Ongoing Progress in Spacecraft Controls

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Edited by
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JULY 1992
The Mars Mission Research Center is a cooperative program shared by North Carolina State University and North Carolina A&T State University to broaden the nation's engineering capability to meet the critical needs of the civilian space program. Its funding is shared by the National Aeronautics and Space Administration and participating industries. The first workshop, held October 1990, was devoted to "Technology for Lunar/Mars Aerobrakes". The second workshop, held 13 January 1992 at Langley Research Center, focused on "Ongoing Progress in Spacecraft Controls". It was jointly sponsored by the NASA Langley Research Center Guidance, Navigation and Control Technical Committee and the Mars Mission Research Center. This publication is a compilation of the papers presented at that workshop. The technical program addressed additional Mars mission control problems that currently exist in robotic missions in addition to human missions. The topics included control system design in the presence of large time delays, fuel-optimal propulsive control, and adaptive control to handle a variety of unknown conditions.

Dave Ghosh, Editor
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Review of Mars Mission Scenarios

Dr. Gerald Walberg
North Carolina State University
HOW SHALL WE GO TO MARS?

- Mission Scenarios
  - Far Term Missions
  - Initial Missions
- Exposure to Reduced Gravity
- Exposure to Space Radiation
- Initial Mass in Low Earth Orbit

Opposition-Class Missions
2002-2015

Mission Times (days)
- Outbound: 172 - 334
- Stopover: 60
- Inbound: 250 - 375
- Total: 531-714

\[ \Delta V's \ (km/s) \]
- TMI: 3.72 - 4.89
- M: 1.78 - 3.93
- TEI: 1.24 - 3.30
- E: 3.77 - 5.20

Entry Velocities (km/s)
- M: 6.42 - 8.58
- E: 11.39 - 12.80
Split Sprint Missions
2002-2015

Mission Times (days)
Outbound 238 - 287
Stopover 30
Inbound 145 - 172
Total 440 - 470

$\Delta V$'s (km/s)
TMI 4.01 - 6.04
Midcourse VSB - 3.47
M 3.93
TEI 1.99 - 4.21
E 3.71 - 4.26

Entry Velocities (km/s)
M 8.57
E 11.32 - 11.87

Low Thrust Mission

Mission Times (days)
Earth Spiral 52
Outbound 510
Mars Spiral 39
Stopover 100
Mars Spiral 23
Inbound 229
Earth Spiral 16
Total 969
Staging From Earth-Moon Libration Point

Mars Staging From Phobos
Visit 1 Trajectory
One Cycling Spacecraft
2001-2016

Mission Times (days)
Outbound 221 - 1101
Stopover 1331 - 1352
Inbound 197 - 1193
Total 1849 - 2545

ΔV's (km/s)
TMI 3.94 - 4.04
M 1.50 - 1.69
TEI 1.50 - 1.69
E 3.94 - 4.04

Entry Velocities (km/s)
M 6.14 - 6.33
E 11.56 - 11.65

Up-Escalator, Down-Escalator Scenario
Two Cycling Spacecraft
2001-2016

Mission Times (days)
Outbound 148 - 169
Stopover ~ 730
Inbound 146 - 170
Round Trip 1020 - 1069

ΔV's (km/s)
TMI 4.47 - 4.72
M 3.11 - 8.06
TEI 3.55 - 7.92
E 4.43 - 4.72

Entry Velocities (km/s)
M 7.75 - 12.70
E 12.04 - 12.33
GROUND RULES

• REDUCED GRAVITY EXPOSURE

3 Criteria Considered:

Cumulative Reduced g Exposure  
Cumulative Zero g Exposure  
Continuous Zero g Exposure

• RADIATION DOSE

- Ignore Van Allen Belts and Nuclear Rocket  
- Consider Galactic Cosmic Rays and Solar Flares  
- GCR's Vary with Solar Cycle  
- Solar Flare Dose Varies Inversely with Distance from Sun  
- One Giant Solar Flare each Year at Worst Time and Place  
- Charged Particle Transport Analyses of Simonsen, Nealy, Townsend and Wilson  
- 25 cm H2O Storm Shelter  
- Shielding by Martian Atmosphere and Terrain

• IMLEO

- Rocket Equation Analysis  
  \[
  \frac{(m_s + m_p)}{m_p} = 1.1  
  \]
  \[
  \frac{(m_{AB} + m_{p/L})}{m_{p/L}} = 1.15  
  \]
  \[
  I_{sp} = 480 \text{ sec and } 960 \text{ sec }
  \]
Crew Exposure to Reduced Gravity

Flight Experience

Cumulative Reduced g

Cumulative Zero g

Continuous Zero g

Exposure Time, Yr.

Blood Forming Organ Radiation Doses

Time Since Earth Departure, Days

Mission Classes
- Opposition
- Conjunction
- Sprint
Earth Departure Masses
Split Sprint Missions

Earth Departure Date for Manned Vehicle

Earth Departure Masses
Fast Transfer Conjunction Missions

Earth Departure Date
SUMMARY

- MANY VARIED SCENARIOS PROPOSED

  - Far Term Mission Candidates:
    - Extraterrestrial Resources
    - Complex Space Infrastructures
    - Advanced Technologies

  - Initial Mission Candidates:
    - Simple Infrastructure
    - Near Term Technology
    - Low Cost
    - Conjunction Class
    - Opposition Class
    - Split Sprint
    - Fast Trans. Conj.

- SCENARIO CHOICE DEPENDS ON REDUCED GRAVITY EXPOSURE CRITERIA, IMLEO AND OTHER COST FACTORS

  - Radiation Dose Important But Not a Mission Discriminator
  
  - IMLEO is Useful (But Incomplete) Mission Cost Indicator

  - Criterion = Cumulative Reduced g:
    $\rightarrow$ Sprint $\rightarrow$ Nuclear Thermal Propulsion

  - Criterion = Cumulative Zero g:
    $\rightarrow$ Sprint or Fast Trans. Conj. $\rightarrow$ Nuclear Thermal or Chemical/AB

  - When Cost Factors Other Than IMLEO are Considered, Chemical/AB and Nuclear Thermal Propulsion are Competitive
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STABILITY AND OPTIMAL CONTROL THEORY OF HEREDITARY SYSTEMS WITH APPLICATIONS FROM OSCILLATING FLYING VEHICLES, MECHANICAL SYSTEMS AND ROBOTICS

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§0. Motivation.

Consider an $n$-dimensional system

$$
\dot{x}(t) = Ax(t),
$$

(0.1)

where $x(t)$ belongs to $E^n$, the Euclidean $n$-space. The aim is to stabilize the rest position $x = 0$ by adding a damping term $A_1 x(t)$. In practice the damping term which is added has a time delay because it does not react instantaneously but only after a time lag $h > 0$. Thus it is more accurately modelled by adding $A_1 x(t - h)$ instead of $A_1 x(t)$. The equation considered is

$$
\dot{x}(t) = Ax(t) + A_1 x(t - h).
$$

(0.2)

The problem then is to stabilize the rest position: find a necessary and sufficient condition for the damped system (2) to be asymptotically stable. This problem is natural in systems where servomechanisms are used to improve performance and efficiency, in ship and aircraft stabilization and automatic steering.

As reported by Minorsky in [7], [8], for ships exposed to turbulent waters the problem encountered is undesirable self-excited oscillations. Here the engineers aim to eliminate the oscillations. An automatic control servomechanism introduces the delayed damping term $A_1 x(t - h)$ to (0.1) to reduce the oscillations. In a similar situation the flap of an airplane wing is regulated by an automatic control. In such systems, controls $u$ are introduced via a control matrix $B$ to yield a system whose dynamics is governed by

$$
\dot{x}(t) = Ax(t) + A_1 x(t - h) + Bu(t)
$$

(0.3)

The aim of the control device is to 'steer' the system (0.3) to an equilibrium position as fast as possible.

The system (0.3) is a special case of the dynamics of the deterministic model of a flying vehicle, which is derived in Kohmanovskii and Nosov [4, pp 120-123]. The equations have the form

$$
\ddot{q}(t) - \int_0^t B_{-1}(s)\ddot{q}(t - s)ds + B_0 \dot{q}(t) + B_1 q(t)
$$

$$
= \int_0^t B_2(s)\dot{q}(t - s)ds + D u(t).
$$
This can be recast in the form

\[ \dot{x}(t) - \int_0^t A_{-1} \dot{x}(t-s) ds = A_0 x(t) \]
\[ + \int_0^t A_1(s)x(t-s) ds + B u(t). \]  \hspace{1cm} (0.4)

The drives of control surfaces are described by $Bu$, where $u$ is the control signal that drives the signals. In the above equation $q$ is a vector of generalized coordinates.

In the scalar case we have the one-dimensional oscillation of the control surface, the so called control surface buzz [4]. If a rigid wing moves in a gas flow and turns with respect to an axis, the angular wing displacement are restricted by an elastic spring of rigidity $k$. The equation of motion is

\[ \ddot{q}(t) + a \dot{q}(t) + bq(t) = k_1 \int_0^\infty J_0(s) \dot{q}(t-s) ds \]
\[ + k_1 \int_0^t J_1(s) \ddot{q}(t-s) ds + q u(t) \]

for some constants $a, b$. In both cases $u$ is a control signal that helps to stabilize the wing.

The problem of interest which is proposed for investigation for (0.4) can be stated as follows: Find an optimal control subject to its constraints such that the solution of (0.4) with this control and with an initial state $x(t) = \phi(t), t \in [-h, 0]$ will hit the zero target in minimum time $T$ and remain there for every after. Another problem we propose to solve is that of minimizing some effort or energy function $E(u) = \int_0^T G(u(t))$ when $u$ is constrained, for the system (0.4) whose dynamics transfers an initial data $\dot{\phi}$ to a final point $\phi$. For physical applications particularly in aerospace where one is interested in the deployment in space of large assemblies of flexible structures, the performance $E(u)$ may be fuel consumption, the maximum thrust available for the control system or the energy. We propose also to explore the problem of time optimality which minimizes fuel consumption.

For the linear (0.3) or (0.4) the classical approach would be solve

(i) the related initial problem of determining conditions for asymptotic stability of

\[ \dot{x}(t) = A_0 x(t) + A_1 x(t-h), \]
or
\[ \ddot{x}(t) - A_1 \dot{x}(t) + A_1 x(t) = A_0 x(t) + A_1 x(t - h); \]

(ii) the constrained controllability problem of (0.3) and (0.4), that is, the conditions needed to transfer any initial function to another using controls subject to its constraints

and finally to construct an optimal feedback control

\[ f : C \to U \]

where \( U \) is the constraint set and \( C = ([{-h,0}], E^n) \) is the space of continuous functions into the \( n \)-dimensional Euclidean space \( E^n \). Instead of \( C \) we can use the space \( W^{(1)}_2 \), the Sobolev space of absolutely continuous functions \( x : [-h,0] \to E^n \) whose derivated \( \dot{x}(t) \in L_2([-h,0], E^n) \).

Though a lot has now been achieved for (0.3) (see the forthcoming book, E. N. Chukwu [1], "Stability and Time Optimal Control of Hereditary Systems"), the time optimal, minimum fuel problem of (0.4) has not yet been fully investigated. We propose to tackle the following problems:

I: The stability problem for (0.4) and its generalization

II: The controllability problem for nonlinear generalization of (0.4).

which can form the base of technological knowledge necessary for the expected deployment of large flexible structures in space.

We now motivate the equations which we propose to study in problems I and II.

§1. Mechanical Systems

We now examine some simplified mechanical problems whose optimal feedback control strategy will be investigated. The linearized equation of motion of a single degree of freedom mechanical subject to a retarded follower force and control is given by

\[ ml^2 \ddot{q}(t) + (s - Fl)q(t) = -F(q(t - h) + u(t)) \quad (1.1) \]
The scalar $q$ is the general coordinate, $s$ is the torsional stiffness at the pin, $m$ is the mass at the end of the light beam, $l$ denotes the length of the beam, and $F$ stands for the constant magnitude of the applied force. The measurable control $u$ is introduced to restore the system to its equilibrium position in infinite time. The constant delay at the angle of the force is $h$.

Figure 1.1

Figure 1.2
If the model has two degrees of freedom the mechanical system has the following dynamics

\[
\begin{pmatrix}
3m_l^2 & m_l^2 \\
m_l^2 & m_l^2 \\
0 & 0 \\
0 & F_l
\end{pmatrix}
\begin{pmatrix}
\dot{q}_1(t) \\
\dot{q}_2(t) \\
q_1(t-h) \\
q_2(t-h)
\end{pmatrix}
+ \begin{pmatrix}
2s - F_l - s \\
-s - F_l \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
q_1(t) \\
q_2(t) \\
q_1(t) \\
q_2(t)
\end{pmatrix}
= \begin{pmatrix}
u_1(t) \\
u_2(t)
\end{pmatrix}
\]

(1.2)

The two systems have nonlinear versions.

§2. Robotics

Problems of the dynamics of Robotics must incorporate delays [9, p. 30]. These can occur in the control system of the robot, in the transmission of information and in the mechanical part of the robot. Delays which occur in the information transmission are crucial in undersea and space teleoperations in [9, p. 131].

Mechanical model of an elastic robot is described by the systems

\[
\begin{bmatrix}
\dot{q}_1(t) \\
\dot{q}_2(t) \\
\dot{v}(t)
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
\alpha^2 & -\alpha^2 & -2k\alpha
\end{bmatrix}
\begin{bmatrix}
q_1(t) \\
q_2(t) \\
v(t)
\end{bmatrix}
+ \begin{bmatrix}
0 & -k & 0 \\
0 & 0 & 0 \\
0 & -2Kk\alpha & 0
\end{bmatrix}
\begin{bmatrix}
q_1(t-h) \\
q_2(t-h) \\
v(t-h)
\end{bmatrix}
\]

(2.1)

where \( \alpha = \sqrt{\frac{1}{m_2}} \) is the natural frequency of the undamped, uncontrolled system and

\[
k = f/(2m_2)
\]

the relative damping factor. One can of course introduce control variables on the right hand of the equation, and go beyond stability to study optimal control of the dynamics.

§3. Controllability Theory

Definition 3.1. The linear control process (4.1.1) is Euclidean controllable on the interval \([\sigma, t_1]\) if for each \( \phi \in C \) and \( x_1 \in E^n \) there is a square integrable controller \( u \) such that \( x_\phi(\sigma, \phi, u) = \phi \) and \( x(t_1, \sigma, \phi, u) = x_1 \). It is Euclidean null-controllable if in the definition \( x_1 = 0 \). It is Euclidean controllable if it is Euclidean controllable on every interval \([\sigma, t_1]\) \( t_1 > \sigma \).
The conditions for Euclidean controllability have been well studied: See Kirillova and Churakova [5], [2], Weiss [11], Manitius and Olbrot [6]. They are all conditions on matrices representing the system.

Characterization of Euclidean controllability in terms of the systems coefficients is available for the autonomous system,

\[ \dot{x}(t) = A_0x(t) + A_1x(t - h) + Bu(t), \]  

(3.1)

where \( A_0, A_1, B \) are constant matrices. First introduce the so-called "determining equations",

\[ Q_k(s) = A_0Q_{k-1}(s) + A_1Q_{k-1}(s - h), \quad k = 1, 2, 3\ldots, \quad s \in (-\infty, \infty). \]

\[ Q_0(s) = \begin{cases} B & s = 0 \\ 0 & s \neq 0 \end{cases} \]  

(3.2)

and define

\[ \bar{Q}_n(t_1) = [Q_0(s), Q_1(s)\ldots Q_{n-1}(s) \; s \in [0, t_1]] \]

We have:

**Theorem 3.1.** The system (3.1) is Euclidean controllable on \([0, t_1]\) if and only if

\[ \text{rank} \; \bar{Q}_n(t_1) = n. \]

**Remarks.** Note that the non-zero elements of \( Q_k(s) \) form the sequence:

\[ s = 0 \quad h \quad 2h \]

\[ Q_0(s) = B_0 \]

\[ Q_1(s) = A_0B \]

\[ Q_2(s) = A_0^2B \quad (A_0A_1 + A_1A_0)B_0 \quad A_1^2B_0 \]

Note that if \( t_1 \leq h \), the only elements in the sequence are the terms \([B, A_0B, \ldots, A_0^{n-1}B]\), so that \( E^n \)-controllability on an interval less than \( h \) implies the full rank of \([B, \ldots, A_0^{n-1}B]\).

If this has less than full rank and \( t_1 > h \), other terms can be added to \( \bar{Q}_n(t_1) \).
§4 Constrained Controllability of Linear Delay Systems

In the last section the controls are big. In this section we consider controllability of

$$\dot{x}(t) = L(t, x_t) + B(t)u(t), \quad (4.1)$$

when the controls are required to lie on a bounded convex set $U$ with non-empty interior. For ease of treatment $U$ will be assumed to be the unit cube

$$C^m = \{u \in E^m : |u_j| \leq 1, j = 1, \ldots, m\}. \quad (4.2)$$

Here $u_j$ denotes the $j$th component of $u \in E^m$. Consistent with our earlier treatment the class of admissible controls is defined by

$$U_{ad} = \{u \in L^\infty([0, t_1] : u(t) \in C^m \text{ a.e. on}[0, t_1]\}$$

**Definition 4.1.** The system (4.1) is null controllable with constraints if for each $\phi \in C$ there is a $t_1 < \infty$ and a control $u \in U_{ad}$ such that the solution $x()$ of (4.1) satisfies

$$x_\sigma(\sigma, \phi, u) = \phi, \quad x_t(\sigma, \phi, u) = 0.$$  

It is locally null controllable with constraints if there exists an open ball $O$ of the origin in $C$ with the following property: For each $\phi \in O$ there exists a $t_1 < \infty$ and a $u \in U_{ad}$ such that the solution $x()$ of (4.1) satisfies

$$x_\sigma(\sigma, \phi, u) = \phi, \quad x_t(\sigma, \phi, u) = 0.$$  

**Theorem 4.1.** Assume that

(i) the systems (4.1) is null controllable;

(ii) the system

$$\dot{x}(t) = L(t, x_t) \quad (4.3)$$

is uniformly asymptotically stable, so that there are constants $k > 0$, $\alpha > 0$ such that for each $\sigma \in E$ the solution $x$ of (4.3) satisfies

$$\|x_t(\sigma, \phi)\| \leq k\|\phi\|e^{-\alpha(t-\sigma)}.$$  

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Then (4.1) is null controllable with constraints.

§5. Optimal Feedback Control

We now consider the problem of the construction of an optimal feedback control needed to reach the Euclidean space origin in minimum time for the linear systems,

\[ \dot{x}(t) = A_0 x(t) + \sum_{j=1}^{N} A_j x(t - \tau_j) + B u(t). \]  

(5.1)

Here \( 0 < \tau < 2\tau < \ldots < \tau N = h \); \( A_i \) are \( n \times n \) and \( B \) is an \( n \times m \) constant matrices. The controls are \( L_\infty \) functions whose values on any compact interval lie in the \( m \)-dimensional unit cube

\[ C^m = \{ u \in E^m : |u_j| \leq 1, j = 1, \ldots, m \}. \]

We shall show that the time optimal feedback system

\[ \dot{x}(t) = \sum_{j=0}^{N} A_j x(t - \tau_j) + B f(x(t)) \]

(5.2)

executes the time-optimal regime for (5.1) in the spirit of Hájek [3] and Yeung[12]. The construction of \( f \) provides a basis for direct design, and it is done for strictly normal systems which we now define.

**Definition 5.1.** Let

\[ J_0 = \{ t = j \tau, j = 0, 1, 2, \ldots \}, \]

and assume \( J_0 \) is finite. Suppose \( U(\varepsilon, t) \) is the fundamental matrix solution of

\[ \dot{x}(t) = A_0 x(t) + \sum_{j=1}^{N} A_j x(t - \tau_j) \]

(5.3)

on some interval \([0, \varepsilon], \varepsilon > 0 \). Note that \( U(\varepsilon, t) \) is piecewise analytic, and its analyticity may break down only at points of \( J_0 \), (see Tadmor [10]). The system (5.1) is strictly normal on some interval \([0, \varepsilon)\) if for any integers

\[ r_j \geq 0, \text{ satisfying } \sum_{j=1}^{M} r_j = n \]
the vectors

\[ Q_{kj}(s) = \sum_{j=0}^{N} A_i Q_{k-1} j(s - r_j) \quad k = 1, 2, \quad s \in (-\infty, \infty), \]

\[ Q_{0j}(s) = \begin{cases} b_j & s = 0 \\ 0 & s \neq 0 \end{cases} \]

\[ j = 1, \ldots, m; \]

are linearly independent;

and \( B = (b_1 \ldots b_m) \).

It follows from Theorem 7.1.4 in [1] that a complete, strictly normal system is normal, and has rank \( B = \min[m, n] \). Indeed, choose any column \( b_j \) of \( B \) and set \( r_j = n, r_i = 0 \) for \( i \neq j \) in the definition.

**Theorem 5.1.** Consider the system

\[ \dot{x}(t) = \sum_{j=0}^{N} A_j x(t - r_j) + Bu(t), \quad (5.1) \]

and assume

(i) The system (5.1) is Euclidean controllable

(ii) The system

\[ \dot{x}(t) = \sum_{j=1}^{N} A_j x(t - r_j), \quad (5.3) \]

is uniformly asymptotically stable.

(iii) The (5.1) is strictly normal.

then there exists an \( \varepsilon > 0 \) and a function \( f : \text{Int } \mathbb{R}(\varepsilon) \rightarrow E^m \) which is an optimal feedback control of (5.1) in the following sense: If

\[ \ddot{z}(t) = \sum_{j=0}^{N} A_j z(t - r_j) + Bf(z(t), \quad z \in \text{Int } \mathbb{R}(\varepsilon) \quad (5.2) \]
then the set of solutions of (5.3) coincides with the set of optimal solutions of (5.1) in Int \( \mathbb{R}(\varepsilon) \). Also \( f(0) = 0 \) for \( x \neq 0 \), \( f(x) \) is among the vertices of the unit cube \( U \). Furthermore \( f(x) = f(x) = -f(-x) \). If \( m \leq n \), \( f \) is uniquely determined by the condition that optimal solutions of (5.1) solve (5.3).

REFERENCES


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SENSOR FILTER DESIGNS FOR LARGE FLEXIBLE SPACECRAFT

by

NANCY A. NIMMO

ONGOING PROGRESS IN SPACECRAFT CONTROLS
JANUARY 13, 1992
1. INTRODUCTION

Problem

Traditional control methods may excite flexible modes causing degraded performance or instability of large flexible space structures (LFSS). Control techniques developed for control of LFSS require a numerical model of the structure and some knowledge of model error. This will be increasingly difficult with the complex space structures planned for the future. If filters could be used to condition the sensor output, control design would be less demanding.
2. CONVENTIONAL LOW-PASS ANALOG FILTER DESIGNS

Objective of low-pass filter design
Preserve desired frequency components and attenuate undesired frequency components

Conventional low-pass filter designs
Filter approximations with frequency response characteristics which satisfy the following specifications:
- Passband frequency, stopband frequency,
- Passband attenuation, stopband attenuation.

TERMINOLOGY
Filter transfer function: \( T(j \omega) = \frac{Y(j \omega)}{U(j \omega)} = |T(j \omega)| \angle \theta(j \omega) \)

Attenuation: \( \alpha = -20 \log |T(j \omega)| \text{ dB} \)

Filter Specifications
- Passband frequency \( \omega_p = 1 \text{ rad/sec} \)
- Stopband frequency \( \omega_s = 2 \text{ rad/sec} \)
- Maximum attenuation \( \alpha_{\text{max}} = 3 \text{ dB} \)
- Minimum attenuation \( \alpha_{\text{min}} = 20 \text{ dB} \)

Filters satisfying specifications
- Butterworth: fourth-order
- Chebyshev: third-order
- Cauer: second-order
- Bessel: none (Bessel filter transition band too wide)
Comparison of Filters

Frequency Response Characteristics

Butterworth: maximally flat magnitude response in passband

Bessel: maximally flat delay characteristics in passband, largest transition band

Chebyshev: ripple in passband, sharp rolloff, nonlinear phase response

Cauer: ripple in stopband and passband, sharpest rolloff nonlinear phase response
3. Controls Structures Interaction (CSI) Test Article

Some objectives of the NASA CSI Program is to develop and validate the technology needed to design, verify, and operate spacecraft in which the structure and control system interact beneficially to meet the requirements of future spacecraft.

A CSI testbed has been developed to validate CSI design methodology, to implement practical sensors and actuators for LFSS control, and to evaluate controller designs.

The Phase-0 CSI Evolutionary Model (CEM) was the test article used for this study.

CSI Evolutionary Model
Phase-Zero
System ID Frequency Response Functions

Y - FREQUENCY RESPONSE FUNCTION

Z - FREQUENCY RESPONSE FUNCTION

experimental data
finite element model
Figure 5.4 Frequency Response of CSI Evolutionary Model with Chebyshev filters
4. CONTROL STRATEGY

Objectives for control of large flexible space structures (LFSS)

Accurate line-of-sight (LOS) pointing
Vibration suppression

Challenges for control design

Model parameter errors
Incomplete model (unmodeled modes)

Robust control methods

Static dissipative control (local velocity control)
Virtual passive control (2nd-order formulation)

5. Closed-Loop Simulation

Objective: Investigate the effect of sensor filtering on performance of feedback controllers

A. Controllers developed without filter dynamics in the design model
   i. Static dissipative control (SD-0)
   ii. Virtual passive control (AVA)

B. Controllers developed WITH filter dynamics in the design model (static dissipative control)
Figure 7.9 Acceleration response of closed-loop system with controller AVA and Butterworth filters (sensor location 1)
6. Closed-Loop Laboratory Experiments

7 Hz mode unstable when sensor filters used in system

Discrepancies between experiment and simulation due to

MODELING ERRORS
- inaccurate parameters
- unmodeled modes

DISCRETIZATION ERRORS
closed-loop system is not a continuous-time system but a hybrid system
Figure 8.7 Experimental acceleration response of a closed-loop system with second-order Butterworth filters and controller AVA
Future Work

Modify filter design: Add zeros to numerator to compensate for phase lag

Formulate filter as a second-order system (corresponding to virtual passive control design)

Stability conditions for hybrid systems

Discretization issues: Method of implementing of digital filters

7. Conclusions and Future Work

Conclusions

- Sensor filters used successfully in closed-loop simulations
- Dynamics of higher-order filters must be considered in control law design
- Instability occurred in closed-loop experiments due to design model inaccuracies and discretization errors
- More accurate models of CSI test articles needed
- Stability conditions for continuous-time systems not valid for hybrid systems
FUEL OPTIMAL PROPULSIVE REBOOST
OF FLEXIBLE SPACECRAFT

by

Jim Redmond

Building 1192C, Room 124
NASA Langley Research Center
Hampton, Virginia
January 13, 1992
Motivation

Unbounded Fuel Optimal Control Problems
- Exact solutions are difficult to obtain for higher order systems.
- Impulsive forces cannot be implemented directly.
- Exact solutions provide basis for judging the optimality of approximate techniques.
- Properties of the exact solution can be used to develop improved approximations.

Floating Harmonic Oscillator
- Model possesses both rigid and flexible body motion characteristic of proposed spacecraft.

Overview

I. Fuel Optimal Propulsive Control

II. Numerical Solution by Adaptive Grid Bisection

III. The Floating Harmonic Oscillator
   a. System Definition
   b. Rigid Body Reboost Class
   c. Vibration Suppression Class
   d. General Reboost Class

IV. Concluding Remarks
Fuel Optimal Control

**System Equation:**
\[ \dot{x}(t) = Ax(t) + \sum_{j=1}^{m} b_j u_j(t) \]

**Control Objective:**
Transfer the system from \( x_0 \) to \( x_1 \) in maneuver time \( T_f \) while minimizing fuel consumption.

**Fuel Function:**
\[ \text{Fuel} = \sum_{j=1}^{m} \int_{0}^{T_f} |u_j(t)| \, dt \]

---

Fuel Optimal Control

**General Solution:**
\[ x(t) = e^{At} \left( x_0 + \sum_{j=1}^{m} \int_{0}^{t} e^{-As} b_j u_j(s) \, ds \right) \]

**Reachable State:**
\[ y = \sum_{j=1}^{m} \int_{0}^{T_f} e^{-A(t-s)} b_j u_j(t) \, dt = e^{-AT_f} x_1 - x_0 \]

**Control Index:**
\[ g_j(\eta, t) = \eta^T e^{-A(t-s)} b_j \quad j=1,2,...,m \]

**Hyperplane Constraint:**
\[ H = \{ \eta : \eta^T x = 1 \} \]

**Index Extremum:**
\[ \alpha^* = \min_{\eta \in H} \max_{1 \leq j \leq m} \sup_{0 \leq t \leq T_f} | g_j(\eta, t) | \]
Fuel Optimal Control

Optimal Control:
\[ u_j^*(t) = \frac{g_j^T c_j}{\alpha^*} \quad (j=1,2,\ldots,m) \]

Impulse Vector:
\[ g_j = [\text{sgn}(g_j(q_j^*, \tau_{1j})) \delta(t-\tau_{1j}) \ldots \text{sgn}(g_j(q_j^*, \tau_{N_{1j}})) \delta(t-\tau_{N_{1j}})]^T \]

Impulse Coefficient Vector:
\[ c_j = [c_{1j} \ c_{2j} \ \ldots \ c_{N_{1j}}]^T \]

Coefficient Constraint:
\[ \mathbf{1} = 1 \times \mathbf{c} \]
\[ \mathbf{1} = [1 \ 1 \ \ldots \ 1]^T \quad \mathbf{c} = [c_1^T \ c_2^T \ \ldots \ c_m^T]^T \]

Optimum Fuel:
\[ \text{Fuel}^* = \frac{1}{\alpha^*} \]

Adaptive Grid Bisection

(1) Generate a square grid \( G \) of normal vectors \( \eta_1, \eta_2, \ldots, \eta_p \) that form a subset of the hyperplane \( H \).

Figure 1 - Square Grid Generated in Step (1) in Which \( n=3 \) and \( L=2 \)
Adaptive Grid Bisection

(2) Determine \( \alpha_{ij} = \sup_{0 \leq t \leq T_f} |g_j(\eta_{ij}, t)| \) for each \( \eta_{ij} \) (i=1,2,...p) and for each (j=1,2,...m). The suprema \( \alpha_{ij} \) are computed to within the error margin \( \varepsilon_1 \).

\[ g_j(\eta, t) \]

![Sample Function for Step (2) Supremum Computation](image)

(3) Determine the grid-optimal normal vector \( \eta^*_i \) for which \( \alpha = \min_{1 \leq i \leq p} \max_{1 \leq j \leq m} \alpha_{ij} \).

(4) Select an updated grid \( G \) of normal vectors \( \eta \), \( \eta_2, ..., \eta_p \) centered about the grid-optimal normal vector \( \eta^*_i \) based on the following:

(a) If \( \eta^*_i \) is an interior grid point, decrease the grid spacing by 50% (bisection).

(b) If \( \eta^*_i \) is a boundary grid point, increase the spacing by 50%.
Adaptive Grid Bisection

(5) Repeat steps (2)-(4) until the grid spacing is within the error margin $\varepsilon_2$. The converged grid-optimal normal vector represents the optimal normal vector $\tilde{n}^*$. 

(6) Determine the number of impulses $N_j$ associated with $u^*_j(t)$ ($j=1,2,...m$), the corresponding impulse times $\tau_{ij}$ ($i=1,2,...N_j$; $j=1,2,...m$) and the sign functions $\text{sgn}(g_j(\tilde{n}^*,\tau_{ij}))$. 

Figure 3 - $\tilde{n}^*_1$ in the Interior Leads to Grid Spacing Decreased by 50% in Step (4)

Figure 4 - $\tilde{n}^*_1$ on the Boundary Leads to Grid Spacing Increased by 50% in Step (4)
Adaptive Grid Bisection

(7) Compute the impulse coefficients $c_{ij}$
($i=1,2,\ldots,N_j; j=1,2,\ldots,m$).

$$P_c = Q \cdot c \geq 0$$

$$P = \left[ \frac{1}{\alpha} \int_0^T e^{-\Lambda t} b_1 g_1^T dt \quad \ldots \quad \frac{1}{\alpha} \int_0^T e^{-\Lambda t} b_m g_m^T dt \right] \rightsquigarrow Q = \begin{bmatrix} y \\ 1 \end{bmatrix}$$

Floating Harmonic Oscillator

System Definition

Equations of Motion:

$$m_1 \ddot{y}_1(t) + k(y_1(t) - y_2(t)) = u_1(t)$$
$$m_2 \ddot{y}_2(t) + k(y_2(t) - y_1(t)) = u_2(t)$$

State Definition:

$$x_1(t) = \frac{m_1 y_1(t) + m_2 y_2(t)}{m_1 + m_2}$$
$$x_2(t) = y_2(t) - y_1(t)$$
$$x_3(t) = \dot{x}_1(t)$$
$$x_4(t) = \dot{x}_2(t)$$
Floating Harmonic Oscillator

**State Space Representation:**

\[ \dot{\chi}(t) = A\chi(t) + \sum_{j=1}^{m} b_j u_j(t) \]

**System Matrices:**

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & -\omega^2 & 0 & 0
\end{bmatrix}
\quad b_1 = \begin{bmatrix}
0 \\
0 \\
\frac{1}{m_1+m_2} \\
-\frac{1}{m_1}
\end{bmatrix}
\quad b_2 = \begin{bmatrix}
0 \\
0 \\
\frac{1}{m_1+m_2} \\
\frac{1}{m_2}
\end{bmatrix}
\]

**Natural Frequency of Oscillation:**

\[ \omega = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}} \]
**Floating Harmonic Oscillator**  
Rigid Body Reboost Class

*Initial Conditions:*  
\[ y_{10} = y_{20} = 1 \]
\[ x_{10} = x_{20} = 0 \]

*Optimal Normal Vector:*  
\[ \tilde{\eta}^* = [ -1 \quad 0 \quad -T_f/2 \quad 0 ]^T \]

*Optimal Control:*  
\[ u_j^*(t) = \frac{m_j}{T_f} [ -\delta(t) + \delta(t-T_f) ], \ (j=1,2) \]

---

**Figure 5 - Rigid-Body Reboost Class:**  
Minimum Fuel versus Maneuver Time
Floating Harmonic Oscillator  Vibration Suppression Class

Initial Conditions:

\[ y_{10} = \frac{-m_2}{m_1+m_2} \quad y_{20} = \frac{m_1}{m_1+m_2} \]

\[ x_{10} = 0 \quad x_{20} = 1 \]

Figure 6 - Vibration Suppression Class: Minimum Fuel versus Maneuver Time

Table 1 - Convergence Example: Vibration Suppression Class, \( m_2 = 2m_1 \) Case, \( T_f = 2.0s \)

<table>
<thead>
<tr>
<th>location</th>
<th>( \eta_1 )</th>
<th>( \eta_3 )</th>
<th>( \eta_4 )</th>
<th>step size</th>
<th>min max sup lb/eq</th>
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<td>0.000000000</td>
<td>0.000000000</td>
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<td>0.000000000</td>
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<td>0.000000000</td>
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<td>0.655029163</td>
<td>-0.013534393</td>
<td>0.000000000</td>
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### Table 2 - Control Parameters:
**Vibration Suppression Class, \( m_2 = 2m_1 \) Case**

<table>
<thead>
<tr>
<th>( T_f (\text{secs}) )</th>
<th>( t_1 (\text{secs}) )</th>
<th>( \sigma_{\text{in}}(g_1) )</th>
<th>( c_1 )</th>
<th>( t_2 (\text{secs}) )</th>
<th>( \sigma_{\text{in}}(g_2) )</th>
<th>( c_2 )</th>
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<td>-</td>
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<td>+</td>
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<td>0.250000</td>
<td>-</td>
<td>0.250000</td>
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</tr>
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</table>

### Figure 7 - Vibration Suppression Class, \( m_2 = 2m_1 \) Case, \( T_f = 0.5s \):
Displacement and Velocity of Each Mass versus Time
Floating Harmonic Oscillator  Vibration Suppression Class

Figure 8 - Vibration Suppression Class, \( m_2 = 2m_1 \) Case, \( T_1 = 2.0 \text{s} \):
Displacement and Velocity of Each Mass versus Time

FLOATING HARMONIC OSCILLATOR  VIBRATION SUPPRESSION CLASS

Figure 9 - Vibration Suppression Class, \( m_2 = 2m_1 \) Case, \( T_1 = 4.0 \text{s} \):
Displacement and Velocity of Each Mass versus Time
Initial Conditions:

\[
Y_{10} = 1 - \frac{m_2}{m_1 + m_2} \quad y_{20} = 1 + \frac{m_1}{m_1 + m_2}
\]

\[
x_{10} = 1 \quad x_{20} = 1
\]

Figure 10 - General Reboost Class:
Minimum Fuel versus Maneuver Time

Table 3 - Control Parameters:
General Reboost Class, \( m_2 = 2m_1 \) Case

<table>
<thead>
<tr>
<th>( T_p(\text{sec}) )</th>
<th>( T_f(\text{sec}) )</th>
<th>( \mu(\text{sec}) )</th>
<th>( c_{1f} )</th>
<th>( c_{2f} )</th>
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Floating Harmonic Oscillator  General Reboost Class

Figure 11 - General Reboost Class, \( m_2 = 2m_1 \) Case, \( T_f = 0.8s \):
Displacement and Velocity of Each Mass versus Time

Floating Harmonic Oscillator  General Reboost Class

Figure 12 - General Reboost Class, \( m_2 = 2m_1 \) Case, \( T_f = 2.0s \):
Displacement and Velocity of Each Mass versus Time
**Figure 13 - General Reboost Class, m_2=2m_1 Case, T = 5.3s:**
Displacement and Velocity of Each Mass versus Time

**Concluding Remarks**

- Adaptive Grid Bisection can be used to solve some fuel optimal propulsive control problems.

  "Fuel Optimal Propulsive Reboost of Flexible Spacecraft"
  "Fuel Optimal Propulsive Reorientation of Axisymmetric Spin-Stabilized Satellites"
  "An Exact Solution to the Fuel Optimal Propulsive Control of a Tutorial Structure"

- Exact fuel optimal solutions provide a basis for assessing the degree of optimality attained with approximate techniques.
  - Linear Quadratic Regulator
  - Independent Modal Space Control
  - Impulse Damping Control

- Knowledge of exact solutions can be used to improve the optimality of existing approximations.
Ongoing Research in Spacecraft Controls

Fuel Optimal Maneuver: Experimental Testbeds

Presented by
John L. Meyer

January 13, 1992
LaRC GNC Technical Committee and Mars Mission Research Center
NASA Langley Research Center
Hampton, Virginia
Objective:

To Conduct Experiments in Fuel Optimal Propulsive Maneuver of Flexible Spacecraft in order to Verify and Extend Recently Developed Theory and Apply to Various Classes of Spacecraft

Ongoing Progress in Spacecraft Controls
I. Related Experimental Efforts

II. Completed Experiments
   A. Experimental Verification of Impulse Damping Control
   B. Impulse Damping of the SCOLE Reflector and Mast
   C. Impulse Damping Control of an Experimental Structure

III. Experiment Preparations
   A. Reaction Control System
   B. Rotating Flexible Beam
   D. IMAGE Testbed
   C. Aerobrake Configuration
IV. Current and Future Experiments

A. Fuel Optimal Control of an Experimental Rotating Flexible Beam

B. Fuel Optimal Aerobrake Maneuver of an Experimental Lunar Orbital Transfer Vehicle

C. Fuel Optimal Propulsive Maneuvers of an Experimental Space Structure Undergoing Translation, Rotation, and Flexible Body Motions

V. Summary
<table>
<thead>
<tr>
<th></th>
<th>Reference</th>
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**Ongoing Progress in Spacecraft Controls**
Related Experimental Efforts


8. NASA Langley Research Center CSI Program, i.e. Phase 0
Completed Experiments

Experimental Verification of Impulse Damping Control

**Completed Experiments**

**Impulse Damping of the SCOLE Reflector and Mast**

**SCOLE Planform**

**SCOLE Reflector and Mast**

3 Axis Angular Rate Sensors

X and Y Accelerometers at Reflecting Center

Air Jets at Reflecting Center

**Ongoing Progress in Spacecraft Controls**

---

**Controlled Response of Mode #1**

Window = 2.0 sec, Yobs = 0.20, Yosh = 1.0

\[ \text{Position} = 1 \times 10^{-3}, \text{Time} = 0.4 \text{ sec} \]

**Strain Gages**

**Mounting Bracket**

Reaction Control Jet

(located at node of 2nd mode)

**Ongoing Progress in Spacecraft Controls**
Completed Experiments
*Impulse Damping Control of an Experimental Structure*

Ongoing Progress in Spacecraft Controls

**Frequencies:**
- $\omega_1 = 0.264$ Hz
- $\omega_2 = 0.723$ Hz
- $\omega_3 = 1.377$ Hz
- $\omega_4 = 2.412$ Hz
Ongoing Progress in Spacecraft Controls
Control of Vertical Beam Under Mode 1 Excitation

$P_s = 35$ psi, Window $= 1.875$ sec, Deadband $= 0.5$ in, $\mu_n = 1.0$

Control of Vertical Beam Under Multi-Mode Excitation

$P_s = 35$ psi, Window $= 1.875$ sec, Deadband $= 0.35$ in, $\mu_n = 0.5$
Effect of Additional Actuators on Same Supply
50 psi Supply Pressure

Load Cell Output (lbf)

Time (sec)

Ongoing Progress in Spacecraft Controls
Supply Line Length Effects

40 psi Supply Pressure / 12 ft. Supply Line

40 psi Supply Pressure / 25 ft. Supply Line
Free Response of IMAGE Structure to Multi-Mode Excitation

Differentiation Yields

Integration Yields

Ongoing Progress in Spacecraft Controls
90° Rotation Maneuver of IMAGE Structure
(30 psi - 1.0° deadband)

Response of IMAGE Structure to 90° Maneuver

Ongoing Progress in Spacecraft Controls
Free Response of IMAGE Structure
to 1st Mode Excitation

Ongoing Progress in Spacecraft Controls
Controlled Response of IMAGE Structure

10 psi - 0.2 in. deadband

30 psi - 0.2 in. deadband

45 psi - 0.5 in. deadband

Elastic Displacements at Actuator #2 (in)

Ongoing Progress in Spacecraft Controls
Control Strategies for Lunar Orbital Transfer Vehicle

Pitch Control and Roll Modulation for Course Adjustments in Aerobrake Corridor

Equation of Motion

\[ m(x) \left( x \frac{\partial^2 \theta(t)}{\partial t^2} + \frac{\partial^2 u(x,t)}{\partial t^2} - u(x,t) \left( \frac{\partial \theta(t)}{\partial t} \right)^2 \right) = -T \frac{\partial u(x,t)}{\partial t} f(x,t) \]

where \( T = \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2}{\partial x^2} \right) \)

Boundary Conditions

\[ u(0,t) = \frac{\partial^2 u(0,t)}{\partial x^2} = \frac{\partial^2 u(1,t)}{\partial x^2} = \frac{\partial^3 u(1,t)}{\partial x^3} = 0 \]

Minimize Fuel

\[ \text{Fuel} = \sum_{j=1}^{m} \int_{0}^{T} \left| u_j(t) \right| \, dt \]

Control of the Form

\[ u_j^*(t) = \alpha \frac{\xi_j}{\sigma_j^*} \left( j=1,2,...,m \right) \]

\[ \xi_j = \text{tan} \left( \frac{\theta_1}{\sigma_j^*} \right) \left( \sigma_j^* \right)^2 \text{tan} \left( \frac{\theta_1}{\sigma_j^*} \right) \left( \sigma_j^* \right)^2 \]

1 Angular Rate
2 Strain Gauge
3 Propulsive Actuators
Ongoing Progress in Spacecraft Controls

Control Form

\[ u_j^*(t) = \frac{e_j^T c_j}{\alpha_j} \text{ (j=1,2,...,m)} \]

\( \alpha \) = Pitch Angle

\[ I_{\alpha} \dot{\alpha} = -\frac{dC_{\alpha}}{d\alpha} \frac{1}{2} \rho \omega^2 V_{\infty}^2 A_{ref} I_{ref} \alpha = k_p \alpha \]

\[ \omega_{\alpha} = \frac{1}{2\pi} \sqrt{\frac{k_p}{I_{\alpha}}} \]

\( \phi \) = Roll Angle (rotation about \( V_{\infty} \))

\[ I_{\phi} \dot{\phi} = -\frac{dC_{\phi}}{d\phi} \frac{1}{2} \rho \omega^2 V_{\infty}^2 A_{ref} I_{ref} \phi \]

\[ \frac{dC_{\phi}}{d\phi} \equiv 0 \therefore \omega_{\phi} \equiv 0 \]

Current and Future Experiments

Fuel Optimal Aerobrake Maneuvers of an Experimental IOTV

Rigid Body Rotation

\[ I_{\alpha} \dot{\alpha} + \frac{g}{L} \sin \theta_x = \frac{M_{\alpha}}{I_{\alpha}} \]

\[ I_{\phi} \dot{\phi} + \frac{g}{L} \sin \theta_y = \frac{M_{\phi}}{I_{\phi}} \]

Rigid Body Translation

\[ X = L \sin \theta_x \equiv L \theta_x \]

\[ Y = L \sin \theta_y \equiv L \theta_y \]

8 Strain Gages
13-Axis Angular Rate Gyro
1 X-Y Translation Sensor
20 Reaction Control Jets

Flexible Body Motion

\[ \ddot{q}_r + \alpha_r^2 q_r = Q_r \text{ (r = 1,2,...,n)} \]

\[ u_k = \sum_{r=1}^{n} \phi_r q_r \]

8 Strain Gages
13-Axis Angular Rate Gyro
1 X-Y Translation Sensor
20 Reaction Control Jets

Ongoing Progress in Spacecraft Controls
## Summary

| ✔ | Experimental Verification of Impulse Damping Control |
| ✔ | Impulse Damping of the SCOLE Reflector and Mast |
| ✔ | Experiment Preparation for RCS, Rotating Flexible Beam, IMAGE, and Aerobrake Configuration |
| ✔ | Impulse Damping Control of an Experimental Structure |
| ☐ | Fuel Optimal Control of an Experimental Rotating Flexible Beam |
| ☐ | Fuel Optimal Aerobrake Maneuver of an Experimental Lunar Orbital Transfer Vehicle |
| ☐ | Fuel Optimal Propulsive Maneuvers of an Experimental Space Structure Undergoing Translation, Rotation, and Flexible Body Motions |

*Ongoing Progress in Spacecraft Controls*
"Initial Investigations of an Adaptive Discrete-Time Controller for Nonlinear Time-Varying Robotic Models"

by

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North Carolina State University
Raleigh, North Carolina 27695-7910
Problem:

Consider a non-linear, time-varying differential equation model of a multi-link rigid robot:

\[ D(q(t)) \ddot{q}(t) + C(q(t), \dot{q}(t)) + G(q(t)) + H(\dot{q}(t)) = u(t) \]

- \( D(\cdot) \) : inertial matrix
- \( C(\cdot) \) : centripetal and Coriolis forces
- \( G(\cdot), H(\cdot) \) : parasitic forces (gravity, friction)
- \( u(t) \) : applied torque
- \( q(t) \) : generalized coordinate vector

Objective:

Design a discrete-time controller which forces

\[ y(t) = f(q(t), \dot{q}(t)) \]

to follow a desired trajectory, when non-linearities may be uncertain.
Note:

\[ S(q^{-1}) u(k) = T(q^{-1}) y^*(k + d) - R(q^{-1}) y(k) \]

Let the desired closed-loop poles be

\[ P(q^{-1}) = 1 + p_1 q^{-1} + \ldots + p_l q^{-l} \]

Then if the controller parameters \( S(q^{-1}), R(q^{-1}) \) satisfy:

\[ A(q^{-1}) S(q^{-1}) + q^d B(q^{-1}) R(q^{-1}) = P(q^{-1}) , \]

the tracking requirement will be satisfied with

\[ T(q^{-1}) = \frac{P(q^{-1})}{B(1)} \]

and

\[ B(1) = \sum_{i=0}^{m} b_i \]

Because \( A(q^{-1}) \) and \( B(q^{-1}) \) may be unknown, some identification method is employed. Here we use the recursive least-squares method,
Transforming the discrete-time model into parametric form:

\[ y(k) = \Phi^T(k - 1) \Theta \]

where \( \Phi^T(k-1) = [-y(k-1),..., -y(k-n), u(k-1),..., u(k-1-m)] \)

\( \Theta^T = [a_1,..., a_n, b_0,..., b_m] \),

the recursive formula to update the estimate \( \hat{\Theta}(k) \) is:

\[
\hat{\Theta}(k+1) = \hat{\Theta}(k) + F(k+1) \Phi(k) \varepsilon^0(k+1)
\]

\[
F(k+1) = \frac{1}{\lambda_1} \left[ F(k) - \frac{F(k) \Phi(k) \Phi^T(k) F(k)}{\lambda_1 + \Phi^T(k) F(k) \Phi(k)} \right]
\]

\[ \varepsilon^0(k+1) = y(k+1) - \Phi^T(k) \hat{\Theta}^T(k) \]

with forgetting factor \( 0 < \lambda_1 < 1 \).

The closed-loop system is:

\[
\left[ A(q^{-1}) S(q^{-1}) + q^{-d} B(q^{-1}) R(q^{-1}) \right] y(k) = B(q^{-1}) T(q^{-1}) y^*(k)
\]

\[ y(k) = \frac{B(q^{-1})}{B(1)} y^*(k) \]

where all parameters are replaced by their estimates.
Issues:

(1) Since the plant is really time-varying, we should construct a
time-varying discrete-time model.

(2) Since the plant is really nonlinear, we need to add a
correction term in the control law for using a linear model.

(3) Since the plant is really continuous-time, we want to add a
correction term in the control law for using a discrete-time
model.
Figure 1: Model-Reference algorithm control structure
Look at one link at a time:

\[ A(q^{-1})y(k) = q^d B(q^{-1}) u(k) \]

where \( q^{-1} \) is the delay operator, \( y(k) \) is the discretized output (assuming a single output per link), \( d \) is the system delay and

\[ A(q^{-1}) \triangleq a_1 + a_1 q^{-1} + ... + q_n q^{-n} \]

\[ B(q^{-1}) \triangleq b_0 + b_1 q^{-1} + ... + b_m q^{-m} . \]

It is desired to track a trajectory \( y^*(t) \) which, when discretized, is denoted by \( y^*(k+d) \). Further, any disturbance due to non-zero initial conditions should be eliminated.
Figure 1: Model-Reference algorithm control structure
Note:

\[ S(q^{-1}) u(k) = T(q^{-1}) y^*(k + d) - R(q^{-1}) y(k) \]

Let the desired closed-loop poles be

\[ P(q^{-1}) = 1 + p_1 q^{-1} + ... + p_l q^{-l} \]

Then if the controller parameters \( S(q^{-1}), R(q^{-1}) \) satisfy:

\[ A(q^{-1}) S(q^{-1}) + q^{-d} B(q^{-1}) R(q^{-1}) = P(q^{-1}), \]

the tracking requirement will be satisfied with

\[ T(q^{-1}) = \frac{P(q^{-1})}{B(1)} \]

and

\[ B(1) = \sum_{i=0}^{m} b_i \]

Because \( A(q^{-1}) \) and \( B(q^{-1}) \) may be unknown, some identification method is employed. Here we use the recursive least-squares method,
Transforming the discrete-time model into parametric form:

\[ y(k) = \Phi^T(k-1) \Theta \]

where \( \Phi^T(k-1) = [- y(k-1), \ldots, - y(k-n), u(k-1), \ldots, u(k-1-m)] \)

\( \Theta^T = [a_1, \ldots, a_n, b_0, \ldots, b_m] \),

the recursive formula to update the estimate \( \hat{\Theta}(k) \) is:

\[ \hat{\Theta}(k+1) = \hat{\Theta}(k) + F(k+1) \Phi(k) \varepsilon^O(k+1) \]

\[ F(k+1) = \frac{1}{\lambda_1} \left[ F(k) - \frac{F(k) \Phi(k) \Phi^T(k) F(k)}{\lambda_1 + \Phi^T(k) F(k) \Phi(k)} \right] \]

\[ \varepsilon^O(k+1) = y(k+1) - \Phi^T(k) \hat{\Theta}(k) \]

with forgetting factor \( 0 < \lambda_1 < 1 \).

The closed-loop system is:

\[ \left[ A(q^{-1}) S(q^{-1}) + q^{-d} B(q^{-1}) R(q^{-1}) \right] y(k) = B(q^{-1}) T(q^{-1}) y^*(k) \]

\[ y(k) = \frac{B(q^{-1})}{B(1)} y^*(k) \]

where all parameters are replaced by their estimates.
Issues:

(1) Since the plant is really time-varying, we should construct a
time-varying discrete-time model.

(2) Since the plant is really nonlinear, we need to add a
correction term in the control law for using a linear model.

(3) Since the plant is really continuous-time, we want to add a
correction term in the control law for using a discrete-time
model.
Figure 2: Self-Tuning adaptive control
Adaptive Algorithm:

Assume the continuous-time plant is described by:

\[ A(k, q^{-1}) y(k) = q^{-d} B(k, q^{-1}) u(k) \]

Then the algorithm can still be used with the following steps:

1. Use the modified RLS to estimate the parameters of the discrete-time model:

\[ y(k) = \Phi^T(k-1) \theta(k) \]

where

\[ \theta^T(k-1) = [a_1(k), ..., a_n(k), b_0(k), ..., b_m(k)] \]

and \( \Phi^T(k-1) \) is as before.

2. Calculate \( T(k,q^{-1}), S(k,q^{-1}), R(k,q^{-1}) \) based upon the estimates \( \hat{A}(k,q^{-1}), \hat{B}(k,q^{-1}) \). The ideal model is:

\[ A(k,q^{-1}) S(k,q^{-1}) + q^{-d} B(k,q^{-1}) R(k,q^{-1}) = P(q^{-1}) \]

\[ T(k,q^{-1}) = \frac{P(q^{-1})}{B(k,1)} \]

\[ B(k,1) = \sum_{i=0}^{m} b_i(k) \]

Note here the closed-loop poles are still assumed to be selected as constants.
For a robotic manipulator, each link may follow the closed-loop pole

\[ P(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2} \]

\[ p_1 = -2e^{-\delta w_n h} \cos(\sqrt{1 - \delta^2} w_n h) \]

\[ p_2 = e^{-2\delta w_n h} \]

where \( h \) is the sampling period, \( \delta \) is the damping coefficient and \( w_n \) is the natural frequency of oscillation, as selected by the designer.

3. Calculate the control law:

\[ S(k, q^{-1}) u(k) = \hat{T}(k,q^{-1}) y^*(k+1) - \hat{R}(k,q^{-1}) y(k) . \]

4. Repeat step 1 until complete time duration.
<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Figure 4</th>
<th>Figure 5</th>
<th>Figure 6</th>
<th>Figure 7</th>
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<td>30</td>
<td>60</td>
<td>30</td>
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</table>

Table 1: Simulation Parameters

Figure 3: Two-link manipulator example
Robot parameters:

\[ d_{11} = a_1 + a_2 \cos \theta_2 \]
\[ d_{12} = d_{22} = a_3 + 0.5 \times a_2 \cos \theta_2 \]
\[ d_{22} = a_3 \]
\[ c_1 = -a_2(\dot{\theta}_1 \dot{\theta}_2 + 0.5 \dot{\theta}_1^2) \sin \theta_2 \]
\[ c_2 = 0.5a_2 \dot{\theta}_1^2 \sin \theta_2 \]
\[ g_1 = a_4 \cos \theta_1 + a_5 \cos (\theta_1 + \theta_2) \]
\[ h_1 = b_1 \dot{\theta}_1 + b_2 \text{sgn}(\dot{\theta}_1) \]
\[ g_2 = a_5 \cos(\theta_1 + \theta_2) \]
\[ h_2 = b_3 \dot{\theta}_2 + b_4 \text{sgn}(\dot{\theta}_2) \]

where
\[ a_1 = m_1 \dot{l}_1^2 + m_2 \dot{l}_1^2 + m_2 \dot{l}_2^2 + I_1 + I_2 = 4.93 \]
\[ a_2 = 2m_2 \dot{l}_1 \dot{l}_2 = 0.94 \]
\[ a_3 = m_2 \dot{l}_2^2 + I_2 = 0.90 \]
\[ a_4 = (m_1 \dot{l}_1 + m_2 \dot{l}_1)g = 68.65 \]
\[ a_5 = m_2 \dot{l}_2 g = 10.64 \]
\[ b_1 = 6.82 \]
\[ b_2 = 3.5 \]
\[ b_3 = 3.91 \]
\[ b_4 = 3.5 \]
Figure 4a: Trajectory of the 1st link

Figure 4b: Trajectory of the 2nd link
Figure 4c: Tracking error
Figure 5a: Trajectory of 1st link

Figure 5b: Trajectory of 2nd link
Figure 5c: Tracking error
Figure 6a: Trajectory of 1st link

Figure 6b: Trajectory of 2nd link
Figure 7a: Trajectory of 1st link

Figure 7b: Trajectory of 2nd link
Figure 7c: Tracking error
To include an error term to compensate for going from continuous-time to discrete-time, consider the model:

\[ A(k-1,q^{-1}) y(k) = q^d B(k-1,q^{-1}) u(k) + e^0(k) \]

or

\[ A(k,q^{-1}) y(k) = q^d B(k,q^{-1}) u(k) + e(k) \]

where \( e^0(k) \) and \( e(k) \) are the a priori prediction error and a posteriori prediction error, respectively.

\[
\begin{align*}
\varepsilon^0(k) &= y(k) - \Phi^T(k-1) \hat{\theta}(k-1) \\
\varepsilon(k) &= y(k) - \Phi^T(k-1) \hat{\theta}(k)
\end{align*}
\]

These errors can be integrated into the closed-loop configuration and the control parameters may be selected to compensate for the errors.

If the pole placement control law is used, the closed-loop system is:

\[
[A(k,q^{-1}) S(k-1,q^{-1}) + q^d B(k-1,q^{-1}) R(k-1,q^{-1})] y(k) = B(k-1,q^{-1}) T(k-1,q^{-1}) y^*(k) + e(k) + e^0(k)
\]
where $e(k)$ is added to account for the time-varying discrete-time system.

That is:

$$e(k) = \sum_{i=0}^{n-1} \Delta A(k-i,q^{-1}) S^{i+1}(k-1,q^{-1}) y(k-1-i)$$

$$- \sum_{i=0}^{n-1} \Delta B(k-i,q^{-1}) S^{i+1}(k-1,q^{-1}) u(k-2-i)$$

$$+ \sum_{i=1}^{m} B^i(k,q^{-1}) \Delta R(k-i,q^{-1}) y(k-1-i)$$

$$+ \sum_{i=1}^{m} B^i(k,q^{-1}) \Delta S^1(k-i,q^{-1}) u(k-2-i)$$

$$- \sum_{i=1}^{m} B^i(k,q^{-1}) \Delta T(k-i,q^{-1}) y^*(k-1) \sum_{i=1}^{n} S_i(k) e^o(k-i)$$

where

$$S^1(k,q^{-1}) = q[S(k,q^{-1}) - S_0(k)]$$

$$\vdots$$

$$S^{i+1}(k,q^{-1}) = q[S^i(k,q^{-1}) - S_i(k)]$$

$$B^i(k,q^{-1}) = q[B^{i-1}(k,q^{-1}) - b_{i-1}(k)]$$

$$\Delta A(k,q^{-1}) = A(k,q^{-1}) - A(k-1,q^{-1}).$$

Note $e(k)$ can be calculated at time $k-1$. 
We consider the control law as

\[ S(k-1,q-1) u(k-1) = T(k-1,q-1) y^*(k) - R(k-1,q-1) y(k-1) + \phi(k-1) \]

The closed-loop system is:

\[ P(q-1) y(k) = B(k-1,q-1) T(k-1,q-1) y^*(k) + e(k) + e^*(k) + B(k-1,q-1) \phi(k-1) \]

In the ideal case, \( B(k-1,q-1) \) is stable and \( e^*(k) \) is given. We can calculate the correction input term \( \phi(k-1) \) as:

\[
\phi(k-1) = \frac{[P(q-1) - B(k-1,q-1) T(k-1,q-1)] y^*(k) - e(k) - e^*(k)}{B(k-1,q-1)}
\]

to cancel out \( e(k) \) and \( e^*(k) \). But, in the actual situation, \( B(k-1,q-1) \) may be unstable and \( e^*(k) \) is not given. We use \( e^*(k-1) \) instead of \( e^*(k) \) to calculate \( \phi(k-1) \) as:

\[
\phi(k-1) = \frac{[P(q-1) - B(k-1,q-1) T(k-1,q-1)] y^*(k) - e(k) - e^*(k-1)}{B(k-1,1)}
\]
Figure 8: Tracking error (including correction input)
Conclusions:

- Developed an adaptive control methodology which addresses three types of errors:
  - errors due to model linearization
  - errors due to controlling a time-varying plant
  - errors due to discretization

- Further work is needed to
  - determine stability range of model parameters ($\delta, \omega_n$)
  - include prediction error term
  - determine efficient methods for tuning correction error term to improve transient characteristics
  - look at implementation on several robotic experimental testbeds
IDENTIFICATION OF LINEAR SYSTEM MODELS AND STATE ESTIMATORS FOR CONTROLS

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OUTLINE

- Introduction
- State Estimation under Unknown Noises
- State Estimation under Unknown System Model and Unknown Noises
- Examples
- Conclusion
• Linear State Feedback Control System

Reasons for state estimator
(1) Number of sensors usually are less than number of states interested.
(2) Interested states are not always directly measurable.
(3) System is affected by process (input) noise.
(4) Outputs are corrupted by measurement (output) noise.

• Kalman Filter State Estimation

Requirements: (1) A state space model of the system
(2) The noise statistics (covariances)
• Some problems in control of flexible space structures

(1) Obtaining a model by ground testing might not be possible
(2) If an analytical model is used, modeling error could cause problem
(3) Working environment is unknown
(4) System characteristics might change (reorientation, structural damage, material deterioration, etc.)

• Techniques in need

(1) On-orbits system identification
(2) Adaptive state estimation
(3) Adaptive state-space system identification

• Problem statement

Given the input/output data of a linear system,

- suppose:

  (1) the system model is known but the noise statistics are unknown,

  (2) both the system model and noise statistics are unknown,

how to conduct state estimation for control purposes?
**State Estimation under Unknown Noises**  
*(Adaptive Kalman Filtering)*

Three approaches:

1. Estimating noise covariances (Q & R)
2. Using weighting least-squares method
3. Estimating Kalman filter gain directly

**Input-output Relationship of a System and of the Kalman Filter**

(1) State-space model of a system

\[ x_{i+1} = Ax_i + Bu_i + w_i \]
\[ y_i = Cx_i + v_i \]
\[ y = [C(zI - A)^{-1}R][u + v + w] \]

(2) State-space innovation model of the Kalman filter

\[ \hat{x}_{i+1} = A\hat{x}_i + Bu_i + AK\epsilon_i \]
\[ y_i = C\hat{x}_i + \epsilon_i \]
\[ (1) y = [C(zI - A)^{-1}R][u + v + w] + C(zI - A)^{-1}K\epsilon_i \]
\[ (2) y = [C(zI - \bar{A})AK] + [C(zI - \bar{A})^T R][u + v + w] + C(zI - \bar{A})^T K\epsilon_i \]

K : Kalman gain  
\( \epsilon_i \) : residual
- Relation between Residual and Stochastic Part of Output

\[ s_i = y_i - \sum_{m} C_k^{m} m_k, s_i = \sum_{m} C_k^{m} e_{i,m} + e_i = \sum_{m} M_i m_k \]  
(MA model)

\[ e_i = \sum_{m} \epsilon_i m_k \]  
(AR model)

\[ N(z^{'}) = M^{'}(z^{'}) \text{ or } M(z^{'}) = N^{'}(z^{'}) \]

\[ M(z^{'}) = I + C(zI-A)^{-1} AK \]
\[ = I + CAKz + CA'Kz + \cdots + CA'Kz \]

- Obtaining Kalman Filter Gain

(1) Invert the whitening filter

\[ N^{'}(z^{'}) = M(z^{'}) = I + CAKz + CA'Kz + \cdots + CA'Kz \]

(2) Form two matrices

\[ G = \begin{bmatrix} CAK \\ CA'R \\ \vdots \\ CA'K \end{bmatrix}, \quad H = \begin{bmatrix} CA \\ CA' \\ \vdots \\ CA' \end{bmatrix} \]

(3) Find the least-squares solution of \( K \)

\[ \hat{K} = HG \quad \text{where} \quad H = (H'H)^{-1}H' \]

(4) Perform state estimation

\[ \hat{x}_t = \hat{A}\hat{x}_{t-1} + \hat{B}u_{t-1} \]
\[ \hat{x}_t = \hat{x}_{t-1} + \hat{K}(y_t - C\hat{x}_t) \]
- Inverse Filter Method for Adaptive State Estimation

State Estimation under Unknown System Model and Unknown Noises

- system identification
- Kalman filter identification
State Estimation under Unknown System Model and Unknown Noises

Two approaches:

1. Identifying a system model first, then the filter gain.
2. Identifying a system model and the filter gain at the same time.

System Identification:

1. Obtaining system mathematical models from input-output data.
2. Frequency-domain and time-domain
3. Model types: transfer function, difference equation, state-space equation, impulse response, etc.

For control purposes, time-domain state-space system identification methods are preferred.

- Input-output Relationship for a System and for the Kalman Filter

\[
\begin{align*}
\text{system:} & \quad y = [C(zI - A)^{-1}B]u + [C(zI - A)^{-1}]w + v \\
\text{filter:} & \quad (2)y = [C(zI - \tilde{A})^TAK]y + [C(zI - \tilde{A})^T]B]u + e
\end{align*}
\]
• Markov Parameters and Eigensystem Realization Algorithm (ERA)

(1) System Markov parameters (impulse response)

\[ y_k = CB_{u_{k-1}} + CAB_{u_{k-2}} + \ldots + CA^{i-1}B_{u_{k-i}} + CA^iK_{e_{k-i}} + \ldots + CA^iK_{e_{k-i-1}} + \epsilon_k \]

\[
\begin{bmatrix}
CB & CAB & \ldots & CA^{i-1}B \\
CAK & CA^2K & \ldots & CA^iK
\end{bmatrix}
\]

(2) Filter Markov parameters

\[ y_k = CAKy_{k-1} + CA^qKy_{k-q-2} + \ldots + CA^qK_{y_{k-q}} + CB_{u_{k-1}} + CA^qB_{u_{k-q-2}} + \ldots + CA^qB_{u_{k-q-1}} + \epsilon_k \]

\[
\begin{bmatrix}
CAK & CA^2K & \ldots & CA^iK \\
CA^qK & CA^qB & \ldots & CA^qB
\end{bmatrix}
\]

(3) ERA (a system identification method)

\[
\begin{bmatrix}
CB & CAB & \ldots & CA^{i-1}B
\end{bmatrix}
\xrightarrow{\text{ERA}}
\begin{bmatrix}
A', B', C
\end{bmatrix}
\]

• Relationship between Filter Markov Parameters and System Markov Parameters

ARX model:

\[ y_k = \sum_{i=1}^{q} CA^iK_{y_{k-i}} + \sum_{i=1}^{q} CA^iB_{u_{k-i}} + \epsilon_k = \sum_{i=1}^{q} M_{y_{k-i}} + \sum_{i=1}^{q} N_{u_{k-i}} + \epsilon_k \]

(1) Markov parameters for stochastic input

\[ CA^kK = M_q + \sum_{i=1}^{q-1} M_q, CA^iK \]

(2) System Markov parameters:

\[ CB = N_q \]

\[ CA^qB = N_{q-1} + \sum_{i=1}^{q-1} M_q, CA^iB \]

\[ \xrightarrow{\text{ERA}} \begin{bmatrix} A', B', C \end{bmatrix} \]
Examples

- Mini-Mast
- Ten-bay structure
Fig. 6.4 Mini-Mast structure.
The identified modal parameters of the Mini-Mast

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<table>
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<th>Mode</th>
<th>freq. (rad/sec)</th>
<th>Damp (%)</th>
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<td>1.20</td>
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<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>38.4473</td>
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Output and residual comparisons (1000 data)
Fig. 7.1 Ten-bay truss structure test configuration.
• Identification of a ten-bay structure

The identified modal parameters

<table>
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<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
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<tr>
<td>3</td>
<td>48.5</td>
<td>0.40</td>
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</table>

• Output and residual comparisons (3750 data)
Conclusions

(1) Viewing the input-output relationship of a system through the Kalman filter provides helpful insights.

(2) If the system state-space model is known, the Kalman filter gain can be obtained by whitening the stochastic part of the output.

(3) A system state-space model and the corresponding Kalman filter can be identified at the same time from the parameters of an ARX (a difference equation) model.

(4) For a stochastic system, a complete state-space realization is \([A, B, C, K]\), where the Kalman filter gain characterizes stochastic property of the system.
Fuzzy Logic Controller for Manipulator Systems: Preliminary Results

Presented by
R. J. Stanley II
H. Roberts

North Carolina State University
Mars Mission Research Center

Project Members:
D. Gerber
H. Roberts
R. J. Stanley II
J. C. Windsor

Project Advisor:
Dr. G. Lee

Mars Mission Research Center, N.C.S.U.
OUTLINE:

1) MOTIVATION

2) FUZZY CONTROL

3) RESULTS AND COMPARISONS

4) FUTURE ACTIVITIES
\[ \sum M = I \ddot{\theta} \]

\[ I \ddot{\theta} = -m \cdot g \cdot L \cdot \sin(\theta) + \tau \]

\[ \tau = \text{Torque applied by motor} \]
\[ I = \text{Inertia} = m \cdot L^2 \]

\[ \tau = -K_p(\theta - \theta_d) - K_d(\dot{\theta} - \dot{\theta}_d) - K_i \int_0^t (\theta - \theta_d) \, ds \]

\[ \theta_d = \text{desired position} \]
\[ \dot{\theta}_d = \text{desired velocity} \]

\[ \ddot{\theta} + \omega_n^2 = -K_p \theta - K_d \dot{\theta} - K_i \int_0^t \theta \, ds \]

\[ \omega_n^2 = \frac{g}{L} \]

\[ y = \int_0^t \theta \, ds \]
\[(s + \gamma) \cdot \left( s^2 + 2\alpha s + \alpha^2 + \beta^2 \right) \]

\[\alpha = \text{Vibration decay rate}\]

\[\beta = \text{Closed-loop frequency of oscillation}\]

\[\gamma = \text{Steady-state error decay rate}\]

\[K_p = \left[ \left( \alpha^2 + \beta^2 + 2\alpha\gamma - \omega_n^2 \right) \right] \cdot I\]

\[K_i = \left[ \left( \alpha^2 + \beta^2 \right) \gamma \right] \cdot I\]

\[K_d = (2\alpha + \gamma) \cdot I\]

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Philosophy:

"The fuzzy algorithm is based on intuition and experience, and can be regarded as a set of heuristic decision rules or 'rules of thumb'.'
Membership Matrix Table

<table>
<thead>
<tr>
<th>Linguistic Sets</th>
<th>Quantized Levels</th>
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<tr>
<td></td>
<td>-4  -3  -2  -1  0  1  2  3  4</td>
</tr>
<tr>
<td>LP</td>
<td>0    0    0    0    0    0   0   0.6  1</td>
</tr>
<tr>
<td>SP</td>
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</tr>
<tr>
<td>ZE</td>
<td>0    0    0    0.6  1   0.6  0   0    0   1</td>
</tr>
<tr>
<td>SN</td>
<td>0    0.6  1   0.6  0   0    0    0    0   1</td>
</tr>
<tr>
<td>LN</td>
<td>1    0.6  0   0    0    0    0    0    0   1</td>
</tr>
</tbody>
</table>

Graphical Representation of linguistic Rule 1

Error (e)  Error change (ê)  Control input (I)

Universe of discourse (U)

(If error is ZE and error change is SP then control input is SN)

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Center of Gravity

\[ I = \sum_{1}^{n} \frac{(u_{n} \times U_{n})}{\sum_{1}^{n} u_{n}} \]

\( u \equiv \) The membership function

\( U \equiv \) The universe of discourse

\( n \equiv \) The number of contributions

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Membership function (u)

Contribution by rule 1

Contribution by rule 2

Contribution by rule 4

Universe of Discourse (U)

Sample Lookup Table

<table>
<thead>
<tr>
<th>Error</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
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<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
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<td>1</td>
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<td>3</td>
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<td>-1</td>
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<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-4</td>
<td>-5</td>
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</tbody>
</table>
Three Joint Displacements With PID
Alpha=0.2303 For All Three Displacements
Three Joint Displacement With Fuzzy Logic Control

Joint Displacement With PID
Alpha = 2.303 For Link One

Displacement (Radians)

Time (Seconds)
Position of Link One vs. Time
With Fuzzy Logic Control

Position of Link Two vs. Time
With Fuzzy Logic Control
Position of Link Three vs. Time
With Fuzzy Logic Control

Joint Displacement With and Without
Parameter Variation (PID)
Joint Displacement With and Without Parameter Variation (Fuzzy Controller)

Joint Displacement With and Without Parameter Variation (Fuzzy Controller)
Future Activities:

- Coordinated Robotic Testbed
  - NASA missions for on orbit assembly.

- Issues:
  - Flexibility in links/joints (R.M.S., A.P.S.)
  - Adaptability to varying inertia
  - Mobility of manipulator system
  - Master / Slave
  - Controller
    a) Tracking
    b) Vibration compensation

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Recursive Least Squares Approximation to a Third Order Polynomial

\[ y = -7.9365 \times 10^{-3} + 1.1720x - 0.41270x^2 + 7.4074e-2x^3 \quad R^2 = 0.837 \]
\[ E(t) :\quad Y(t) - R(t) \]

\[ R(t) := \text{Reference Position} \]

\[ U(t) := \text{Control Command} \]
The Universe Of Discourse Of The Functions

The Error, $E(t)$, ranges from -1000 to 1000

The Error change, $\Delta E(t)$, ranges from -100 to 100

The Gain, $A(t)$, ranges from 0.2 to 0.707

The following rules were implemented.

1. If $E(t)$ is LP and $\Delta E(t)$ is any, then $A(t)$ is LP
2. If $E(t)$ is MP and $\Delta E(t)$ is LP, then $A(t)$ is LP
3. If $E(t)$ is SP and $\Delta E(t)$ is MP, then $A(t)$ is MP
4. If $E(t)$ is ZF and $\Delta E(t)$ is any, then $A(t)$ is SP
5. If $E(t)$ is SN and $\Delta E(t)$ is MN, then $A(t)$ is MP
6. If $E(t)$ is MN and $\Delta E(t)$ is LN, then $A(t)$ is LP
7. If $E(t)$ is LN and $\Delta E(t)$ is any, then $A(t)$ is LP

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Defining the Fuzzy subsets.

For -1000 to -875 \( E(t) \) is -1000

For -875 to -675 \( E(t) \) is -750

For -675 to -475 \( E(t) \) is -500

For -475 to -275 \( E(t) \) is -250

For -275 to 275 \( E(t) \) is 0

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<table>
<thead>
<tr>
<th>QUANTIZED VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(t) )</td>
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<td>-------------------</td>
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</table>
### COARSE LOOK-UP TABLE

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*Three Joint Displacements With PID
Alpha=.2303 For All Three Displacements*
Joint Displacement With PID
Alpha = .2303 For Link One

Displacement (Radians)

Time (Seconds)

Three Joint Displacement With
Fuzzy Logic Control

Displacement (Radians)

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Joint Displacement With and Without Parameter Variation (Fuzzy Controller)
Joint Displacement With and Without Parameter Variation (Fuzzy Controller)
Future Activities:

- The number of rules
- Overlapping of subsets
- Matrix membership (shape)
- Calibration Techniques
- Predictor (time delays)
Editor's Summary of the Panel Discussion

The Panel was moderated by the Editor, Dr. Dave Ghosh, and consisted of six members, three from NASA Langley Research Center and three from the Mars Mission Research Center. The members from NASA Langley were:

Dr. Raymond C. Montgomery
Mr. Jerry R. Newsom
Mr. Lawrence W. Taylor

and the members from the Mars Mission Research Center were:

Dr. Lawrence Silverberg
Dr. Ethelbert Chukwu
Dr. Gordon K. F. Lee

Each panel member was given an opportunity to express his views concerning research needs and opportunities related to control systems technology for accomplishing a manned Mars mission. After this the floor was opened for discussion. This section of the proceedings is the editors independent interpretation of the comments made during the course of the panel discussions.

It was felt that though theoretical development is essential, especially in the initial phases of a new control problem, experiments must be carried out in conjunction to establish a thorough understanding of problems and refinement of theories. In short, experimental activity is an essential element of research. In this regard Langley testbeds should be developed and made available to test new ideas emanating from the MMRC. It was recognized that in an university environment faculty tend to work independently and isolate themselves from practical problems. It was also recognized that 'design' is an important part of engineering curricula and there should be a balance between research and design. In view of this, MMRC is trying to integrate faculty from different areas to work in a design framework. It was also felt that research plans for Mars Mission should reflect its long term nature and should not pander to groups looking for quick results.

Several suggestions were made for inclusion in the university research activity. Students should work on simple and fundamental problems. Computationally exact solutions, now available, should be used to revisit old problems and to throw light on new problems. On-orbit
assembly issues involving human and robots especially in the presence of time delay, should be explored. Also, human operator models should be developed and used in control system synthesis, analysis, and design.
This publication is a collection of papers presented at the Mars Mission Research Center workshop on Ongoing Progress in Spacecraft Controls held at the NASA Langley Research Center, Hampton, Virginia, on January 13, 1992. It was jointly sponsored by the NASA Langley Research Center Guidance, Navigation, and Control Technical Committee and the Mars Mission Research Center. The technical program addressed additional Mars mission control problems that currently exist in robotic missions in addition to human missions. Topics include control system design in the presence of large time delays, fuel-optimal propulsive control, and adaptive control to handle a variety of unknown conditions.