SEMI-ANNUAL STATUS REPORT
NASA Grant NAG5-557

Bandwidth Efficient CCSDS Coding Standard Proposals

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May, 1992

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Glossary

signal set The set of signals that are actually transmitted over the channel, e.g., 8PSK. An element of the signal set is called a *signal point* or simply a *signal*.

modulation interval The amount of time required to transmit one signal. Denoted by T.

signal constellation The set of signals to which the encoder output is mapped, e.g., Lx8PSK. An element of the constellation is called a *constellation point*. (The signal constellation is the same as the signal set when L=1.) If the constellation is Lx8PSK, then the signal set is 8PSK and it requires LxT seconds to transmit a single constellation point.

symbol An element in the Galois field over which a Reed-Solomon code is constructed. If the Reed-Solomon code is constructed over $GF(2^8)$, then a symbol is a block of 8 bits.

spectral efficiency The average number of information bits transmitted per signal in a coded or uncoded system. Denoted by η with units of bits/signal.

rotational invariance The minimum rotation, in degrees, of the received signal set for which the decoder can still decode correctly (with the help of differential encoding). Note that the smaller the rotational invariance of a code the better. By definition, if a code is 45° rotationally invariant it is also 90° and 180° rotationally invariant.

constraint length The total number of memory elements in an encoder. Denoted by ν. Note that a trellis code with constraint length ν has 2^ν states.

uncoded bits For trellis codes, uncoded bits are information bits that are not coded by the convolutional encoder contained in the trellis encoder.

rate The ratio of the number of inputs bits to the number of output bits of a code. For trellis codes, uncoded bits are included. Denoted by R.

minimum squared Euclidean distance The minimum distance between any two codewords in a trellis code calculated using the squared Eulidean distance metric. The Minimum Squared Eulidean Distance (MSED) is often called the *free Euclidean distance*. For trellis codes with certain symmetry properties, the MSED may be found by considering distances only from the all zero codeword.

distance spectrum A complete enumeration of the squared Euclidean distance between any two possible codewords in a trellis code. For trellis

codes with certain symmetry properties, the distance spectrum may be found by considering only distances between nonzero codewords and the all zero codeword.

coding gain The reduction in signal to noise ratio for a coded system to achieve the same performance as an uncoded system of the same spectral efficiency.

signal to noise ratio The ratio of the average transmitted signal energy per information bit, E_b, to the one sided power spectral density of an additive white Gaussian noise channel, N_0.

asymptotic coding gain The coding gain of a code as the signal to noise ratio approaches infinity. This is a function only of the minimum squared Euclidean distance of the code.

real coding gain The coding gain of a code at a particular bit error rate. Usually determined by computer simulation. The real coding gain is less than or equal to the asymptotic coding gain and is a function of the distance spectrum of the code.

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1 System Constraints

The basic concatenated coding system for the space telemetry channel consists of a Reed-Solomon (RS) outer code, a symbol interleaver/deinterleaver, and a bandwidth efficient trellis inner code. A block diagram of this configuration is shown in Figure 1. The system may operate with or without the outer code and interleaver.

In this recommendation, the outer code remains the (255, 223) RS code over $GF(2^8)$ with an error correcting capability of $t = 16$ eight bit symbols. This code's excellent performance and the existence of fast, cost effective, decoders justify its continued use.

The purpose of the interleaver/deinterleaver is to distribute burst errors out of the inner decoder over multiple codewords of the outer code. This utilizes the error correcting capability of the outer code more efficiently and reduces the probability of an RS decoder failure. Since the space telemetry channel is not considered bursty, the required interleaving depth is primarily a function of the inner decoding method. A diagram of an interleaver with depth 4 that is compatible with the (255, 223) RS code is shown in Figure 2. Specific interleaver requirements are discussed after the inner code recommendations.

Previous coding standards [1] have utilized the optimal, R = 1/2, constraint length $\nu = 6$, binary convolutional code with QPSK modulation and Viterbi decoding for the inner code. Bandwidth constraints and the desire for increased data transmission rates motivate the consideration of bandwidth efficient trellis codes for the inner code. A reasonable set of system constraints affecting the choice of the inner code is listed below.

1. Spetral efficiency $\eta \geq 2.0$ bits/signal.
2. Compatibilty with the RS outer code.
3. Viterbi decoding complexity ≤ 64 states (parallel implementation).
4. High speed decoding.
5. Constant envelope modulation (for compatiblity with traveling wave tube amplifiers (TWTA's)).
6. Invariance of the trellis code to rotations of the signal set.

In the following sections, recommendations for the inner code are discussed. Brief summaries of all the recommended inner codes are included as an appendix to this report.

2 Inner Code Performance

The system constraints given in Section 1 limit the choice of the inner code to trellis codes with LxMPSK constellations and either suboptimal sequential decoding or optimal Viterbi decoding. Of these, four possibilities stand out:

- Periodically Time Varying Trellis Codes (PTVTC's) [2].
- Multi-Level MPSK Trellis Codes (MLTC's) [3].
- The Viterbi Pragmatic Trellis Code [4].
- Ungerboeck [5] and Pietrobon, et. al. [6] MPSK Trellis Codes.

In the remainder of this section, the performance of these four classes of codes is discussed. The performance measure used to compare codes is the signal to noise ratio (SNR) required for an information bit error rate (BER) of 10^{-5} on the additive white Gaussian noise channel (AWGN). The real coding gain of a particular code is the reduction in SNR required to achieve a BER of 10^{-5} compared to an uncoded system with the same spectral efficiency.

Periodically time varying trellis codes were introduced in [2] as a means of achieving fractional spectral efficiencies, η, defined as the average number of information bits transmitted with each 2-dimensional signal. This is accomplished by using a time varying convolutional code in the trellis encoder. Figure 3 shows simulation results of two PTVTC's and two comparable multidimensional 3x8PSK Pietrobon codes (labeled SSP in this and subsequent figures). In both cases, the PTVTC's lose $\approx$ 0.5dB in real coding gain at 10^{-5}. In addition, the PTVTC's are only 180° rotationally invariant compared to 90° and 45° invariance for the $\eta = 2.67$ and $\eta = 2.33$ 3x8PSK Pietrobon codes.

In certain instances, multi-level trellis codes using multi-stage decoding (MSD) have a performance/complexity advantage over single level trellis codes with Viterbi decoding. Figure 4 shows simulation results for a 3-level 8PSK code constructed by Wu, et. al. [3] with MSD. This code has a spectral efficiency of $\eta = 1.997$ bits/signal and the MSD uses 18 states. Compared to the Ungerboeck 16 state, 8PSK code with $\eta = 2.0$ bits/signal, the MLTC loses $\approx$ 0.2dB at 10^{-5}. In addition, the MSD requires buffering of data between levels and the rotational invariance properties of these MLTC's are unknown.

Viterbi, et. al. [4] suggested using the standard (2, 1, 6) binary convolutional code as the basis of a set of "pragmatic" 2-dimensional trellis codes. With 8PSK modulation, this results in a suboptimal 64 state trellis code with $\eta = 2.0$ bits/signal. The pragmatic trellis code has a practical advantage in that a large number of single chip Viterbi decoders already exist for the binary (2, 1, 6) code. Presumably, these existing chips can easily be modified to decode the pragmatic trellis codes. A union bound on the performance of this code is shown in Figure 5 compared to simulation results for a 64 state, 2x8PSK, Pietrobon code with the same rate. The pragmatic code has $\approx$ 0.32dB less coding gain at 10^{-5} and loses more coding gain as the SNR is increased. Also, the pragmatic code is only 180° rotationally invariant compared to 90° invariance for the 2x8PSK code.

From the discussion of the four types of codes using Viterbi decoding, it appears that the Ungerboeck [5] and Pietrobon et. al. [6] MPSK codes are the most promising as inner codes in a concatenated coding system. In the next section, sequential decoding of the inner code is discussed.

2.1 Sequential Decoding

It has been shown that sequential decoding [7] is a good alternative to Viterbi decoding for trellis codes [8]. Sequential decoding performs almost as well as Viterbi decoding and the computational complexity is essentially independent of the code constraint length. Thus, larger coding gains are possible when larger constraint length codes are used with sequential decoding. However, the Ungerboeck and Pietrobon et. al. MPSK codes are constructed by exhaustive search and thus only small constraint length codes have been found. These codes can be used as the inner code with Viterbi decoding, but to take full advantage of sequential decoding large constraint length codes must be constructed.

In [9], a new approach to constructing good large constraint length trellis codes for use with sequential decoding was proposed. The procedure begins by randomly choosing a relatively small set of codes. The error performance of each of these codes is evaluated using sequential decoding and the code with the best performance among the chosen set is retained. The performance of many of the randomly chosen codes is quite good and often approaches the perfromance of the best known codes. This approach can be justified by the well known fact that a randomly chosen code is very likely to be a good code. All of the recommendations for sequential decoding discussed in this report use codes that were constructed using this approach.

The cut-off rate, R_0, defined as the maximum rate at which the average number of computations for sequential decoding is bounded, is a fundamental performance limit for sequential decoding. R_0 is also regarded as the maximum rate for which reliable communication can be achieved with reasonable complexity regardless of the specific method of decoding being used. Practically, sequential decoding has been shown to be capable of approaching R_0 [8],[9], i.e., good performance can be achieved at the SNR for which $R_0 = \eta$.

Encoders with some uncoded information bits simplify code construction and decoding complexity, but limit the achievable free distance, which is the predominant parameter determining the code performance of large constraint length codes. Hence, all the information bits in large constraint length trellis codes should be encoded. For some short constraint length codes, however, encoders with uncoded bits may give optimum free distance codes, even though they introduce parallel transitions in the code trellis. For a trellis code with only one coded bit, the parallel transitions limit the potential asymptotic coding gain to 3.0 dB, while for codes with 2 and 3 coded bits the potential asymptotic coding gains are limited to 6.0 dB and

9.0 dB, respectively. To approach R_0, the maximum asymptotic coding gain is required, so all the information bits should be encoded when searching for good trellis codes for use with sequential decoding.

2.1.1 Recommendation 1

For sequential decoding, a $\nu = 17$, 1x8PSK code with a spectral efficiency of 2.0 bits/signal is proposed. This code was specifically constructed for use with sequential decoding [9]. Following the notation of [5], the parity check coefficients of the code in octal form are $h^0 = 674241$, $h^1 = 174116$, and $h^2 = 041642$.

A modified version of the Fano Algorithm (FA) [7] is used to decode the inner code. Since the computational effort of sequential decoding is a random variable with a Pareto distribution, a buffer must be used to store the incoming data. A finite buffer may overflow, however, resulting in data loss (erasures). The Modified Fano Algorithm (MFA)[8] proposed here guarantees erasurefree decoding.

This constraint length 17, 1x8PSK trellis code with a decoder speed factor of $\mu = 4$ and a buffer size of 32K signals achieves a real coding gain of $4.86g$dB. This code is 180^0 rotationally invariant. The performance of this code is shown is shown in Figure 6 compared to uncoded QPSK.

2.2 Viterbi Decoding

The Ungerboeck [5] and Pietrobon, et. al. [6], MPSK codes appear to be the most promising option for the inner code with Viterbi decoding. To select a code, it is necessary to specify the following parameters:

1. ν, the constraint length of the code.
2. η, the spectral efficiency of the code.
3. M, the size of the 2-dimensional signal set.
4. L, the number of 2-dimensional signal sets in the LxMPSK constellation.

Each of these parameters affects the performance and complexity of the code.

When Viterbi decoding is used, the constraint length, ν, is the predominant factor in code performance and decoder complexity for a given spectral efficiency. For example, with $\eta = 2.0$ bits/signal and a 1x8PSK constellation, increasing the constraint length by 1 increases the asymptotic coding gain by approximately 0.5dB and doubles the number of states in the decoder. With current technology, the best tradeoff between code performance and implementation complexity of a fully parallel Viterbi decoder appears to be with $\nu = 6$.

Changing the spectral efficiency of a code with a fixed constraint length allows a tradeoff between coding gain and information rate. Increasing η decreases the coding gain and increases the information rate. Conversely, decreasing η increases the coding gain and decreases the information rate. The system constraints suggest spectral efficiencies from 2 to 3 bits/signal with 2.0 bits/signal being the best choice.

The size, M, of the 2-dimensional signal set is essentially determined by the desired spectral efficiency and the practical difficulty of detecting the phase in large MPSK signal sets. In order to achieve a spectral efficiency of 2.0 or more information bits per 2-dimensional signal in a coded system, M must be at least 8. Similarly, a spectral efficiency of 3.0 or more information bits per 2-dimensional signal requires M to be at least 16. However, it is generally considered too difficult to reliably detect a 16PSK signal and thus M=8 is stressed in this recommendation. Spectral efficiencies of 3.0 or more bits will require either that the detection of 16PSK signals be made feasible or that nonconstant envelope QAM signal sets be used and the associated TWTA problem be solved.

The primary effect of the choice of L is on the decoder speed and complexity and the rotational invariance properties of the code. For Lx8PSK, at least L=2 is required for the code to be better than 180° rotationally invariant. The rotational invariance is also affected by the constraint length and spectral efficiency. As L increases, the speed, in bits per second, of the decoder also increases, since L$\times\eta$ information bits are decoded on each branch of the trellis. The importance of decoder speed suggests that L=4 be used if at all possible. Decoder complexity is discussed in detail in a later section.

2.2.1 Recommendation 2

In light of the previous discussion, the best code to use in conjunction with Viterbi decoding is the $\nu = 6$, 4x8PSK Pietrobon code with a spectral efficiency of $\eta = 2.0$ bits/signal. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 107$, $h^1 = 036$, $h^2 = 016$, $h^3 = 044$, and $h^4 = 034$.

This code achieves a real coding gain of 3.85dB and is 45° (fully) rotationally invariant. For this spectral efficiency and constraint length, full rotational invariance cannot be achieved with L less than 4. The performance of this code is shown in Figure 7 compared to Viterbi's pragmatic code and uncoded QPSK.

2.2.2 Recommendation 3

If a higher spectral efficiency is desired and the increased power requirement is tolerable, the $\nu = 6$, 4x8PSK Pietrobon code with $\eta = 2.5$ bits/signal could be used. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 103$, $h^1 = 042$, $h^2 = 024$, and $h^3 = 014$.

This code achieves a real coding gain of 3.76dB and is 45° (fully) rotationally invariant. The 4x8PSK performance is slightly better than the 2x8PSK code with the same spectral efficiency and constraint length, and offers increased decoding speed with only a modest increase in complexity. The performance of this code is shown in Figure 8 compared to an uncoded system with the same spectral efficiency.

3 Decoder Speed and Complexity

3.1 Sequential Decoding with the Modified Fano Algorithm

The complexity of the MFA is about the same as the well known FA. If a 32 K signal buffer is used with an 8-bit quantization scheme, then the buffer must store 256Kbits. This is the main storage requirement for a sequential decoder. Also, a ROM with 8×256 entries (for an 8-bit quantization scheme) is needed to store the metric table.

In Viterbi decoders, a survivor must be selected at each state. The add-compare-select (ACS) operation needed to perform such a selection is defined as a computation. Thus, 2^{ν} computations are needed to decode one branch. Similarly, a forward look in the MFA may be defined as a computation of comparable complexity to the ACS operation. A typical computation in the MFA involves regenerating code branches, finding the branch metrics, computing the path metrics, and choosing the path with the best metric. The number of computations that a sequential decoder can perform during a modulation interval (T, the time needed to transmit one signal) is defined as the decoder speed factor μ. The decoding speed that can be achieved depends on μ. Generally, μ sequential decoders in parallel are required to achieve the same decoding speed as a parallel implementation of a Viterbi decoder, assuming that identical logic is used in both decoders. When $\mu < 2^{\nu}$, the hardware complexity needed to implement μ sequential decoders in parallel, however, is less than the hardware complexity needed in a parallel implementation of a Viterbi decoder.

Assuming that a 300 MHz channel capable of transmitting 2-dimensional 8-PSK signals at a baud rate of 300 M signals/second is available, the inner decoder must process 8-PSK signals at the same 300 M signals/second rate. In practice, this is achieved by using several decoders, each operating at a lower rate, in parallel. A Viterbi decoder implemented in parallel using current technology can achieve a decoding speed of 25 million ACS opereations per second. Thus, 12 parallel Viterbi decoders are needed to achieve a decoding speed of 300 M signals/second. If $\mu = 4$, then approximately 48 parallel sequential decoders would be required to achieve the same decoding speed. However, the hardware complexity of each sequential decoder would be much less than that of one of the parallel Viterbi decoders with 64 states.

3.2 Parallel Viterbi Decoding

As noted above, a fully parallel implementation of a single Viterbi decoder can perform 25 million add-compare-select (ACS) operations per second per state. Thus, the $\nu = 6$, 1x8PSK, Ungerboeck code would require twelve 64 state Viterbi decoders each operating at 25MHz to fully utilize the 300MHz channel.

The PTVTC's, MLTC's, and the pragmatic trellis codes also require twelve decoders each operating at 25MHz to achieve 300MHz. The multidimensional Lx8PSK codes, however, would require 12 decoders each operating at 25/L MHz, since each ACS operation chooses L 8PSK signals. The decrease in ACS speed comes at the expense of increased parallel transition decoding complexity. For example, the 1x8PSK code requires a 25MHz ACS, but has no parallel transitions. The 2x8PSK code requires a 12.5 MHz ACS and has 2 parallel transitions, and the 4x8PSK code requires only a 6.25MHz ACS but has 16 parallel transitions. Alternatively, an Lx8PSK code could use a 25MHz ACS and reduce the number of decoders needed to use the full 300MHz channel by a factor of L. In this case, the 2x8PSK code would need 6 decoders with a 25MHz ACS, and the 4x8PSK code only 3 decoders with a 25MHz ACS. If a faster channel of 600MHz is available, then 24 1x8PSK decoders with a 25MHz ACS are needed, versus 12 2x8PSK decoders or 6 4x8PSK decoders with a 25MHz ACS.

The problem of increasing numbers of parallel transitions can be substantially mitigated by using ROM's to do table lookup decoding of the parallel transitions. If each received 2-dimensional 8PSK signal is quantized to b bits, then each Lx8PSK signal requires bL bits. This translates into a ROM with 2^{bL} entries or, equivalently, bL address bits. One ROM is required for each of the $2^{\tilde{k}}$ possible sets of parallel transitions. Each ROM entry would store the $k - \tilde{k}$ bits of the decoded parallel transition and the corresponding branch metric.

Using this method, the $\nu = 6$, 4x8PSK code with $k = 8$, $\tilde{k} = 4$, and $b = 5$-bit quantization would require $2^{\tilde{k}} = 16$ ROM's each with $2^{bL} = 2^{20} \approx$ 1Mbyte of storage. Since each byte must store $k - \tilde{k} = 4$ information bits for the decoded parallel transition, a ROM with 8-bit bytes would have only 4 bits left over for the branch metric.

4 Alternative Inner Codes

The codes presented in section 2 of this report are the primary recommendations for the new CCSDS inner code. In this section, several alternative inner codes are discussed.

4.1 Alternatives for 8PSK with Viterbi Decoding

The codes discussed in Recommendations 2 and 3 are the most desirable codes for Viterbi decoding from a pure performance perspective. That is,

they achieve the largest coding gains and highest decoding speeds and are fully rotational invariant. However, it may be that they are considered too complex to implement with current technology.

Recommendations 2 and 3 dealt with $\nu = 6$, 4x8PSK trellis codes with Viterbi decoding and spectral efficiencies of 2.0 and 2.5 bits/signal, respectively. These codes may be considered too complex for a high speed parallel implementation of a Viterbi decoder because of the constraint length or the large ROM's required for the parallel transition decoding due to the large 4x8PSK constellation. These problems can be addressed by using smaller constraint length codes with less coding gain or smaller constellations with slower decoder speeds. Recommendations 4 through 6 are alternatives to Recommendation 2. A similar set of alternatives also exist for Recommendation 3 and can be found in [6].

4.1.1 Recommendation 4

If a constraint length of 6 is prohibitive, the $\nu = 4$, 4x8PSK Pietrobon code with a spectral efficiency of 2.0 bits/signal is a suitable replacement for Recommendation 2. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 21$, $h^1 = 03$, $h^2 = 02$, $h^3 = 04$, and $h^4 = 10$.

This code has a real coding gain of 3.02dB and is 45° (fully) rotationally invariant. Assuming a 25MHz ACS, this code achieves the same 200Mbps decoding speed as the 64 state code of Recommendation 2, but with 0.8dB less coding gain. The performance of this code is shown in Figure 9 compared to the code of Recommendation 2 and uncoded QPSK.

4.1.2 Recommendation 5

If a 4x8PSK constellation is prohibitive, the $\nu = 6$, 2x8PSK Pietrobon code with a spectral efficiency of 2.0 bits/signal is a suitable replacement for Recommendation 2. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 125$, $h^1 = 004$, $h^2 = 050$, and $h^3 = 012$.

This code has a real coding gain of 3.8dB, but is only 90° rotationally invariant. Thus, though it achieves the same real coding gain as the 4x8PSK code of Recommendation 2, it has a decoding speed of only 100Mbps assuming a 25MHz ACS. However, if it becomes practical to use a 4x8PSK constellation at a later date, the 2x8PSK code may be converted to a sub-optimal 4x8PSK code. This can be done by simply changing the parallel transition ROM's of the decoder from 1Kbyte by 8-bits to 1Mbyte by 12 or 16-bits. (This assumes that the original decoder is built with enough address lines to address the larger ROM.) The converted 4x8PSK code loses 0.67dB compared to the optimal 4x8PSK code at 10^{-5}. The performance of this code is shown in Figure 10 compared to the code of Recommendation 2 and uncoded QPSK.

4.1.3 Recommendation 6

If a constraint length of 6 and a 4x8PSK constellation are prohibitive, the $\nu = 4$, 3x8PSK Pietrobon code with a spectral efficiency of 2.0 bits/signal is the best replacement for Recommendation 2. For $\nu = 4$ and $\eta = 2.0$ bits/signal, reducing the size of the constellation any further would involve changing the rotational invariance properties to at least 90°. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 27$, $h^1 = 04$, and $h^2 = 12$.

This code has fewer states, lower trellis branch complexity, and smaller parallel transition ROM's than Recommendation 2 in return for reduced coding gain and slower decoding speeds. It has a real coding gain of 2.94dB and is 45° (fully) rotationally invariant. Assuming a 25MHz ACS, it can achieve a decoding speed of 150Mbps. The performance of this code is shown in Figure 11 compared to the code of Recommendation 2 and uncoded QPSK.

4.1.4 The Implemented Code

One alternative to Recommendation 3 is the $\nu = 4$, 2x8PSK code with a spectral efficiency of 2.5 bits/signal. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 23$, $h^1 = 12$, and $h^2 = 16$.

This code has a real coding gain of 3.31dB and is 45° rotationally invariant. A prototype of a fully parallel implementation of a Viterbi decoder for this code has been built and will be tested at the White Sands Missile Range in New Mexico in 1992. The performance of this code is shown in Figure 12 compared to the code of Recommendation 3 and an uncoded system with the same spectral efficiency.

4.2 Higher Spectral Efficiency Codes

It was mentioned in section 2 that to get spectral efficiencies of 3.0 to 4.0 bits/signal it is necessary to use either Lx16PSK or Lx16QAM constellations. These constellations are not yet feasible on existing satellite links. However, good codes for these constellations have been found [9], [10], and [11]. In the remainder of this section, recommendations for Lx16PSK and Lx16QAM codes with sequential decoding and Viterbi decoding are presented.

4.2.1 Recommendation 7: 1x16PSK with Sequential Decoding

The $\nu = 16$, 1x16PSK trellis code with a spectral efficiency of 3.0 bits/signal in this recommendation is constructed for use with sequential decoding [9]. Following the notation of [5], the parity check polynomials of this code in octal form are $h^0 = 255005$, $h^1 = 106076$, and $h^2 = 161140$.

This constraint length 16, 1x16PSK trellis code with a decoder speed factor of $\mu = 4$ and a buffer size of 32K signals achieves a real coding gain of 4.55dB and is 90° rotationally invariant. The performance of this code is shown is shown in Figure 13 compared to uncoded 8PSK.

4.2.2 Recommendation 8: 2x16PSK with Viterbi Decoding

For Viterbi decoding, a $\nu = 6$, 2x16PSK trellis code with a spectral efficiency of 3.0 bits/signal is recommended. Following the notation of [5], the parity check polynomials of this code in octal form are $h^0 = 107$, $h^1 = 016$, and $h^2 = 044$.

This code achieves a real coding gain of 3.52dB and is 45° rotationally invariant. For this spectral efficiency and constraint length, full rotational invariance (22.5°) requires a 3x16PSK or 4x16PSK constellation and a code with increased trellis branch complexity and reduced minimum squared Euclidean distance! The performance of this code is shown in Figure 14 compared to uncoded 8PSK.

4.2.3 Recommendation 9: 1x16QAM with Sequential Decoding

The $\nu = 16$, 1x16PSK trellis code with a spectral efficiency of 3.0 bits/signal in this recommendation is constructed for use with sequential decoding [9]. Following the notation of [5], the parity check coefficients of this code in octal form are $h^0 = 242123$, $h^1 = 165172$, and $h^2 = 064140$.

This constraint length 16, 1x16QAM trellis code with a decoder speed factor of $\mu = 4$ and a buffer size of 32K signals achieves a real coding gain of 5.44dB and is 180° rotationally invariant. The performance of this code is shown is shown in Figure 15 compared to an uncoded 16QAM system with the same spectral efficiency.

4.2.4 Recommendation 10: 1x16QAM with Viterbi Decoding

For Viterbi decoding, a nonlinear $\nu = 6$, 1x16QAM trellis code with a spectral efficiency of 3.0 bits/signal is recommended. Following the notation of [5], the parity check polynomials of this code in octal form are $h^0 = 103$, $h^1 = 014$, and $h^2 = 020$. A detailed encoder diagram for this code showing the nonlinear element is given in Figure 16.

This codes achieves a real coding gain of 4.15dB and is 90° rotationally invariant. The use of a nonlinear code allows full rotationally invariance to be achieved with reduced branch complexity and a smaller constellation without sacrificing coding gain. The performance of this code is shown in Figure 17 compared to an uncoded 16QAM system with the same spectral efficiency and the optimal linear $\nu = 6$, 1x16QAM trellis code.

4.3 The Shannon Graph for the Inner Codes

The bit error rate performance of the codes discussed in sections 2 through 4 is summarized in the Shannon graph of Figure 18. This figure is a plot of the SNR (in E_b/N_0) required to achieve a BER of 10^{-5} versus spectral efficiency, η. A detailed explanation of Shannon graphs can be found in Appendix B.

Each *point* on the graph represents the performance of a single code and has a label of the form (n, k, ν), where n is the total number of bits out of the encoder, k is the total number of information bits input to the encoder including uncoded bits (see the Glossary), and ν is the code constraint length, followed by the constellation and any other relevant information. For example, the point labelled "(3, 2, 17) 8PSK" in Figure 18 is the code of Recommendation 1 and it requires $E_b/N_0 = 4.94$dB to achieve a BER of 10^{-5}.

The real coding gain at a BER of 10^{-5} of any code in the figure is also easily determined by comparing its SNR value to the SNR value of the uncoded system with the same spectral efficiency. For example, from Figure 18 it is seen that uncoded QPSK requires a SNR of $E_b/N_0 = 9.80$dB (simulation) to achieve a BER of 10^{-5} and thus the real coding gain at 10^{-5} of the (3, 2, 17) 8PSK code is 4.86dB.

The four curves shown in Figure 18 represent the theoretical performance limits on any communications system. A point on a curve represents the minimum required SNR for reliable communication at a particular spectral efficiency regardless of complexity. Thus, from the curve labelled "8PSK Bound", a minimum of 2.8dB is required to achieve reliable communication using 8PSK with $\eta = 2.0$. The code of Recommendation 1 requires 2.14dB more SNR than the theoretical minimum.

5 Compatiblity with the Outer Code

In this section, the interleaver requirements and performance of the recommended inner codes with the NASA standard (255, 223) Reed-Solomon outer code are discussed. For the concatenated coding system of Figure 1 using a t−error correcting (N, K) RS code over $GF(2^b)$, the overall BER performance is well approximated by

$$p_b = \frac{2t+1}{N} \sum_{i=t+1}^{N} \binom{N}{i} p_s^i (1-p_s)^{N-i}, \tag{1}$$

where p_s is the b−bit *symbol error rate* (SER) out of the inner decoder and ideal interleaving is assumed. The standard method for determining the performance of a concatenated coding system of this type is to find p_s for a particular inner code by computer simulation and then to evaluate (1).

5.1 Sequential Decoding

Recommendation 1 employs a $\nu = 17$, 1x8PSK trellis code with a spectral efficiency of 2 bits/signal as the inner code and the (255,223) Reed-Solomon code as the outer code. Simulation results for the 8-bit SER of this inner code are shown in Figure 19. Under normal channel conditions, the inner code operates at a BER of 10^{-5} with $E_b/N_0 = 4.94$dB. At this SNR, the 8-bit SER is $p_s = 5.0 \times 10^{-5}$ and, using (1), the BER of the concatenated coding system is $p_b = 6.48 \times 10^{-49}$!

For this code, the bits between the inner and outer code are interleaved as follows. First, a block of 223×8 information bits is divided into 223 symbols of 8 bits each, and each 8-bit symbol is regarded as an element of the Galois field $GF(2^8)$. The 223 symbols are the input to the outer RS encoder and the output is a 255 symbol codeword. The 255 symbols are then temporarily stored in an 8×255 interleaver array as shown in Figure 20. The array of Figure 20 is called an interleaver of depth 8. After 8 codewords have been stored in the interleaver, the bits are then read out column by column and encoded by the inner trellis encoder.

For sequential decoding with the modified Fano algorithm, the input to the inner trellis encoder is divided into a sequence of finite blocks called frames. With a frame size of 2×255 bits, each interleaver array consists of 32 frames with 255 8PSK signals per frame. The output of the inner trellis encoder is transmitted over the channel and decoded frame by frame. The output of the inner decoder is then reloaded into an interleaver array at the receiver.

The receiver interleaver array is then read row by row by the Reed-Solomon decoder with each row corresponding to a 255 symbol RS codeword. If one or two of the 32 frames in a single interleaver array fail, *at most* 16 symbols in any single RS codeword will be in error. Since the (255,223) outer RS code can correct up to 16 symbol errors in a single codeword, the output of the outer decoder will be correct in this case.

5.2 Viterbi Decoding

In recommendation 2, a $\nu = 6$, 4x8PSK trellis code with a spectral efficiency of 2 bits/signal is the inner code and the (255,223) Reed-Solomon code is the outer code. Simulation results for the 8-bit SER of this inner code are shown in Figure 21. The inner code achieves a BER of 10^{-5} with $E_b/N_0 = 5.95$dB. At this SNR, the 8-bit SER is $p_s = 2.38 \times 10^{-5}$ and, using (1), the BER of the concatenated coding system is $p_b = 2.16 \times 10^{-54}$!

The required interleaving depth for Viterbi decoding of the inner code can be estimated as follows. For a trellis code with constraint length ν, the shortest path with minimum distance is typically of length $\nu + 1$ branches, or 1 branch if the minimum distance occurs along a parallel transition. The longest path with minimum distance is typically on the order of 5ν branches. For high SNR's, where the minimum distance path is the most likely error

event, the inner decoder will output bursts of errors of length 1 branch to 5ν branches.

For the proposed $\nu = 6$, 4x8PSK code, this would result in bursts of errors up to 30 branches long. Since each branch is labelled with 8 information bits, this results in a maximum burst of length 240 bits. For the depth 4 interleaver of Figure 22, this would result in a burst of *at most* 31 8-bit RS symbols in error with a maximum of 8 symbols per RS code block. Up to two such bursts could occur in a single interleaver array without exceeding the error correcting capability of the (255, 223) RS code. Assuming that 50 percent of the 240 bits in a burst are actually in error, 3 bursts occurring in a single interleaver array would correspond to a bit error rate of approximately 4.41×10^{-2} out of the inner decoder. In practice, the inner decoder is operated at a much lower BER and an interleaving depth of 4 should suffice.

5.3 Shannon Graphs for the Concatenated System

Under normal channel conditions, the inner code is expected to operate at a BER of 10^{-5} without the outer code. The outer code is then used to provide virtually error free performance, i.e., $p_b \approx 10^{-50}$. This indeed is the situation with the codes of Recommendations 1 and 2.

In addition, under adverse channel conditions the inner codes by themselves may fall short of the desired BER performance. In this case, the outer (255, 223) RS code is used to bolster the performance of the inner code. The performance of the concatenated coding system in this mode of operation is examined by determining the required p_s out of the inner code to achieve a particular overall p_b using equation (1). The various inner codes can then be compared based on the SNR necessary for each code to reach the required p_s.

With the (255, 223) RS outer code, the inner code must operate with a SER of $p_s = 1.00675 \times 10^{-2}$ for the overall BER to be $p_b = 10^{-10}$. The performance of the 10 recommended inner codes is shown in the Shannon graph of Figure 23. In this graph, the x-axis is in terms of E_b/N_0, where E_b is the average energy per information bit for the inner encoder. (Since the input to the inner encoder has already been encoded by the outer RS encoder, this E_b is lightly less than the overall average energy per information bit.) Similarly, the y-axis is the spectral efficiency of the inner code and does not reflect the reduction in spectral efficiency due to the RS encoder. For example, the $\nu = 17$, 1x8PSK code of Recommendation 1 with $\eta = 2.0$ bits/signal achieves $p_s = 1.00675 \times 10^{-2}$ with $E_b/N_0 = 4.0$dB and hence is plotted at (2.0,4.0) in Figure 23.

Shannon graphs for an overall $p_b = 10^{-20}$ and $p_b = 10^{-30}$ are shown in Figure 24 and Figure 25, respectively. With the (255, 223) RS outer code, the inner code must operate with $p_s = 2.34575 \times 10^{-3}$ for the overall BER to be $p_b = 10^{-20}$ and $p_s = 5.9152 \times 10^{-4}$ for the overall BER to be $p_b = 10^{-30}$. The points in these figures are plotted in the same manner as those in Figure

23.

Close examination of Figures 23 to 25 reveals an interesting aspect to those inner codes that use sequential decoding, namely recommendations 1, 3, and 9. The performance of these large constraint length codes is not affected by the outer code to the same degree as the short constraint length codes with Viterbi decoding. This is consistent with the earlier observation that the performance of large constraint codes with sequential decoding approaches R_0, a fundamental performance limit of digital communications systems. Since there is little margin for improvement, it is not surprising that the outer RS code contributes less in this case. If it were practical to implement Viterbi decoders for large constraint length codes, a similar effect would be observed.

6 Conclusion

The codes discussed in this report represent state of the art consstruction of bandwidth efficient trellis codes for use with sequential and Viterbi decoding. All of the recommendations offer increased spectral efficiency and better performance compared to the current CCSDS inner code standard.

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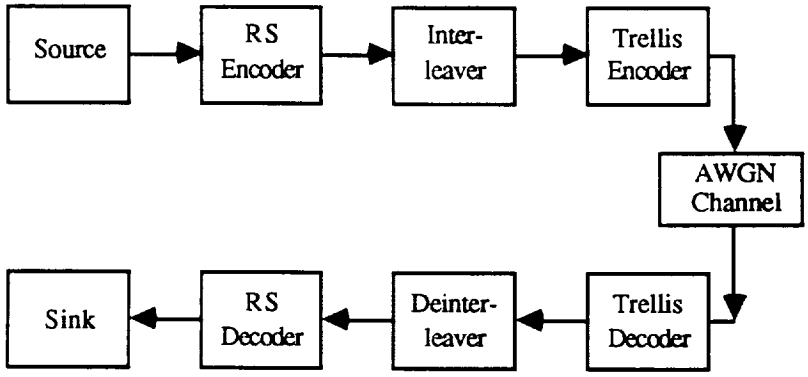


Figure 1: Typical Concatenated Coding System

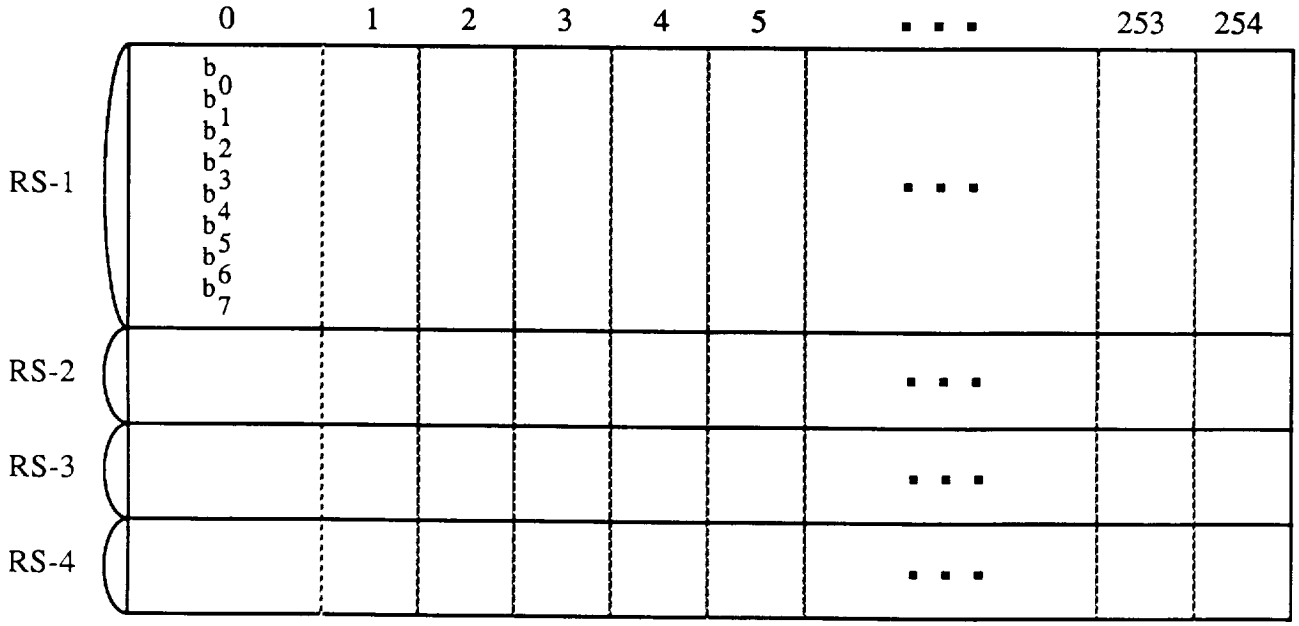


Figure 2: Depth Four Symbol Interleaver

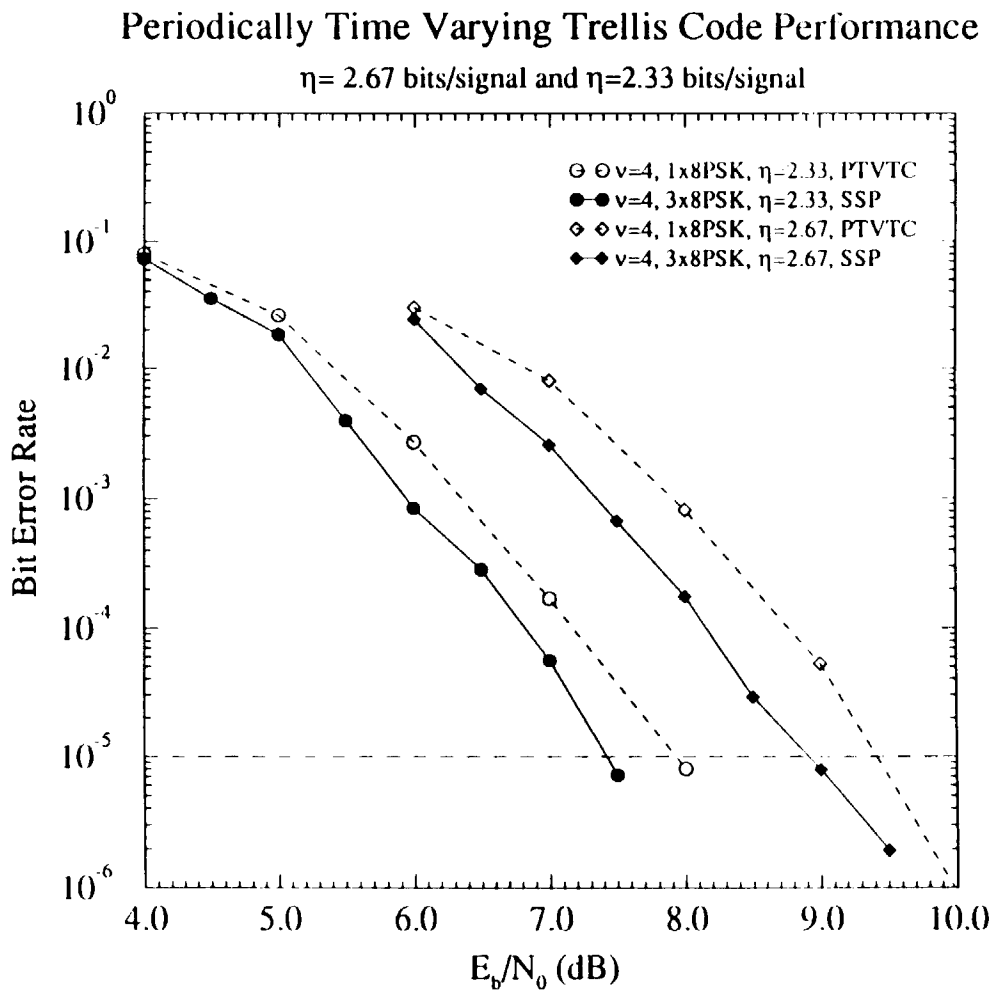

Periodically Time Varying Trellis Code Performance
η= 2.67 bits/signal and η=2.33 bits/signal
v=4, 1x8PSK, η=2.33, PTVTC
v=4, 3x8PSK, η=2.33, SSP
v=4, 1x8PSK, η=2.67, PTVTC
v=4, 3x8PSK, η=2.67, SSP
Bit Error Rate
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
4.0
5.0
6.0
7.0
8.0
9.0
10.0
E_b/N_0 (dB)

Figure 3

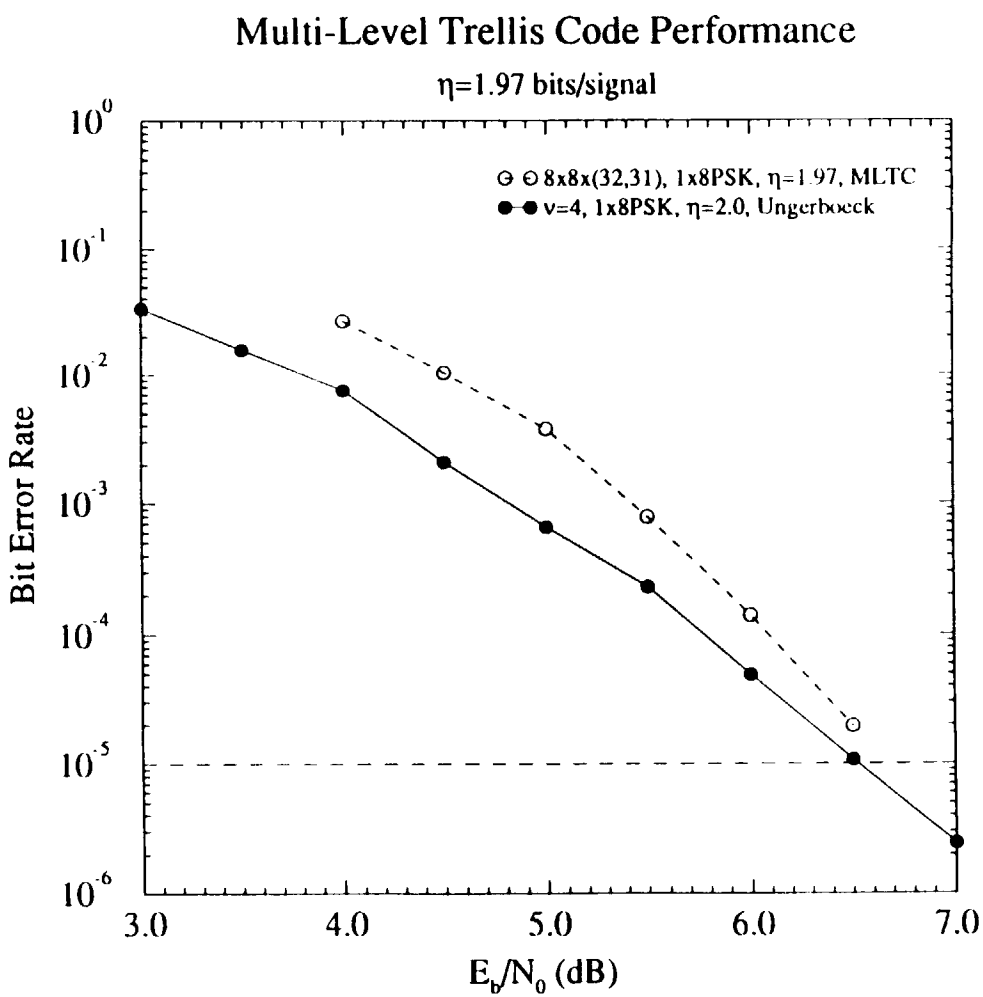

Multi-Level Trellis Code Performance
η=1.97 bits/signal
8x8x(32,31), 1x8PSK, η=1.97, MLTC
ν=4, 1x8PSK, η=2.0, Ungerboeck
Bit Error Rate
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
3.0
4.0
5.0
6.0
7.0
E_b/N_0 (dB)

Figure 4

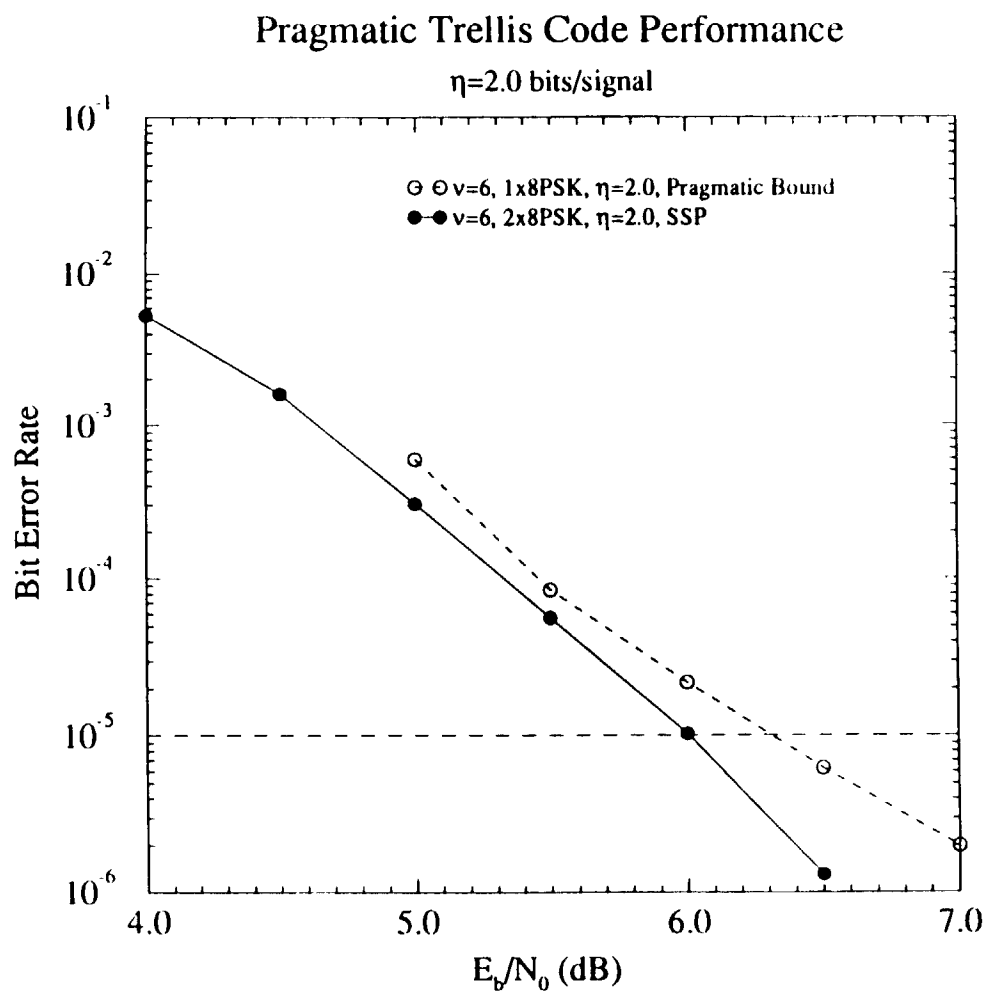

Pragmatic Trellis Code Performance
η=2.0 bits/signal
ν=6, 1x8PSK, η=2.0, Pragmatic Bound
ν=6, 2x8PSK, η=2.0, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
E_b/N_0 (dB)

Figure 5

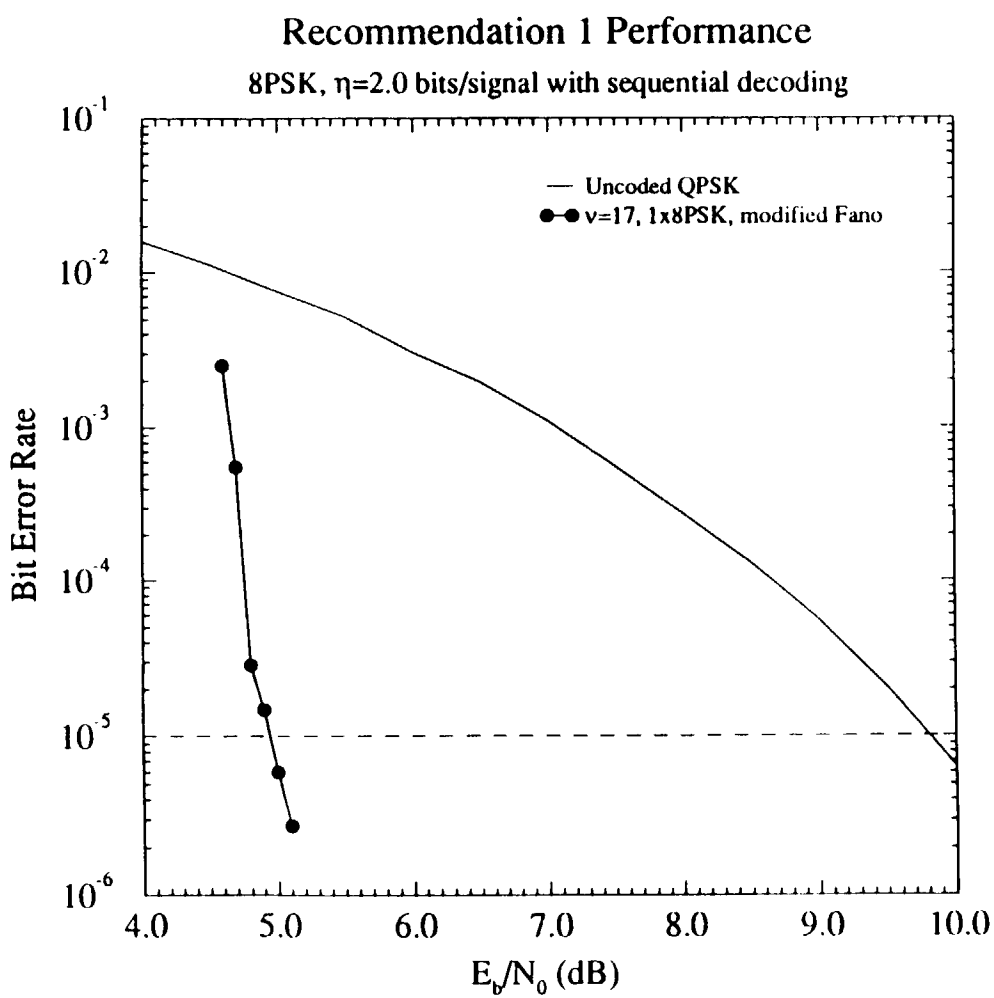

Recommendation 1 Performance
8PSK, η=2.0 bits/signal with sequential decoding
Uncoded QPSK
ν=17, 1x8PSK, modified Fano
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
8.0
9.0
10.0
E_b/N_0 (dB)

Figure 6

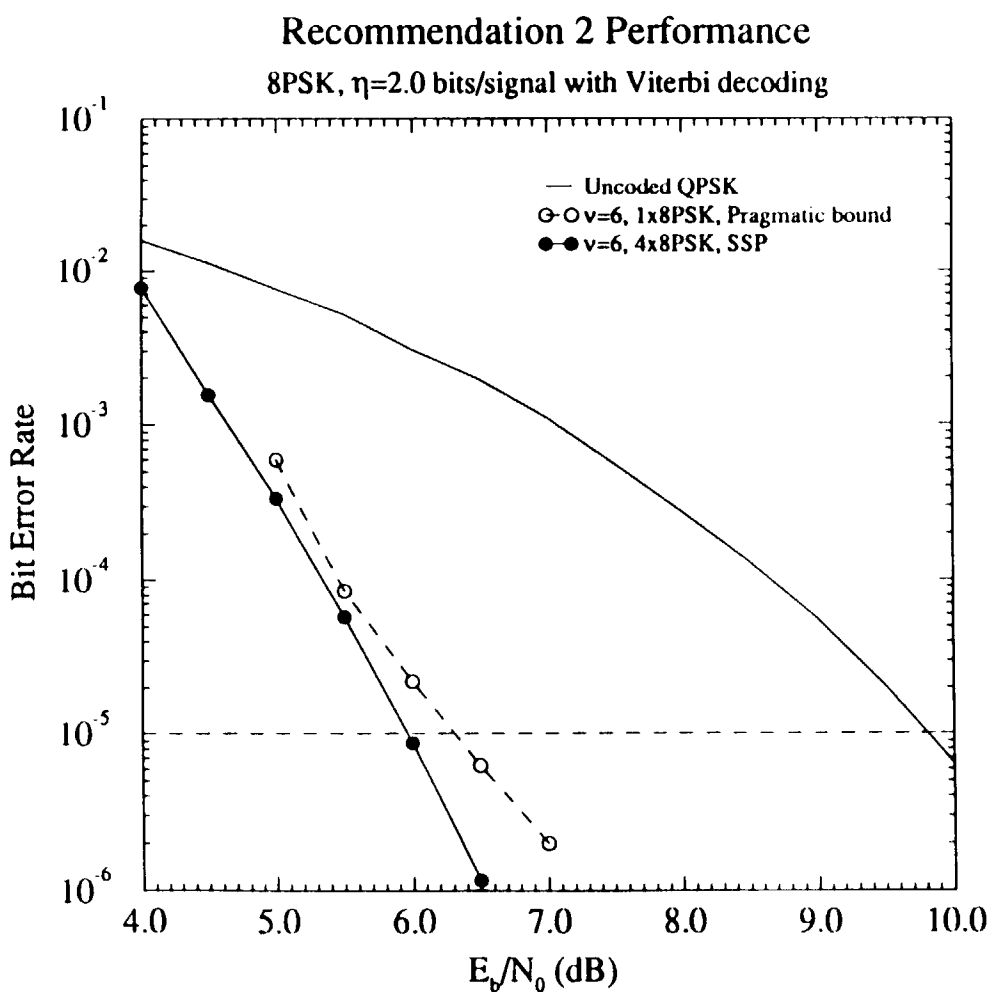

Recommendation 2 Performance
8PSK, η=2.0 bits/signal with Viterbi decoding
Uncoded QPSK
v=6, 1x8PSK, Pragmatic bound
v=6, 4x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
8.0
9.0
10.0
E_b/N_0 (dB)

Figure 7

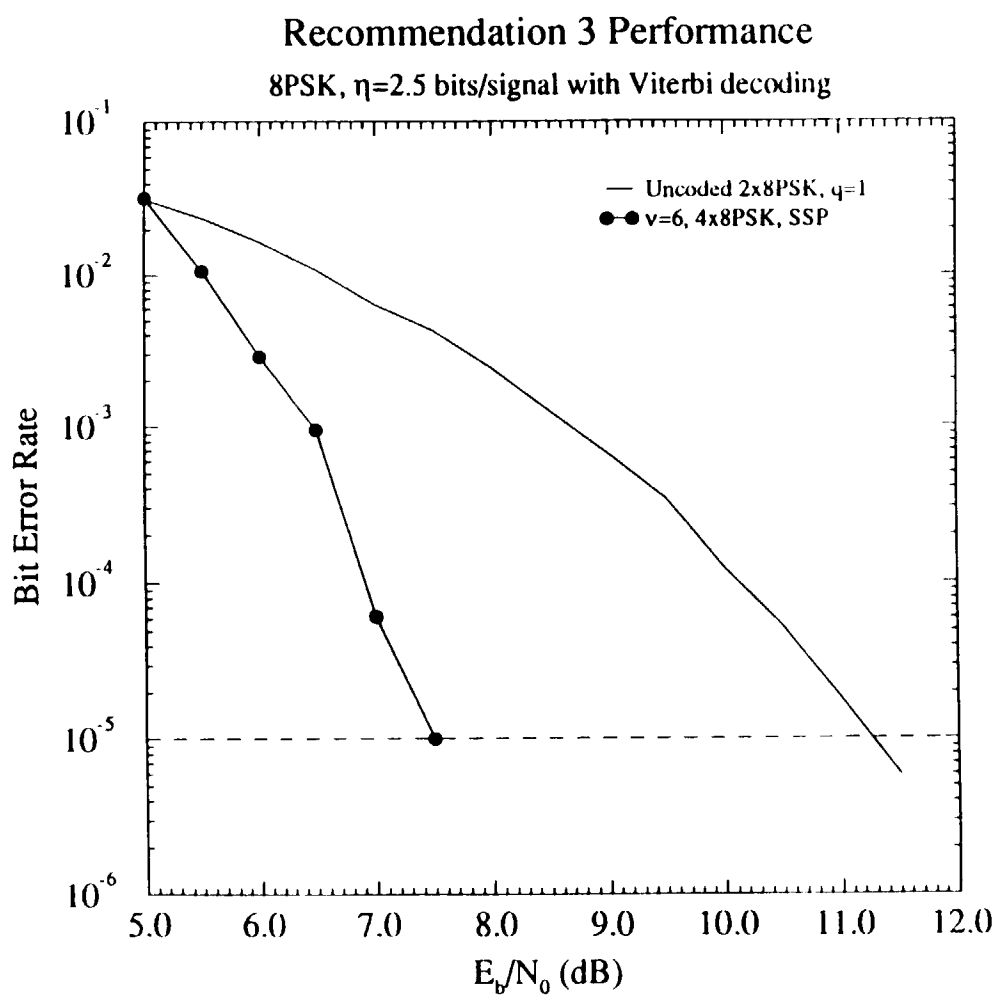

Recommendation 3 Performance
8PSK, η=2.5 bits/signal with Viterbi decoding
Uncoded 2x8PSK, q=1
v=6, 4x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
E_b/N_0 (dB)

Figure 8

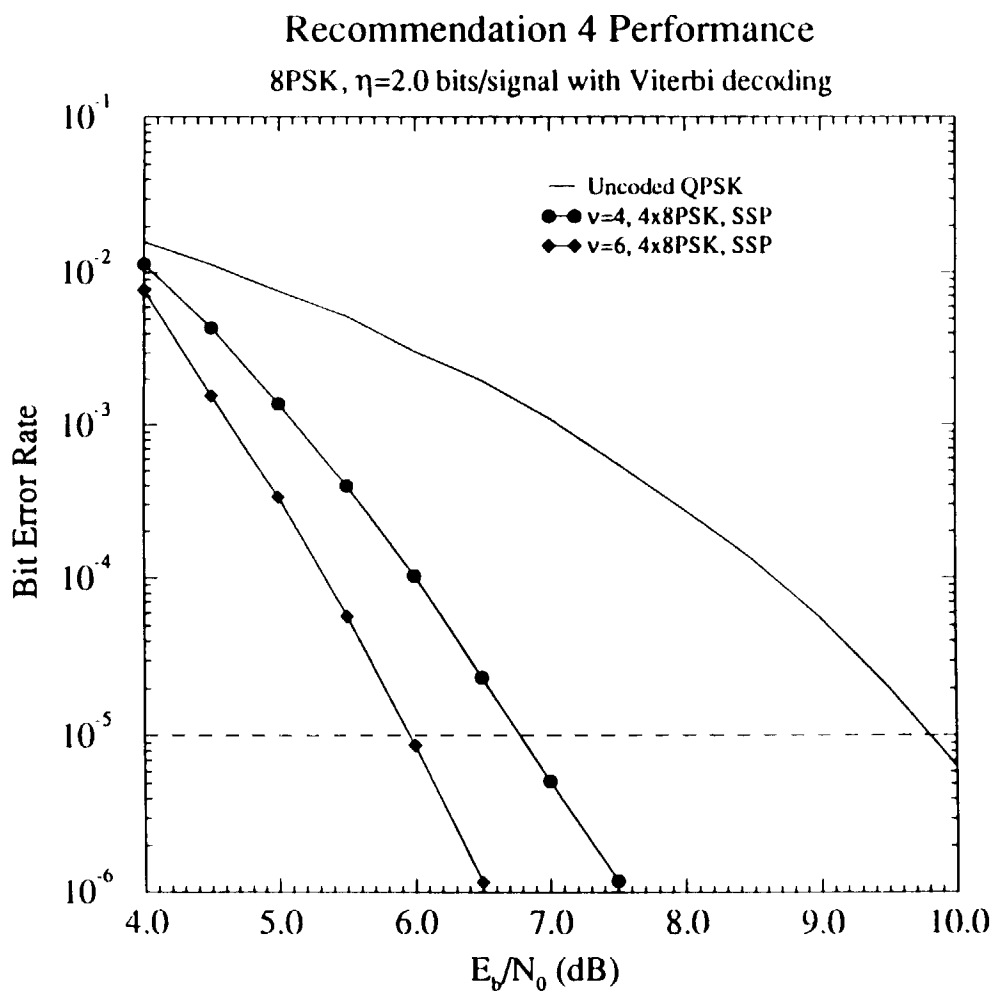

Recommendation 4 Performance
8PSK, η=2.0 bits/signal with Viterbi decoding
Uncoded QPSK
v=4, 4x8PSK, SSP
v=6, 4x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
8.0
9.0
10.0
E_b/N_0 (dB)

Figure 9

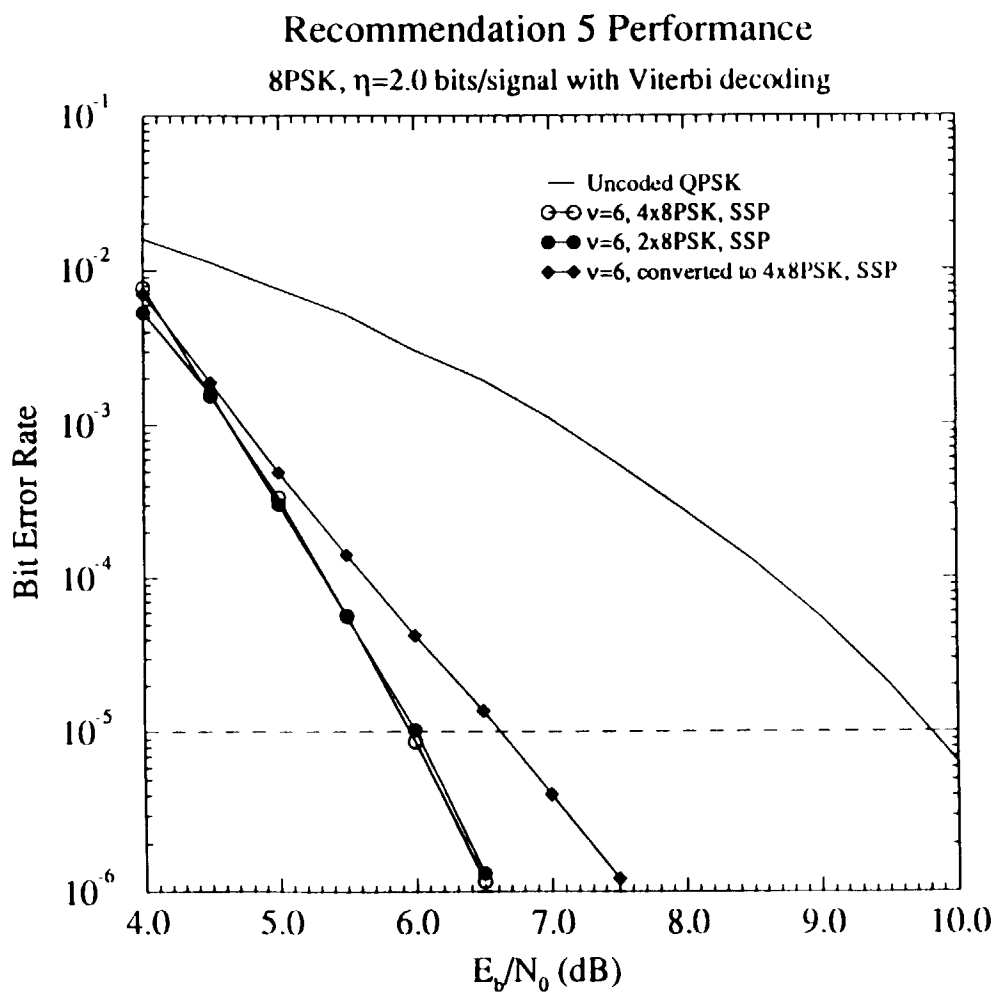

Recommendation 5 Performance
8PSK, η=2.0 bits/signal with Viterbi decoding
Uncoded QPSK
v=6, 4x8PSK, SSP
v=6, 2x8PSK, SSP
v=6, converted to 4x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Eb/N0 (dB)

Figure 10

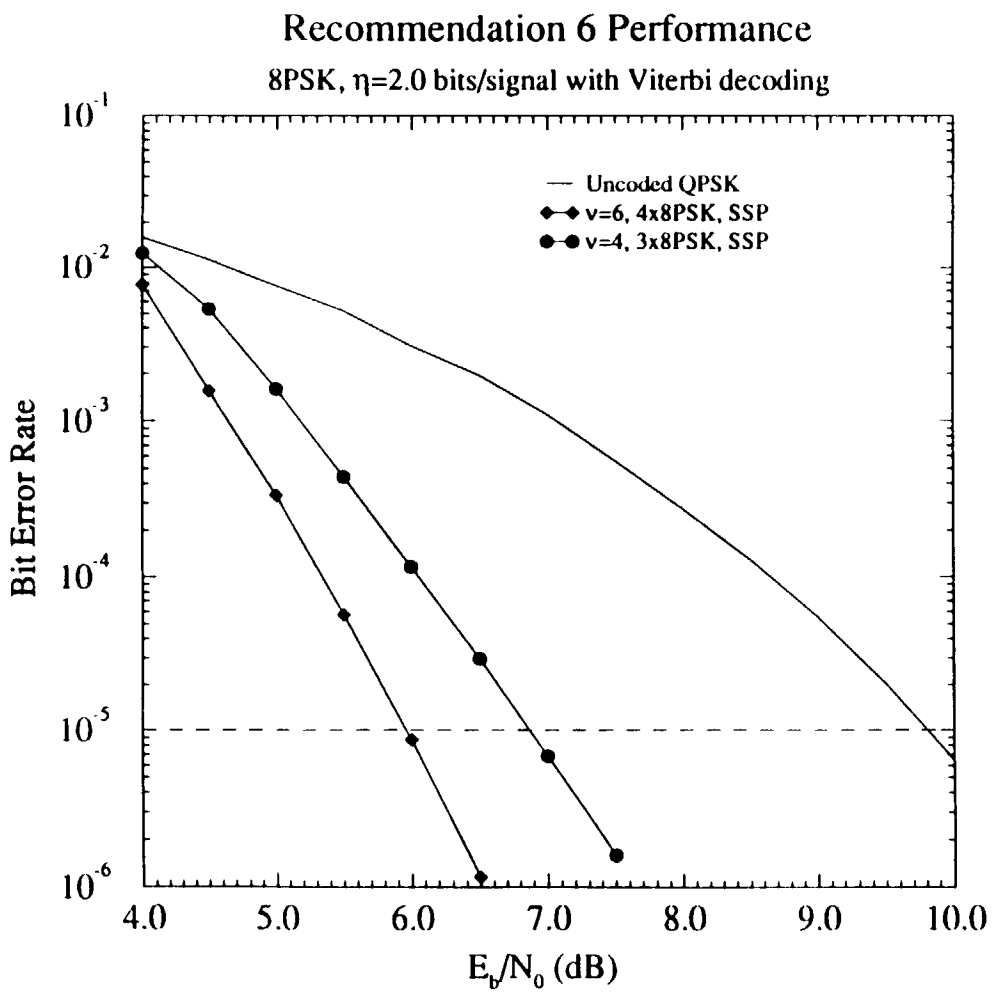

Recommendation 6 Performance
8PSK, η=2.0 bits/signal with Viterbi decoding
Uncoded QPSK
v=6, 4x8PSK, SSP
v=4, 3x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
8.0
9.0
10.0
E_b/N_0 (dB)

Figure 11

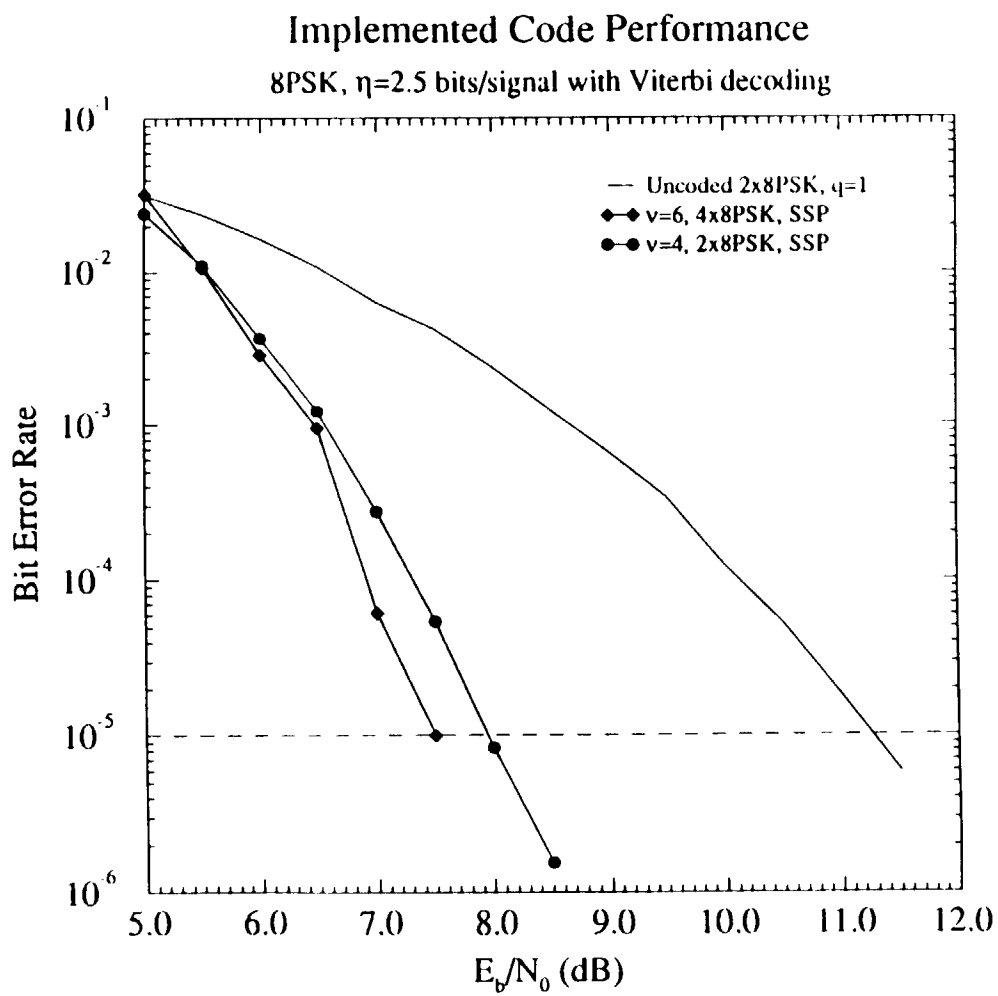

Implemented Code Performance
8PSK, η=2.5 bits/signal with Viterbi decoding
Uncoded 2x8PSK, q=1
v=6, 4x8PSK, SSP
v=4, 2x8PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
Eb/N0 (dB)

Figure 12

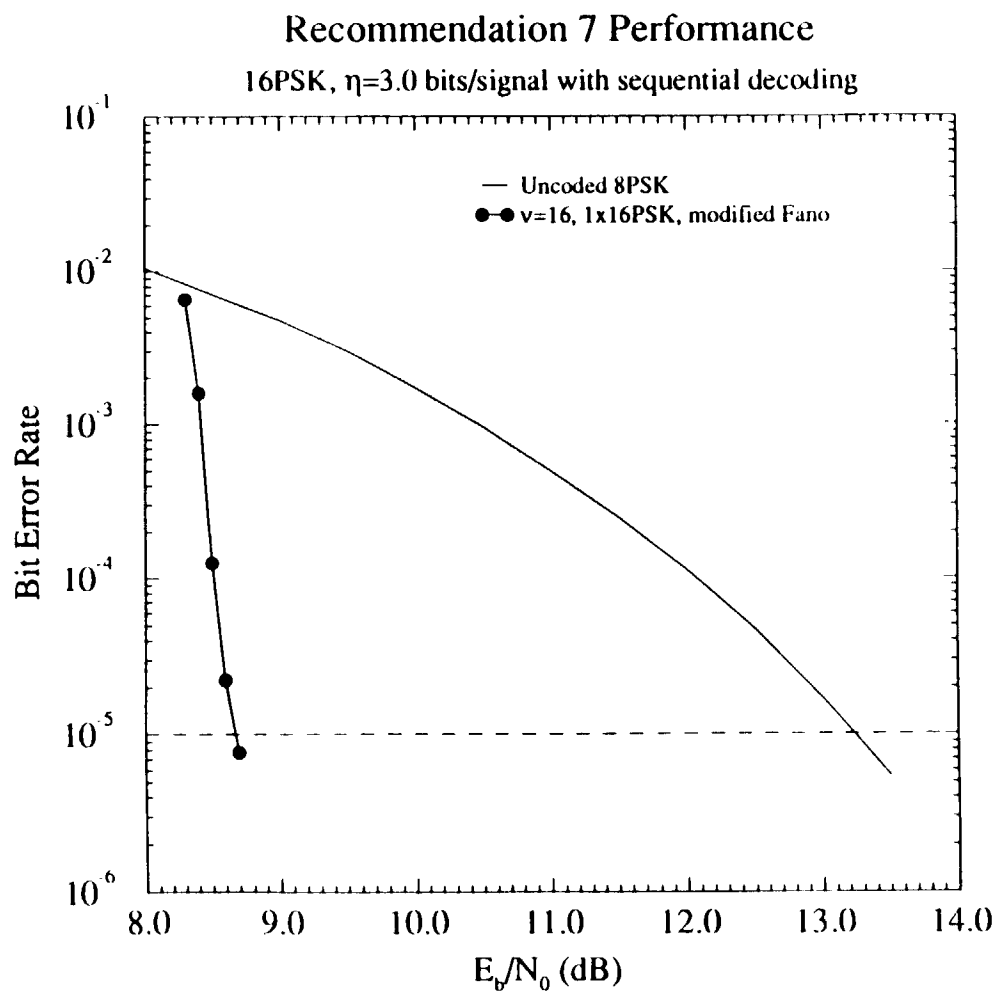

Recommendation 7 Performance
16PSK, η=3.0 bits/signal with sequential decoding
Uncoded 8PSK
v=16, 1x16PSK, modified Fano
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
8.0
9.0
10.0
11.0
12.0
13.0
14.0
Eb/N0 (dB)

Figure 13

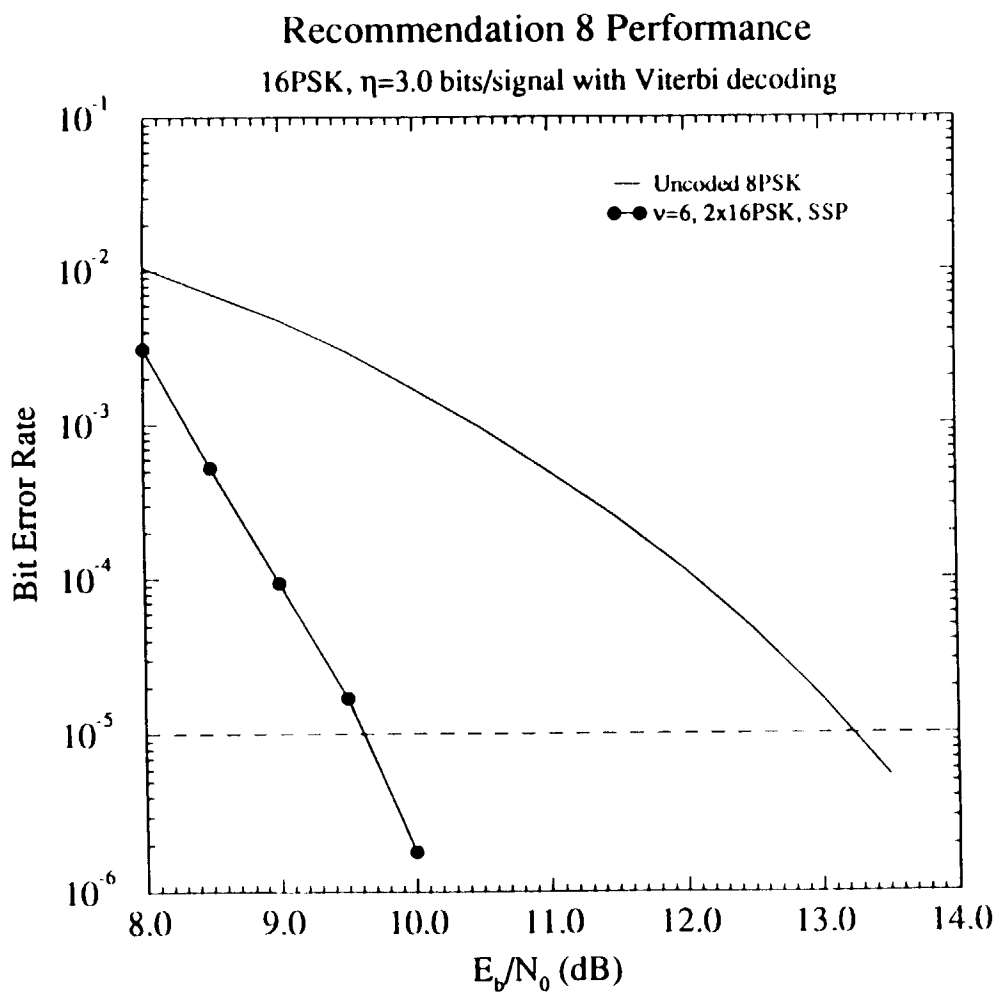

Recommendation 8 Performance
16PSK, η=3.0 bits/signal with Viterbi decoding
Uncoded 8PSK
ν=6, 2x16PSK, SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
8.0
9.0
10.0
11.0
12.0
13.0
14.0
E_b/N_0 (dB)

Figure 14

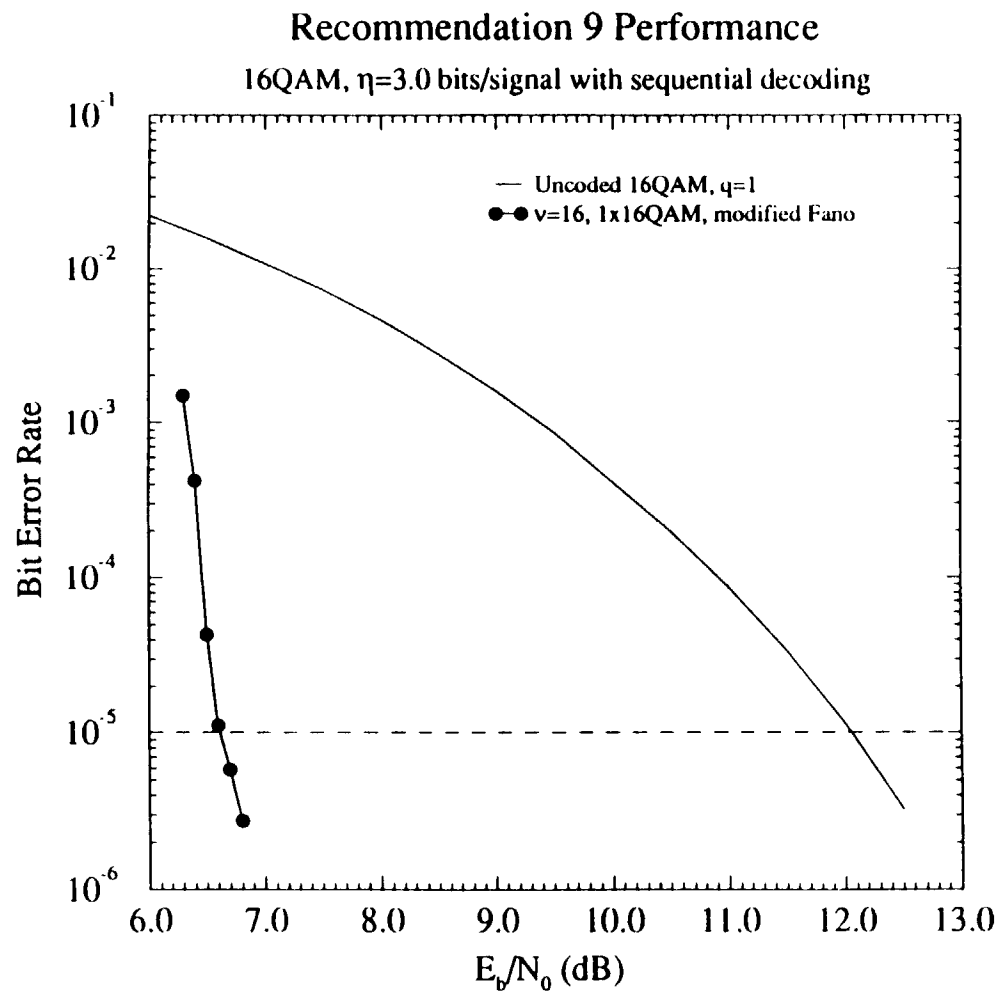

Recommendation 9 Performance
16QAM, η=3.0 bits/signal with sequential decoding
Uncoded 16QAM, q=1
v=16, 1x16QAM, modified Fano
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
Eb/N0 (dB)

Figure 15

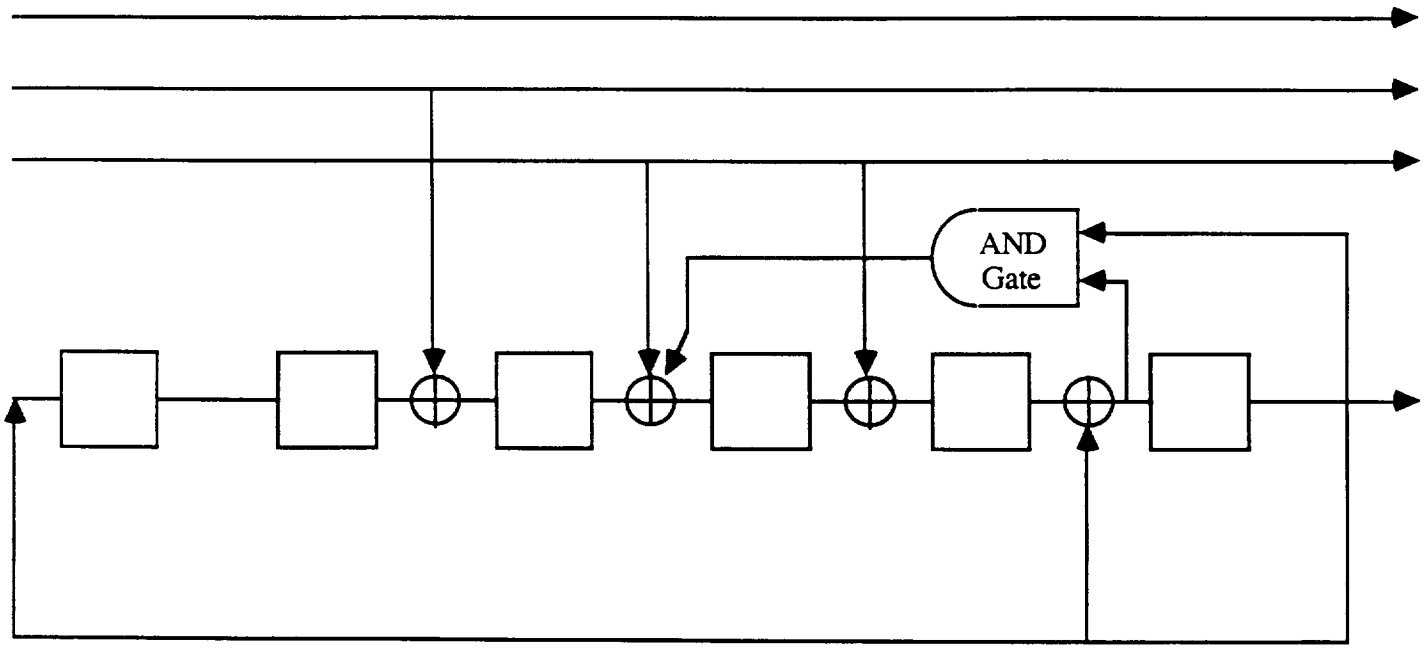


Figure 16: Nonlinear Encoder for Recommendation 10.

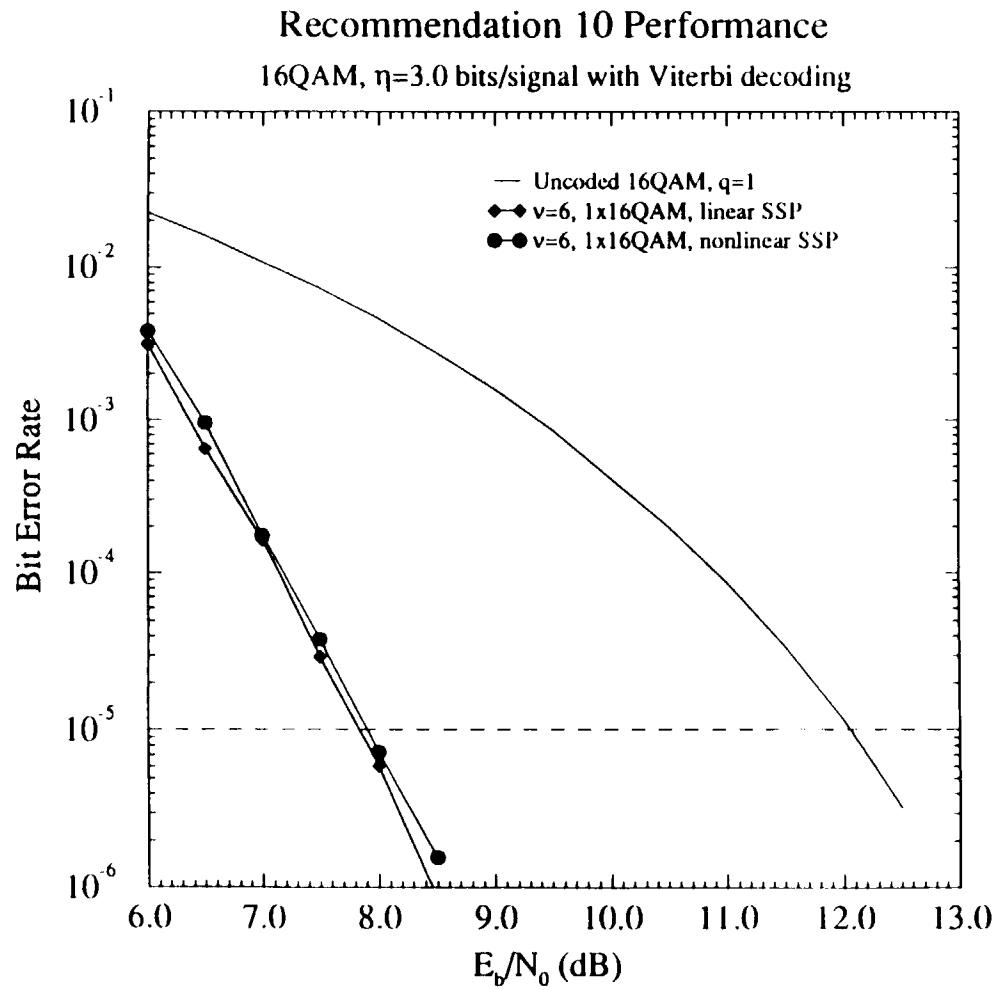

Recommendation 10 Performance
16QAM, η=3.0 bits/signal with Viterbi decoding
Uncoded 16QAM, q=1
ν=6, 1x16QAM, linear SSP
ν=6, 1x16QAM, nonlinear SSP
Bit Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
6.0
7.0
8.0
9.0
10.0
11.0
12.0
13.0
E_b/N_0 (dB)

Figure 17

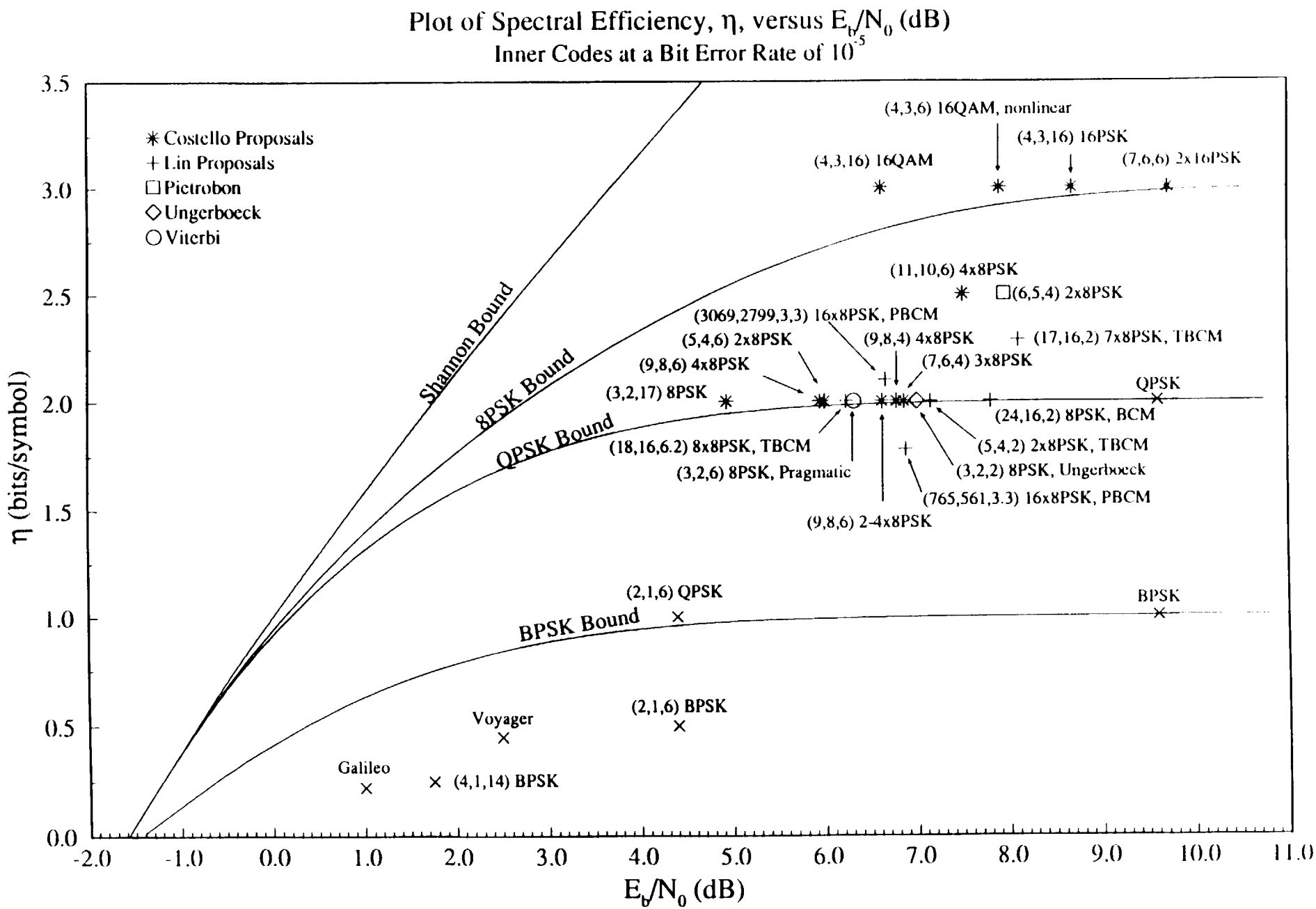

Plot of Spectral Efficiency, η, versus E_b/N_0 (dB)
Inner Codes at a Bit Error Rate of 10^{-5}
Costello Proposals
Lin Proposals
Pietrobon
Ungerboeck
Viterbi
Shannon Bound
8PSK Bound
QPSK Bound
BPSK Bound
(4,3,6) 16QAM, nonlinear
(4,3,16) 16PSK
(4,3,16) 16QAM
(7,6,6) 2x16PSK
(11,10,6) 4x8PSK
(6,5,4) 2x8PSK
(3069,2799,3,3) 16x8PSK, PBCM
(5,4,6) 2x8PSK
(9,8,4) 4x8PSK
(17,16,2) 7x8PSK, TBCM
(9,8,6) 4x8PSK
(7,6,4) 3x8PSK
(3,2,17) 8PSK
QPSK
(24,16,2) 8PSK, BCM
(18,16,6.2) 8x8PSK, TBCM
(5,4,2) 2x8PSK, TBCM
(3,2,6) 8PSK, Pragmatic
(3,2,2) 8PSK, Ungerboeck
(765,561,3.3) 16x8PSK, PBCM
(9,8,6) 2-4x8PSK
(2,1,6) QPSK
BPSK
(2,1,6) BPSK
Voyager
Galileo
(4,1,14) BPSK
η (bits/symbol)
E_b/N_0 (dB)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

Figure 18

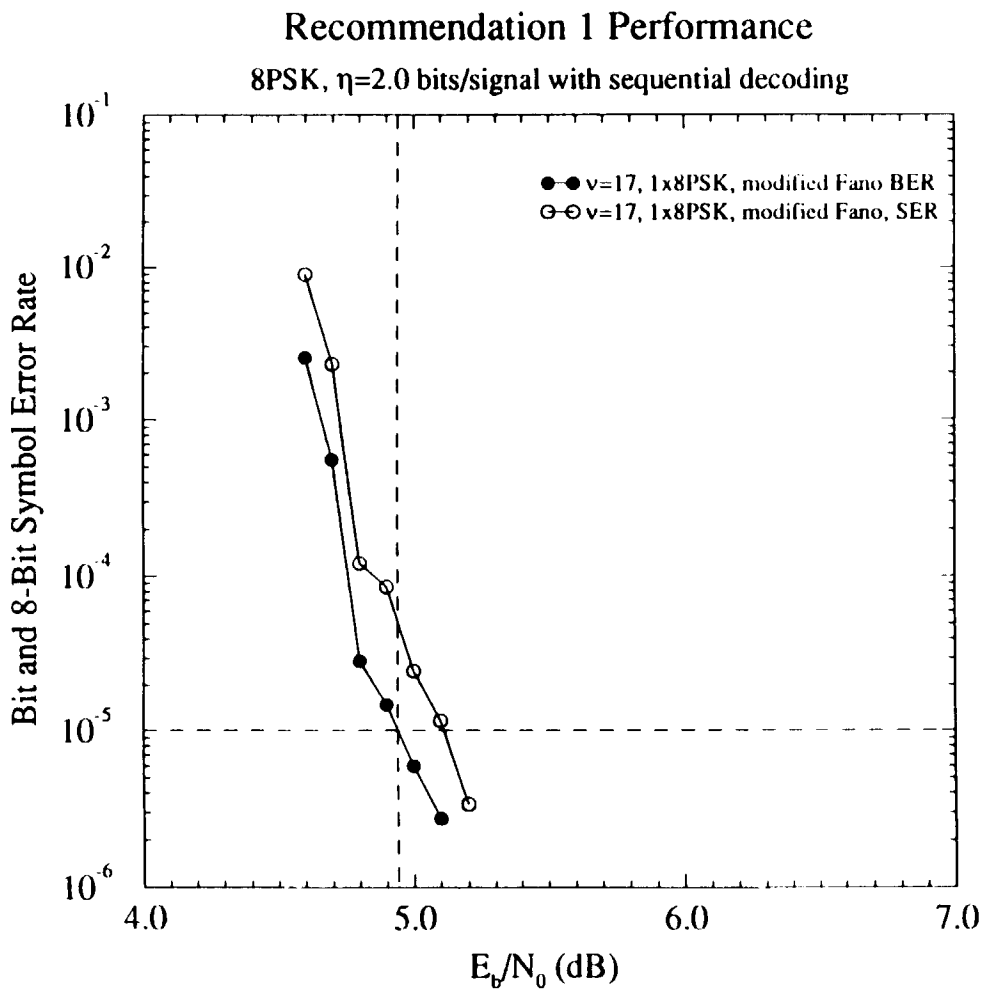

Recommendation 1 Performance
8PSK, η=2.0 bits/signal with sequential decoding
v=17, 1x8PSK, modified Fano BER
v=17, 1x8PSK, modified Fano, SER
Bit and 8-Bit Symbol Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
4.0
5.0
6.0
7.0
E_b/N_0 (dB)

Figure 19

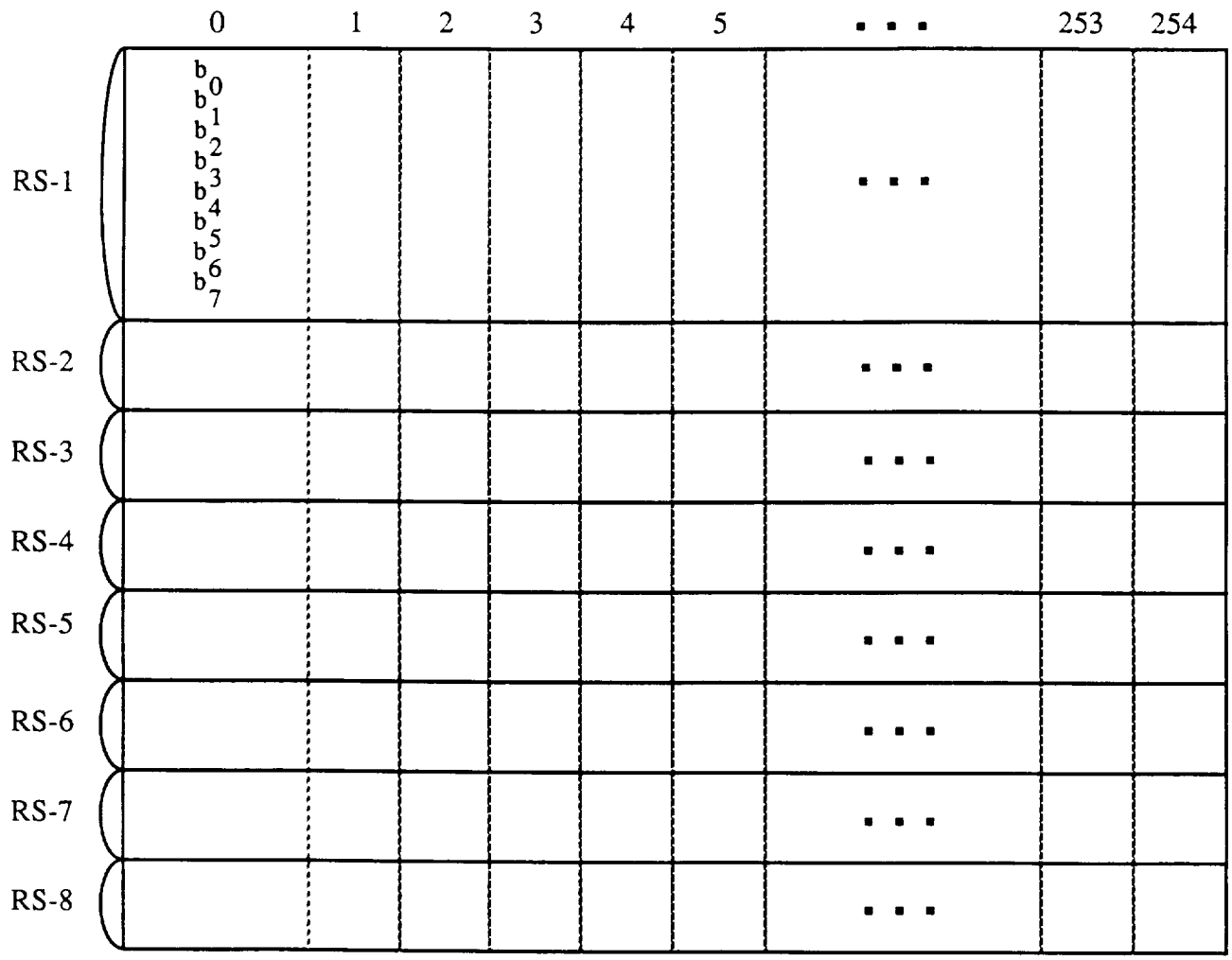


Figure 20: Depth Eight Interleaver for Recommendation 1.

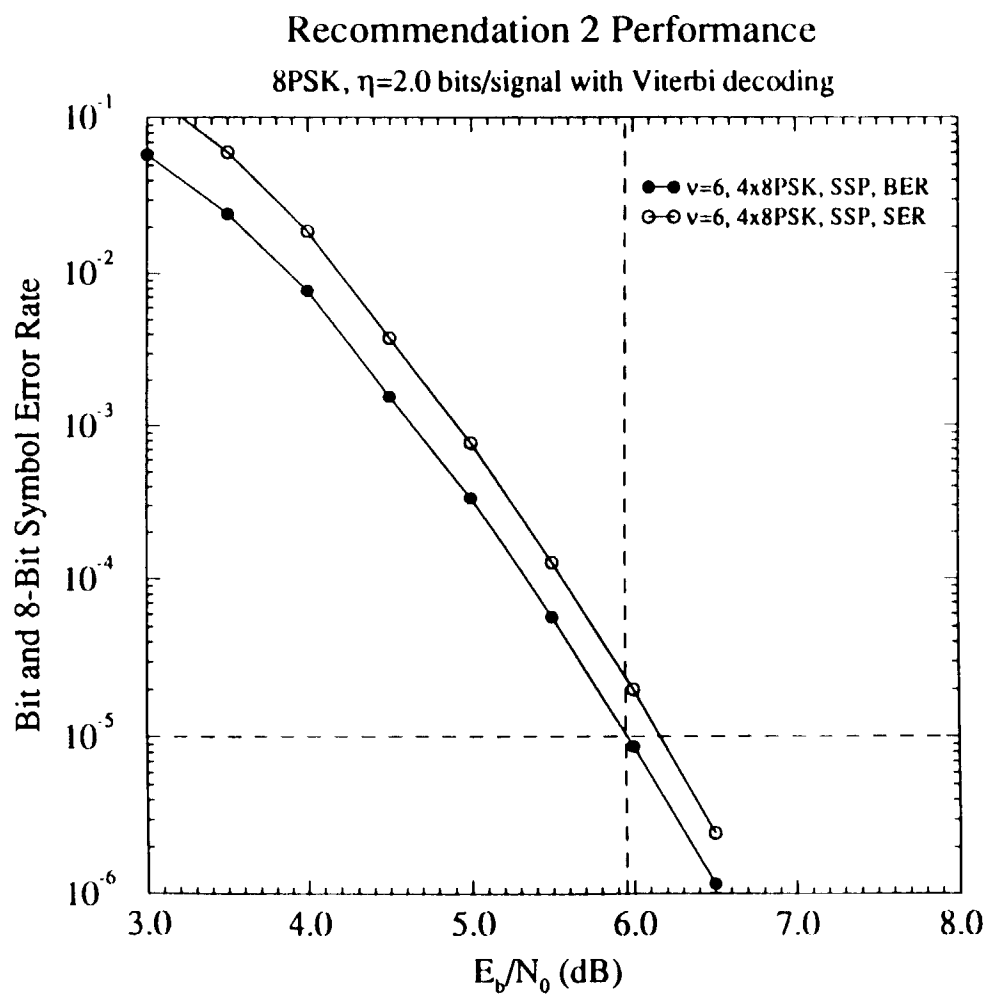

Recommendation 2 Performance
8PSK, η=2.0 bits/signal with Viterbi decoding
v=6, 4x8PSK, SSP, BER
v=6, 4x8PSK, SSP, SER
Bit and 8-Bit Symbol Error Rate
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
3.0
4.0
5.0
6.0
7.0
8.0
E_b/N_0 (dB)

Figure 21

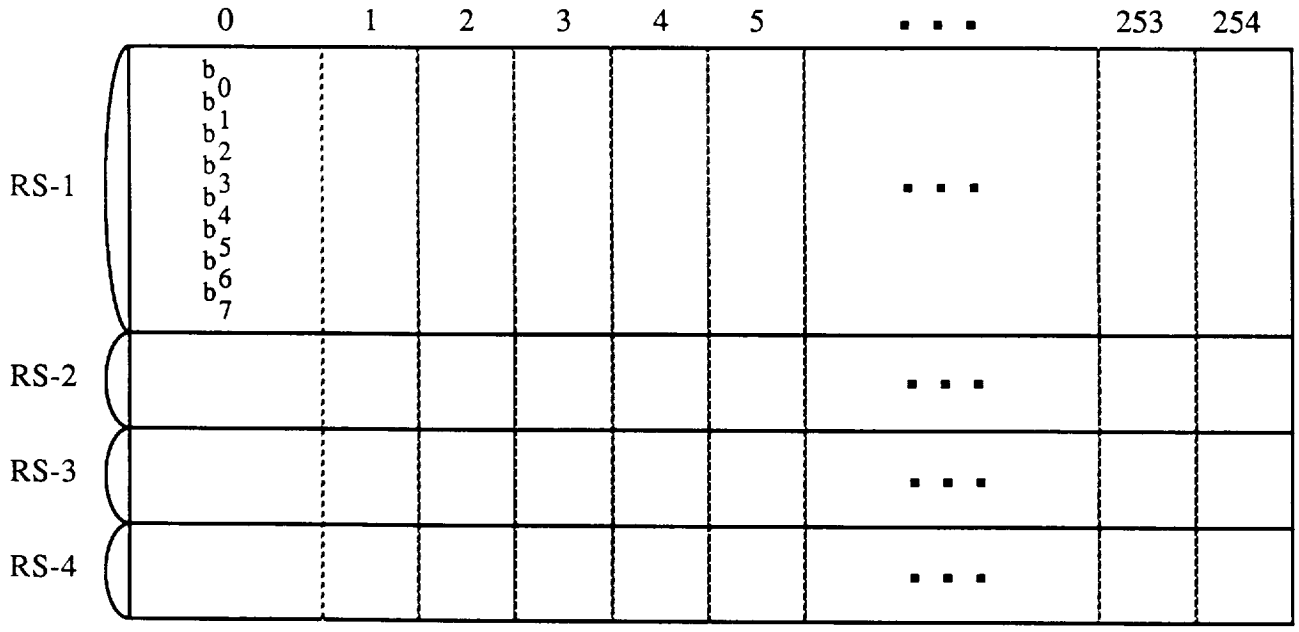


Figure 22: Depth Four Interleaver for Recommendation 2.

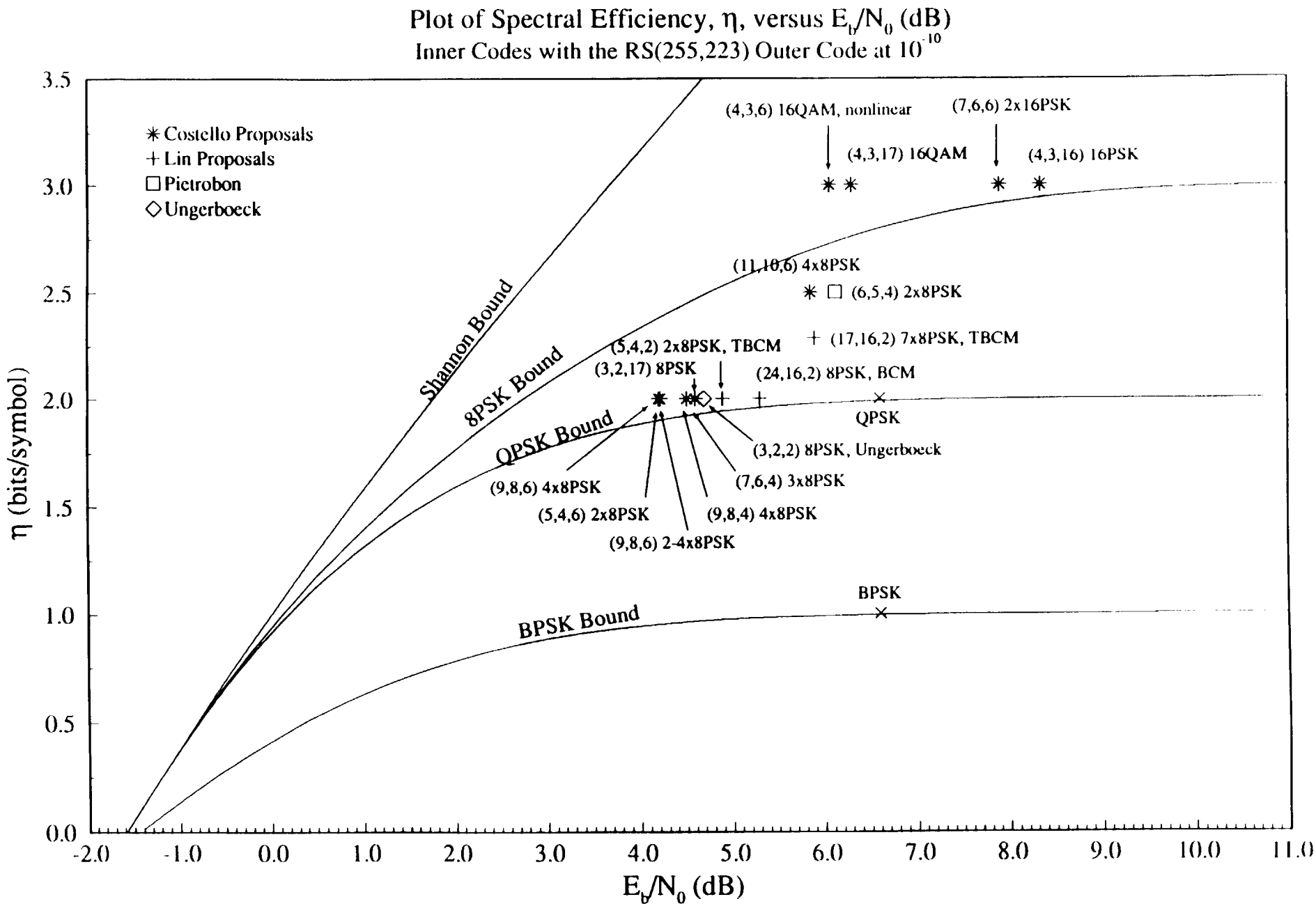

Plot of Spectral Efficiency, η, versus E_b/N_0 (dB)
Inner Codes with the RS(255,223) Outer Code at 10^{-10}
✳ Costello Proposals
+ Lin Proposals
□ Pietrobon
◇ Ungerboeck
Shannon Bound
8PSK Bound
QPSK Bound
BPSK Bound
(4,3,6) 16QAM, nonlinear
(7,6,6) 2x16PSK
(4,3,17) 16QAM
(4,3,16) 16PSK
(11,10,6) 4x8PSK
(6,5,4) 2x8PSK
(17,16,2) 7x8PSK, TBCM
(5,4,2) 2x8PSK, TBCM
(3,2,17) 8PSK
(24,16,2) 8PSK, BCM
QPSK
(3,2,2) 8PSK, Ungerboeck
(7,6,4) 3x8PSK
(9,8,6) 4x8PSK
(5,4,6) 2x8PSK
(9,8,4) 4x8PSK
(9,8,6) 2-4x8PSK
BPSK
η (bits/symbol)
E_b/N_0 (dB)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

Figure 23

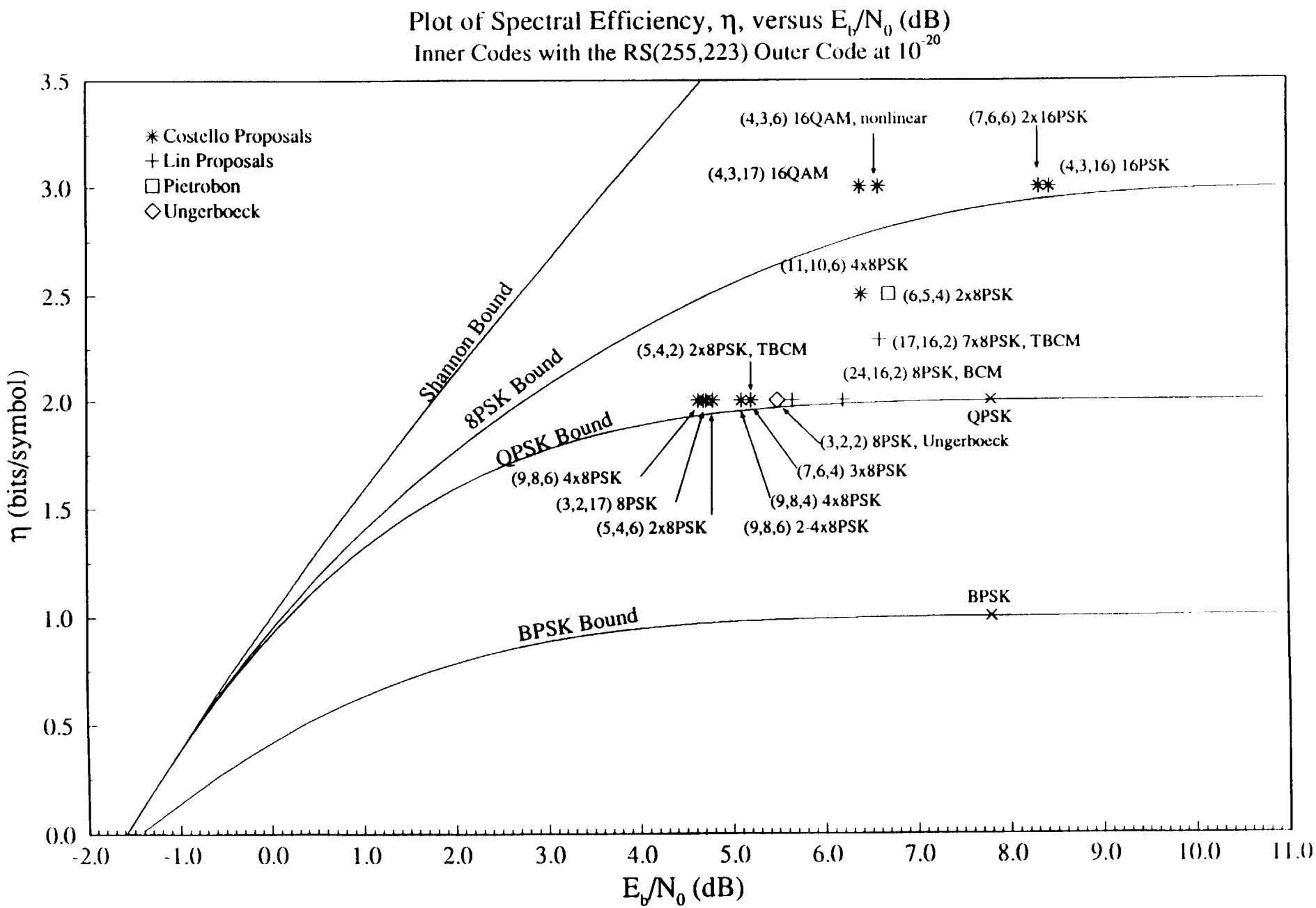

Plot of Spectral Efficiency, η, versus E_b/N_0 (dB)
Inner Codes with the RS(255,223) Outer Code at 10^{-20}
Costello Proposals
Lin Proposals
Pietrobon
Ungerboeck
Shannon Bound
8PSK Bound
QPSK Bound
BPSK Bound
(4,3,6) 16QAM, nonlinear
(7,6,6) 2x16PSK
(4,3,17) 16QAM
(4,3,16) 16PSK
(11,10,6) 4x8PSK
(6,5,4) 2x8PSK
(17,16,2) 7x8PSK, TBCM
(5,4,2) 2x8PSK, TBCM
(24,16,2) 8PSK, BCM
QPSK
(3,2,2) 8PSK, Ungerboeck
(7,6,4) 3x8PSK
(9,8,6) 4x8PSK
(3,2,17) 8PSK
(9,8,4) 4x8PSK
(5,4,6) 2x8PSK
(9,8,6) 2-4x8PSK
BPSK
η (bits/symbol)
E_b/N_0 (dB)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

Figure 24

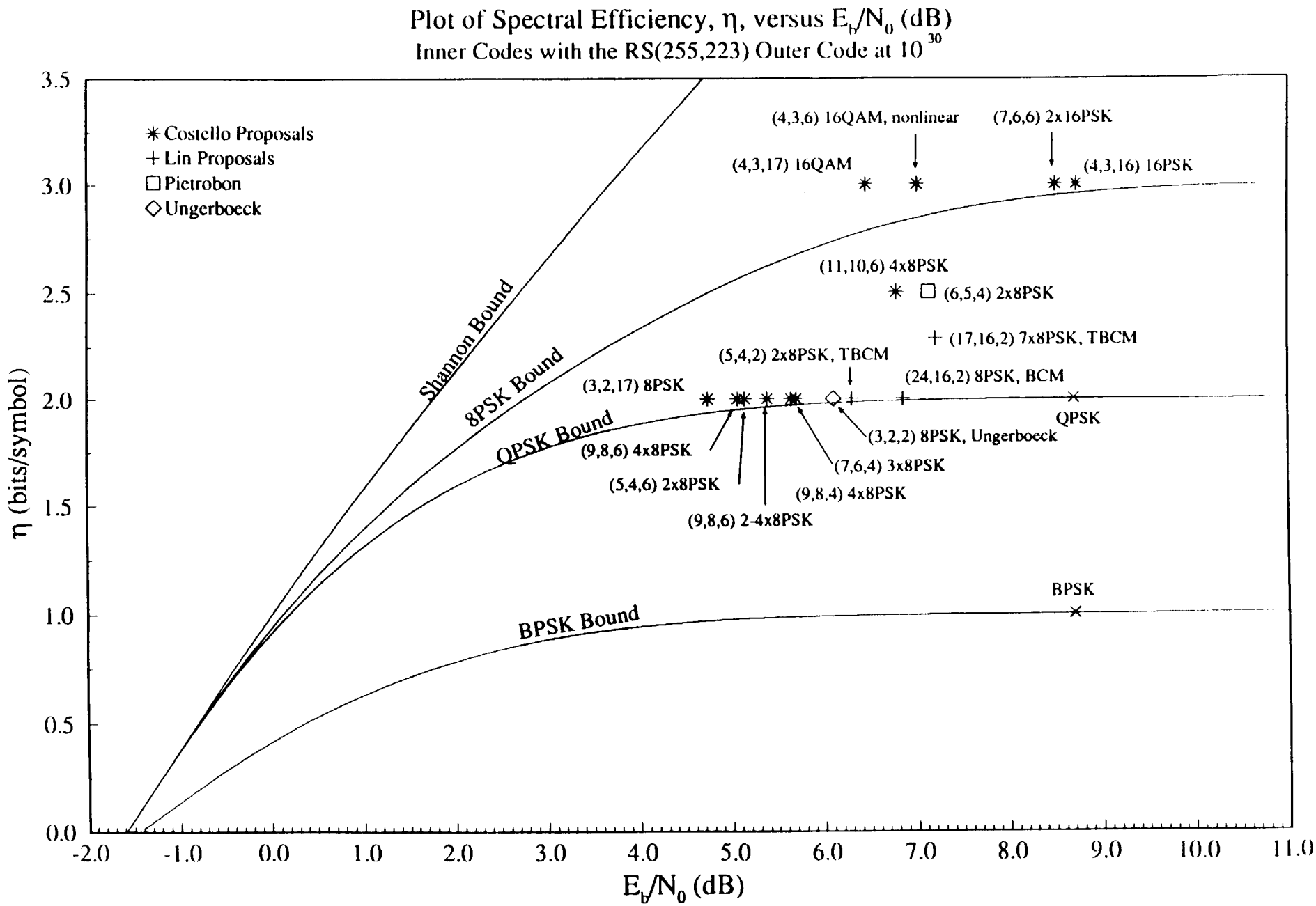

Plot of Spectral Efficiency, η, versus E_b/N_0 (dB)
Inner Codes with the RS(255,223) Outer Code at 10^{-30}
Costello Proposals
Lin Proposals
Pietrobon
Ungerboeck
Shannon Bound
8PSK Bound
QPSK Bound
BPSK Bound
(4,3,6) 16QAM, nonlinear
(7,6,6) 2x16PSK
(4,3,17) 16QAM
(4,3,16) 16PSK
(11,10,6) 4x8PSK
(6,5,4) 2x8PSK
(17,16,2) 7x8PSK, TBCM
(5,4,2) 2x8PSK, TBCM
(24,16,2) 8PSK, BCM
(3,2,17) 8PSK
QPSK
(3,2,2) 8PSK, Ungerboeck
(9,8,6) 4x8PSK
(7,6,4) 3x8PSK
(5,4,6) 2x8PSK
(9,8,4) 4x8PSK
(9,8,6) 2-4x8PSK
BPSK
η (bits/symbol)
E_b/N_0 (dB)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0

Figure 25

A Recommendation Summaries

This appendix contains 2 to 3 page summaries of the 10 recommendations for the inner code discussed in the main body of this report. The summaries appear in the following order:

1. $\nu = 17$, η=2.0 bits/signal 1x8PSK with sequential decoding.
2. $\nu = 6$, η=2.0 bits/signal 4x8PSK with Viterbi decoding.
3. $\nu = 6$, η=2.5 bits/signal 4x8PSK with Viterbi decoding.
4. $\nu = 4$, η=2.0 bits/signal 4x8PSK with Viterbi decoding.
5. $\nu = 6$, η=2.0 bits/signal 2x8PSK with Viterbi decoding.
6. $\nu = 4$, η=2.0 bits/signal 3x8PSK with Viterbi decoding.
7. $\nu = 16$, η=3.0 bits/signal 1x16PSK with sequential decoding.
8. $\nu = 6$, η=3.0 bits/signal 2x16PSK with Viterbi decoding.
9. $\nu = 16$, η=3.0 bits/signal 1x16QAM with sequential decoding.
10. $\nu = 6$, η=3.0 bits/signal 1x16QAM, nonlinear with Viterbi decoding.

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 1

Spectral Efficiency:	2.0 bits/signal
Constellation:	1x8PSK

Complexity

Constraint Length:	17
Tree Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	N/A
Decoder Speed Factor:	4
Buffer Size:	32k signals
Decoder:	Modified Fano Alg., soft decisions

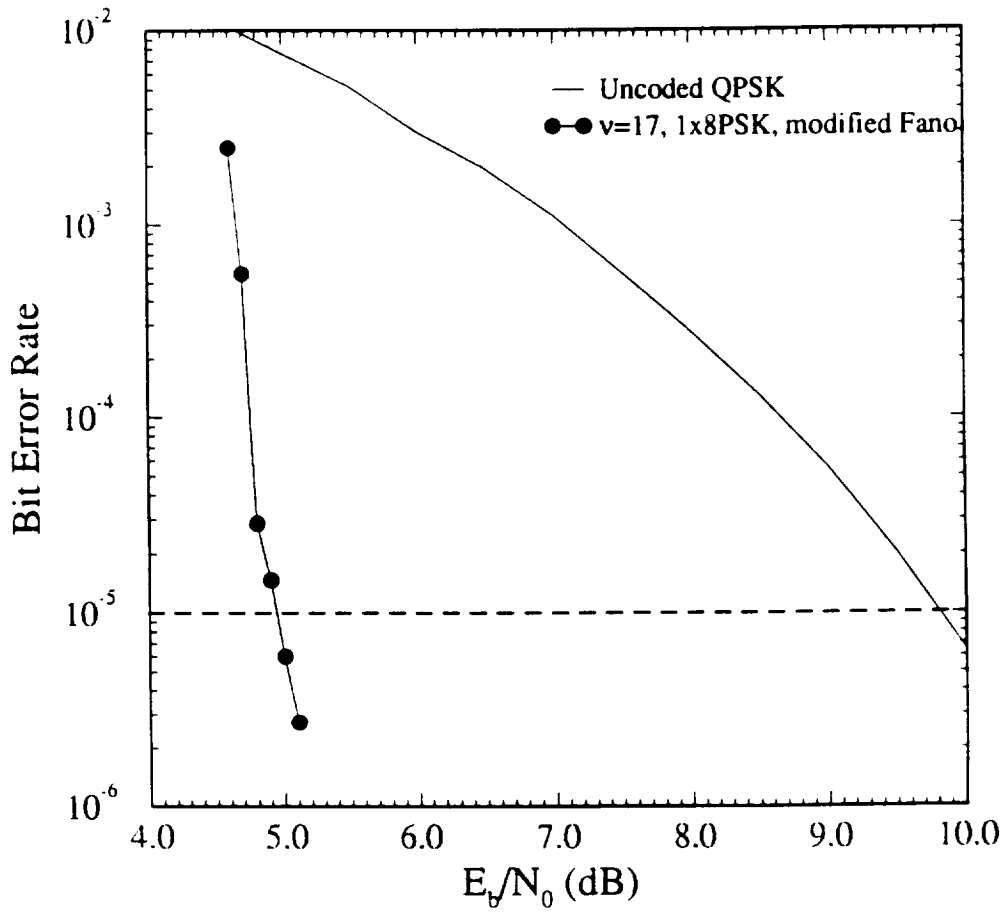
Performance

Rotational Invariance:	180°
Free Euclidean Distance:	Unknown ($\approx$ 8.686)
Real Coding Gain:*	4.86 dB at a BER of 10^{-5}
Decoder Speed:	12.5 Mbits with 25Mhz computation

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 1

8PSK, η=2.0 bits/signal



Bandwidth Efficient CCSDS Coding Standard Recommendation No. 2

Spectral Efficiency:	2.0 bits/signal
Constellation:	4x8PSK

Complexity

States:	64
Trellis Branch Complexity:	$2^{\tilde{k}} = 16$
Parallel Transitions:	$2^{k-\tilde{k}} = 16$
Decoder:	Parallel Viterbi, soft decisions

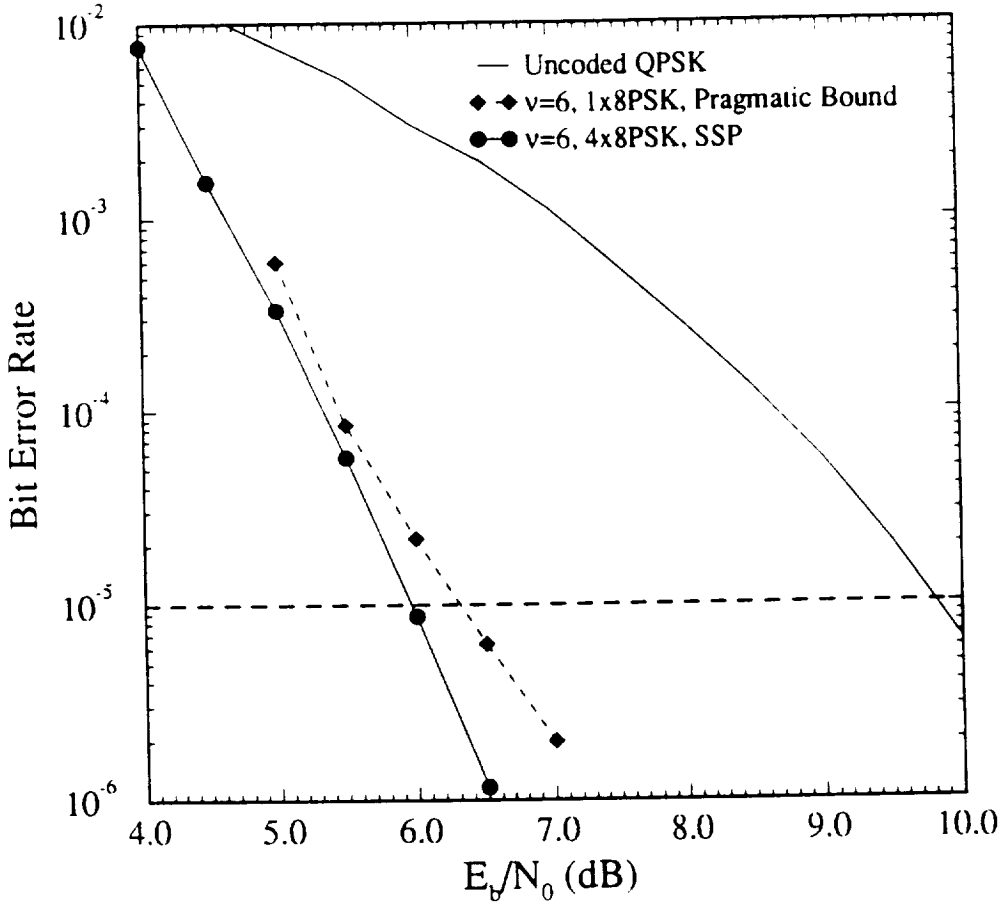
Performance

Rotational Invariance:	45° (Full)
Free Euclidean Distance:	6.686
Real Coding Gain:*	3.85 dB at a BER of 10^{-5}
Decoder Speed:	200 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendaion No. 2

8PSK, η=2.0 bits/signal



- The 4x8PSK code has 0.37 dB more coding gain than Viterbi's pragmatic code at a BER of 10^{-5} and is 45° rotationally invariant compared to 180° for the pragmatic code.
- The 4x8PSK code also has 4 times the decoded bit rate of the pragmatic code, assuming a 25Mhz ACS operation.

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 3

Spectral Efficiency:	2.5 bits/signal
Constellation:	4x8PSK

Complexity

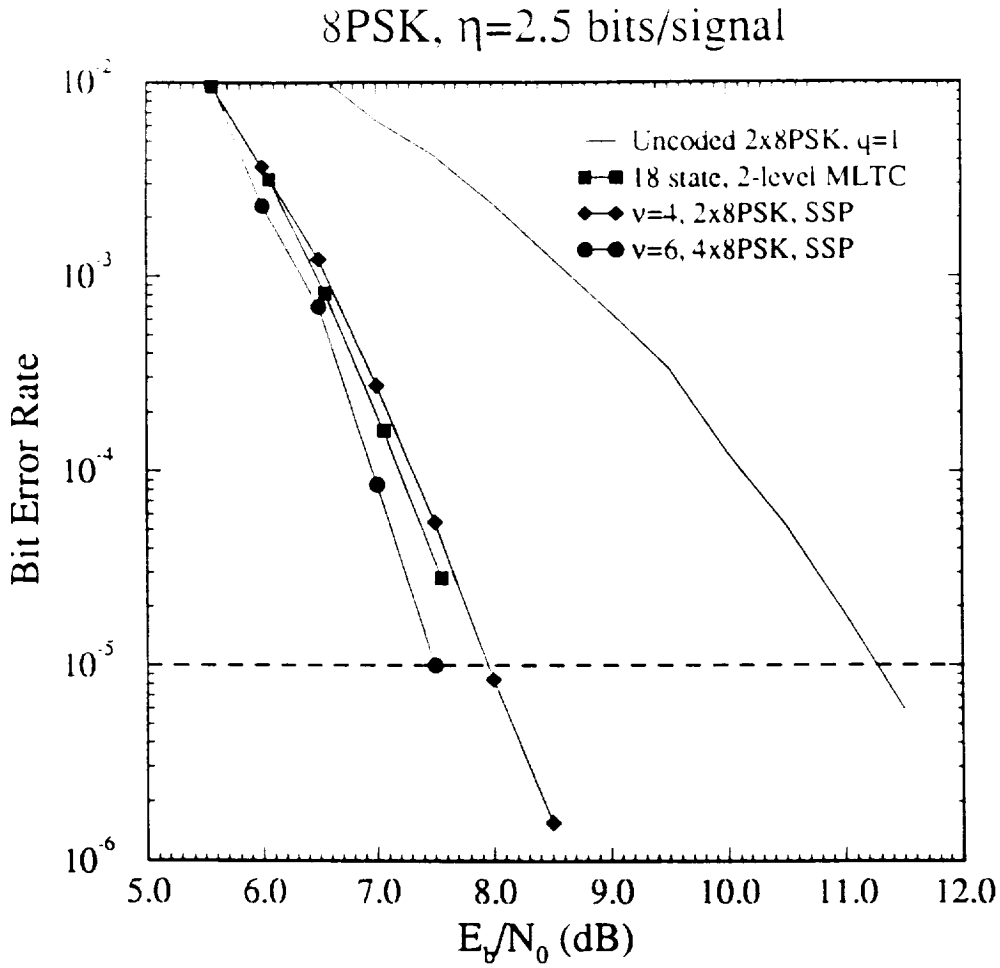
States:	64
Trellis Branch Complexity:	$2^{\tilde{k}} = 8$
Parallel Transitions:	$2^{k-\tilde{k}} = 128$
Decoder:	Parallel Viterbi, soft decisions

Performance

Rotational Invariance:	45° (Full)
Free Euclidean Distance:	4.0
Real Coding Gain:*	3.76 dB at a BER of 10^{-5}
Decoder Speed:	250 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 3



- High rate two level multilevel trellis codes (MLTC's) do not have a significant performance/complexity advantage over multidimensional trellis codes.
- The MLTC shown above has a rate of 2.48 bits/T and a decoder complexity of 18 states, but only performs a few tenths of dB better than a 2x8PSK code with a spectral efficiency of 2.5 bits/signal and sixteen states.
- The 16 state 2x8PSK code has been implemented in hardware and is being tested at White Sands Missile Range in New Mexico in 1992.

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 4

Spectral Efficiency:	2.0 bits/signal
Constellation:	4x8PSK

Complexity

States:	16
Trellis Branch Complexity:	$2^{\tilde{k}} = 16$
Parallel Transitions:	$2^{k-\tilde{k}} = 16$
Decoder:	Parallel Viterbi, soft decisions

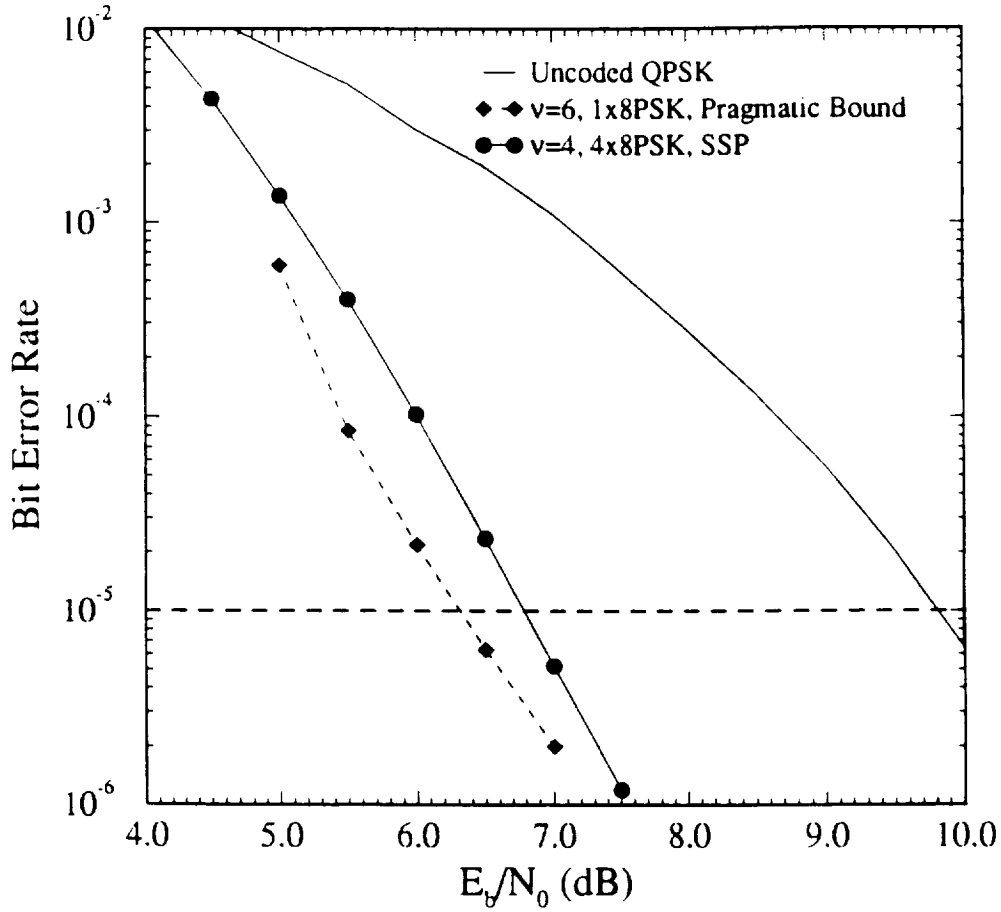
Performance

Rotational Invariance:	45° (Full)
Free Euclidean Distance:	4.686
Real Coding Gain:*	3.02 dB at a BER of 10^{-5}
Decoder Speed:	200 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 4

8PSK, η=2.0 bits/signal



- The 16 state, 4x8PSK code has only 0.46 dB less coding gai than Viterbi's pragmatic code at a BER of 10^{-5} and is 45° ro tationally invariant compared to 180° for the pragmatic code
- The 16 state, 4x8PSK code also has 4 times the decoded bi rate of the pragmatic code, assuming a 25Mhz ACS operation

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 5

Spectral Efficiency:	2.0 bits/signal
Constellation:	2x8PSK

Complexity

States:	64
Trellis Branch Complexity:	$2^{\tilde{k}} = 8$
Parallel Transitions:	$2^{k-\tilde{k}} = 2$
Decoder:	Parallel Viterbi, soft decisions

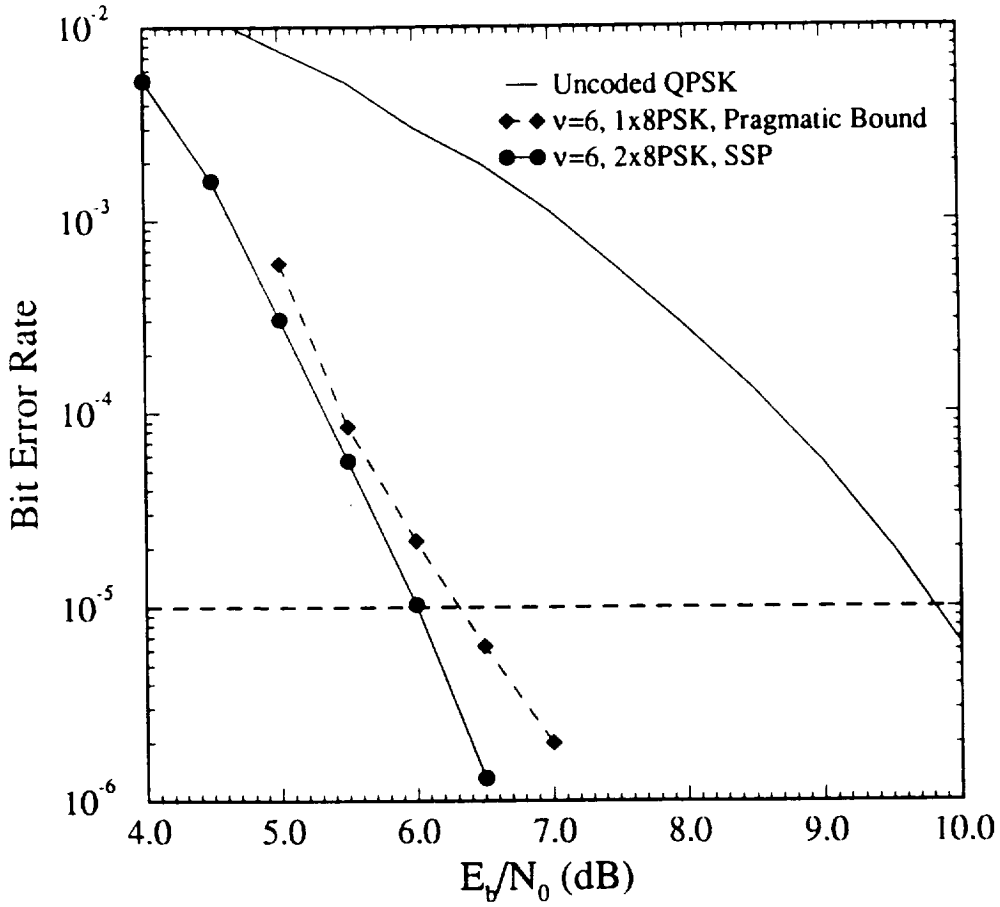
Performance

Rotational Invariance:	90°
Free Euclidean Distance:	6.343
Real Coding Gain:*	3.8 dB at a BER of 10^{-5}
Decoder Speed:	100 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 5

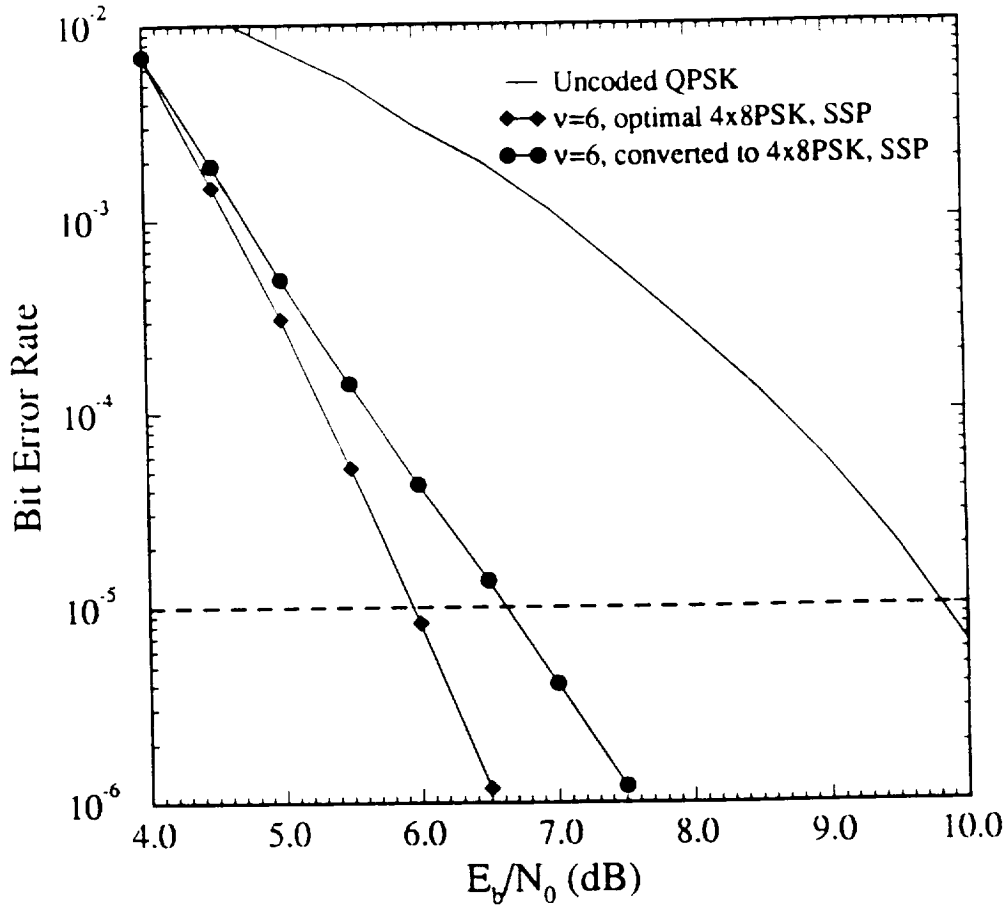
8PSK, η=2.0 bits/signal



- The 2x8PSK code has 0.32 dB more coding gain than Viterbi's pragmatic code at a BER of 10^{-5} and is 90° rotationally invariant compared to 180° for the pragmatic code.
- The 2x8PSK code also has twice the decoded bit rate of the pragmatic code, assuming a 25Mhz ACS operation.

CCSDS Recommendation No. 5

8PSK, η=2.0 bits/signal



- The 2x8PSK code may be converted into a suboptimal 4x8PS code when larger ROM's are available by simply changing th parallel transition ROM's.
- The converted code loses 0.67 dB compared to the optima 4x8PSK code (which may be considered too complex to im plement at this time) at a BER of 10^{-5}, but would be capabl of operating at up to 200 Mbits.

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 6

Spectral Efficiency:	2.0 bits/signal
Constellation:	3x8PSK

Complexity

States:	16
Trellis Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	$2^{k-\tilde{k}} = 16$
Decoder:	Parallel Viterbi, soft decisions

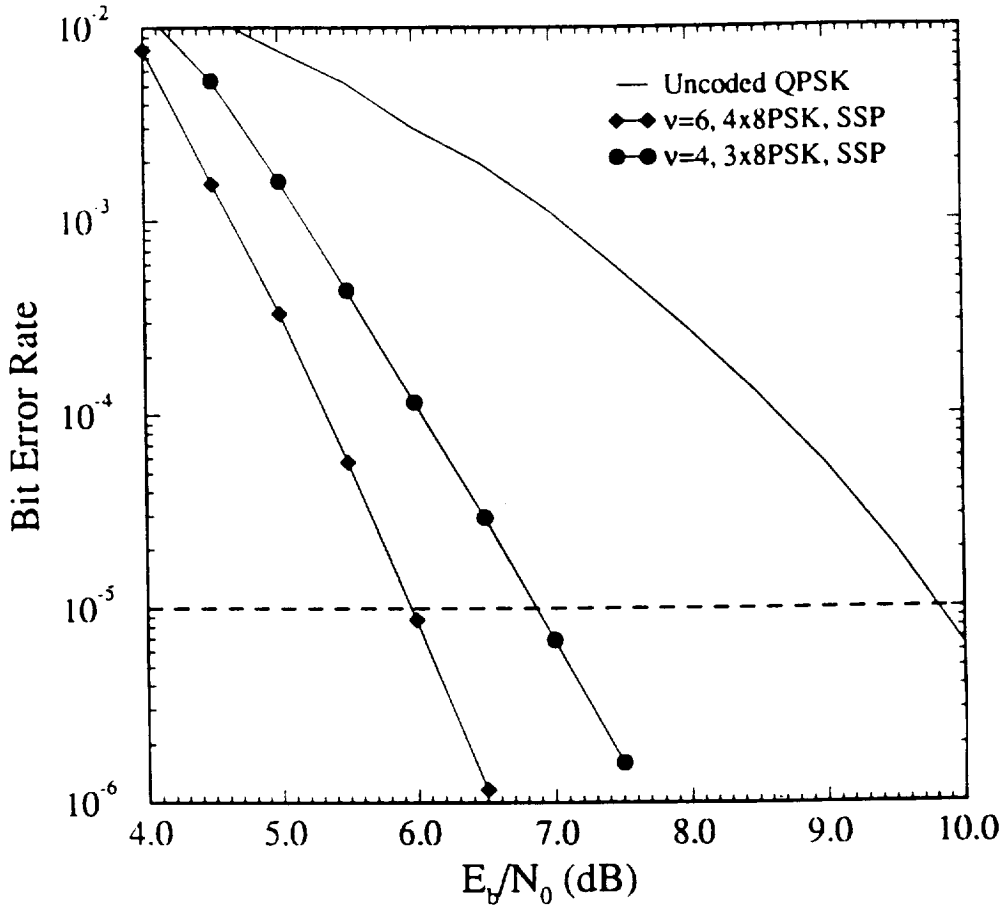
Performance

Rotational Invariance:	45° (Full)
Free Euclidean Distance:	4.0
Real Coding Gain:*	2.94 dB at a BER of 10^{-5}
Decoder Speed:	150 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 6

8PSK, η=2.0 bits/signal



Bandwidth Efficient CCSDS Coding Standard Recommendation No. 7

Spectral Efficiency:	3.0 bits/signal
Constellation:	1x16PSK

Complexity

Constraint Length:	16
Tree Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	$2^{k-\tilde{k}} = 2$
Decoder Speed Factor:	4
Buffer Size:	32k signals
Decoder:	Modified Fano Alg., soft decisions

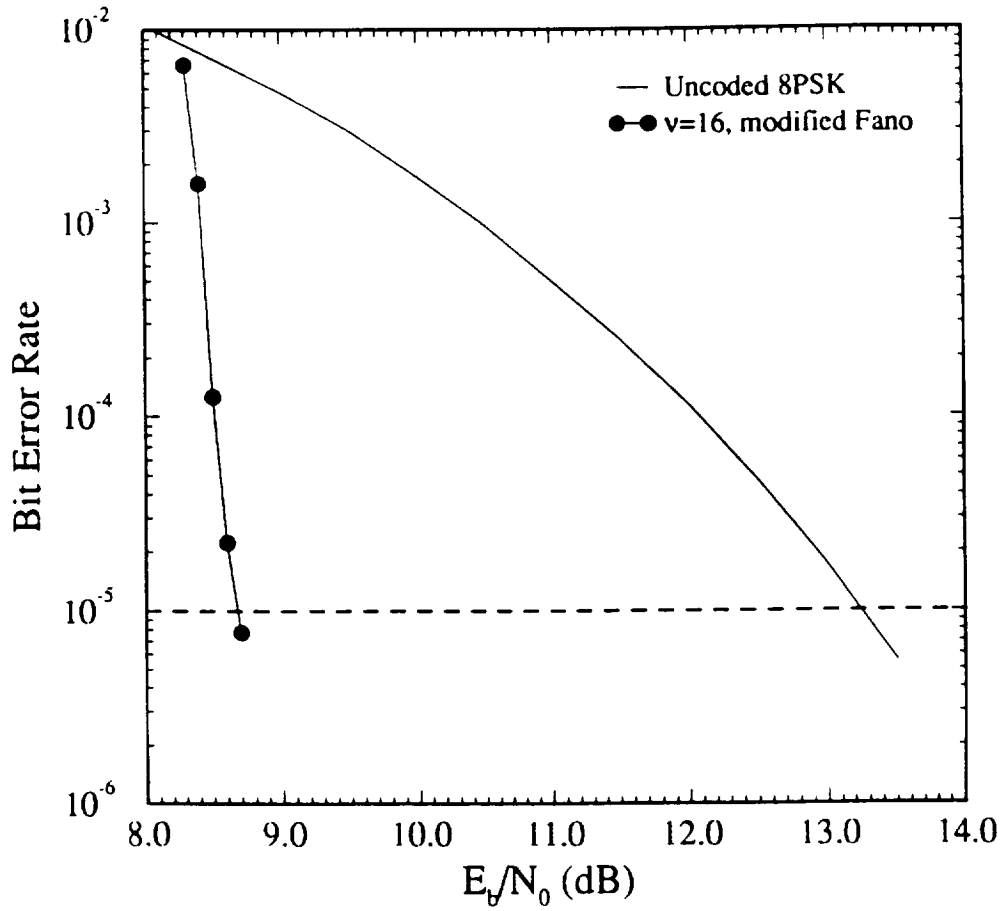
Performance

Rotational Invariance:	Unknown, possible 180°
Free Euclidean Distance:	Unknown (≈ 2.5)
Real Coding Gain:*	4.55 dB at a BER of 10^{-5}
Decoder Speed:	18.75 Mbits with 25Mhz computatior

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 7

16PSK, η=3.0 bits/signal



Bandwidth Efficient CCSDS Coding Standard Recommendation No. 8

Spectral Efficiency:	3.0 bits/signal
Constellation:	2x16PSK

Complexity

States:	64
Trellis Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	$2^{k-\tilde{k}} = 16$
Decoder:	Parallel Viterbi, soft decisions

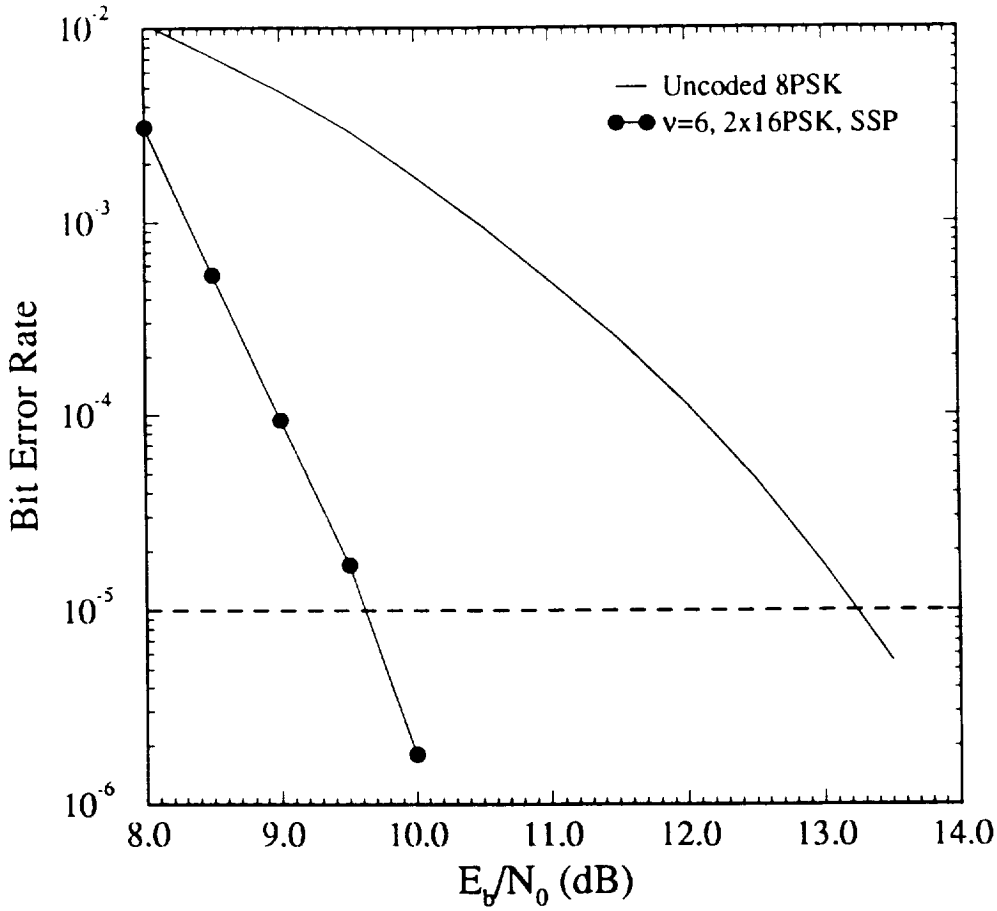
Performance

Rotational Invariance:	45°
Free Euclidean Distance:	2.0
Real Coding Gain:*	3.52 dB at a BER of 10^{-5}
Decoder Speed:	300 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 8

16PSK, η=3.0 bits/signal



- The 2x16PSK code has the same real coding gain as the 1x16PSK code and allows a higher decoded bit rate.
- To get full rotationally invariance, 22.5°, with a linear code requires a 3x16PSK or 4x16PSK signal set, increased trellis branch complexity, and a reduction in free Euclidean distance!
- Full rotational invariance can be achieved with a nonlinear 1x16PSK code, but the free Euclidean distance is reduced to 1.781.

Bandwidth Efficient CCSDS Coding Standard Recommendation No. 9

Spectral Efficiency:	3.0 bits/signal
Constellation:	1x16QAM

Complexity

Constraint Length:	16
Tree Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	$2^{k-\tilde{k}} = 2$
Decoder Speed Factor:	4
Buffer Size:	32k signals
Decoder:	Modified Fano Alg., soft decisions

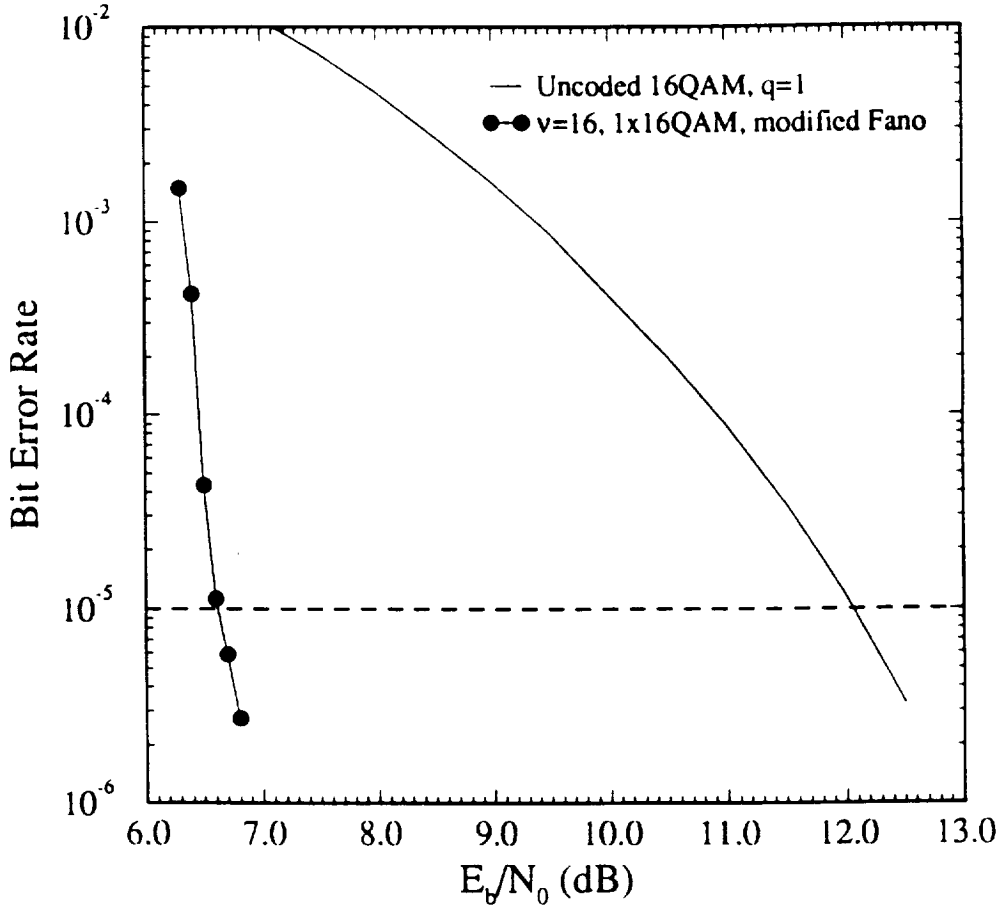
Performance

Rotational Invariance:	Unknown, possible 180°
Free Euclidean Distance:	Unknown (≈ 8.0)
Real Coding Gain:*	5.44 dB at a BER of 10^{-5}
Decoder Speed:	18.75 Mbits with 25Mhz computatior

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 9

16QAM, η=3.0 bits/signal



Bandwidth Efficient CCSDS Coding Standard Recommendation No. 10

Spectral Efficiency:	3.0 bits/signal
Constellation:	1x16QAM, nonlinear code

Complexity

States:	64
Trellis Branch Complexity:	$2^{\tilde{k}} = 4$
Parallel Transitions:	$2^{k-\tilde{k}} = 2$
Decoder:	Parallel Viterbi, soft decisions

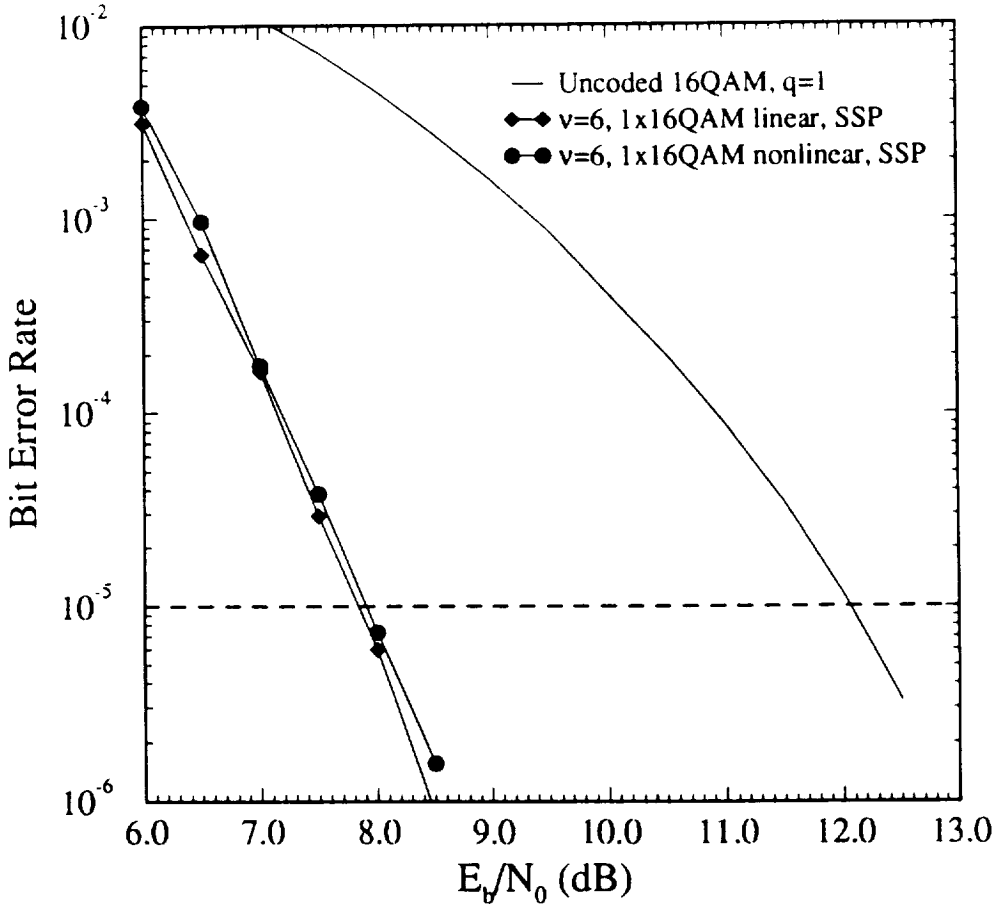
Performance

Rotational Invariance:	90° (Full)
Free Euclidean Distance:	7.0
Real Coding Gain:*	4.15 dB at a BER of 10^{-5}
Decoder Speed:	75 Mbits with 25Mhz ACS

*Compared to an uncoded system of the same spectral efficiency.

CCSDS Recommendation No. 10

16QAM, η=3.0 bits/signal



- The optimum linear 1x16QAM code with 64 states has the same free Euclidean distance, but is only 180° rotationally invariant.
- The linear code has 0.07 dB more real coding gain due to a slightly smaller number of nearest neighbors.
- To get a 90° rotationally invariant *linear* code requires a 4x16QAM signal set.
- The optimum linear 4x16QAM code with 64 states has a free Euclidean distance of 8.0, but has a trellis branch complexity of 32 and 142 nearest neighbors!

B The Shannon Graph

With his 1948 paper "*The Mathematical Theory of Communication*," Claude E. Shannon stimulated a body of research that has evolved into the two modern fields of Information Theory and Communication Theory. That one paper should spawn two active research areas is extraordinary and, as will become apparent, a direct consequence of the nature of the results. The fundamental philosophical contribution of this seminal treatise was the formal application of probability theory to the study and analysis of communication systems. The theoretical contribution of Shannon's work was a useful definition of "information" and several "channel coding theorems" which gave explicit upper bounds, called the channel capacity, on the rate at which "information" could be transmitted reliably on a given communications channel.

In the context of current research in coded modulation, the result of primary interest is the "noisy channel coding theorem for continuous channels with average power limitations." This theorem states that the capacity, C, of a continuous additive white Gaussian noise (AWGN) channel with bandwidth B is given by

$$C = B \log_2 \left(1 + \frac{E_s}{N_0}\right) \tag{B1}$$

where E_s is the average signal energy in each signalling interval, T, and $N_0/2$ is the two sided noise power spectral density. This theorem is both profound in its implications and, fortunately so for communication engineers, frustrating in its ambiguity.

It is profound, because it states unequivocally that for any transmission rate, R, less than or equal to the channel capacity, C, there exists a coding scheme that achieves an arbitrarily small probability of error; conversely, if R is greater than C, no coding scheme can achieve reliable communication. The field of Information Theory is, in a strict sense, an effort to apply Shannon's definition of information and methods of analysis to different channels and problems, such as cyptography. It is frustrating, because like most existence theorems it gives no hint as to how to find the appropriate coding scheme or how complex it must be. Communication engineers and coding theorists make their living trying to create schemes that achieve the levels of performance promised by Shannon's results. The following figure is both a measure of how close they have come and how much better they can possibly do.

The bound of equation (B1) can be put into a form more useful for the present discussion by introducing the parameter η called the spectral efficiency That is, η represent the average number of information bits transmitted per signalling interval. Assuming perfect Nyquist signalling, then

$$0 \leq \eta \leq C/B$$

and

$$E_s/N_0 = \eta E_b/N_0,$$

where E_b is the average energy per information bit. Substituting the above relations into equation (B1) and performing some minor manipulations yields

$$E_b/N_0 \geq \frac{2^\eta - 1}{\eta}, \tag{B2}$$

which relates the spectral efficiency, η, to the signal-to-noise ratio (SNR), E_b/N_0. This bound, labelled Shannon's Bound, is plotted in the figure and represents the absolute best performance possible for a communications system on the AWGN channel.

In this form, Shannon's bound gives the minimum signal-to-noise ratio required to achieve a specific bandwidth efficiency with an arbitrarily small probability of error. For example, if one wants to transmit $\eta = 1$ information bits per channel signal, then there exists a coding scheme that operates reliably with an SNR of 0dB. Conversely, any coding scheme, no matter how complex, sending $\eta = 1$ information bits per signal with an SNR *less than* 0dB will be unreliable. The bound of equation (B2) also manifests the fundamental tradeoff between bandwidth efficiency and SNR. That is, increased bandwidth efficiency can be reliably achieved only with a corresponding increase in minimum SNR. At this point, it is important to reiterate that Shannon's results do not suggest what code or what type of signalling is necessary to achieve this bound, and consequently it can be a discouraging measure of a system's performance.

In real communication systems, there are many practical considerations that take precedence over Shannon's bound in design decisions. For example, satellite communication systems that use nonlinear travelling wave tube amplifiers (TWTA's) require constant envelope signalling such as M-ary phase shift keying (MPSK). Thus, even if Shannon's results firmly stated that capacity at a spectral efficiency of $\eta = 3$ bits per signal can be achieved with a rate 3/4, constraint length $\nu = 8$, convolutional code using 16 quadrature amplitude modulation (QAM), it would not be feasible to do so on the TWTA satellite link.

It therefore seems reasonable to ask what the minimum SNR required to achieve reliable communication is *given* a modulation scheme and a bandwidth efficiency, η. For the discrete input, continous output, memoryless AWGN channel with M-ary one dimensional, e.g., amplitude modulation

(AM), or two dimensional (PSK, QAM) modulation and assuming equiprobable signalling, the capacity bound becomes

$$\eta^* = \log_2(M) - \frac{1}{M}\sum_{i=0}^{M-1} E\left\{\log_2 \sum_{j=0}^{M-1} \exp\left[\frac{|a^i + n - a^j|^2 - |n|^2}{N_0}\right]\right\}, \quad \text{(B3)}$$

where a is the channel signal, n is a Gaussian distributed noise random variable with mean 0 and variance $N_0/2$, and E is the expectation operator. The bound of equation (B3) is plotted in the figure for BPSK, QPSK, and 8PSK modulation.

For a specified signalling method and spectral efficiency, this bound represents the minimum SNR required to achieve reliable communication. For example, to send $\eta = 1$ information bits per signalling interval using QPSK modulation requires a minimum SNR of $E_b/N_0 = 0.5$dB. Any system using QPSK modulation with $\eta = 1$ and operating with a SNR lower than 0.5dB will not be reliable, regardless of complexity.

Also depicted on the figure is the performance of a number of real coded communications systems with a variety of bandwidth efficiencies. These points are plotted by determining, either analytically or by simulation, the SNR required for the system to achieve a information bit error rate (BER) of 10^{-5}. (Thus, a BER of 10^{-5} is chosen as the "arbitrarily small probability of error.") By comparing these points to the corresponding bound with the same bandwidth efficiency and modulation, it can be seen how close to the ultimate performance a system is. For example, the well known rate $R = 1/2$, $\nu = 6$, convolutional code sends $\eta = 1$ information bits per QPSK signal with a BER of 10^{-5} at an SNR of $E_b/N_0 = 4.4$dB. This is 3.9dB away from the QPSK bound and 4.4dB from Shannon's bound. The performance of a number of recent TCM schemes are also shown on the figure. For a spectral efficiency of 2 bits per signal, the Ungerboeck $R = 2/3$, $\nu = 6$, 8PSK trellis code is 3.0dB from the bound and performs 0.4dB better than the $R = 2/3$, $\nu = 6$, 8PSK pragmatic trellis code suggested by Viterbi. It should be noted that the previous comment reflects performance at a BER of 10^{-5}; the Ungerboeck code has an asymptotic coding gain of 5.0dB compared to 3.0dB for the pragmatic code. To achieve a spectral efficiency of 3 bits per signal with constant envelope signalling, 16PSK can be used. The best known $R = 3/4$, $\nu = 6$, 16PSK trellis code achieves a BER of 10^{-5} with a SNR of $E_b/N_0 = 9.6$dB and, as shown, is about 6.0dB from the Shannon Bound.

If constant envelope signalling is not required, then quadrature amplitude modulation (QAM) offers improved performance at high spectral efficiencies, i.e. more bits per signal. The performance of three $R = 3/4$, 16QAM, trellis codes are shown in the figure. The $\nu = 4$, 16QAM convolutional code proposed by TRW performs 0.5dB better than the 16PSK code even though it has fewer states. Further improvement is available if 16QAM TCM is used. A linear, $\nu = 4$, 16QAM trellis code is 1.1dB better than the

16PSK code and the nonlinear, $\nu = 6$, 16QAM code is 1.8dB better. The latter code also has the advantage of being fully rotationally invariant.

Recent advances in coding theory, including coded modulation and constellation shaping, and the technological feasibility of increasingly complex coding schemes have brought the bounds of Shannon and other information theorists within sight. In fact, it has been suggested that with sophisticated shaping techniques, complex codes and large lattice theoretic constellations capacity may be achieved in some specialized systems in the near future. This figure illustrates the progress made toward that goal.