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A Letter of Intent for an Experiment to Study Strong Electromagnetic Fields at RHIC via Multiple Electromagnetic Processes.

M.Fatyga

Physics Department
Brookhaven National Laboratory
Upton,NY 11973

and

John W.Norbury

Department of Physics
Rider College
Lawrenceville, NJ 08648

ABSTRACT

An experimental program at RHIC which is designed to study non-perturbative aspects of electrodynamics is outlined. Additional possibilities for new studies of electrodynamics via multiple electromagnetic processes are also described.

1. Introduction

This letter is being submitted in response to the call for letters of intent for small experiments at RHIC. In it, we present an outline of an experimental program which is designed to study multiple electromagnetic processes in the quasi-elastic scattering of relativistic heavy ions. By quasi-elastic scattering we mean these events in which particles are produced out of the vacuum due to electromagnetic interactions alone. Two beam ions are required to pass one another within a distance which is greater than the sum of their nuclear radii. We believe that such measurements belong to the exclusive domain of relativistic heavy ion colliders and constitute a unique test of electrodynamics under the conditions of strong electromagnetic fields. Following the submission of the letter we intend to work on the design of an apparatus. To this end, John Norbury is presently working on calculations which will establish a set of parametrized probability distributions for various electromagnetic processes. These parametrizations will include the dependence of probabilities on an invariant mass of di-lepton pairs, the rapidity of pairs or mesons and the impact parameter in a collision. Following the completion of this work, we should be able to establish fast event generators which can then be used for realistic Monte Carlo simulations of an apparatus. By formulating a program of measurements we are also hoping to increase the theoretical activity in this area. We will work towards appraising our theoretical colleagues about opportunities which will be presented by RHIC in this domain. We are anxious to receive as much theoretical feedback as possible.
Finally, we are very interested in hearing the opinion of the Committee on the merits of our proposal. Encouragement would be most helpful in searching for new collaborators.

Having had to work with a very short deadline, we would like to apologize for all of the defects and shortcomings of this letter. It has been put together in haste. We will be glad to answer any questions that the committee may have after reading it.

2. Physics Motivation

It has been recognized for some time now, that relativistic heavy ions can be treated as a good source of virtual photons due to the enhancement of a virtual photon flux (per ion) by a factor which is *approximately* equal to the square of a charge of an ion. [1] The cross sections for two-photon processes in a heavy ion collider are greater than those in e^+e^- machines by a factor which is *approximately* equal to the beam charge to the fourth power. It also has been pointed out however, that this gain in flux of virtual photons is largely offset by limitations in the luminosity of heavy ion machines. [2] In fact, this loss in luminosity is about equal in magnitude to the gain in the flux of virtual photons. Therefore (it was concluded), heavy ions do not offer any significant advantage in the magnitude of cross sections for two photon processes. At the same time, significant new difficulties due to hadronic backgrounds are introduced . We have no quarrel with these assertions. There exists however, an *entire class* of interesting processes exclusive to Relativistic Heavy Ion Colliders which involve a production of particles by electromagnetic fields. These are multiple electromagnetic processes, in which two or more distinct objects (lepton pairs, neutral hadrons, hadron pairs) are created through the electromagnetic polarization of a vacuum. If one approximates the probability of a double process by the convolution of probabilities for simple two photon processes, the cross section for a double electromagnetic process depends on the charge of beams like Z^8. Consequently, relativistic heavy ion colliders have no competition from other machines in this field of study. The purpose of this section is to discuss the physics significance of such processes as well as the program of measurements for exploring opportunities in this area. Specifically, we will discuss a production of multiple lepton pairs (e^+e^- and $\mu^+\mu^-$), a production of π^0 or η^0 in coincidence with e^+e^- pairs and a coincidence of all these processes with beam rapidity neutrons (protons) which are emitted due to a Giant Dipole Excitation of beam ions.

2.1. e^+e^- pair production

The production of e^+e^- pairs in the quasi-elastic scattering of relativistic heavy ions will exhibit strong non-perturbative effects. In fact, it has been shown by many authors, that the perturbative estimate of the probability to produce a pair violates unitarity at low invariant masses. [3] It seems particularly attractive to study these effects at RHIC, due to its variety of available beams. All the effects of strong fields can in essence be turned on and turned off by a mere change of beams. Most calculations of a non-perturbative pair production emphasize masses just above a production threshold ($(1-2)\frac{MeV}{c^2}$), concluding that strong non-perturbative effects can be seen predominantly in the production of pairs with very low invariant masses. From an experimental point of view, such an observation implies that it would be necessary to measure electron pairs at extremely low invariant masses (and perhaps trigger a measurement on them) in order to explore the non-perturbative regime. We will seek to demonstrate that in a properly designed experiment this region of non-unitarity can be extended up to invariant masses of the order of $(10-20)\frac{MeV}{c^2}$. As an added benefit, we will show that such an experiment can be provided with a relatively background-free trigger.

In a perturbative approximation, the probability to produce a pair with an invariant mass M and at an impact parameter b is a function of the product Mb. One therefore observes, that if the unitarity is violated in the production of a threshold pair at an impact parameter equal to the compton wavelength of an electron (385fm), one expects the same violation of unitarity in a production of pairs with invariant masses of the order of $20\frac{MeV}{c^2}$ at the smallest allowed impact parameter prior to the nuclear contact (15.2fm for gold+gold collision). In fig.(1) we show a Weizsacker-Williams calculation of the probability to produce an e^+e^- pair in a gold+gold collision at top RHIC energy (100GeV/nucleon), and at a minimum allowed impact parameter. [4] This calculation clearly shows a violation of unitarity in the production of pairs with invariant masses of the order of $10\frac{MeV}{c^2}$. However, there remains a significant difficulty in applying this observation to a realistic measurement. First, a *minimum bias* $10\frac{Mev}{c^2}$ pair can be produced in a wide range of impact parameters. As an illustration, in Fig(2) we show the impact parameter dependence of the probability to produce a $10\frac{MeV}{c^2}$ pair. It is clear that the decrease of the probability due to an increase of an impact parameter is quite slow. This slow drop will cause a minimum bias cross section to be dominated or at least severely contaminated by less interesting distant collisions, simply because of a phase space ($2\pi b db$) factor. Moreover, it may be very difficult to trigger an experiment in a collider environment on pairs of a low mass.

To remedy both difficulties, we designed a measurement which allows for the selection of collisions with a small impact parameter and simultaneously provides a distinct and a relatively background free trigger. Namely, we propose to trigger the experiment on a high invariant mass e^+e^- pair and measure all other pairs associated with such an event. A veto on hadronic interactions can be provided by two multiplicity detectors positioned in a forward-backward direction. A leading hadronic background at the trigger level should be a topological equivalent of a minimum bias p-p collision. Harder hadronic interactions will produce large multiplicities and should not be difficult to suppress. When an invariant mass of a trigger pair approaches that of a WW limit (about $3\frac{GeV}{c^2}$ at RHIC at a minimum allowed impact parameter), only collisions with small impact parameters will be selected. Moreover, there should be no particular difficulties in triggering an experiment on a single pair in a few hundred Mev range of invariant masses. By varying the invariant mass of a trigger pair one can change the range of impact parameters being selected, thus studying an evolution of e^+e^- pair production with a varying strength of electromagnetic fields. Since we have the freedom of choosing the charge of beams, the same evolution can be studied by independent means (changing the charge of beams). For a completeness of this discussion, one should point out that an interpretation of the coincidence measurement which was described here is still dependent on our understanding of the impact parameter dependence of the probability to produce a trigger pair. At high invariant masses such probability distributions should be calculable reliably via perturbative approximations. *In addition*, there exists a set of auxiliary, supporting measurements which will test the accuracy of our predictions. A list of such measurements with a rudimentary description of their interpretation will be provided later in this letter.

In summary, we propose to use a massive e^+e^- pair as an experimental trigger on elastic heavy ion collisions at impact parameters close to the nuclear contact. Such a trigger should allow us to study a fully non-perturbative production of e^+e^- pairs at fairly high invariant masses.

We believe that there exists yet another regime of invariant masses in which measurements of a production of multiple e^+e^- pairs is of interest. Namely, we are considering a study of the production of two or more pairs in an invariant mass range of $(20-100)\frac{MeV}{c^2}$. A motivation for such a study is relatively simple. Let us observe, that the compton wavelength which is associated with a pair in this range of masses is in a $4-20fm$ range. Hence, formation times of such pairs are not much smaller than the collision time, and volumes necessary to produce them are also large. For these two reasons, we believe that higher order corrections to the probability to produce two pairs (relative to a simple convolution of first order probabilities) will be most pronounced in the $20-100\frac{MeV}{c^2}$ range of invariant masses. We will study a feasibility of such a measurement through Monte Carlo simulations in a near future. In this letter, we will only evaluate event rates by using a convolution of probability distributions for a single pair which were obtained in a first order perturbative approximation.

2.2. π^0,η^0 in coincidence with e^+e^-

As a logical extension of a multiple pair measurement, we will now consider a process in which a neutral meson is produced in coincidence with e^+e^- pairs. In the first step, one would like to verify that the dependence of total integrated cross sections for the production of π^0 and η^0 on the charge of beams shows no anomalies. The observation of an anomaly could indicate a change in the rate of production of strange quarks relative to up and down quarks when the strength of electromagnetic fields increases. In the second step, one measures the probability that an e^+e^- pair is produced in coincidence with a meson. One also measures the properties of such pairs, namely their invariant mass and rapidity distributions. An interpetation of the coincidence rates can be twofold. First, they probe the impact parameter dependence of the probability to produce π^0 and an η^0. Pairs of a relatively low invariant mass will be particularly sensitive to the shape of these distributions. Second, when the invariant mass of an electron pair increases up to the $40-100\frac{MeV}{c^2}$ range, the energy density which is necessary to produce such a pair becomes comparable to that of a π^0 or even an η^0. Consequently, an observation of an anomaly in these coincidence rates could indicate an interference between the formation of a meson and the formation of an electron pair. Once again we will invoke an argument, that the relatively large compton wavelength of an electron pair in this mass range implies that an overlap between formation times and formation volumes of a meson and a pair is likely. One should also note that the preceding measurement of a multiple pair production can be used as a calibration measurement of sorts which will facilitate an understanding of the coincidence between pairs and mesons.

2.3. An electromagnetic excitation of the Giant Dipole Resonance

Finally, we discuss yet another coincidence experiment that can accompany all the measurements which were discussed above. Namely, in an elastic collision of relativistic heavy ions, either one of the two ions can experience an excitation of the Giant Dipole Resonance due to an interaction with the electromagnetic field of its counterpart in a collision. A leading de-excitation mode is the emission of a neutron or a proton with an energy of the order of a few MeV in the center of mass system of an ion. Such a free nucleon appears in the laboratory system with a beam rapidity, thus having a kinetic energy which is centered on 100GeV at a top RHIC energy. Beam rapidity neutrons can be quite easily detected in a zero degrees calorimeter positioned downstream from the first bending magnet, whereas protons can be detected after the same bend at an angle which is a bit larger than twice the bending angle of the beam. In fig(3) we show a probability to detect a beam rapidity neutron as a function of the impact parameter in a collision. The functional form of this probability distribution can be very well approximated by the form $P(b) = \frac{P_0}{b^2}$. We can see that in a gold+gold collision at the minimum allowed impact parameter a probability of emitting a single neutron from either ion is of the order of 40%. This large probability implies that coincidences with beam rapidity neutrons will be frequent for all processes. We will now show how such a coincidence measurement can be used to constrain calculations of the impact parameter dependence of the probability to produce a trigger pair in a multiple pair measurement. To illustrate the method, let us assume that a calculation of a probability to produce the neutron is correct. We will further assume that the probability to produce an e^+e^- pair of a high invariant mass (defined as a trigger pair in a multiple pair measurement) is described by a $P(b) = \frac{P_t}{b^n}$ formula, where n is an adjustable parameter (to be deduced from a measurement). For simplicity , we will limit the discussion to pairs with rapidity zero. In fig(4) we show the probability to observe a neutron in coincidence with a pair for three values of a parameter n, where $n = 0, 2, 4$. This probability is plotted against a parameter $A = \frac{b_{max}}{b_{min}}$, where b_{min} is the minimum allowed impact parameter, and b_{max} is related to an invariant mass of a pair due to an adiabatic cutoff of the virtual photon spectrum in a Weizsacker-Williams approximation (b_{max} in units of [fm]):

$$b_{max} = \frac{2 * \gamma * 197.3}{M}$$

It is quite clear, that such a coincidence measurement has a significant level of sensitivity to the functional form of the probability to produce a pair. Naturally, the actual functional form of the probability distribution can be more complicated than $\frac{1}{b^n}$. We assumed such a simple form to illustrate the sensitivity of this method. In the final analysis, the best available calculation for both probability distributions (for a neutron emission and for a production of a pair) must be used in calculation and then compared to a measurement. A complete experiment requires a consistency test for three independent observables: the total cross section for an emission of a neutron, the cross section for producing an electron pair of an invariant mass M, and the cross section for a coincidence between a neutron and a pair. All three observables should be measured with beams of different charges and energies.

An emission of a single beam rapidity neutron will be accompanying all processes which were discussed in previous sections, and with a high probability of occurence. It can be treated as an additional factor in efforts to understand multiple electromagnetic processes.

2.4. A summary of proposed measurements.

Let us list now a set of measurements which should allow for a reasonably exhaustive analysis of phenomena which were discussed above:

- the total cross section for e^+e^- pairs as a function of the charge and energy of beams.

- the invariant mass and rapidity distributions of e^+e^- pairs as a function of the charge and the energy of beams.

- the total cross section for π^0 and η^0 as a function of the charge and the energy of beams.

- the rapidity distribution of π^0 and η^0 as a function of the charge and the energy of beams.

- the multiple pair production with a trigger pair of a large invariant mass. All measurements should be done in a widest possible range of beam charges and energies (cross sections will be vanishing very fast with decreasing charge and energy of the beams).

- the multiple pair production with two (or more) pairs in a $(20-100)\frac{MeV}{c^2}$ range of invariant masses. All measurements should be done in a widest possible range of beam charges and energies (cross sections will be vanishing very fast with decreasing charge and energy of the beams).

- the $\pi^0 e^+ e^-$ and $\eta^0 e^+ e^-$ coincidence cross sections. Invariant mass and rapidity distributions of pairs, a comparison between pairs accompanying π^0 and pairs accompanying η^0. All measurements should be done in a widest possible range of beam charges and energies (cross sections will be vanishing very fast with decreasing charge and energy of beams).

3. Estimates of count rates.

3.1. Assumptions

A number of assumptions will be made to calculate event rates for the purpose of this letter:

1. all probabilities and cross sections are obtained by a simple convolution of first order probabilities (usually obtained in a Weizsacker-Williams approximation).
2. a following formula is being used to calculate cross sections for multiple processes:

$$\sigma_{mp} = P_1(b_{min}) * P_2(b_{min}) * \ldots P_i(b_{min}) * 7.6 \; [barns]$$

're $P_i(b_{min})$ isthe probability of a single process at the minimum allowed impact parameter. A deriviation of this formula is given in an appendix 1.

3. all rate estimates refer to a collision of two gold ions at an energy of 100GeV/nucleon at a luminosity of $2 * 10^{26} cm^{-2} s^{-1}$.[5]

3.2. Multiple e^+e^- pairs

The following approximate formula was used to calculate the probability to produce a single pair of an invariant mass M_i at the minimum allowed impact parameter b_{min}:

$$P_i(M) = \frac{10^2}{M_i^2}$$

where M_i is an invariant mass expressed in MeV. This probability is normalized to unity at $M_i = 10\frac{MeV}{c^2}$ (Fig(1)), and describes the probability of producing a pair in an interval of invariant masses $[M_i(1-\alpha), M_i(1+\alpha)]$, with $\alpha = 0.05$. Consequently, the probability to produce n pairs at a minimum allowed impact parameter is:

$$P_n = \frac{10^{2n}}{M_1^2 * M_2^2 * \ldots . M_i^2}$$

A deriviation of these formulas is provided in an appendix 2. A summary of results is presented in three figures, Fig(5) - Fig(7). In Fig.(5) we show the event rate for two pair events, with the mass of one of the two pairs fixed at $100\frac{MeV}{c^2}$ and the second one taken as a variable. In Fig.(6) we show the event rate for events with two pairs, each within an invariant mass range which is centered on $M_0 = 100\frac{MeV}{c^2}$, plotted against a parameter α describing a width of a mass range for each pair. Hence, each pair has a mass within a range $[M_0(1+\alpha), M_0(1-\alpha)]$. Finally, in Fig.(7) we show event rates for events with 2 pairs of the same invariant mass, with α fixed at a value of $\alpha = 0.05$. The event rate is plotted against a value of invariant masses ($M_1 = M_2 = 20, 30, 40, 50, \ldots\ldots \frac{MeV}{c^2}$). Since no acceptance cuts were put into these calculations, event rates should be treated as an upper limit (within limitations of our approximations). One might add, that changing the beam to uranium (from gold), would increase all rates by approximately a factor of 2.5.

3.3. Events with π^0 and η^0

In fig(8) we show the results of a calculation of event rates for events with one neutral meson (π^0, η^0) and an e^+e^- pair. Event rates are plotted against the invariant mass of an e^+e^- pair.

4. Technical considerations

4.1. Triggering

In the following section we discuss an outline of trigger configurations which can satisfy the requirements of measurements which were proposed in previous sections. Monte Carlo studies of trigger configurations will continue until the final proposal can be formulated.

First, let us remind the reader that we wish to limit our measurements to quasi-elastic collisions of relativistic heavy ions, *with no hadronic interactions*. A successfull trigger will be capable of vetoing all hadronic interactions. Events of interest are characterized by the low multiplicity, as well as the survival of both ions completely (or at least nearly) intact. We must allow for some exceptions to the last requirement due to decays of electromagnetically induced nuclear resonances. These resonances will be dominated by the Giant Dipole Resonance which can result in an emission of a neutron or a proton at a beam rapidity.

As an example of a trigger configuration let us first discuss a study of multiple e^+e^- pairs, with one pair of a large invariant mass. A trigger setup for this measurements should include following elements:

- One massive e^+e^- pair in which a total energy measurement is coupled to a simple tracking arrangement.
- No multiplicity being detected on either side of an interaction region. Shapes and locations of multiplicity vetoes should be chosen so that a maximum rejection is obtained for a minimum bias p-p collision. Detector arrangements used for luminosity measurements in p-p or p-pbar colliders can be used as an example of such a veto.
- At most one beam rapidity neutron or a beam rapidity proton can be seen on either side of an interaction region. Neutrons can be detected by hadronic calorimeters positioned at zero degrees past the first bending magnet, downstream from an interaction region. Protons can be detected at twice the deflection angle of a beam past the same first bending magnet. It should be noted that at high energies one may have to allow for events with two beam rapidity nucleons emitted from a single ion. Although a probability of such events occuring due to electromagnetic interaction alone should be very low, one may choose to study events with two (or more) nucleons as a function of a beam energy to make sure that no significant bias is introduced by rejecting two-nucleon events.
- A crude multiplicity veto around the interaction vertex can be added to veto on central collisions which do not activate downstream multiplicity vetoes. We feel that a veto of this kind would be redundant, but a possibility of adding it will be studied further.

As an invariant mass of a trigger pair decreases towards a $(10 - 100)\frac{Mev}{c^2}$ range, one expects increasing difficulties in triggering. Various designs of a trigger must be studied to cope with these difficulties. At this time we can only list some elements of a trigger setup which should definitely be considered:

A segmented electromagnetic calorimeter with a low noise (probably crystals with photomultipliers) capable of detecting low energy depositions at a trigger level. A simulataneous requirement of a total energy and a hit multiplicity can then be imposed. An independent multiplicity detector should accompany this setup as a third element in a trigger. Standard hadronic vetoes are alwys imposed.

Triggering on the intermediate mass range will be one of the main objects of study in a design of an electron spectrometer for this experiment.

5. Compatibility with large experiments

At this time we believe that our program of measurements is not compatible with large experiments which are being designed to search for the quark-gluon plasma. Reasons for a poor compatibility can be summarized as follows:

5.1. Differences in the transverse momentum acceptance

Most designs that we are familiar with involve significant p_t thresholds for all produced objects (di-leptons or mesons). These thresholds are introduced in an attempt to hold down costs and avoid a need for a 2π detector coverage in φ. A study of electromagnetic phenomena must use a detector system with an acceptance which is centered on $p_t = 0$. Having said this, we do not imply that a study of this kind requires a very elaborate detector system. We must stress again, that an event structure which we are considering here is much simpler than that of a central collision. We are considering events with a low multiplicity (hence a low occupancy rate), without a problem of discriminating against an intense pion background. These two requirements seem to be primary factors in determining costs of an experimental apparatus for a study of central collisions.

5.2. An energy range for single particles.

The dynamic range for energies of single particles in our apparatus is very different from the dynamic range in a typical RHIC experiment. With the exception of a massive trigger pair, we are interested in electrons and photons in the few to few hundred MeV energy range. In a multiple pair measurement a specialized low energy electron spectrometer is probably called for. In a photon measurement a crystal detector positioned close to the beam line with a maximum possible coverage in solid angle should be considered. Both the electron and the photon parts of an apparatus must be succesfully merged. Although a careful study of the design of an apparatus must be performed before final conclusions are reached, it is our intuition that the set of requirements which is necessary for a good measurement of electromagnetic processes will be contradictory to those imposed by large experiments. We are particularly refering to dead material surrounding the beamline, as well as space constraints imposed by magnets and other elements of a very complex apparatus.

6. Summary

In conclusion, we have submitted this letter of intent with following goals in mind:

- To demonstrate that it is possible to design an experiment in which non-perturbative production of e^+e^- pairs can be studied in the $(10-20)\frac{MeV}{c^2}$ energy range, rather than $(1-2)\frac{MeV}{c^2}$ energy range. We understand that detecting electrons of such low energies still poses a significant technical challenge. At the same time, a ten-fold increase in the energy of electrons significantly improves prospects for tracking and reduces problems which are associated with multiple scattering. We believe that the physics payoff is worth the effort.

- To demonstrate that many new possibilities will exist in a study of electromagnetic phenomena in the $(10-100)\frac{MeV}{c^2}$ range of invariant masses, and sufficient event rates exist to consider experiments in this area. Such a field of a study also represents a unique domain of relativistic heavy ion colliders.

- Finally, we would like to obtain the advice and the opinion of the Committee, whether such a direction of study can be supported at RHIC. All the work which served as a basis for this letter had been accomplished with marginal resources. Before committing significantly more time to the preparation of an experimental design, we would like to get a better understanding of the prospects for submitting a successfull proposal in this area. Given some encouragement from the committee we will work towards developing the final design of an apparatus. We will also seek new collaborators.

We are convinced that the physics motivation for this proposal is compelling. Electrodynamics constitutes one of the pillars of modern physics. We should take advantage of any opportunity to devise meaningful new tests of our understanding of electromagnetic phenomena. understanding of it should be taken advantage of. In our opinion, relativistic heavy ion colliders will provide us with such new opportunities, perhaps leading to the creation of a completely new field of study.

Appendix I.

A rate calculation for multiple electromagnetic processes

In order to allow for the expeditious calculation of rates for this letter we derived few formulas based on simplifying assumptions:

Let us consider two electromagnetic processes, each with a probability distribution with respect to an impact parameter which is described by a formula $P_i = \frac{1}{b^{n_i}}$, $i = 1, 2$. A cross section for a convolution of these two processes can be calculated as follows:

$$\sigma_{mp} = \int_{b_{min}}^{b_{max}} \frac{P_1}{b^{n1}} \frac{P_2}{b^{n2}} 2\pi b db \qquad (I.1)$$

where b_{min} is a minimum allowed impact parameter prior to the nuclear contact, b_{max} is a maximum impact parameter determined by an adiabatic cutoff for one of the two processes (whichever is smaller).

The result of an integration can be expressed as follows:

$$\sigma_{mp} = \frac{P_1 * P_2 * 2\pi}{(n_1 + n_2 - 2)} * \kappa \qquad (I.2)$$

with

$$\kappa = \left[\frac{1}{b_{min}^{n_1+n_2-2}} - \frac{1}{b_{max}^{n_1+n_2-2}} \right] \qquad (I.3)$$

For simplicity, we now assume that b_{max} is much larger than b_{min}, a true statement for most of the processes which we consider. Under this assumption we can rewrite the result as:

$$\sigma_{mp} = \frac{P_1}{b_{min}^{n_1}} * \frac{P_2}{b_{min}^{n_2}} * \frac{2}{n_1 + n_2 - 2} * \pi b_{min}^2 \qquad (I.4)$$

An expression πb_{min}^2 desrcibes a total hadronic cross section, which for gold+gold collision is approximately equal to 7.6 barns. For final rate estimates we take $n_1 = n2 = 2$ for all processes considered (an approximation again. In reality, n is somewhat less than 2). We believe that our rate estimates should be good to within a factor of two. More exact rate estimates will be performed for a proposal, based on calculations which are now in progress.

Appendix II.

The dependence of e^+e^- probabilities on the invariant mass.

The dependence of the probability to produce an e^+e^- pair on its invariant mass can be well described by a $\frac{1}{M^3}$ dependence. This dependence is well illustrated in fig.(9), where the probability to produce a pair is integrated over a fixed interval of invariant masses, $1\frac{Mev}{c^2}$. To make our discussion more realistic from the experimental point of view, we chose to present these probabilities in a somewhat modified form. Namely, we integrate the probability to produce a pair of an invariant mass M_0 over a mass interval which is described by a fraction of M_0. In other words, we integrate over an invariant mass range $[M_0(1-\alpha); M_0(1+\alpha)]$, where α is a parameter. Hence, we obtain:

$$P_\alpha = \int\limits_{M_0(1-\alpha)}^{M_0(1+\alpha)} \frac{1}{M^3}\,dM \qquad \text{(II.1)}$$

which leads to:

$$P_\alpha = \frac{2}{M_0^2}\left[\frac{\alpha}{(1-\alpha^2)^2}\right] \qquad \text{(II.2)}$$

In most rate estimates $\alpha = 0.05$ was taken. A typical dependence of results on the value of α is shown in fig.(6) . The reader should bear in mind that using our formulas with large values of a parameter α may be somewhat misleading, due to a very steep dependence of probabilities on the invariant mass. For example, a rate estimate for events with two $100\frac{MeV}{c^2}$ pairs, using $\alpha = 0.4$, will be dominated by events with two $60\frac{MeV}{c^2}$ pairs. A value of α should be adjusted to fit a particular physics topic which is being discussed, as well as capabilities of the experimental apparatus.

For the purpose of rate estimates in this letter we normalized a probability to produce a $10\frac{MeV}{c^2}$ pair, at a minimum allowed impact parameter to unity. We use $\alpha = 0.05$ in this approximation. The actual value of a probability which was obtained via a Weizsacker-Williams calculation is 1.04.

Appendix III.

An average mass of the coincident pair. A simple example

To provide the reader with an example of qualitative features of the multiple pair measurement we made a *simple* estimate of an average mass of the coincident pair as a function of the invariant mass of the trigger pair. First, we assume that the average mass of a pair produced in a collision with the impact parameter b is:

$$M_b = \frac{M_0}{b}$$

with M_0 adjusted so that $M_b = 10\frac{MeV}{c^2}$ at $b = b_{min}$. Second, we calculate the average mass of a coincident pair by using a probability distribution for the trigger pair which is expressed as $P_e = \frac{1}{b^2}$. The average mass of a coincident pair can then be calculated as follows:

$$M_c = \left[\int_{b_{min}}^{b_{max}} \frac{P_e}{b^2} \frac{M_0}{b} 2\pi b db \right] * N^{-1} \qquad \text{(III.1)}$$

where

$$N = \int_{b_{min}}^{b_{max}} \frac{P_e}{b^2} 2\pi b db \qquad \text{(III.2)}$$

and b_{max} is determined by the adiabatic cutoff in the Weizsacker-Williams spectrum of virtual photons:

$$b_{max} = \frac{2 * \gamma * 197.3}{M}$$

Results of this calculation are summarized in fig.(10). An average mass of a coincident pair shows a fairly steep dependence on the mass of a trigger pair, thus demonstrating a sensitivity of the coincidence measurement to the range of impact parameters which is spanned by a trigger pair. The actual value of an average mass will be higher than indicated in fig(10). after the perturbative component in mass distribution is taken into account.

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Figure Captions

Fig.1) A probability to produce an e^+e^- pair of the invariant mass M_0, calculated for a collision at a minimum allowed impact parameter (prior to the nuclear contact). An invariant mass of a pair is contained within the $(M_0 * 0.95; M_0 * 1.05)$ range. Gold + Gold, 100GeV/nucleon.

Fig.2) A probability to produce an e^+e^- pair of the invariant mass $M_0 = 10\frac{MeV}{c^2}$, plotted as a function of an impact parameter. An invariant mass of a pair is contained within the $(M_0 * 0.95; M_0 * 1.05)$ range. Au + Au, 100GeV/nucleon.

Fig.3) A probability to produce a beam rapidity neutron in an elastic scattering of two gold ions plotted as a function of the impact parameter in a collision. Au + Au, 100GeV/nucleon.

Fig.4) A probability to produce a beam rapidity neutron in a coincidence with a second electromagnetic process, plotted as a function of a ratio between an impact parameter of an adiabatic cutoff for a coincidence process and a minimum allowed impact parameter $(A = \frac{b_{max}}{b_{min}})$. A probability of a second (unspecified) process is assumed to depend on the impact parameter like $\frac{1}{b^n}$. Results of calculations using three different values of the parameter n are showed.

Fig.5) Event rates for events with two e^+e^- pairs. An invariant mass of one pair is fixed at $100\frac{MeV}{c^2}$. Rates are plotted against a value of an invariant mass of a second pair. One day indicates 24 hours of running with a design luminosity of $L = 2 * 10^{26} cm^{-2} s^{-1}$,

Fig.6) Event rates for events with two e^+e^- pairs. An invariant mass of each pair is contained within a range of $(M_0 * (1 + \alpha); M_0 * (1 - \alpha))$, with M_0 fixed at a value of $100\frac{MeV}{c^2}$.Rates are plotted against a value of a parameter α. One day indicates 24 hours of running with a design luminosity of $L = 2 * 10^{26} cm^{-2} s^{-1}$.

Fig.7) Event rates for events with two e^+e^- pairs. An invariant mass of each pair is contained within a range of $(M_0 * (1+\alpha); M_0 * (1-\alpha))$, with α fixed at a value of 0.05. Rates are plotted against a value of a parameter M_0. One day indicates 24 hours of running with a design luminosity of $L = 2 * 10^{26} cm^{-2} s^{-1}$.

Fig.8) Event rates for events with a neutral meson and an e^+e^- pair. An invariant mass of a pair is contained within a range of $(M_0 * (1+\alpha); M_0 * (1-\alpha))$, with α fixed at a value of 0.05. Rates are plotted against a value of a parameter M_0. One day indicates 24 hours of running with a design luminosity of $L = 2 * 10^{26} cm^{-2} s^{-1}$.

Fig.9) A probability to produce an e^+e^- pair of the invariant mass M, calculated for a collision at a minimum allowed impact parameter (prior to the nuclear contact). A probability is plotted against $\frac{1}{M^3}$ to illustrate its dependence on M. An invariant mass of a pair is contained within the 1MeV range. Gold + Gold, 100GeV/nucleon.

Fig.10) A correlation between the masses of a trigger pair and a coincident pair. Gold + Gold, 100GeV/nucleon.

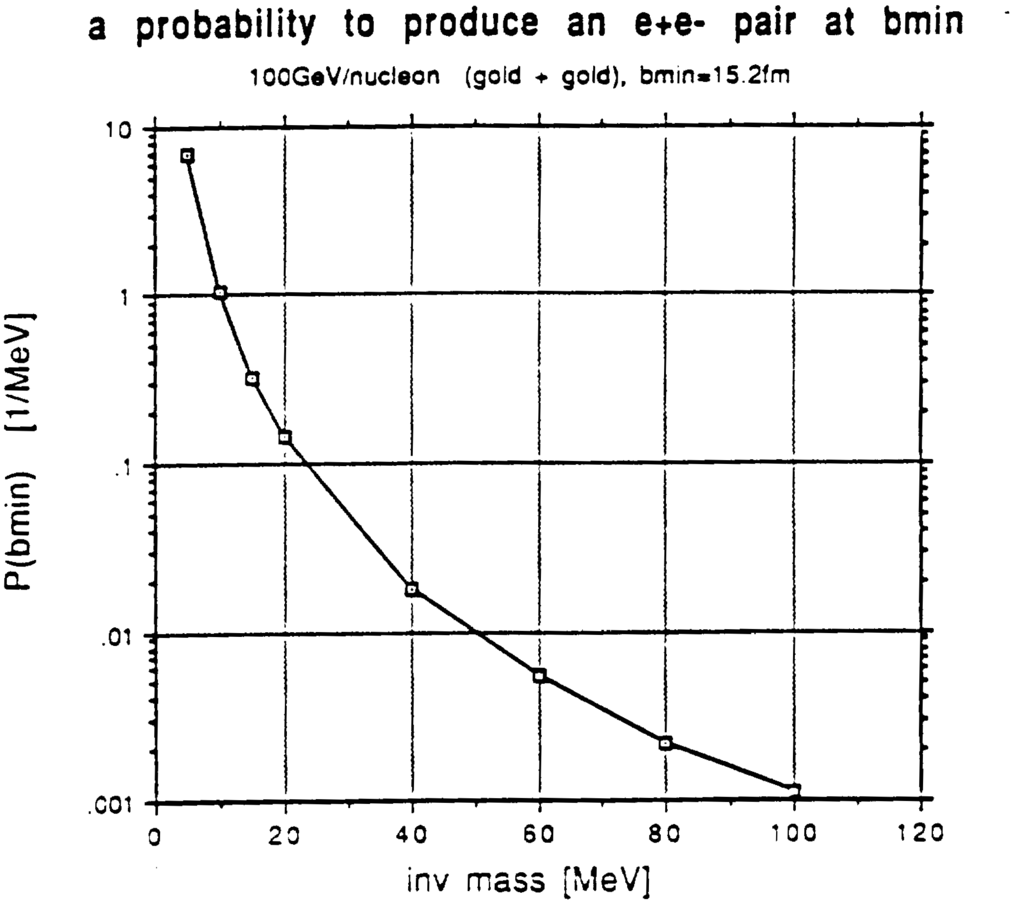

a probability to produce an e+e- pair at bmin
100GeV/nucleon (gold + gold), bmin=15.2fm
P(bmin) [1/MeV]
10
1
.1
.01
.001
0
20
40
60
80
100
120
inv mass [MeV]

fig. 1

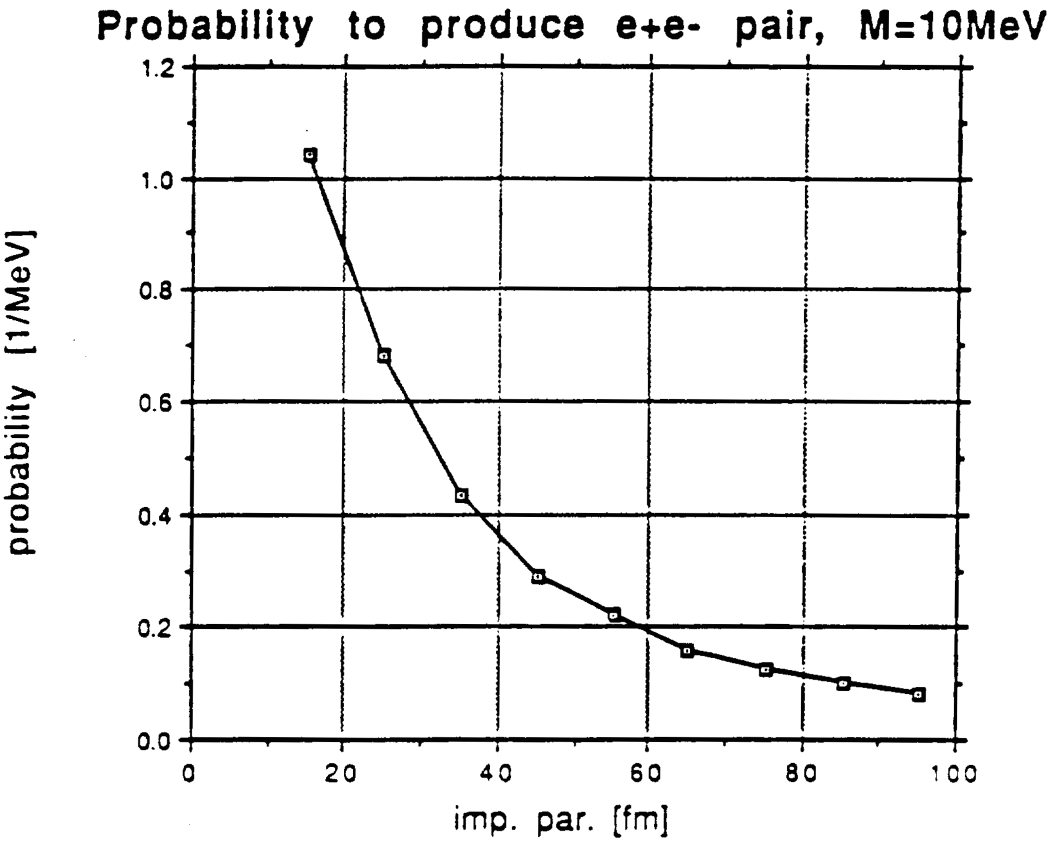
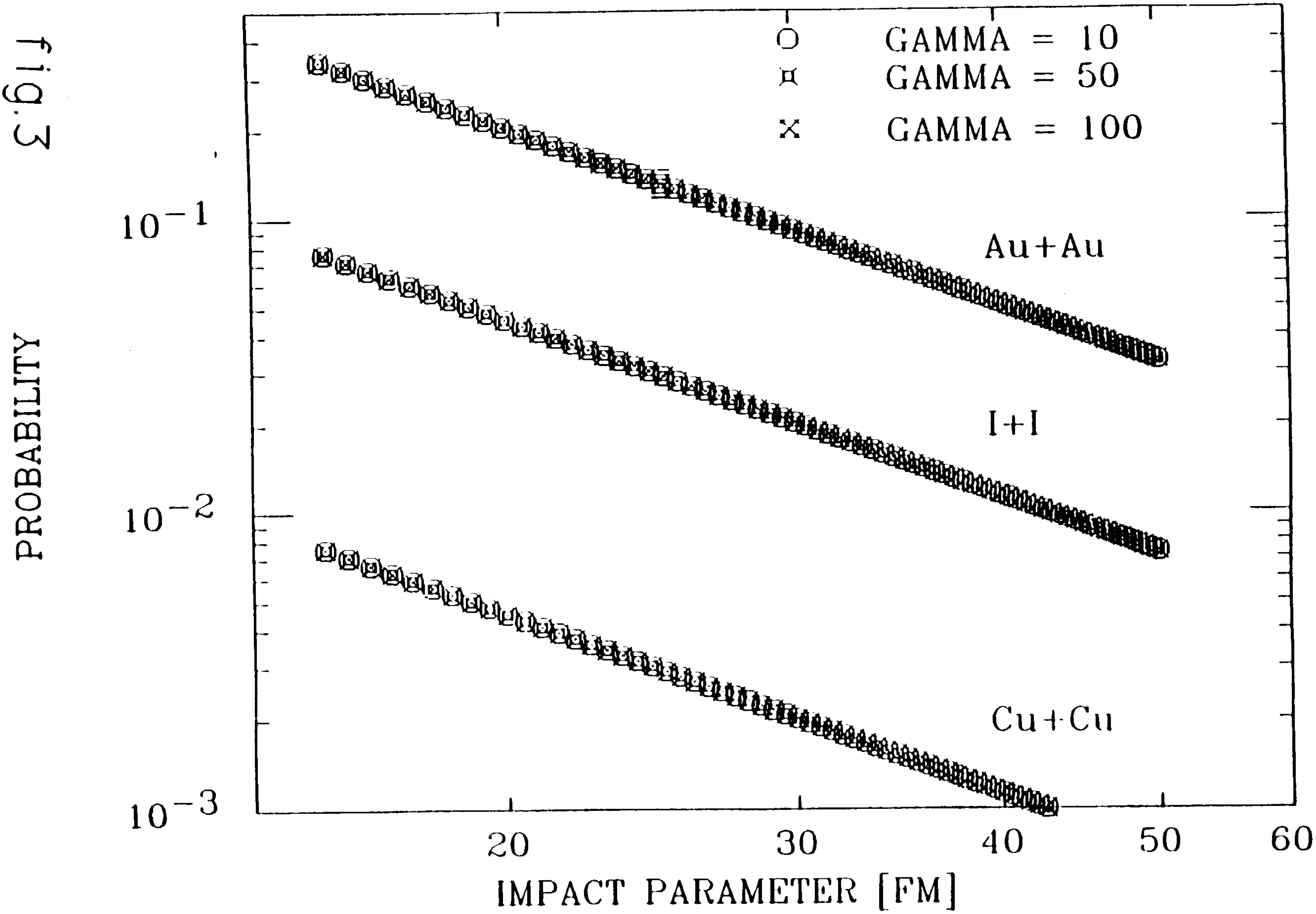

Probability to produce e+e- pair, M=10MeV
probability [1/MeV]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
imp. par. [fm]

fig.2

fig.3


○ GAMMA = 10
¤ GAMMA = 50
× GAMMA = 100
Au+Au
I+I
Cu+Cu
PROBABILITY
10^{-1}
10^{-2}
10^{-3}
20
30
40
50
60
IMPACT PARAMETER [FM]

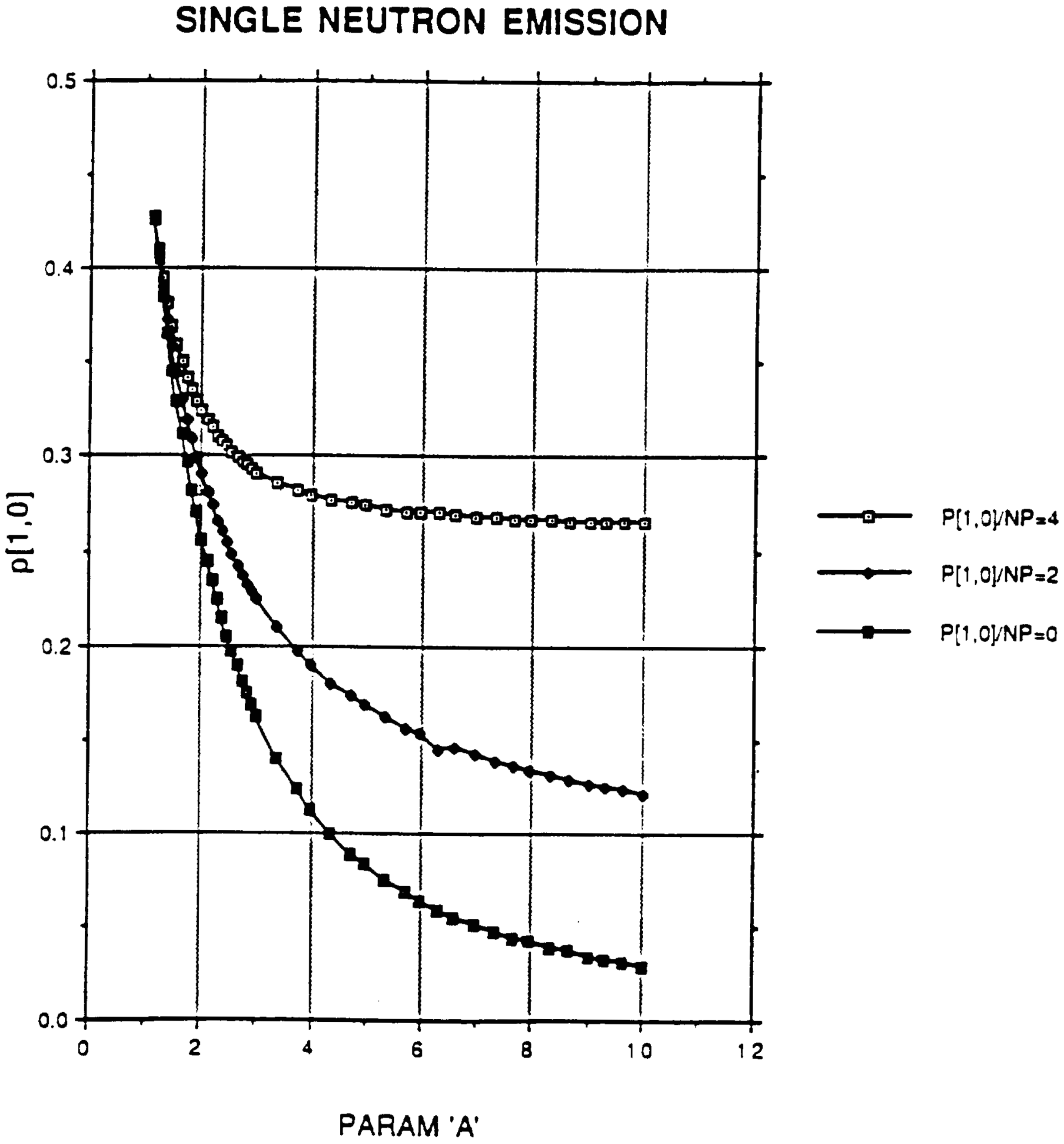

SINGLE NEUTRON EMISSION
0.5
0.4
0.3
0.2
0.1
0.0
ρ[1,0]
0
2
4
6
8
10
12
PARAM 'A'
P[1,0]/NP=4
P[1,0]/NP=2
P[1,0]/NP=0

fig.4

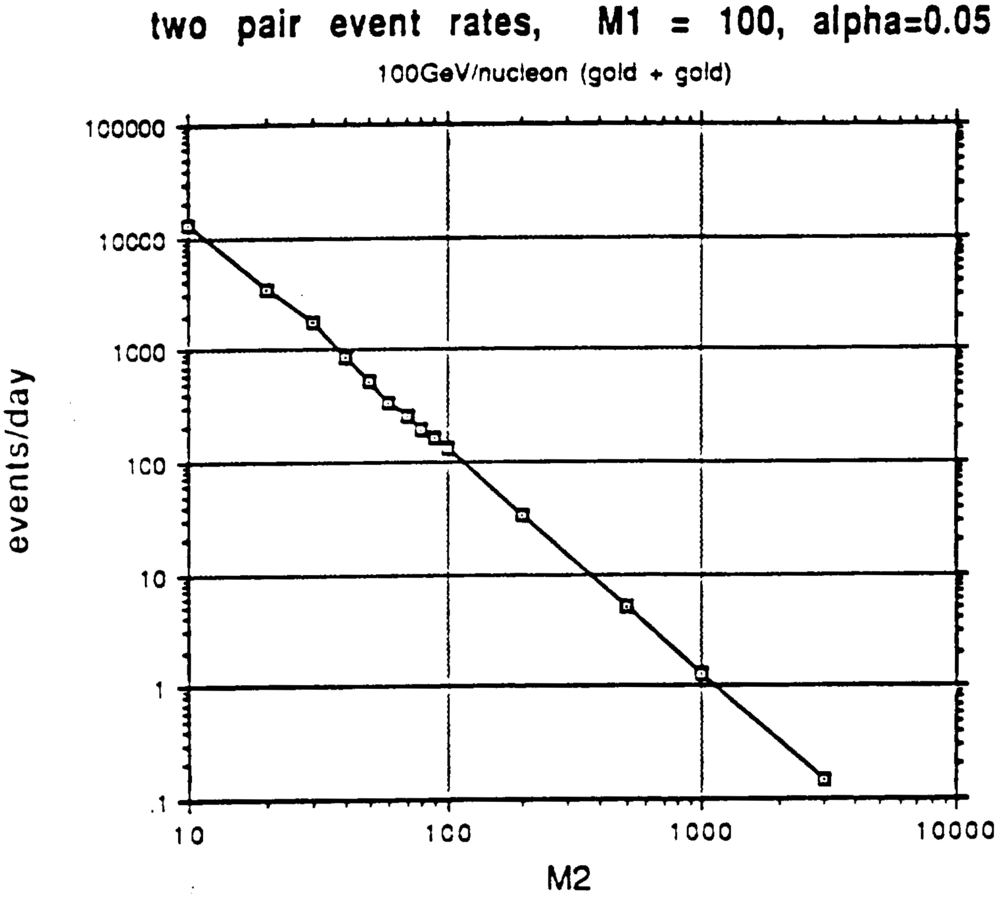

two pair event rates, M1 = 100, alpha=0.05
100GeV/nucleon (gold + gold)
events/day
100000
10000
1000
100
10
1
.1
10
100
1000
10000
M2

fig.5

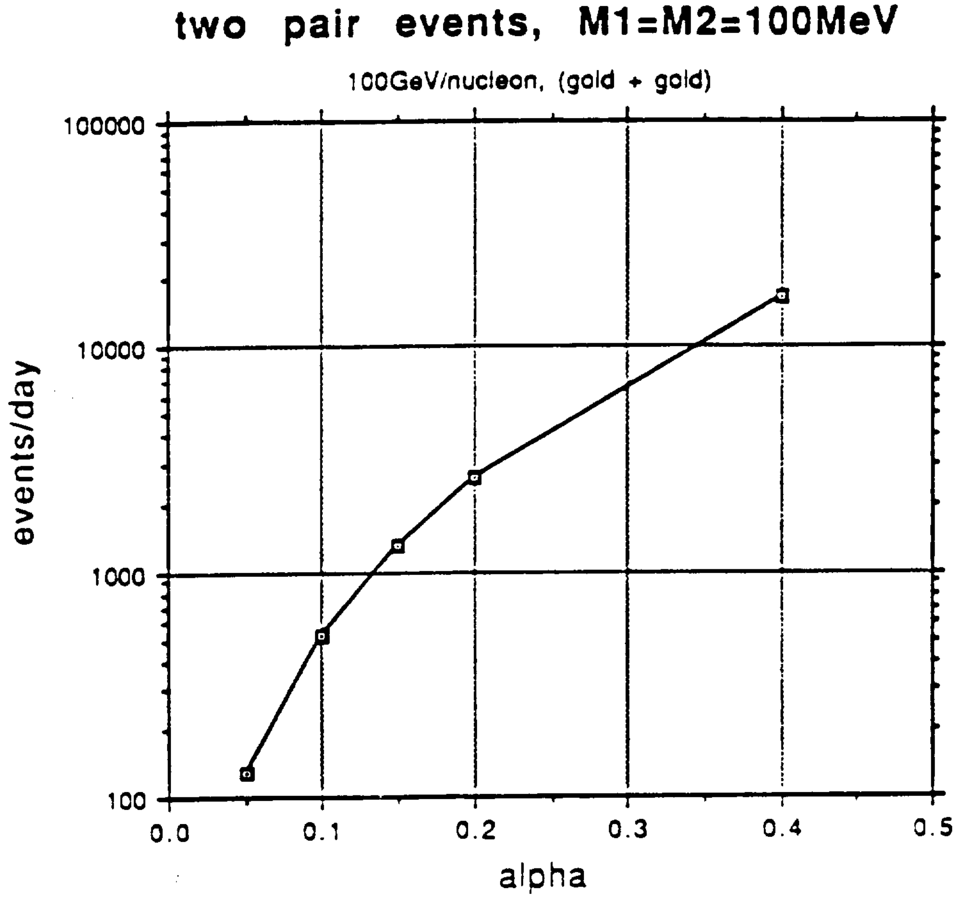

two pair events, M1=M2=100MeV
100GeV/nucleon, (gold + gold)
events/day
100000
10000
1000
100
0.0
0.1
0.2
0.3
0.4
0.5
alpha

fig.6

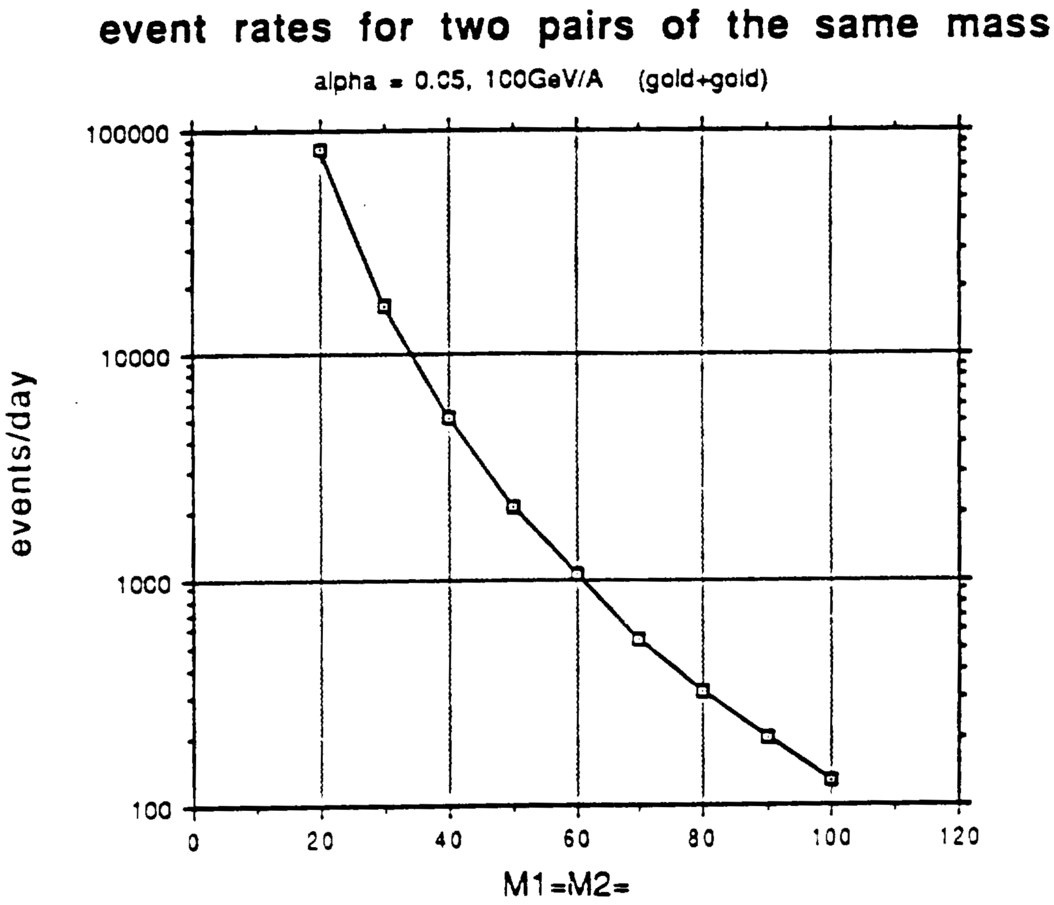

event rates for two pairs of the same mass
alpha = 0.05, 100GeV/A (gold+gold)
events/day
100000
10000
1000
100
0
20
40
60
80
100
120
M1=M2=

fig.7

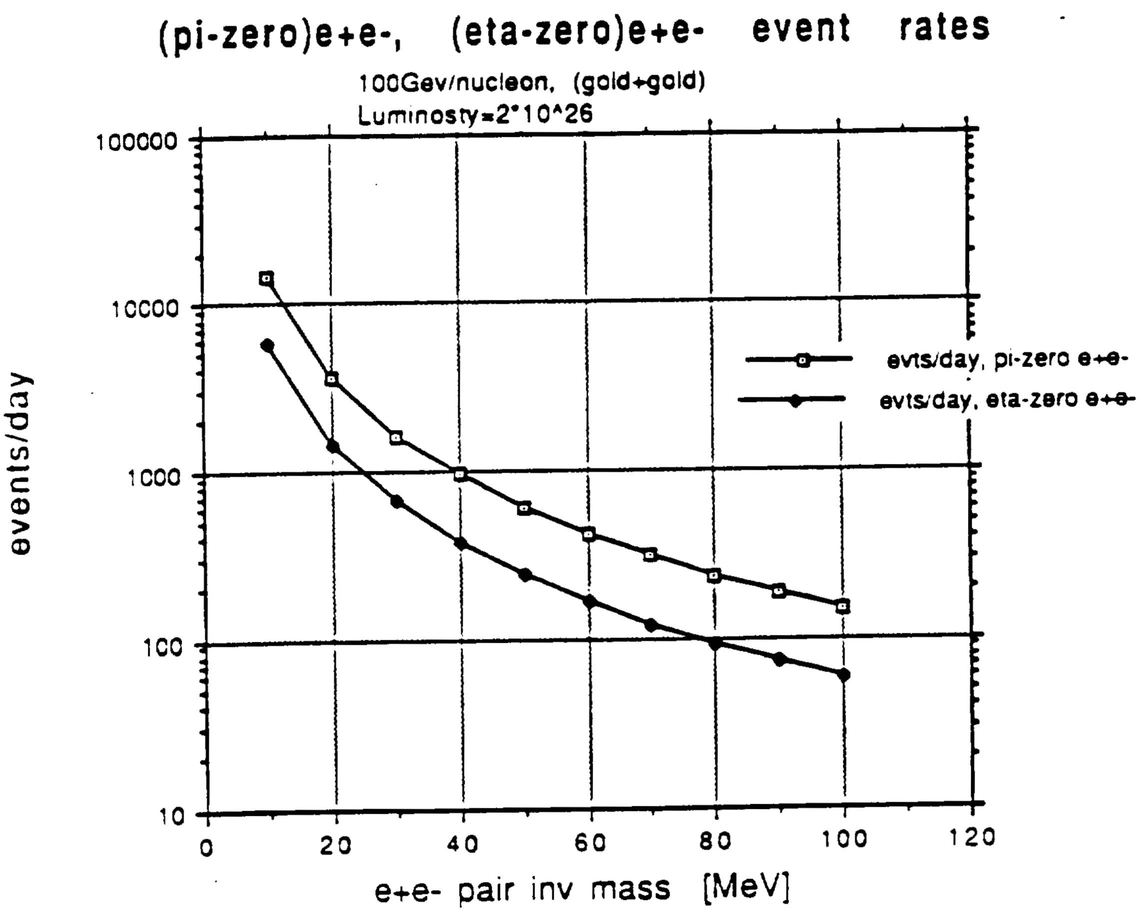
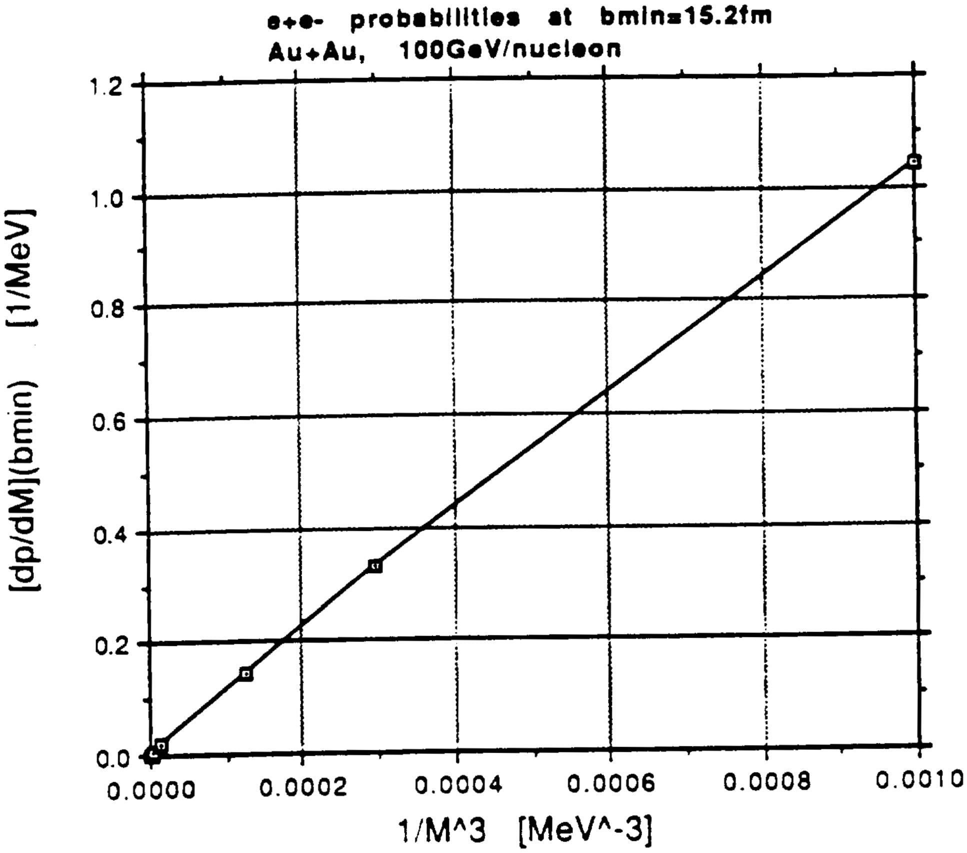
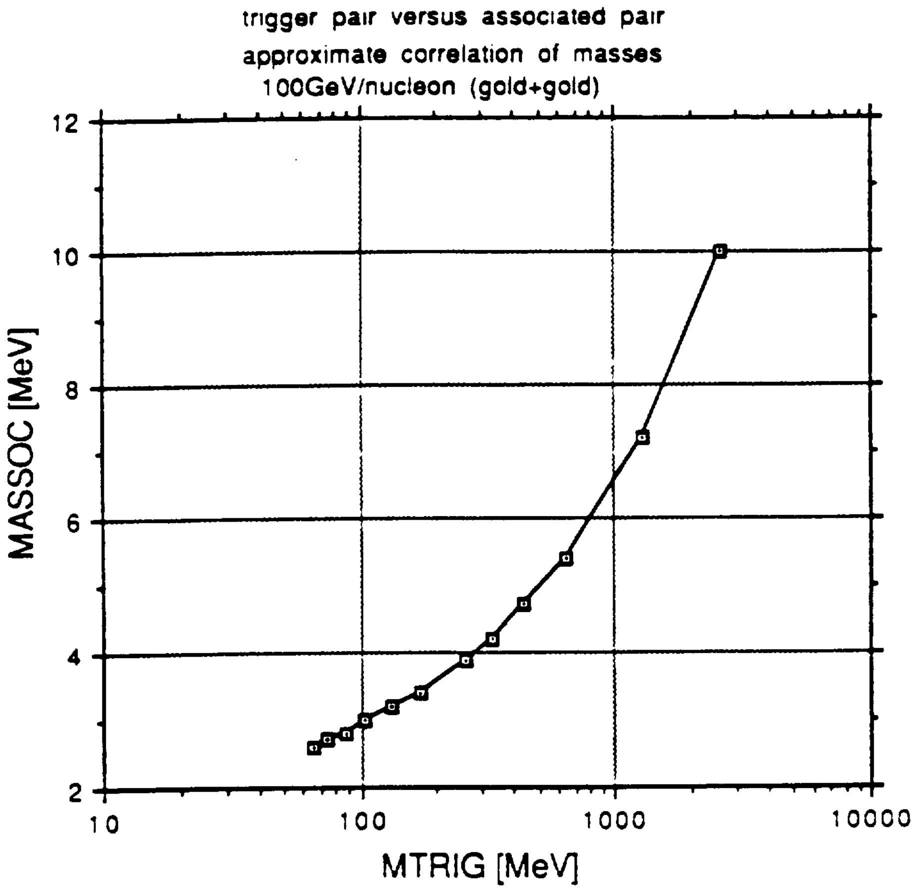

(pi-zero)e+e-, (eta-zero)e+e- event rates
100Gev/nucleon, (gold+gold)
Luminosty=2*10^26
100000
10000
1000
100
10
events/day
0
20
40
60
80
100
120
e+e- pair inv mass [MeV]
evts/day, pi-zero e+e-
evts/day, eta-zero e+e-

fig.8



e+e- probabilities at bmin=15.2fm
Au+Au, 100GeV/nucleon
[dp/dM](bmin) [1/MeV]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
1/M^3 [MeV^-3]

fig.9



trigger pair versus associated pair
approximate correlation of masses
100GeV/nucleon (gold+gold)
MASSOC [MeV]
2
4
6
8
10
12
10
100
1000
10000
MTRIG [MeV]

fig.10