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Effects of Retardation in Relativistic Equations with Confining Interaction

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Abstract

A method has been developed for solving two-body relativistic bound state equations in momentum space with a confining interaction. A total of six different three-dimensional reductions of the Bethe-Salpeter equation are studied with particular emphasis placed on the competing roles of relativistic kinematics and retardation. The results indicate that these two effects counteract each other and this sheds some light on why non-relativistic models of meson spectroscopy have been quite successful.

Many theoretical studies of meson spectroscopy[1,2] have been performed in a non-relativistic framework with a confining plus Coulomb-like potential. The confining term prevents the quarks from escaping to large distances and the Coulomb term simulates the short range behavior of the one gluon exchange force. Motivated by the studies of lattice gauge theories[3], most work in this area uses a linearly rising potential to provide confinement. Relativity has also been introduced into the problem by different authors with various prescriptions[4,5]. Although the best way to do meson physics in the two-body framework would be to solve the Bethe-Salpeter(BS)[6] equation, it is more practical and economical to solve a three-dimensional reduction of it. However it is well known that there exist, in principle, infinitely many possible three-dimensional reductions of the BS equation[7] and generally speaking there is no reason to prefer one reduction to another, although in some special cases the physical problem itself suggests the use of a particular reduction scheme. For example, in the case of a system of one heavy quark and one light quark one might prefer the Gross equation[8] since the heavy quark can be put on mass-shell with some justification. Therefore for the general $q\bar{q}$ problem it would seem useful to carry out a systematic study of the various reductions of the BS equation. We have developed a method for solving bound state equations in momentum space with the singular kernel that arises from the linear confining potential[9] and in this letter we generalize the nonrelativistic linear potential to the relativistic case, and compare solutions for the scalar and spinor $q\bar{q}$ system obtained using six representative three dimensional reductions of the BS equation.

The nonrelativistic linear confining potential can be written as

$$V(r) = \lim_{\eta \to 0} k r e^{-\eta r} \tag{1}$$

In momentum space this becomes

$$V(\mathbf{q}) = -\lim_{\eta \to 0} \frac{k}{2\pi^2} \frac{\partial^2}{\partial \eta^2} \frac{1}{\mathbf{q}^2 + \eta^2} \tag{2}$$

The relativistic generalization of this potential has been obtained by replacing 3-vector $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ by 4-vector q, so that $q^2 = \mathbf{q}^2 - q_0^2$. This would appear to be the most natural generalization of the non-relativistic linear potential, and indeed yields the non-relativistic

potential exactly when retardation effects are neglected. One can see that the momentum space potential has a singularity in the limit of $\eta \to 0$. One way of avoiding this singularity problem [5] is to carry out the calculation for a small finite value of eta. However this does not produce true confinement. We have previously studied how to extract the *exact* $\eta \to 0$ limit for the nonrelativistic case[9]. We have also generalized this limiting procedure to the relativistic case. Complete mathematical details of this procedure will be communicated elsewhere. In this letter we describe the main ideas concentrating rather on a discussion of the results and the effects of retardation and relativistic kinematics.

In the following we study these effects in two model systems, one containing scalar particles and the other containing spinors. For scalar " quarks " we consider the Minimal Relativity(MR) equation,the Blankenbecler-Sugar(BBS) equation[10] and the Kadyshevsky equation[11] with and without retardation (K and K0). For spinor quarks the Gross equation[8](G) (with retardation) and the Thompson (T) equation[12] (without retardation) are studied. These equations are the same set that were considered in the work of Woloshyn and Jackson[7] where the scattering of scalar particles was studied.

All six equations can be written in the generic form in C.M. frame as

$$D_i \phi(\mathbf{p}) = -\int V(p', p)\phi(\mathbf{p}')d\mathbf{p}' \tag{3}$$

where the operators D_i are listed in Table 1.

The singularity that arises from the non-relativistic confining potential in momentum space has been handled by a subtraction procedure [9] similar in spirit, but very different in detail, to that developed for the Coulomb potential[13]. For the relativistic generalization of the linear potential considered herein, the singularity structure of the relativistic kernel remains the same as the non-relativistic case. Thus we obtain the extremely useful result that the relativistic singularity can be handled by subtracting a term propotional to the nonrelativistic kernel. The $\eta \to 0$ limit is taken in the same way as the non relativistic case[9], so that we obtain in the case of $l = 0$ and for equal mass particles:

$$D_i \phi_0(p) = -\frac{k}{\pi p^2} \mathrm{P} \int_0^\infty [Q_0'(\bar{y})\phi_0(p') - (\frac{E_p}{m})^2 Q_0'(y)\phi_0(p)]dp'. \tag{4}$$

Here $\bar{y}$ and y are defined as

$$\bar{y} = \frac{p^2 + p'^2 - (E_p - E_{p'})^2}{2pp'} \tag{5}$$

$$y = \frac{p^2 + p'2}{2pp'} \qquad (6)$$

P denotes the principal value integral, Q_0 and Q'_0 are the Legendre function of the second kind and its first derivative respectively and $E_p = \sqrt{m^2 + \mathbf{p}^2}$.

Using the relativistic generalization of the method developed in reference 9, these equations are solved for the total energy W for the s-wave and particles of equal mass m. Only coupling to the positive energy channels is retained. The usefulness of these relativistic equations depends on the extent to which they reproduce global properties of the spectrum characterised by the dependence of the energy E_n on the principal quantum number n. This dependence is most easily revealed by studying the ratio E_n/E_1. E_n is related to the total energy W_n through $E_n = W_n - 2m$. Table (2) contains the results for the ratio E_n/E_1 for the equations listed above for a reasonable choice of mass and coupling parameters.

Consider first the equations which have no retardation effect, (BBS, K0, T). One sees that in all three cases the energy ratios are significantly smaller than the non-relativistic result (which is independent of mass) and furthermore that this difference is more important for small quark masses which is as one would expect for a purely kinematic effect. In addition, the higher radial excitations show more pronounced relativistic corrections, which is consistent with the virial theorem [2] for a positive power law potential which requires larger kinetic energies for orbits with greater average radii.

A result of considerable interest is that when retardation is included, as in equations (MR, K, G), the effect of relativistic kinematics described above is counteracted, in that the energy ratios move back towards the non-relativistic values rather than continuing to become smaller. This provides one possible explanation as to why non-relativistic equations have been quite successful in describing meson spectroscopy. Notice that the differences between MR and BBS, K and K0 equations is retardation. By comparing the differences between MR column and BBS column to the differences between K and K0 column in table 2 we notice that the effect of retardation is more pronounced in the Kadyshevsky equation than in the Minimal Relativity equation.

In conclusion we have solved the two-body relativistic bound state problem for a

relativistic confining interaction which is a generalization of the non-relativistic linear potential. We have considered six different 3-dimensional relativistic equations, four for scalars particles and two for spinor quarks. In all cases we have studied, we have found that the effects of relativistic kinematics and retardation counteract each other. Future work will be devoted to including spinors and coupling to the negative energy channels in all six equations so that detailed comparisons to experiment can be carried out.

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Table 1

D_i operators for relativistic equations

G and T equations are describing pseudoscalar mesons with spinor quarks

The other four relativistic equations are for scalar quarks

i	Name	D_i	Retardation
MR	Minimal Relativity	$4E_p(E_p^2 - W^2/4)$	Yes
BBS	Blankenbecler Sugar	*same as* MR	No
K	Kadyshevsky	$2E_p^2(E_p - W/2)$	Yes
K0	Kadyshevsky	*same as* K	No
G	Gross	$2E_p - W$	Yes
T	Thompson	*same as* G	No

Table 2

Energy ratios $\frac{E_{n+1}}{E_1}$

for the six relativistic equations discussed in the text

G and T equations are for spinor quarks with $k = 0.2GeV^2$

The other four relativistic equations are for scalar quarks with $k = 0.2GeV^4$

The nonrelativistic(NR) equation is with $k = 0.2GeV^2$

n	MR	BBS	K	K0	G	T	NR	mass (GeV)
1	1.73	1.71	1.74	1.72	1.79	1.72	1.75	1.5
2	2.31	2.27	2.35	2.30	2.47	2.30	2.36	1.5
3	2.81	2.75	2.88	2.80	3.09	2.80	2.90	1.5
1	1.58	1.50	1.68	1.54	1.90	1.67	1.75	0.5
2	2.00	1.82	2.21	1.89	2.73	2.18	2.36	0.5
3	2.35	2.08	2.65	2.16	3.52	2.62	2.90	0.5
1	1.51	1.41	1.66	1.44	1.98	1.63	1.75	0.3
2	1.87	1.65	2.13	1.69	2.91	2.11	2.36	0.3
3	2.18	1.84	2.52	1.89	3.83	2.51	2.90	0.3

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