first to study the electromagnetic bias theoretically. He based his work upon specular point theory [Barrick, 1968] and a model for the joint height-slope probability density function based on the work of Longuet-Higgins [1963]. The later work of Huang [1984], Barrick and Lipa [1985] and Srokosz [1986] continued the use of specular point theory while improving the model used for the joint height-slope probability density function.

During the EM bias experiments by Melville et al. [1990,1991] a wire wave gauge was used to obtain the modulation of the high frequency waves by the low frequency waves. It became apparent that the EM bias was primarily caused by the modulation of the short waves [Arnold et al., 1990]. This report will present a theory using physical optics scattering and an empirical model of the short wave modulation to predict the EM bias. The predicted EM bias will be compared to measurements at C and Ku bands.

PREDICTION OF EM BIAS

The back scattered power from a small patch on the ocean surface depends on the displacement of the patch under observation from mean sea level. It has been observed that more power is reflected from the troughs of waves than from their crests. A typical measurement of the relative back scatter coefficient as a function of displacement is shown in figure 1.

The EM bias can be defined mathematically as the ratio of the first two moments of the back scatter coefficient profile given by

$$\epsilon = \frac{E[\eta \sigma^0(\eta)]}{E[\sigma^0(\eta)]}$$  \hspace{1cm} (1)
The task then is to develop a theory to predict the back scatter coefficient profile from which the EM bias can be calculated.

Physical optics scattering will be used to predict the back scatter coefficient. The physical optics integral will be evaluated at the frequency of interest rather than the high frequency limit. This will have the potential of predicting the electromagnetic frequency dependence of the EM bias.

This theory will assume the primary cause of the EM bias at C and Ku bands to be the modulation of the short wave amplitudes by the long waves. This effect will be described by a semi-empirical model. The wave number modulation of the short waves will be neglected allowing the short wave spectrum to be described by a constant spectral shape. The short wave spectrum will be assumed to be unidirectional with a $k^{-3}$ shape. The short wave amplitude modulation will be found empirically by measuring the energy in the short waves as a function of long wave displacement by using a wire wave gauge.

**PHYSICAL OPTICS SCATTERING THEORY**

Physical optics or the Kirchhoff approximation is a well known scattering theory having been used for rough surface scattering by Beckman and Spizzichino [1963], Hagfors [1966], Fung and Moore [1966] and Holliday et al. [1986]. Barrick [1970] argued that the only valid use of physical optics was in the high frequency limit resulting in geometric optics or specular point theory. Specular point theory was discussed by Kodis [1966], Barrick [1968], Tyler [1976] and Barrick and Bahar [1981]. Fung and Chan [1971], Fung and Eom [1981] and Chen and Fung [1988] showed using method of moment calculations that the high frequency restriction of Barrick [1970] is too restrictive. The validity of the Kirchhoff approximation requires only the radius of curvature of the surface to be large compared to the electromagnetic wavelength.
The high frequency portion of the ocean wave spectrum causes the average radius of curvature of the ocean surface to be small. This apparently renders invalid the use of either specular point or physical optics scattering theory. However Tyler [1976] showed that the high frequency features of a surface should be smoothed prior to application of specular point theory. The common practice is to include only the portion of the ocean surface with wavelengths longer than the electromagnetic wavelength [Valenzuela, 1978; Barrick and Lipa, 1985]. By numerically evaluating the physical optics integral a filter function as proposed by Tyler [1976] does not need to be applied for the employed power law spectrum. Physical optics scattering estimates will be compared to method of moment calculations for typical conditions of the employed power law spectrum. Physical optics scattering estimates obtained without the use of a filter function will be shown to be acceptable thus establishing the validity of the scattering theory.

The physical optics integral for a unidirectional surface is given by (for development see Beckman and Spizzichino [1963] or Tsang et al. [1985])

\[
\sigma^0 = \left( \frac{k^2 L^2}{\pi} \right) \int_{-1}^{1} du (1-|u|) e^{-4\sigma^2 k^2 [1-C_p(k_0 L u)]} \tag{2}
\]

where \( \sigma^0 \) is the back scatter coefficient, \( k \) is the electromagnetic wavenumber, \( L \) is the illumination length, \( \sigma \) is the rms wave height, \( k_0 \) is the dominant wave length of the surface spectrum and \( C_p \) is the surface correlation coefficient given by

\[
C_p(k_0 L u) = \int_{k_0}^{\infty} dk (p-1) k_0^{p-1} k^{-p} \cos(kL u) \tag{3}
\]

This transform can be performed giving a series solution as

\[
C_{p \neq \text{integer}}(u) = 1 + (p-1) \cos\left[ \frac{\pi}{2} (p-1) \right] \Gamma(1-p) |u|^{p-1}
\]

\[
+ \sum_{m=1}^{\infty} \frac{(-1)^m u^{2m}}{(2m-p+1) (2m)!}
\tag{4}
\]

\[
C_{p=2i}(u) = 1 + \frac{(-1)^i u^{2i-1}}{2 (2i-2)!} + (1-2i) \sum_{m=1}^{\infty} \frac{(-1)^m u^{2m}}{(2m-2i+1) (2m)!}
\tag{5}
\]

\[
C_{p=2i+1} = 1 + \frac{(-1)^i u^{2i}}{(2i-1)!} \left[ -\gamma + \sum_{n=1}^{2i} \frac{1}{n} - \ln|u| \right]
\tag{6}
\]

\[
-2i \sum_{m=1, m \neq i}^{\infty} \frac{(-1)^m u^{2m}}{2(m-i) (2m)!}
\]
The physical optics integral can be evaluated numerically. Figure 2 shows the backscatter coefficient as a function of $\sigma k$ for $k_0L = 2\pi$ and $\rho=3$. Figure 2 also compares the physical optics results with exact method of moment results. Good agreement is achieved over the region tested demonstrating the validity of the scattering theory at $C$ band. Method of moment code will be improved in the future to allow surfaces with larger $\sigma k$ to be tested.

GULF OF MEXICO EXPERIMENT

An experiment to measure the EM bias at $C$ and $Ku$ bands (the frequencies of the Topex/Poseidon altimeters) was conducted from December 1989 through May 1990 [Melville et al, 1990] from a Shell Offshore production complex (Brazos-19) in 40 meters of water off the coast of Texas in the Gulf of Mexico. Nadir looking coherent scatterometers at 5 and 14 GHz and a Thorn/EMI IR wave gauge were mounted 18 meters above sea level in the middle of a 60 meter bridge joining two platforms. For short periods of the experiment, a capacitance wire wave gauge was mounted adjacent to the footprints of the scatterometers. The wind speed and direction, air and sea temperature, humidity and rain fall were measured by an R. M. Young instrument package. The data contained in this report comes from a week of data taken from May 10 through May 17, 1990.

The EM bias was measured using the back scatter and doppler of the $C$ and $Ku$ band scatterometers. The wave displacement was obtained by integrating the doppler centroid, which is proportional to the vertical wave velocity, over time to give the displacement. This method of obtaining the wave displacement was compared with the measurements of the Thorn/EMI IR wave gauge and the wire wave gauge yielding close agreement. The simultaneous measurements of back scatter and wave displacement were then used to calculate the EM bias.
The capacitance wire wave gauge was used to measure the short wave modulation. The short wave RMS height was measured by calculating the energy in the high pass filtered wave gauge output. The wave gauge output was high pass filtered at 1.2 Hz corresponding to a one meter separation wave length (assuming a dispersion relation of $\omega^2 = gk$.) The short wave modulation can then be described by the short wave RMS height as a function of displacement from mean sea level. Physical optics scattering and the measured short wave RMS height profile can then be used to predict the back scatter coefficient profile from which the EM bias can be calculated.

**EM BIAS AND POWER PREDICTION RESULTS**

The wave displacement as measured by the capacitance wire wave gauge is shown at the top of Figure 3 for a typical record. This record was recorded on day 6 or May 16, 1990 at 09:58. The envelope of the high passed filtered wave displacement (short waves) is shown at the bottom of Figure 3. It is easily observed that the short waves are being modulated by the long waves with the peak modulation occurring near the crests of the long waves.

The short wave modulation can be described by the RMS short wave height as a function of wave displacement. This was calculated for a ten minute record, the wave displacement shown in Figure 3 was part of this record, and is shown in Figure 4. This clearly shows that the short waves have larger amplitudes on the crests of the waves than in their troughs.

Physical optics scattering and the measured short wave RMS height profile can be used to predict the back scatter coefficient profile at C and Ku bands. Figure 5 shows the predicted and measured back scatter coefficient profiles at C and Ku bands. The predicted back scatter coefficient profiles can be seen to be in close agreement with the measured profiles.
Hourly averages of significant wave height, wind speed, C and Ku band EM bias are shown in Figure 6 for an interval of one week. The predicted bias was calculated from the predicted back scatter coefficient profiles. There is close agreement between the measured and predicted EM biases. A comparison between the measured and predicted biases is shown in Figure 7. The prediction was accurate to within ±2 cm over the range of observed bias.

A comparison between the C and Ku band biases is shown in Figure 8. The C and Ku band biases are the same at small biases but the C band bias becomes larger than the Ku band bias as the bias increases. The predicted bias accurately predicts the electromagnetic frequency dependence of the measured EM bias at C and Ku bands.

The measured back scatter coefficients are compared to the predicted coefficients. Figure 9 shows measured back scatter coefficients at C and Ku bands plotted versus the measured rms short wave height. The measurements are compared to the predicted relationship given by the solid line.

The assumption of a unidirectional surface causes the predicted coefficients to be higher than the measured coefficients. A constant gain was applied to the measurements to account for this difference. A constant gain difference in the back scatter coefficient has no effect on the bias. The bias only depends on the relative relationship between the back scatter coefficient and the rms short wave height.

As seen in figure 9 the measured relationship between the back scatter coefficients and the rms short wave heights is well predicted by physical optics scattering. This provides strong experimental evidence to the validity of the employed physical optics scattering theory.

**EM BIAS MODEL**

A model relating the EM bias to the important parameters describing the ocean surface will be developed. It will be shown that the bias can be described by a simple relationship between wave height, a short wave modulation strength parameter and a term depending on the electromagnetic frequency.
The short wave modulation profile is approximated linearly by

\[ \sigma^2(\eta) = (\sigma_m)^2 \left( 1 + m \frac{\eta}{\sqrt{\eta^2}} \right) \]  

(7)

where \( \sigma_m \) is the rms short wave height and \( m \) is a measure of the strength of the modulation. The radar cross section profile can be computed using the physical optics integral giving

\[ \sigma^0(\eta) = \frac{k^2 L^2}{\pi} \int_{-1}^{1} du (1 - |u|) e^{-4(\sigma_m k)^2 (1 + m \eta / \sqrt{\eta^2}) [1 - C_p(k_0 L u)]} \]  

(8)

Approximating the rcs profile by the linear relation

\[ \sigma^0(\eta) = \sigma^0 \left[ 1 + \left( \frac{\epsilon}{\sqrt{\eta^2}} \right) \frac{\eta}{\sqrt{\eta^2}} \right] \]  

(9)

the bias can be determine from

\[ \frac{\epsilon}{\sqrt{\eta^2}} = \frac{\sqrt{\eta^2}}{\sigma^0(\eta)} \left. \frac{\partial \sigma^0(\eta)}{\partial \eta} \right|_{\eta=0} \]  

(10)

The preceding expression can be computed giving the bias in terms of a frequency dependent term \( \alpha \), a short wave modulation strength parameter \( m \) and the wave height as

\[ \epsilon = -\alpha m \sqrt{\eta^2} \]  

(11)

where

\[ \alpha = \frac{\int_{0}^{1} du (1 - u) 4(\sigma_m k)^2 [1 - C_p(k_0 L u)] e^{-4(\sigma_m k)^2 [1 - C_p(k_0 L u)]}}{\int_{0}^{1} du (1 - u) e^{-4(\sigma_m k)^2 [1 - C_p(k_0 L u)]}} \]  

(12)

The frequency dependent term \( \alpha \) is shown in figure 10 for \( k_0 L = 2\pi \) and \( \rho = 3.0 \). Data from figure x give \( \sigma k \) between 1 and 2 for \( C \) band and between 3 and 6 for \( K_u \) band. Thus the frequency dependent term \( \alpha \) is higher at \( C \) band than at \( K_u \) band causing the \( C \) band bias to be larger than the \( K_u \) band bias.
The relationships given by the analytical result of equation (11) are very important. This result presents for the first time the specific dependence of the EM bias on ocean surface parameters and the electromagnetic frequency. The linear dependence of bias on wave height as predicted by this relationship has been well known from experimental observations for some time [Walsh et al., 1989]. A correspondence between the short wave modulation profile and EM bias was established by Arnold et al. [1990] and reviewed in the last section. Equation (11) specifies the correspondence by showing the bias to be proportional to the short wave modulation strength with a proportionality constant depending on electromagnetic frequency.

The data from the one week experiment as described earlier can be used to test the theory. Figure 11 shows the $\alpha$ parameter, modulation strength $m$, and wave height normalized $C$ and $K'U$ band EM biases.

The $\alpha$ parameter for $C$ band exhibits large fluctuations when the short wave height is small. This is due to the omission of the long waves in the model spectrum. By including the long waves in the model spectrum the transition region between small and large short wave height as shown in figure 2 becomes smoother causing the parameter to also become smoother. The effect of the long waves can be incorporated in the model and this will be done in future work.

The short wave modulation strength parameter $m$ should exhibit the same trends as the normalized EM biases. An inspection of the time series in figure 11 shows this to be the case. The modulation has more variability than the biases but it clearly has the same trends.

The wave height, wind speed, measured biases, and biases predicted using equation (11) are shown in figure 12. The results are similar to those of figure 6 of the last section, but with more variability. The increase of variability is due to the linear approximations made for the short wave modulation profile and the relative RCS profile. Figure 13 show a comparison of the measured to predicted bias.
SUMMARY

Direct measurements of the ocean surface were used in conjunction with physical optics scattering to predict the measured EM bias. The predicted bias was accurate to within ± 2 cm over the range of observed bias. This establishes the short wave modulation as the predominant cause of the EM bias at C and Ku bands.

The EM bias has experimentally been found to depend on wave height, wind speed and electromagnetic frequency [Melville et al., 1990, 1991]. The developed EM bias model correctly predicted the dependence on wave height, wind speed via the short wave modulation strength parameter, and the electromagnetic frequency via the α parameter.

REFERENCES


FIGURE CAPTIONS

1. Typical measured K\textsuperscript{u} band relative back scatter coefficient profile.

2. Physical optics and method of moments back scatter coefficient for $k_0L = 2\pi$ and $p = 3$ versus $\sigma k$.


4. Short wave RMS height profile.

5. Measured and predicted back scatter coefficient profiles at $C$ and $K\textsuperscript{u}$ bands.

6. Hourly averages of significant wave height, wind speed, measured and predicted $C$ and $K\textsuperscript{u}$ band EM bias.

7. Comparison of measured and predicted EM bias.

8. Comparison of $C$ and $K\textsuperscript{u}$ band EM bias.

9. Measured and predicted back scatter coefficients versus $\sigma k$.

10. Frequency dependent term $\alpha$ for $k_0L = 2\pi$ and $p = 3$.

11. Hourly averages of $\alpha$ parameter, modulation strength $m$, and normalized $C$ and $K\textsuperscript{u}$ band biases.

12. Hourly averages of significant wave height, wind speed, measured biases and model predictions.

13. Comparison of measured bias and model predictions.
Figure 1. Typical measured Ku band relative back scatter coefficient profile.
Figure 2. Physical optics and method of moments back scatter coefficient for $k_0L = 2\pi$ and $p = 3$ versus $\sigma k$. 
Figure 3. Modulation of short waves.
Figure 4. Short wave RMS height profile.
Figure 5. Measured and predicted back scatter coefficient profiles at $C$ and $K_u$ bands.
Figure 6. Hourly averages of significant wave height, wind speed, measured and predicted C and Ku band EM bias.
Figure 7. Comparison of measured and predicted EM bias.
Figure 8. Comparison of C and Ku band EM bias.
Figure 9. Measured and predicted back scatter coefficients versus $\sigma k$. 
Figure 10. Frequency dependent term $\alpha$ for $k_0L = 2\pi$ and $p = 3$. 
Figure 11. Hourly averages of $\alpha$ parameter, modulation strength $m$, and normalized $C$ and $Ku$ band biases.
Figure 12. Hourly averages of significant wave height, wind speed, measured biases and model predictions.
Figure 13. Comparison of measured bias and model predictions.