Validation of Finite Element and Boundary Element Methods for Predicting Structural Vibration and Radiated Noise

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ABSTRACT

Analytical and experimental validation of methods to predict structural vibration and radiated noise is presented in this paper. A rectangular box excited by a mechanical shaker was used as a vibrating structure. Combined finite element method (FEM) and boundary element method (BEM) models of the apparatus were used to predict the sound radiated from the box. The FEM was used to predict the vibration, and the surface vibration was used as input to the BEM to predict the sound intensity and sound power. Vibration predicted by the FEM model was validated by experimental modal analysis. Noise predicted by the BEM was validated by sound intensity measurements. Three types of results are presented for the total radiated sound power: (1) sound power predicted by the BEM model using vibration data measured on the surface of the box, (2) sound power predicted by the FEM/BEM model, and (3) sound power measured by a sound intensity scan. The sound power predicted from the BEM model using measured vibration data yields an excellent prediction of radiated noise. The sound power predicted by the combined FEM/BEM model also gives a good prediction of radiated noise except for a shift of the natural frequencies that are due to limitations in the FEM model.

INTRODUCTION

The prediction of noise in the design stage is important for building low-noise and high performance machines. Two steps are involved in predicting noise radiated by a machine: prediction of machine vibration; and prediction of noise based on the predicted vibration or on the vibration obtained from other approaches (e.g., experimental data). Analytical methods and the finite element method (FEM) are used to predict machine vibration. To predict machine radiated noise, analytical methods, the finite element method, and the boundary element method (BEM) are used.

Perreira and Dubowsky (1979, 1980) used a combined analytical-numerical method to model simply shaped machine elements. In their work, a machine link was modeled as a vibrating beam in an infinite, rigid baffle, and the Rayleigh integral was used to calculate the radiated noise. The major advantage of using the Rayleigh integral is its solution efficiency because it does not require a simultaneous equation solution; the sound pressure is determined by direct integration of the known boundary normal velocities. Certain simple machine elements can be modeled well with such treatments; however, the assumptions required to use the Rayleigh integral are rarely met by realistic vibrating structures.

The acoustic finite element method has been used successfully for interior problems in which the acoustic field is calculated within an enclosed volume, such as printer enclosures and vehicle cabins. Bernhard and Takeo (1988) used the FEM to model small cavity enclosures with acoustical treatment materials, sound sources, and apertures. The sound pressure and sound intensity inside the cavity were predicted. The sensitivity of two acoustic design objective functions, the radiated sound power through apertures and the total energy in the cavity, to the surface acoustic treatments were also calculated. Sung and Nefske (1984) used a coupled structural-acoustic finite element model to predict vehicle cabin vibration and noise. The predicted structural response and sound pressure were verified by experiments. For exterior problems, however, one encounters difficulties using the FEM, such as where to stop the domain discretization, and the substantial computational effort required because the three-dimensional acoustic field must be discretized.

The BEM requires substantially less computational effort for exterior problems because only the boundary needs to be discretized rather than the whole acoustic domain, as with the FEM. Termination of domain discretization and attendant numerical closure, problems commonly encountered when using the domain methods, do not appear when using the BEM. Also, the unknown variables on the surface are found directly from the BEM surface solution without having to solve for the values at other points in the exterior region.

Various researchers, including Copley (1967), Schenck (1968), and Meyer et al. (1978), have verified the radiated noise predicted by the BEM by using spheres, cylinders, boxes, etc., where analytical solutions exist. Smith and Bernhard (1988) also have verified predicted noise, using the BEM and the Rayleigh integral equation from measured vibration, with sound pressure measurements in a semianechoic chamber. Oppenheimer (1988) used the FEM and the BEM to predict the sound power and sound pressure of a machine-like enclosure. The predicted sound power and sound pressure were then validated by experiments. The sound power levels were computed from an average of sound pressure level measurements, assuming a diffuse sound field. Single frequency excitation was used for the acoustic measurements, however, such excitations may not excite enough room modes to approximate a true diffuse field.

This paper presents a combined numerical and experimental validation of methods to predict structural vibration and radiated noise. The modal superposition method is used to predict the vibration, which was validated by experimental modal analysis. A modified Helmholtz integral equation for bodies sitting on an infinite plane (Seybert and Wu, 1989) is used to predict the radiated noise, which was validated by sound intensity measurements. Three types of results are presented for the total radiated sound power: (1) sound power predicted by the BEM model using measured vibration data, (2) sound power predicted by the FEM/BEM model, and (3) sound power measured by the sound intensity method.
EXPERIMENTAL APPARATUS AND MEASUREMENTS

Preliminary Considerations

In most experimental/computational validation studies of the type reported herein, the experimental portion is the more difficult part of the study. Instrumentation variability, drift, calibration, dynamic range, signal-to-noise ratio, and a host of other issues, make it difficult to obtain highly repeatable data from an experiment. In addition, the time required to obtain the quantity and type of data (vibration, force inputs, sound pressure, and sound intensity) needed for such a validation is lengthy, thereby increasing the likelihood that experimental conditions will change during the course of the test.

The apparatus chosen for the validation study was a simple rectangular box excited by an electromechanical shaker. The simplicity of this apparatus, with its very controlled excitation as compared to an actual machine (e.g., engine or pump) resulted in a relatively high degree of repeatability without an undue amount of experimental data to be acquired and processed. Even so, most of the discrepancies between the experimental and predicted data were ascribed to the limitations of the experiment, as described in the following section.

Vibration Measurements

The experimental apparatus is shown schematically in Fig. 1. The structure is a rectangular box measuring 279 by 305 by 298 mm. Only the top plate is flexible, whereas the other five surfaces are more massive and stiffer and are assumed rigid. The top plate is aluminum with a thickness of 1.8 mm, and the other five surfaces are steel, each with a thickness of 12.7 mm. Four strips of 12.7-mm square steel rod attach the top plate to the edges of the four side plates to approximate clamped boundary conditions.

A shaker driven by random noise was mounted inside the structure to excite the top plate. An impedance head was used to measure the applied force and the driving point acceleration. An accelerometer, with a mass of 1.5 g, was used to measure the acceleration at various points on the top plate. To ensure that the force was applied in the direction perpendicular to the top plate, a stinger made of music wire connected the impedance head to the top plate. This connection was very effective in minimizing the excitations in directions other than perpendicular to the top plate. The experimental apparatus was set on a concrete floor to create a half-space radiation condition.

A two-channel dynamic signal analyzer was used to collect calibrated acceleration data normalized by the input force of the shaker. Two charge amplifiers were used to amplify and condition the signals before they entered the analyzer. The data in the analyzer were then transferred to a PC and written to disk. The acceleration data were used in two ways: to validate the acceleration data predicted by FEM and as the boundary condition to the BEM program BEMAP (Seybert et al., 1990). An interface program was written to transfer the vibration data to a standard BEMAP input file.

Sound Intensity

Two methods may be used to determine the sound power with sound intensity measurements. In the first, more accurate method, sound intensity measurements are obtained at discrete points; in the second method, the sound intensity measurements are obtained by scanning. Scanning measurements were used in the present study because they are quicker and more convenient than fixed point measurements. A 508- by 508- by 508-mm wire frame was used to define a rectangular scanning surface. The signal analyzer was set to ensemble-average during the scan. The scan included two sets of parallel line sweeps, back and forth and up and down on each of the five sides of the rectangular scanning surface.

A commercial sound intensity measurement system was used to measure the sound intensity and to calculate the total radiated sound power. The measured sound intensity and total radiated sound power were normalised by dividing by the square of the input force. The input force was measured by the impedance head processed by the signal analyzer, and transmitted to the PC. This made it possible to make direct comparison of measured sound power with that predicted by the FEM/BEM model.

FEM/BEM MODELS

Finite Element Model

Two flexible top plate of the apparatus in Fig. 1 was modeled using the FEM program ANSYS. Clamped boundary conditions were used along the edge of the plate. The top plate was modeled using 100 quadrilateral quadratic thin-shell elements. A unit force was applied to excite the plate at a point approximately 50 mm from the geometric center of the plate. The mass of the shaker and impedance head were also included in the FEM model.

![Diagram of experimental apparatus](image)

Figure 1.—Experimental apparatus.
The mass of the accelerometer (1.3 g) was neglected in the FEM model because it did not significantly affect the structural dynamics in the frequency range of interest, limited here to 100 to 500 Hz by the FEM model.

The modal superposition method was used to compute the harmonic response of the FEM model. Figure 2 shows the acceleration normalized by the applied force at the driving point of the top plate of the box. The difference between the experimental data and the FEM data is the result of a shifting of the resonance frequencies. The largest shift (about 5 percent) occurs at the fourth resonance frequency. The major reason for the discrepancy is that clamped boundary conditions for the top surface were used in the FEM model. The boundary conditions of the real system are not perfectly clamped, but are between simple support and clamped. Thus, the FEM model is stiffer than the real system.

The experiment determined the equivalent viscous damping of the structure for each mode by using the "half-power bandwidth" method (Thomson, 1981). Figure 2 demonstrates that the equivalent viscous damping used in the FEM model resulted in the correct peak response for that particular measurement point. However, at other points, the measured values of equivalent viscous damping did not agree as well as that shown in Fig. 2.

There are four modes in the frequency range 100 to 500 Hz. All the mode shapes from the FEM model share the same trend and shape with their counterparts from the experiment. For example, Fig. 3(a) is the mode shape of the third mode calculated from the FEM model, and Fig. 3(b) is the mode shape of the third mode obtained from a sand pattern experiment. Figures 3(c) and (d) are mode shapes of the fourth mode from the FEM and the experiment, respectively.

![Figure 2](image_url)

Figure 2.—Transfer function using modal superposition method.

![Figure 3](image_url)

Figure 3.—Comparison of predicted and measured modes of the top plate.
Boundary Element Model

The BEM was used to predict the noise from the vibrating structure. Two BEM models were used to model the vibrating structure: one for the measured vibration; the other, for the vibration calculated by the FEM model. The mesh used for the measured vibration was more coarse (i.e., less conservative) than the one used for FEM-calculated vibration to reduce the amount of time needed to acquire the vibration data. For the measured vibration, the top plate of the structure was modeled by 36 quadrilateral quadratic boundary elements, resulting in 85 nodes at which the normal surface velocity was measured. These vibration data were measured using the accelerometer shown in Fig. 1. The velocities at the grid points along the edge of the top plate were set to zero since the clamped boundary conditions were used in the FEM model. The other five surfaces were each modeled by 36 quadrilateral quadratic boundary elements (Fig. 4(a)). The total number of nodes for this BEM model was 650. All the grid point velocities on the four side plates were assumed to be zero, which is an approximate assumption for the structure in the present study; a quick check showed that the magnitude of the vibration of the side plates was less than one-tenth of that of the top plate.

For the FEM-calculated vibration, the BEM mesh consisted of 100 quadrilateral quadratic elements on the top plate, resulting in 341 nodes. The four side surfaces were each modeled by 35 elements (116 nodes); the bottom surface was modeled by 25 elements (96 nodes). The total number of nodes for this model was 737. The velocity of the nodes at the edge of the top plate and the velocity of each point of the side plates and the bottom plate were set to zero.

![Image](image.png)

(a) Model used with measured vibration.

(b) Model used with FEM predicted vibration.

Figure 4.—BEM models.

For a BEM model, the mesh size is required to be some fraction of either the acoustical or structural wavelength, whichever is smaller, at the highest frequency of interest. For quadratic boundary elements, this fraction should be one-half of a wavelength or less, and for linear boundary elements, one-quarter of a wavelength or less. The acoustical wavelength in air at 500 Hz is approximately 69 cm whereas the structural wavelength of the highest mode is about 25 cm (see Fig. 5). Consequently, a mesh size of approximately 12 cm would have been appropriate for the present radiation study. However, to be conservative, meshes of 5 cm for the measured vibration input and 6 cm for the FEM-calculated vibration were used. The BEM mesh on the top surface for the FEM-calculated vibration was made finer (3 cm) to correspond to the FEM mesh on the top surface.

Formulation

The theoretical background for the BEM is well-known and is documented in the literature (Brebbia, 1978; Brebbia and Walker, 1980; Brebbia et al., 1984; and Banerjee and Butterfield, 1981). However, for a body sitting on a reflecting surface, the theory must be extended slightly, as discussed next.

![Image](image.png)

Figure 5.—Nomenclature for a body sitting on an infinite plane.

For a body sitting on an infinite reflecting plane $S_H$ (Fig. 5) the boundary of the body $S$ can be divided into two parts: $S_p$, which is in contact with $S_H$; and $S_B$, which is exposed to the acoustic medium $B'$. The boundary integral equation for acoustic radiation can be written as (Seybert and Wu, 1989)

$$C(P)\phi(P) = \int_{S_p} \left[ \Phi_H(P,Q) \frac{\partial \Phi(Q)}{\partial n} - \frac{\partial \Phi_H(P,Q) \Phi(Q)}{\partial n} \right] dS(Q)$$

(1)

where $\phi$ is the velocity potential satisfying the Helmholtz equation in $B'$ and the Sommerfeld radiation condition in the far field; $n$ is the inward normal, and $\Phi_H$ is the half-space Green function which takes the form

$$\Phi_H = \frac{e^{-ikr}}{r} + R_H \frac{e^{-ikr_1}}{r_1}$$

(2)

where $R_H$ is the reflection coefficient of the infinite plane; $r$ is the distance between a point $Q$ on $S_B$ and a point $P$ either in $B'$ or $B$, or on $S_p$; and $r_1$ is the distance between $Q$ and the image point of $P$ with respect to $S_B$. The reflection coefficient $R_H$ is equal to 1 for a rigid, infinite plane or -1 for a soft infinite plane. In the present study, the floor that supported the apparatus in Fig. 1 was considered rigid; thus $R_H = 1$.

The coefficient $C(P)$ in Eq. (1) is $4\pi$ for $P$ in $B'$, and 0 for $P$ in $B$. If $P$ is on $S_B$, but not in contact with $S_H$, $C(P)$ can be evaluated by

$$C(P) = 4\pi - \int_{S_B \cup S_C} \frac{\partial \phi}{\partial n} dS$$

(3)
If $P$ is on $S_0$, and also in contact with $S_H$, $C(P)$ can be evaluated by (Seybert and Wu, 1989)

$$C(P) = (1 + R_H) \int_{2\pi} \int_{\delta s} \frac{1}{\delta s} dS$$

where $\int_{2\pi} \int_{\delta s} \frac{1}{\delta s} dS$ is a function of the geometry and the frequency of the vibration. The vibration data are obtained from measured vibration or from a finite element analysis. In the present study, both measured and predicted vibration data were used as input to BEMAP.

To simplify the transfer of vibration data and surface geometry into BEMAP, a number of software interfaces were written. An interface for the BEMAP program was used to perform the acoustic analysis. The input to BEMAP includes the surface geometry of the structure, the vibration of the structure, and the frequency of the vibration. The vibration data are used as input to BEMAP to calculate the sound pressure and sound intensity at far field points.

RESULTS

The program BEMAP calculates the sound pressure and sound intensity at field points in the near and far fields of the source as well as the sound pressure and sound intensity on the source itself. The sound intensity is integrated over the source to yield the sound power. Although all of these quantities can be validated by measurement, some measurements are more accurate than others. For example, sound pressure measurements are generally repeatable because of inaccuracies in microphone position and "dither" in the sound directivity caused by slight changes in source radiation. Consequently, the sound power of the source was used for validation in this study. The results fall in three categories: measured sound power; numerical prediction of sound power; and prediction of sound power from measured vibration. In each case, the sound power is normalised by dividing by the square of the applied force.

To validate the acoustic portion of the model, measured vibration data were used as input data in the BEMAP program shown in Fig. 4(a). The total radiated sound power was measured using the sound intensity method. The results are compared in Fig. 6. The shape of the measured and predicted sound power traces are very similar, but the measured sound power is generally, slightly greater than predicted values.

The experimental data far below the peaks (i.e., the data below 90 dB) are not smooth and are generally higher than predicted values. Two reasons for this discrepancy are as follows: (1) In the BEM model, we have assumed zero vibration from the sides of the box. Because, the sides are much stiffer than the top and the excitation is applied directly to the top rather than the sides, this is not a bad assumption. However, there is some vibration and, hence, sound radiation from the sides; and (2) Phase mismatch in the microphones used in the intensity probe introduces a residual intensity error which becomes significant for frequencies at which the measured intensity is very low compared to the peak values.

Even though the results in Fig. 6 agree reasonably well, there are significant discrepancies at the resonance frequencies. During the 4-hr period required to obtain the vibration spectrum, the amplitude and frequencies of the resonances shifted around randomly. Thus, the vibration spectrum acquired at any given point on the top plate was slightly different than the vibration at other points. The sound power predicted from the measured vibration tended to average this effect and resulted in resonance peaks that were slightly lower (and broader) than those in the measured sound power (Fig. 6). The measured sound power, on the other hand, was determined at a specific time, usually at the beginning or at the end of the experiment.

Figure 7 shows the total radiated sound power obtained by measurement (i.e., a replication of the measured data in Fig. 6) and that obtained by using the FEM/BEM model in Fig. 4(b). Except for a shift in the resonance frequencies (due to the FEM model, see Fig. 2), the combined FEM/BEM model yields a reasonable prediction of the radiated noise.

Most of the problems previously described can be attributed to the high-Q of the resonances and the difficulty of estimating the actual damping of the apparatus. In many respects, the apparatus used in this study is a worst-case scenario; most practical "built-up" structures will have more damping than is present in the experimental apparatus used here. Table I summarises the results at the resonance peaks.
and in Table II show that frequency averaging or summing mitigates to a
4
FEM/BEM prediction yields the
[29x520]The measured sound power has the
[29x557]one-third
[30x547]power,
[30x575]on the basis of sound pressure levels in one-third octave bands, full octave
[52x128]Experimental
TABLE II. TOTAL RADIATED SOUND POWER LEVEL

<table>
<thead>
<tr>
<th>Mode</th>
<th>Sound power, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
</tr>
<tr>
<td>1</td>
<td>115±0.5</td>
</tr>
<tr>
<td>2</td>
<td>111±1</td>
</tr>
<tr>
<td>3</td>
<td>107.5±0.5</td>
</tr>
<tr>
<td>4</td>
<td>109.5±0.5</td>
</tr>
</tbody>
</table>

For noise control programs in industry, most design decisions are made
on the basis of sound pressure levels in one-third octave bands, full octave
bands, or overall sound level. The data in Figs. 6 and 7 are plotted in
one-third octave bands in Fig. 8. Table II shows the total radiated sound
power, both Linear and A-weighted, in the frequency range 100 to 500 Hz.
The measured sound power has the highest value, and the combined
FEM/BEM prediction yields the lowest value, with a difference of 2.5 and
4 dB for the Linear and A-weighted values, respectively. The data in Fig. 8
and in Table II show that frequency averaging or summing mitigates to a
large extent the errors associated with shifting resonances.

![Figure 8](image-url)

**Figure 8.** One-third octave band for sound power.

**TABLE II. TOTAL RADIATED SOUND POWER LEVEL**

<table>
<thead>
<tr>
<th>Total radiated sound power</th>
<th>Linear, dB</th>
<th>A-weighted, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>120.5</td>
<td>113.5</td>
</tr>
<tr>
<td>Predicted measured vibration</td>
<td>119.0</td>
<td>112.0</td>
</tr>
<tr>
<td>Predicted FEM vibration</td>
<td>118.0</td>
<td>109.5</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Analytical and experimental methods were used to validate the
predictions of the boundary element method (BEM) acoustic computer code
BEMAP. A finite element method (FEM) study was performed to predict the
vibration of a simple rectangular box. Vibration measurements were
compared with the FEM predictions. Sound power radiation from the box
was predicted based on both the predicted and measured vibration. Sound
power predictions were compared to measured values. The results show the
following:

1. Measured and predicted values of sound power show good agreement
   in one-third octave or wider frequency bands.

2. Limitations in the FEM model and in the estimation of damping
   cause discrepancies between measured and predicted sound power when
   viewed on a narrow-band basis. Because accurate vibration data are required
   for the accurate prediction of sound power, limitations in the vibration
   portion of the FEM/BEM model can affect significantly the sound power
   results.

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