THE OPTIMIZATION OF FORCE INPUTS FOR ACTIVE STRUCTURAL ACOUSTIC CONTROL USING A NEURAL NETWORK

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JUNE 1992

NASA Technical Memorandum 107627
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INTRODUCTION

Recent analytical and experimental work¹,² has successfully demonstrated active noise and vibration control in cylinders using extended piezoelectric actuators. Extended piezoelectric actuators, consisting of pre-positioned patches of piezoelectric material, have the potential of reducing control spillover, without necessarily increasing the number of control degrees of freedom. In the cases considered previously, the distribution of optimum actuator input voltage was determined for each discrete frequency of interest. There are several limitations with this type of control implementation. First, the optimum location and number of actuators in an extended array are a function of the excitation frequency. That is, in order to maintain control over the structural/acoustic modes excited at different frequencies, it is necessary to relocate and/or change the number of actuators. Secondly, actuator arrays with more control elements require controllers with additional degrees of freedom.

To overcome these implementation problems, this paper proposes using a neural network³ to determine which piezoelectric actuators, in an extended array, are activated for control at a particular frequency. The concept is demonstrated using a cylinder/cavity model on which the control forces, produced by piezoelectric actuators¹, are applied in order to reduce the interior noise. The two layer neural network, driving the piezoelectric actuators, consists of an input layer, a hidden layer, and an output layer of processing nodes. The interconnection nodal weights, determining the contribution of each processing node, are calculated by a standard back-propagation technique, such that minimum pressure is achieved at a number of error microphones located inside the cylinder. Predicted results are compared with the results calculated by a conventional, least-squares optimization analysis¹. The ability of the neural network to accurately and efficiently control actuator activation for interior noise reduction is therefore demonstrated.

ANALYSIS

The structural acoustic model¹, shown in figure 1, consists of an aluminum cylinder of diameter, \( D = 1.68m \), length, \( L = 3.66m \), and thickness, \( h = 1.7mm \). The cylinder is closed with hard (rigid) end caps and is assumed to have simply-supported boundary conditions. The offending primary noise field inside the cylindrical cavity is produced by the wall vibrations excited by a single, harmonically varying, acoustic monopole externally located at \( x = \frac{L}{2}, \; r = 0.6D, \) and \( \theta = 0^\circ \). For illustrative purposes, figure 1 also shows an extended array of four piezoelectric actuators. All piezoelectric arrays considered in this paper were distributed around the circumference of the cylinder at \( x = \frac{L}{2} \). Each actuator had an axial length of 3.8cm and a circumferential length of 6.4cm.
A schematic of the neural network is shown in figure 2. The unknown, complex interconnection weights $W_{ji}$ and $T_{kj}$, which link the nodes, are randomly initialized and then iteratively adjusted by a back-propagation (gradient) procedure. The complex outputs $y_j$ and $c_k$ of each network layer can then be determined. Starting with the complex reference inputs $r_i$, the output $y_j$ of the first layer is

$$y_j = f \left( \sum_{i=1}^{N_r} W_{ji} r_i + a_j \right)$$

where $N_r$ is the number of reference inputs. Herein, it is assumed that all reference inputs are unity, that is, $r_i = 1$ ($i = 1, 2, ..., N_r$). Usually a sigmoid (nonlinear) function is assumed for $f(x)$, although in this work, because it is not needed, the identity function $f(x) = x$ is substituted. For the same reason, all complex offset biases $a_j$ are nulled.

$$c_k = g \left( \sum_{j=1}^{N_h} T_{kj} y_j + b_k \right)$$
where \( N_h \) is the number of hidden nodes (outputs from first layer). Again, instead of a sigmoid function, it is assumed that \( g(z) = z \) and all biases \( b_k \) are again nulled. The unknown interconnection weights \( T_{kj} \) for the second layer are also determined by the back-propagation process. When this is done the controller inputs \( c_k \) \( (k = 1, 2, \ldots, N_c) \) to each piezoelectric actuator can be determined. It is assumed in this paper that the number of reference inputs, the number of hidden units, and the number of actuators are equal, that is, \( N_r = N_h = N_c \).

The cylinder and piezoelectric actuator model is represented by the third and last layer of the network configuration. The elements of the complex transfer matrix \( H_{mk} \) represent the acoustic pressure at the \( m^{th} \) error microphone due to a unit control input \( c_k = 1 \) on the \( k^{th} \) piezoelectric actuator. The elements of \( H_{mk} \) were computed a priori by a known analytical procedure\(^1\), although the elements of \( H_{mk} \) could have been determined by experiment. The total acoustic pressure, at a single error microphone, inside the cylinder is therefore a superposition of the offending primary noise field \( p_m \) and the acoustic pressure (control field) produced by the control actuator forces\(^1\). The total pressure \( \Lambda_m \) at an error microphone is thus given by

\[
\Lambda_m = \sum_{k=1}^{N_c} H_{mk} c_k + p_m
\]

where \( N_c \) is the number of actuators, \( p_m \) is the primary field at microphone \( m \), and the summation represents the response due to all control actuator forces. For the calculations presented in this paper, three arrays of error microphones, located at \( z = \frac{L}{4}, \frac{5L}{8}, \) and \( z = \frac{3L}{4} \), were utilized. Each array contained 36 error microphones, uniformly distributed around the inside cylinder wall.

The cost function \( E \), which will be minimized by iteratively updating the interconnection weights \( W_{ji} \) and \( T_{kj} \) of the neural network, is defined as the sum of the squares of the microphone pressures \( \Lambda_m \), plus an additional penalty term\(^4\). Therefore,

\[
E = (1 - \lambda) \sum_{m=1}^{N_p} |\Lambda_m|^2 + \lambda \sum_{k=1}^{N_c} \frac{c_k c_k^*}{1 + c_k c_k^*}
\]

(4a)

where \( N_p \) is the number of error microphones and \( * \) indicates a complex conjugate. The first term in equation (4a) is the pressure term and the second term is the penalty term. The value for \( \lambda \) determines the relative weighting of the pressure and penalty terms during adaptation. If \( \lambda = 0 \), then the microphone responses (pressure term) are minimized without constraint (penalty term). However, if \( \lambda \neq 0 \), then the total cost function is minimized and, depending on the value of \( \lambda \), a penalty is imposed for actuator effort. A value of \( \lambda = 0.001 \) was used for all calculations presented in this paper.

The purpose of the penalty term (second term in eq. 4a) is to determine (identify) those actuators not contributing significantly to the reduction of interior noise and to effectively null the outputs of these actuators. In this way, the degrees of freedom of the controller are reduced. Disregarding \( \lambda \), the penalty term is bounded between 0 and 1.0. Thus, a large actuator effort which provides a significant reduction in the pressure term will not be offset by an increase in the penalty term. However, for a small actuator effort that does not provide a significant cost reduction through the pressure term, the penalty term will
have a greater effect. In this case, the penalty term will tend to drive the outputs of the ineffective actuators to zero.

The cost reduction in decibels is defined as follows:

$$E_{dB} = 10.0 \log \left( \frac{\sum_{m=1}^{N_p} |A_m|^2}{\sum_{m=1}^{N_p} |p_m|^2} \right)$$ (4b)

The interconnection weights in the neural network are adjusted using a modified version of the complex, back-propagation algorithm.\(^3,4\) The following equations are found by taking the partial derivative of the cost function with respect to the adjustable weights in the neural network. If an error at the piezoelectric actuators is defined as

$$\delta_k^{(1)} = g'(z_k) \left[(1 - \lambda) \sum_{m=1}^{N_p} A_m H_{mk}^* + \lambda \frac{c_k}{(1 + c_k e_k)^2} \right]$$ (5a)

then the weights are updated according to

$$T_{kj}^{n+1} = T_{kj}^n - \varepsilon \delta_k^{(1)} y_j^*$$ (5b)

where $\varepsilon$ controls the rate of learning and $n$ is the iteration index. Continuing backwards through the network, the $W_{ji}$ interconnection weights of the first layer are updated in a similar manner. First, let

$$\delta_j^{(2)} = f'(z_j^*) \sum_{k=1}^{N_p} \delta_k^{(1)} T_{kj}^*$$ (6a)

which, in essence, accounts for the back-propagation of the error at the actuators to the hidden nodes. The weights are then updated according to

$$W_{ji}^{n+1} = W_{ji}^n - \varepsilon \delta_j^{(2)} r_i^*$$ (6b)

The complex offset values $b_k$ and $a_j$ can be adjusted iteratively in a similar manner, however, in this work these quantities were nulled.

DISCUSSION OF RESULTS

Results are summarized in Tables I and II for the cylinder model and neural net controller illustrated in figures 1 and 2. The excitation frequency is 200Hz, which is very near the resonant frequency of a $\cos 2\theta$ cavity mode. Hence, a strong $\cos 2\theta$ modal response is excited in the cavity by the cylinder wall vibrations\(^1\).

Table I presents the cavity noise reduction achieved with four actuators equally spaced around the midsection of the cylinder. It should be noted that this is a near optimum actuator configuration for this ($\cos 2\theta$) response situation. The least-squares results\(^1\), which achieves a 17.5dB noise reduction, represents an exact solution of this optimization problem. The unpencilized ($\lambda = 0$) and penalized ($\lambda = 0.001$) neural net solutions (eqs. 5 and 6), which are nearly identical, converge very close to the exact solution. Convergence of the neural net solution was not further studied in this paper. All calculations are based
on 50,000 learning iterations and a learning rate parameter of \( \epsilon = 0.1 \). The cost reduction based on integrating the square of the interior pressure over the interior volume\(^1\) was also calculated. A value of 17.5\(dB\) was again obtained, thus establishing that the noise reduction occurred over the total enclosed volume of the cylinder.

In Table II results are presented for an eight actuator configuration. In this case the cost reduction remains 17.5\(dB\) for all solution types. The exact, least-squares solution indicated that the amplitudes for the additional actuators, located at 45\(^\circ\), 135\(^\circ\), 225\(^\circ\), and 315\(^\circ\), are significantly smaller than the amplitudes for the other actuators. The unpenalized neural net solution (eqs. 5 and 6, \( \lambda = 0 \)) again agrees very closely with the exact, least-squares solution.

Also presented in Table II are the neural net results using a penalty term in the cost function (eq. 4a) with \( \lambda = 0.001 \). This approach penalizes nodes in the neural net which provide negligible actuator outputs and eliminates these nodes from the solution. The noise reduction remains constant at 17.5 decibels. The actuator amplitudes for all three solutions are in excellent agreement. In this case the neural net effectively eliminates the actuators at 45\(^\circ\), 135\(^\circ\), 225\(^\circ\), 315\(^\circ\) from the back-propagation solution.

![SPL, dB](image)

(A) NO CONTROL  (B) NEURAL NET, \( \lambda = 0.001 \)  (C) LEAST-SQUARES

Figure 3. Comparison of noise reduction solutions, source plane \( (z = \frac{L}{2}) \).

Finally, figure 3 compares the noise reductions produced in the source plane \( (z = \frac{L}{2}) \) by the neural net, part (b), and least-squares solutions, part (c). The no control case, part (a), is shown for reference. The neural net solution is seen to be nearly identical to the exact, least-squares solution, as might well be inferred from the actuator amplitudes given in Table II.

CONCLUDING REMARKS

The purpose of this paper was to demonstrate the feasibility of coupling neural net controllers with piezoelectric actuators in order to implement active, structural acoustic control. The application was directed at using a neural network to activate the minimum number of actuators in a large, pre-positioned array necessary to produce the most effective noise reductions.
A neural network, by nulling the controller inputs to the least effective actuators, adaptively configures the array in an optimal positional sense. Furthermore, from the analytical modeling approach used in this paper, it appears that back-propagation solutions, used with neural networks, may be numerically more robust than are the direct (matrix inversion) solutions used with quadratic optimization problems. This is particularly true when the optimal solution lies in a low-gradient region of the cost function. This typically occurs when the controller has redundant degrees of freedom. In such cases, the quadratic optimization solution may yield a system matrix which is nearly singular. However, with a neural network the redundant degrees of freedom are effectively nulled.

With regards to future work, the convergence properties of neural networks for active, structural acoustic, control applications need to be further studied, particularly with regards to the effects of learning rate, the number of learning cycles, and types of penalty functions. The use of nonlinear sigmoid functions and other on/off threshold switching schemes should also be investigated.

REFERENCES

Table I. Interior Noise Reduction for Four Actuators

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Cost reduction dB</th>
<th>Actuator amplitude and phase versus location</th>
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<tr>
<td>Eqs. (5), (6)</td>
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<td></td>
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<tr>
<td>$\lambda = 0$</td>
<td>17.5</td>
<td>0°  90°  180°  270°</td>
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<td></td>
<td>52.9</td>
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<td>26.9°</td>
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<td>Eqs. (5), (6)</td>
<td>17.5</td>
<td>0°  90°  180°  270°</td>
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<td>Least squares$^1$</td>
<td>17.5</td>
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<td></td>
<td>53.9</td>
<td>41.1  35.3  41.1</td>
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<tr>
<td></td>
<td>27.0°</td>
<td>-149.3° 30.3°  -149.3°</td>
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Table II. Interior Noise Reduction for Eight Actuators

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Cost reduction dB</th>
<th>Actuator amplitude and phase versus location</th>
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<tr>
<td>Eqs. (5), (6)</td>
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<td></td>
</tr>
<tr>
<td>$\lambda = 0$</td>
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<td></td>
<td>52.1</td>
<td>2.7  41.4  1.7  35.7  1.9  42.0  2.8</td>
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<td>51.2</td>
<td>0.0  37.1  0.0  30.3° -149.4° 0.0</td>
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<td></td>
<td>27.7°</td>
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<td></td>
<td>28.3° 85.3°</td>
<td>-149.2° -84.6° 28.2° -84.6° -149.2° 85.3°</td>
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The Optimization of Force Inputs for Active Structural Acoustic Control using a Neural Network

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Unclassified—Unlimited
Subject Category—71

This paper investigates the use of a neural network to determine which force actuators, of a multi-actuator array, are best activated in order to achieve structural-acoustic control. The concept is demonstrated using a cylinder/cavity model on which the control forces, produced by piezoelectric actuators, are applied with the objective of reducing the interior noise. A two-layer neural network is employed and the back propagation solution is compared with the results calculated by a conventional, least-squares optimization analysis. The ability of the neural network to accurately and efficiently control actuator activation for interior noise reduction is demonstrated.

Structural-Acoustic Control; Piezoelectric Actuators; Neural Networks; Active Control; Smart Structures