CROSS SECTION PARAMETRIZATIONS FOR COSMIC RAY NUCLEI
I. SINGLE NUCLEON REMOVAL.

by

John W. Norbury

Physics Department
Rider College
Lawrenceville, NJ 08648

and

Lawrence W. Townsend

NASA Langley Research Center
Hampton, VA 23665
ABSTRACT

Paramterizations of single nucleon removal from the electromagnetic and strong interactions of cosmic rays with nuclei are presented. These parametrizations are based upon the most accurate theoretical calculations available to date. They should be very suitable for use in cosmic ray propogation through interstellar space, the Earth's atmosphere, lunar samples, meteorites, spacecraft walls and lunar and martian habitats.

PACS: 97.70.S, 96.40.De
I. INTRODUCTION

Galactic cosmic rays (Shapiro 1983, Friedlander 1989) are very high energy particles confined to the region of our Milky Way galaxy. They consist of about 98% fully stripped nuclei including protons and alpha particles, and about 2% electrons and positrons (Simpson 1983). Of the nuclear component, about 87% is hydrogen, about 12% is helium and the other 1% consists of heavier nuclei. Of these heavier nuclei, the CNO group and Fe are the most abundant with a typical energy of about 1 GeV/N. Even though these heavy nuclei are not very abundant, they are very penetrating due to their large mass and high speed.

An understanding of the interactions of galactic cosmic ray nuclei with not only hydrogen and helium but also with heavier nuclei is important for several reasons:

1. Knowledge of the cosmic ray spectrum at the top of the Earth's atmosphere and knowledge of the composition of the interstellar medium and heliosphere enables one to determine the cosmic ray spectrum at the source (Simpson 1983). The interstellar medium (Field 1986) consists primarily of hydrogen and helium so that cosmic ray interactions with these nuclei are the most important (Austin 1981; Ferrando et al 1988). However carbon, nitrogen and oxygen are also present in the interstellar medium (Morton 1975; Karttunen, Kroger, Oja, Poutanen and Donner 1987) and one anticipates that the understanding of cosmic ray interactions with these heavier nuclei may be needed in the future.

2. Knowledge of the spectrum at high altitude and knowledge of the composition of the Earth's atmosphere enables one to determine the cosmic ray spectrum at the top of the
atmosphere (Wilson, Townsend and Badavi 1987a).

3. The radiation environment inside a spacecraft, due to solar and galactic cosmic rays may be determined (Wilson and Townsend 1988). Such knowledge is important for lunar and martian habitats and other long duration space flights (National Council on Radiation Protection and Measurements 1989; Joselyn and Whipple 1990; Rester and Trombka 1989).

4. Studies of the history of extraterrestrial matter (such as lunar samples, meteorites and cosmic spherules and dust found in deep sea sediments) and also of the history of cosmic rays themselves can be made with the knowledge of the production rate of various nuclides (Reedy 1987; Reedy, Arnold and Lal 1983).

5. Cross section parametrizations of cosmic ray nuclei interacting with arbitrary target nuclei including those targets heavier than helium are required in the interpretation of emulsion data and in the the interactions of cosmic rays with air. (Gaisser 1990; Gaisser, Stanev, Freier and Waddington 1982; Gaisser and Stanev 1983, Shapiro and Silberberg 1970).

The basic nucleus-nucleus interaction that a cosmic ray undergoes can occur mainly via the Strong or Electromagnetic (EM) force. (Actually the study of nucleus-nucleus collisions began in cosmic ray studies (Goldhaber and Heckman 1978; Bradt and Peters 1948, 1949, 1950; Kaplan, Peters, Reynolds, and Ritson 1952).) Strong interaction processes (Goldhaber and Heckman 1978; Gyulassy 1981; Benesh, Cook and Vary 1989) have been studied extensively and quite recently the study of Electromagnetic processes in
high energy collisions has begun (Bertulani and Baur 1988).

To study the propagation of cosmic rays through interstellar space, the Earth's atmosphere or a spacecraft wall it is not enough to have a good understanding of the nucleus-nucleus interaction cross section as input to a transport computer code (Wilson, Townsend, Schimmerling, Khandelwal, Khan, Nealy, Cucinotta, Simonsen, Shinn and Norbury 1991). These codes can be very complex and therefore require simple expressions for the cross sections rather than the use of large data bases or complicated theoretical models (Wilson and Townsend 1988). Thus there has been a considerable effort to parameterize the cross section expressions so that the only required inputs are the nuclear energies and charge and mass numbers (Letaw, Silberberg and Tsao 1983; Silberberg and Tsao 1973, 1990; Townsend and Wilson 1986; Norbury, Cucinotta, Townsend and Badavi 1988; Wilson, Townsend and Badavi 1987a,b).

In order to understand cosmic ray transport through the interstellar medium, the early work on parametrizations (Rudstam 1966; Letaw, Silberberg and Tsao 1983; Silberberg and Tsao 1973, 1990) concentrated primarily on proton-nucleus interactions due to the fact that the interstellar medium consists primarily of hydrogen. However based on the 5 items listed above it would also be very useful for a wide variety of cosmic ray studies to have accurate parametrizations for any nucleus-nucleus interaction. It is the aim of the present work to provide such a parametrization. Actually such parametrizations (Wilson, Townsend and Badavi 1987b) have already been formulated and give good results for the removal of many nucleons. However for removal of only a few nucleons from heavy nuclei, the parametrizations (Wilson, Townsend and Badavi 1987b) sometimes give poor results. In fact a whole new approach to the parametrization of few-nucleon removal cross sections in nucleus-nucleus interactions is required. In the present paper an
accurate parametrization of single-nucleon removal cross sections is presented. Future work will discuss the removal of more nucleons. When this program is completed we will have available accurate parametrizations of few-nucleon removal cross sections in nucleus-nucleus interactions. When combined with the many-nucleon removal parametrizations (Silberberg, Tsao and Shapiro 1976; Wilson, Townsend and Badavi 1987b) and proton-nucleus parametrizations (Letaw, Silberberg and Tsao 1983; Silberberg and Tsao 1973, 1990), there will be available accurate cross section parametrizations for arbitrary cosmic ray species interacting with arbitrary media. See also the work of Webber et al (1990).

One approach to the parametrization of cross sections is to simply take all the available experimental data and fit a curve through it. However such an approach often requires a large number of adjustable parameters and may not be applicable to regimes where experiments have not been performed. A much more satisfying approach is to base one's parametrization on a physical theory or model that successfully describes the experimental data as well. This will be the approach of the present work. The various models and theories that have been developed will be collected together and parameterized. The whole method will require only one adjustable parameter \(X_d\) in equation 29. Furthermore this parameter is not essential. Good results are obtained without it. It is only introduced to provide some fine tuning.

A preliminary parametrization of the EM process has already been presented (Norbury, Cucinotta, Badavi 1988), which utilizes the Weizsacker-Williams (WW) method of virtual quanta (Bertulani and Baur 1988; Jackson 1975). However, since then the theory has been improved to include the effects of both electric dipole (E1) and electric quadrupole (E2) interactions (Bertulani and Baur 1988; Norbury 1990a,b), which will henceforth be referred to as multipole theory in contrast to WW theory. In addition Benesh, Cook and
Vary (1989) have recently provided a parametrization of the strong interaction single nucleon removal cross section.

II. STRONG INTERACTION PARAMETRIZATION

The parametrization due to Benesh, Cook and Vary (1989) is

\[ \sigma(N) = \frac{N}{A} \sigma_G P_{esc} \] \hspace{1cm} (1a)

for single neutron removal where \( N \) is the number of neutrons and \( A \) is the number of nucleons and

\[ \sigma(Z) = \frac{Z}{A} \sigma_G P_{esc} \] \hspace{1cm} (1b)

for single proton removal where \( Z \) is the number of protons. See also Norbury and Townsend (1990). \( \sigma_G \) is the reaction cross section given by

\[ \sigma_G = 2\pi (b_e - \frac{\Delta b}{2}) \Delta b \] \hspace{1cm} (2)

where

\[ \Delta b = 0.5 \text{ fm} \] \hspace{1cm} (3)

and the critical impact parameter for single nucleon removal is
\[ b_c = 1.34 \text{ fm} \left[ A_p^{1/3} + A_T^{1/3} - 0.75(A_p^{1/3} + A_T^{1/3}) \right] \] (4)

with \( A_p \) and \( A_T \) being the projectile and target nucleon numbers respectively. The single nucleon escape probability is

\[ P_{esc} = (1 - f) + f \exp^{-v} \] (5)

with

\[ f = \frac{1}{2} (1 - \cos \theta_{\text{max}}) \] (6)

and

\[ \sin \theta_{\text{max}} = \frac{b'_c - \Delta b}{b'_c} \] (7)

and

\[ v = \frac{A \sigma_{NN}}{\pi b_c^2} \] (8)

where \( A \) is the nucleon number of the nucleus from which the nucleon is being removed, and \( b'_c \) is the critical impact parameter for the single nucleon escaping and is given by \( b_c \) in equation (4) but with \( A_T = 1 \) (or \( A_p = 1 \)) if the nucleon is escaping from the projectile (or target). Thus the escape probability is independent of \( A_T \) (or \( A_p \)) as one would expect. \( \sigma_{NN} \) is the nucleon-nucleon cross section which has been parameterized as (Wilson, Townsend, Nealy, Chun, Hong, Buck, Lamkin, Ganapol, Khan and Cucinotta 1989)

\[ \sigma_{NN} = (1 + \frac{5}{T_{\text{lab}}}) \{ 40 + 109 \cos(0.199\pi E/180) \exp[-0.451(T_{\text{lab}} - 25)^{0.258}] \} \text{ mb} \] (9)
for $T_{lab} \geq 25$ MeV and as

$$\sigma_{NN} = \exp [6.51 \exp (T_{lab}/134)^{0.7}] \text{ mb}$$ (10)

for $T_{lab} < 25$ MeV.

Note that the energy dependence of the strong interaction cross section is totally contained in equation (9). Because of the exponential factor in (5) this energy dependence is rather weak as one would expect.

III. ELECTROMAGNETIC THEORY

The EM theory has already been discussed extensively (Bertulani and Baur 1988; Norbury 1989, 1990a,b) and only a few relevant details will be given here. The total nucleus-nucleus EM cross section is written as

$$\sigma = \sigma_{E1} + \sigma_{E2} = \int \left[ N_{E1}(E)\sigma_{E1}(E) + N_{E2}(E)\sigma_{E2}(E) \right] dE$$ (11)

where $N_{Ei}(E)$ is the virtual photon spectrum (of energy $E$) of a particular multipolarity $i$ due to the projectile nucleus and $\sigma_{E1}(E) + \sigma_{E2}(E)$ is the photonuclear reaction cross section of the target nucleus. (In principle the above equation should include other EM multipoles, but their effect is much less important.) A less exact expression is given by WW theory as
\[ \sigma_{WW} = \int N_{WW}(E) [\sigma_{E1}(E) + \sigma_{E2}(E)] \, dE \]  

(12)

where \( N_{WW}(E) \) is the WW virtual photon spectrum. Bertulani and Baur (1988) have shown that

\[
N_{WW}(E) = N_{E1}(E) = \frac{1}{E} \frac{\pi}{2} Z^2 \alpha \frac{1}{\beta^2} \left[ \xi K_0 K_1 - \frac{1}{2} \xi^2 \beta^2 (K_1^2 - K_0^2) \right] 
\]  

(13)

and

\[
N_{E2}(E) = \frac{1}{E} \frac{\pi}{2} Z^2 \alpha \frac{1}{\beta^4} \left[ 2(1 - \beta^2)K_1^2 + \xi(2 - \beta^2)^2 K_0 K_1 - \frac{1}{2} \xi^2 \beta^4 (K_1^2 - K_0^2) \right] 
\]  

(14)

with

\[ \xi = \frac{E \, b_{\text{min}}}{\gamma \beta (hc)} \]  

(15)

where all of the modified Bessel functions \( K \) are functions of \( \xi \). In the above equations \( E \) is the virtual photon energy, \( Z \) is the nuclear charge, \( \alpha \) is the EM fine structure constant, and \( b_{\text{min}} \) is the minimum impact parameter, below which the collision occurs via the Strong interaction. Also \( \beta = \frac{\gamma}{c} \) and \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \) where \( c \) is the speed of light and \( \gamma \) is the speed of the cosmic ray. The minimum impact parameter is given by

\[ b_{\text{min}} = b_c + \frac{\pi a_0}{2 \gamma} \]  

(16)

where
\[ a_0 = \frac{Z_p Z_T e^2}{m_0 v^2} \]  

allows for deviation of the trajectory from a straight line (Aleixo and Bertulani 1989).

In equation (11) the photonuclear cross sections satisfy the following sum rules (Bertulani and Baur 1988):

\[ \int \sigma_{E1}(E) \, dE = 60 \frac{N Z}{A} \text{ MeV mb} \]  

(18)

and

\[ \int \sigma_{E2}(E) \frac{dE}{E^2} = F \frac{0.22 Z A^{2/3}}{\text{MeV}} \frac{\mu b}{\text{MeV}} \]  

(19)

where \( F \) is the fractional exhaustion of this energy-weighted sum rule. The latter expression is the sum rule for the isoscalar \( E2 \) giant resonance. The isovector \( E2 \) resonance is ignored as it decays mainly by 2-nucleon emission (Bertulani and Baur 1988).

**IV. ELECTROMAGNETIC PARAMETRIZATION**

Because the photonuclear cross sections \( \sigma_{E1}(E) \) and \( \sigma_{E2}(E) \) are Lorentzian shaped, they behave somewhat like delta functions. The integrals of equation (11) can be approximated by taking \( N_{E1}(E) \) and \( N_{E2}(E) \) outside the integrals as (Bertulani and Baur 1988):
The integrals are evaluated using the sum rules in equations (18) and (19). In the above equation $E_{\text{GDR}}$ and $E_{\text{GQR}}$ are the central energies of the E1 and E2 photonuclear cross sections given by (Westfall, Wilson, Lindstrom, Crawford, Greiner and Heckman 1979)

\[
E_{\text{GDR}} = \frac{\hbar c}{8 J} \left( \frac{m^* c^2 R_0^2}{8 J} \right) \left( 1 + u \cdot \frac{1 + \varepsilon + 3u}{1 + \varepsilon + u} \right)^{-1/2}
\]

with

\[
u = \frac{3J}{Q'} A^{-1/3}
\]

and

\[R_0 = r_0 A^{1/3}\]

where $\varepsilon = 0.0768$, $Q' = 17$ MeV, $J = 36.8$ MeV, $r_0 = 1.18$ fm, and $m^*$ is 7/10 of the nucleon mass. Note that other expressions for $E_{\text{GDR}}$ such as $80 A^{-1/3}$ (Bertulani and Baur 1988) provide very inaccurate results for light nuclei. Equation (21) is accurate for all mass regions. The central energy of the E2 resonance is simply

\[E_{\text{GQR}} = \frac{63}{A^{1/3}} \text{ MeV}\]

In addition the fractional exhaustion of the Energy-Weighted Sum Rule in equation (19) is given by (Bertrand 1976)

\[\sigma = N_{E1}(E_{\text{GDR}}) \int \sigma_{E1}(E) dE + N_{E2}(E_{\text{GQR}}) E_{\text{GQR}}^2 \int \sigma_{E2}(E) \frac{dE}{E^2}\]
\[ f = 0.9 \text{ for } A > 100 \]
\[ \quad = 0.6 \text{ for } 40 < A \leq 100 \]
\[ \quad = 0.3 \text{ for } 40 \leq A \]

(25)

Finally, to obtain the reaction cross section for proton or neutron removal the above cross sections must be multiplied by the proton or neutron branching ratios. The proton branching ratio has been parameterized by Westfall, Wilson, Lindstrom, Crawford, Greiner and Heckman (1979) as

\[ g_p = \min \left( \frac{Z}{A}, 1.95 \exp(-0.075Z) \right) \]  
(26)

where \( Z \) is the number of protons and the minimum value of the two quantities in square brackets is to be taken. Assuming that only single nucleon removal occurs, the neutron branching ratio is

\[ g_n = 1 - g_p \]  
(27)

For light nuclei however the following branching ratios are used instead of equation (26)

\[ g_p = 0.5 \text{ for } Z < 6 \]
\[ \quad = 0.6 \text{ for } 6 \leq Z \leq 8 \]
\[ \quad = 0.7 \text{ for } 8 < Z < 14 \]  
(28)

Lastly, an adjustable parameter (the only one in the whole parametrization!) is introduced as \( x_d = 0.25 \) where
in place of equation (16).

Finally, if one is interested in a very quick calculation for estimation purposes we shall write down an approximate "pocket" formula which does not require the evaluation of the Bessel functions in (13) and (14). Using the low and high frequency approximations for the dipole photon spectrum (Jackson, 1975) and ignoring quadrupole effects, equation (20) can be written approximately as

\[
\sigma = \frac{1}{E_{GDR}} \frac{2}{\pi} Z^2 \alpha \frac{1}{\beta^2} \left[ \ln \left( \frac{1.123 \gamma c \beta}{E_{GDR} b_{min}} \right) - \frac{1}{2} \beta^2 \right] \quad \text{for} \quad E_{GDR} < \frac{\gamma c \beta}{b_{min}}
\]

\[
= \frac{1}{E_{GDR}} Z^2 \alpha \frac{1}{\beta^2} (1 - \frac{1}{2} \beta^2) \exp(-2E_{GDR}b_{min}/\gamma c \beta) \quad \text{for} \quad E_{GDR} \geq \frac{\gamma c \beta}{b_{min}}
\]

This EM formula, combined with the Strong interaction parameterization, gives a very simple "pocket" formula which may also be useful in complicated versions of transport codes that have CPU time at a premium. However to get a good fit to data one must use \( x_d = -0.1 \) in (30).

V. RESULTS AND CONCLUSIONS

The cross section parametrizations are compared with the existing nucleus-nucleus experimental data in Tables I - III. It can be seen that the overall agreement is extremely good for a very wide variety of projectiles, targets and energies. There are however a few notable discrepancies particularly for \(^{197}\text{Au}\) targets in Table II. It should be
noted however that these discrepancies are not due to the parametrization per se. Similar discrepancies are observed in comparisons between the original theory and experiment (Norbury 1989, 1990a, 1990b, Norbury and Townsend 1990, Benesh, Cook and Vary 1989, Hill, Wohn, Schwellenbach and Smith 1991). It is not clear whether these discrepancies are due to theoretical or experimental problems and their resolution is a matter of ongoing research.

In summary a parametrization of single nucleon removal cross sections for nucleus-nucleus collisions has been developed which accurately reproduces the experimental data for a wide range of nuclear species and energies. Future work will be devoted to few nucleon removal. Combining this with the many nucleon removal parametrizations (Wilson, Townsend and Badavi 1987b) and the proton-nucleus parametrizations (Letaw, Silberberg and Tsao 1983; Silberberg and Tsao 1973, 1990) provides a very useful parametrization of arbitrary cosmic ray species interacting with an arbitrary medium.

Acknowledgements

This work was supported in part by NASA grant number NAG-1-1134. We wish to thank F. Cucinotta for many useful discussions. JWN would also like to thank W. Cheung, M. Boytos and D. Goldstein who have participated in various aspects of this work.
REFERENCES


National Council on Radiation Protection and Measurements 1989, Guidance on Radiation
Received in Space Activities, NCRP Report No. 98, Bethesda, Maryland.


Rudstam, G. 1966, Zs. Naturforschung, 21a, 1027.


Wilson, J.W., Townsend, L.W., and Badavi, F.F. 1987a, Radiation Research, 109, 173.


Table I  Electromagnetic (EM) Cross Sections for single neutron and single proton removal. \( \sigma_{\text{expt}}^{\text{EM}} \) are the experimental EM cross sections from Olson et al 1981, Heckman and Lindstrom 1976, Barrette et al 1990 and Hill 1988. \( \sigma_{\text{param}}^{\text{EM}} \) is the parameterized EM cross section discussed in the text. Values in parentheses use the EM pocket formula.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>( T_{\text{lab}} ) (GeV/N)</th>
<th>Final State</th>
<th>( \sigma_{\text{expt}}^{\text{EM}} ) (mb)</th>
<th>( \sigma_{\text{param}}^{\text{EM}} ) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Pb</td>
<td>2.1</td>
<td>( ^{11}\text{C} )</td>
<td>51 ± 18</td>
<td>48 (57)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Pb</td>
<td>2.1</td>
<td>( ^{11}\text{B} )</td>
<td>50 ± 25</td>
<td>72 (86)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Pb</td>
<td>1.05</td>
<td>( ^{11}\text{C} )</td>
<td>39 ± 24</td>
<td>26 (17)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Pb</td>
<td>1.05</td>
<td>( ^{11}\text{B} )</td>
<td>50 ± 25</td>
<td>39 (26)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Pb</td>
<td>2.1</td>
<td>( ^{15}\text{O} )</td>
<td>50 ± 24</td>
<td>71 (85)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Pb</td>
<td>2.1</td>
<td>( ^{15}\text{N} )</td>
<td>96 ± 26</td>
<td>106 (127)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Ag</td>
<td>2.1</td>
<td>( ^{11}\text{C} )</td>
<td>21 ± 10</td>
<td>20 (24)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Ag</td>
<td>2.1</td>
<td>( ^{11}\text{B} )</td>
<td>18 ± 13</td>
<td>30 (36)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Ag</td>
<td>1.05</td>
<td>( ^{11}\text{C} )</td>
<td>21 ± 10</td>
<td>12 (12)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Ag</td>
<td>1.05</td>
<td>( ^{11}\text{B} )</td>
<td>25 ± 19</td>
<td>18 (18)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Ag</td>
<td>2.1</td>
<td>( ^{15}\text{O} )</td>
<td>26 ± 13</td>
<td>29 (35)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Ag</td>
<td>2.1</td>
<td>( ^{15}\text{N} )</td>
<td>30 ± 16</td>
<td>43 (53)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Cu</td>
<td>2.1</td>
<td>( ^{11}\text{C} )</td>
<td>10 ± 7</td>
<td>9 (11)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Cu</td>
<td>2.1</td>
<td>( ^{11}\text{B} )</td>
<td>4 ± 8</td>
<td>13 (16)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Cu</td>
<td>1.05</td>
<td>( ^{11}\text{C} )</td>
<td>9 ± 8</td>
<td>6 (6)</td>
</tr>
<tr>
<td>( ^{12}\text{C} )</td>
<td>Cu</td>
<td>1.05</td>
<td>( ^{11}\text{B} )</td>
<td>5 ± 8</td>
<td>8 (10)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Cu</td>
<td>2.1</td>
<td>( ^{15}\text{O} )</td>
<td>9 ± 8</td>
<td>13 (16)</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>Cu</td>
<td>2.1</td>
<td>( ^{15}\text{N} )</td>
<td>15 ± 8</td>
<td>19 (23)</td>
</tr>
<tr>
<td>Projectile</td>
<td>Target</td>
<td>$T_{\text{lab}}$ (GeV/N)</td>
<td>Final State</td>
<td>$\sigma_{\text{expt}}^\text{EM}$ (mb)</td>
<td>$\sigma_{\text{param}}^\text{EM}$ (mb)</td>
</tr>
<tr>
<td>------------</td>
<td>--------</td>
<td>---------------------</td>
<td>-------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>Al</td>
<td>2.1</td>
<td>$^{11}\text{C}$</td>
<td>0 ± 5</td>
<td>2 (3)</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>Al</td>
<td>2.1</td>
<td>$^{11}\text{B}$</td>
<td>0 ± 5</td>
<td>3 (4)</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>Al</td>
<td>1.05</td>
<td>$^{11}\text{C}$</td>
<td>1 ± 6</td>
<td>2 (2)</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>Al</td>
<td>2.1</td>
<td>$^{15}\text{O}$</td>
<td>0 ± 5</td>
<td>3 (4)</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>Al</td>
<td>2.1</td>
<td>$^{15}\text{N}$</td>
<td>-1 ± 9</td>
<td>5 (6)</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>C</td>
<td>2.1</td>
<td>$^{11}\text{C}$</td>
<td>-2 ± 5</td>
<td>1 (1)</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>C</td>
<td>2.1</td>
<td>$^{11}\text{B}$</td>
<td>-1 ± 4</td>
<td>1 (1)</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>C</td>
<td>1.05</td>
<td>$^{11}\text{C}$</td>
<td>-2 ± 5</td>
<td>0 (1)</td>
</tr>
<tr>
<td>$^{12}\text{O}$</td>
<td>C</td>
<td>1.05</td>
<td>$^{11}\text{B}$</td>
<td>-2 ± 5</td>
<td>1 (1)</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>C</td>
<td>2.1</td>
<td>$^{15}\text{O}$</td>
<td>-1 ± 4</td>
<td>1 (1)</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>C</td>
<td>2.1</td>
<td>$^{15}\text{N}$</td>
<td>-1 ± 4</td>
<td>1 (1)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>Ti</td>
<td>1.7</td>
<td>$^{17}\text{O}$</td>
<td>8.7 ± 2.7</td>
<td>8 (10)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>Ti</td>
<td>1.7</td>
<td>$^{17}\text{N}$</td>
<td>-0.5 ± 1.0</td>
<td>12 (15)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>Pb</td>
<td>1.7</td>
<td>$^{17}\text{O}$</td>
<td>136 ± 2.9</td>
<td>69 (79)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>Pb</td>
<td>1.7</td>
<td>$^{17}\text{N}$</td>
<td>20.2 ± 1.8</td>
<td>103 (118)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>U</td>
<td>1.7</td>
<td>$^{17}\text{O}$</td>
<td>140.8 ± 4.1</td>
<td>82 (92)</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>U</td>
<td>1.7</td>
<td>$^{17}\text{N}$</td>
<td>25.1 ± 1.6</td>
<td>123 (138)</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>$^{197}\text{Au}$</td>
<td>200</td>
<td>$^{196}\text{Au}$</td>
<td>1120 ± 160</td>
<td>1274 (1297)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{27}\text{Al}$</td>
<td>13.7</td>
<td>1p</td>
<td>37 ± 5</td>
<td>25 (28)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{27}\text{Al}$</td>
<td>13.7</td>
<td>1n</td>
<td>15 ± 4</td>
<td>12 (13)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{120}\text{Sn}$</td>
<td>13.7</td>
<td>1p</td>
<td>313 ± 4</td>
<td>325 (370)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{120}\text{Sn}$</td>
<td>13.7</td>
<td>1n</td>
<td>136 ± 6</td>
<td>151 (172)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{208}\text{Pb}$</td>
<td>13.7</td>
<td>1p</td>
<td>743 ± 27</td>
<td>822 (942)</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$^{208}\text{Pb}$</td>
<td>13.7</td>
<td>1n</td>
<td>347 ± 18</td>
<td>383 (438)</td>
</tr>
</tbody>
</table>
Table II  Total (= EM + Nuclear) Cross Sections for single neutron removal. \( \sigma_{\text{expt}}^{\text{Tot}} \) are the experimental total cross sections from Hill, Wohn, Winger and Smith 1988, Hill, Wohn, Winger, Khayat, Leininger and Smith 1988, Hill, Wohn, Schwellenbach and Smith 1991, Smith et al 1988 and Loveland et al 1988. \( \sigma_{\text{param}}^{\text{Tot}} \) is the parameterized total cross section discussed in the text. Values in parentheses use the EM pocket formula.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>( T_{\text{lab}} ) (GeV/N)</th>
<th>Final</th>
<th>( \sigma_{\text{expt}}^{\text{Tot}} ) (mb)</th>
<th>( \sigma_{\text{param}}^{\text{Tot}} ) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12C</td>
<td>238U</td>
<td>2.1</td>
<td>237U</td>
<td>173 ± 22</td>
<td>191 (195)</td>
</tr>
<tr>
<td>20Ne</td>
<td>238U</td>
<td>2.1</td>
<td>237U</td>
<td>192 ± 16</td>
<td>286 (300)</td>
</tr>
<tr>
<td>12C</td>
<td>197Au</td>
<td>2.1</td>
<td>196Au</td>
<td>178 ± 7</td>
<td>172 (175)</td>
</tr>
<tr>
<td>20Ne</td>
<td>197Au</td>
<td>2.1</td>
<td>196Au</td>
<td>268 ± 11</td>
<td>249 (260)</td>
</tr>
<tr>
<td>40Ar</td>
<td>197Au</td>
<td>1.8</td>
<td>196Au</td>
<td>463 ± 30</td>
<td>458 (491)</td>
</tr>
<tr>
<td>56Fe</td>
<td>197Au</td>
<td>1.7</td>
<td>196Au</td>
<td>707 ± 52</td>
<td>748 (812)</td>
</tr>
<tr>
<td>139La</td>
<td>197Au</td>
<td>1.26</td>
<td>196Au</td>
<td>2130 ± 120</td>
<td>2187 (2295)</td>
</tr>
<tr>
<td>139La</td>
<td>197Au</td>
<td>0.15</td>
<td>196Au</td>
<td>765 ± 48</td>
<td>729 (883)</td>
</tr>
<tr>
<td>238U</td>
<td>197Au</td>
<td>0.96</td>
<td>196Au</td>
<td>3440 ± 210</td>
<td>3997 (3486)</td>
</tr>
<tr>
<td>16O</td>
<td>197Au</td>
<td>60</td>
<td>196Au</td>
<td>400 ± 20</td>
<td>383 (389)</td>
</tr>
<tr>
<td>16O</td>
<td>197Au</td>
<td>200</td>
<td>196Au</td>
<td>560 ± 30</td>
<td>458 (462)</td>
</tr>
<tr>
<td>12C</td>
<td>89Y</td>
<td>2.1</td>
<td>88Y</td>
<td>115 ± 6</td>
<td>117 (119)</td>
</tr>
<tr>
<td>20Ne</td>
<td>89Y</td>
<td>2.1</td>
<td>88Y</td>
<td>160 ± 7</td>
<td>148 (154)</td>
</tr>
<tr>
<td>40Ar</td>
<td>89Y</td>
<td>1.8</td>
<td>88Y</td>
<td>283 ± 11</td>
<td>223 (240)</td>
</tr>
<tr>
<td>56Fe</td>
<td>89Y</td>
<td>1.7</td>
<td>88Y</td>
<td>353 ± 14</td>
<td>319 (351)</td>
</tr>
<tr>
<td>12C</td>
<td>59Co</td>
<td>2.1</td>
<td>58Co</td>
<td>89 ± 5</td>
<td>99 (101)</td>
</tr>
<tr>
<td>20Ne</td>
<td>59Co</td>
<td>2.1</td>
<td>58Co</td>
<td>132 ± 7</td>
<td>119 (122)</td>
</tr>
<tr>
<td>56Fe</td>
<td>59Co</td>
<td>1.7</td>
<td>58Co</td>
<td>194 ± 9</td>
<td>212 (229)</td>
</tr>
<tr>
<td>139La</td>
<td>59Co</td>
<td>1.26</td>
<td>58Co</td>
<td>450 ± 30</td>
<td>433 (461)</td>
</tr>
</tbody>
</table>
Table II continued

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>( T_{lab} ) (GeV/N)</th>
<th>Final State</th>
<th>( \sigma_{\text{expt}}^{\text{Tot}} ) (mb)</th>
<th>( \sigma_{\text{param}}^{\text{Tot}} ) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12C</td>
<td>12C</td>
<td>2.1</td>
<td>11C</td>
<td>61 ± 1</td>
<td>65 (66)</td>
</tr>
<tr>
<td>20Ne</td>
<td>12C</td>
<td>1.05</td>
<td>11C</td>
<td>78 ± 2</td>
<td>73 (73)</td>
</tr>
<tr>
<td>56Fe</td>
<td>12C</td>
<td>1.7</td>
<td>11C</td>
<td>94 ± 2</td>
<td>100 (101)</td>
</tr>
<tr>
<td>139La</td>
<td>12C</td>
<td>1.26</td>
<td>11C</td>
<td>148 ± 2</td>
<td>134 (135)</td>
</tr>
<tr>
<td>28Si</td>
<td>12C</td>
<td>13.7</td>
<td>11C</td>
<td>73.5 ± 3.5</td>
<td>85 (85)</td>
</tr>
</tbody>
</table>
Table III  Nuclear Cross Sections for single neutron and single proton removal. $\sigma_{\text{expt}}^{nuc}$ are the experimental nuclear cross sections from Fig. 4 of Barrette et al 1990. $\sigma_{\text{param}}^{nuc}$ is the parameterized nuclear cross section discussed in the text.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>$T_{\text{lab}}$ (GeV/N)</th>
<th>Final State</th>
<th>$\sigma_{\text{expt}}^{nuc}$ (mb)</th>
<th>$\sigma_{\text{param}}^{nuc}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Al</td>
<td>13.7</td>
<td>1p</td>
<td>140 ± 14</td>
<td>87</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Al</td>
<td>13.7</td>
<td>1n</td>
<td>100 ± 10</td>
<td>87</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Sn</td>
<td>13.7</td>
<td>1p</td>
<td>220 ± 22</td>
<td>120</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Sn</td>
<td>13.7</td>
<td>1n</td>
<td>145 ± 15</td>
<td>120</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Pb</td>
<td>13.7</td>
<td>1p</td>
<td>300 ± 30</td>
<td>136</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>Pb</td>
<td>13.7</td>
<td>1n</td>
<td>180 ± 18</td>
<td>136</td>
</tr>
</tbody>
</table>