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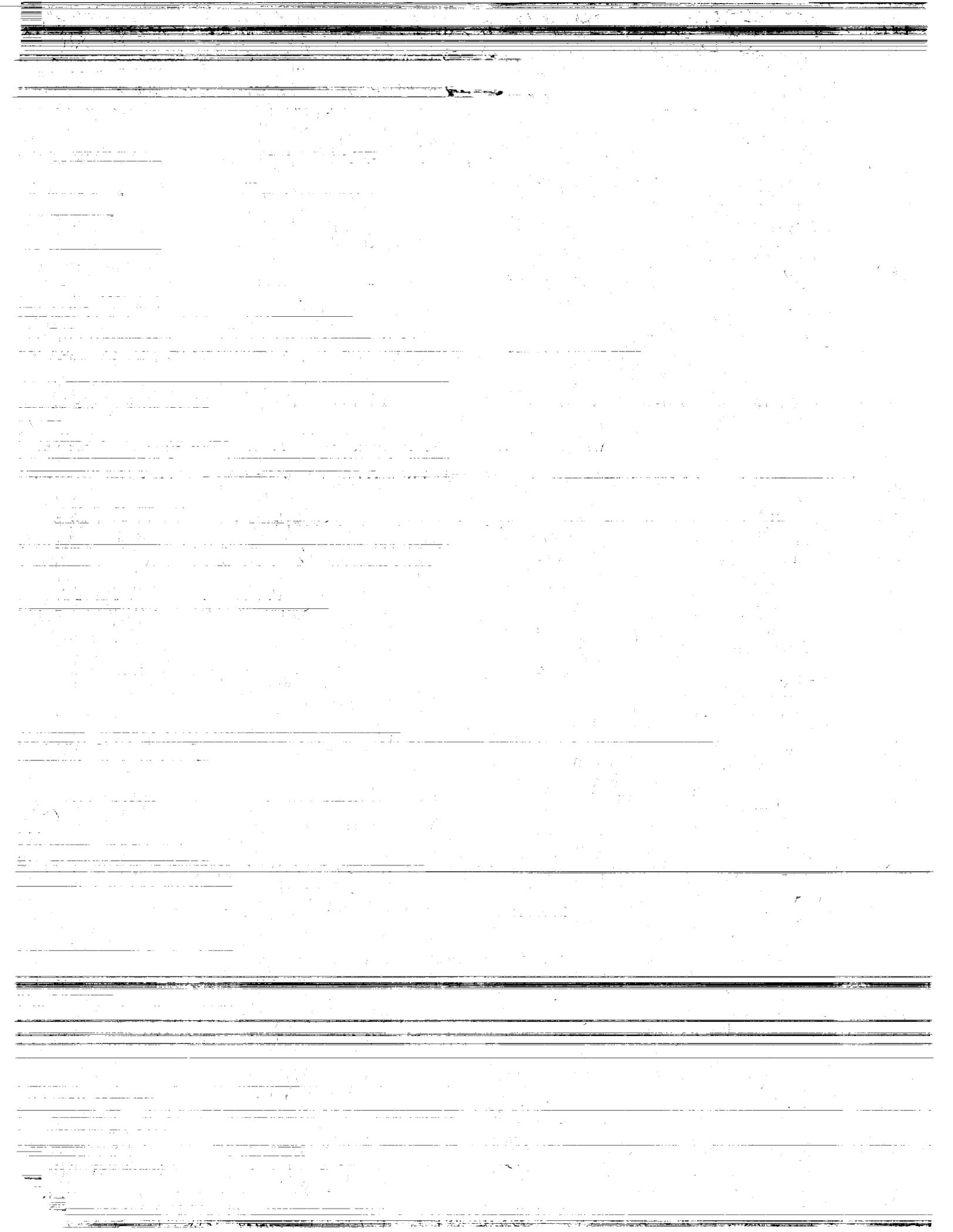
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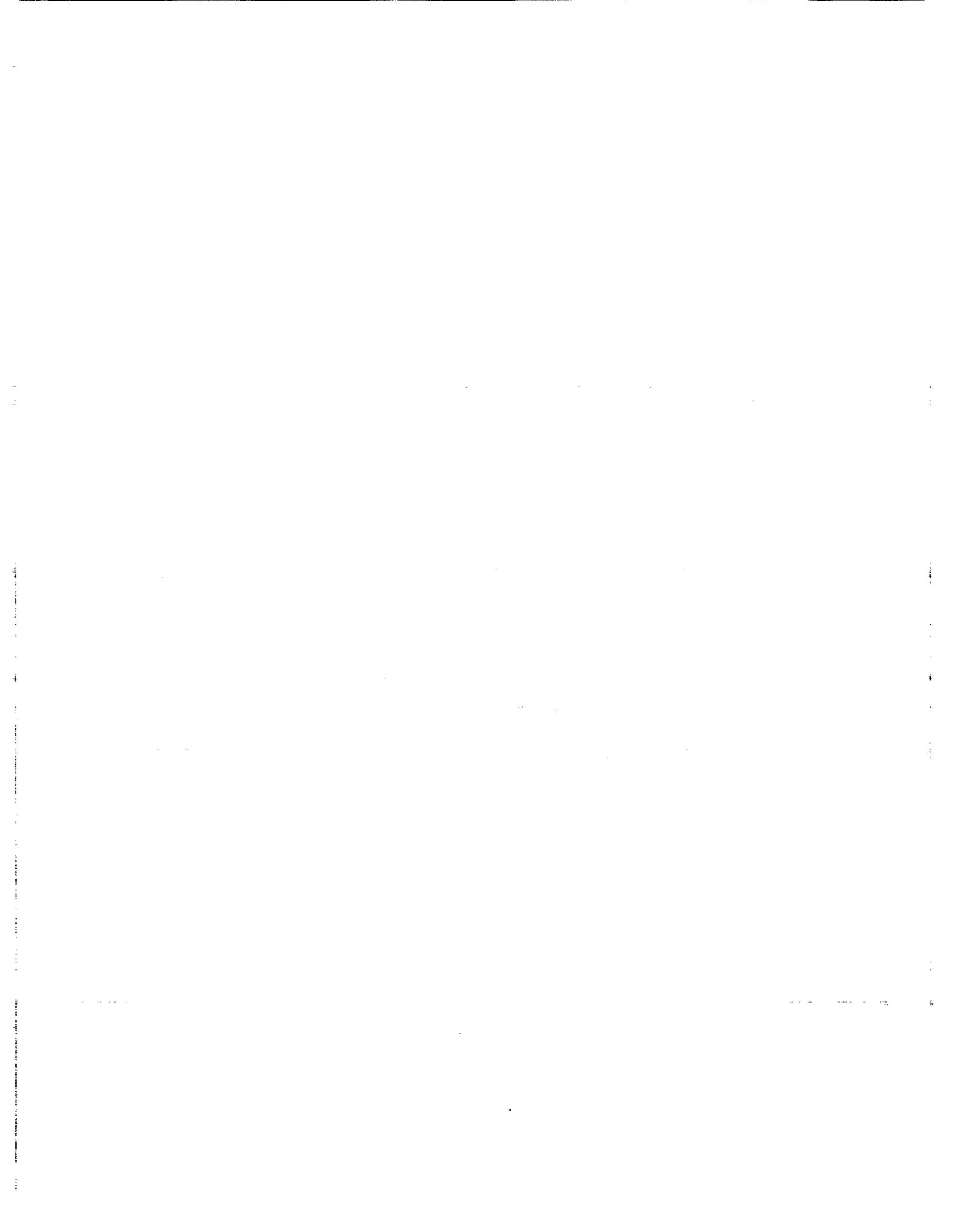
Steady Induction Effects in Geomagnetism

*Part IA: Steady Motional Induction
of Geomagnetic Chaos*

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1. INTRODUCTION

The source of the broad-scale, slowly varying, large-amplitude portion of the geomagnetic field observed at and near Earth's surface is widely held to be electric current flowing in Earth's electrically conducting core and, to a lesser extent, electrically resistive mantle and magnetically permeable crust. Maintenance of this current against resistive dissipation is usually attributed to motional induction in the liquid outer core: motion of the magnetized fluid relative to the solid mantle and crust generates electromotive force and thus electric current (LARMOR, 1919). If the curl of this electromotive force field is neither zero nor perfectly balanced by magnetic diffusion, then the fluid motion will induce slow or secular change of the geomagnetic field in accord with Faraday's law (ELSASSER, 1946a,b, 1947). Indeed, if an initial magnetic field, the relevant material properties, and the evolving fluid velocity field were all known within the Earth, then a straightforward, classically deterministic prediction of geomagnetic secular change could be made. The inverse problem of estimating the fluid motion within the Earth given real observations of the magnetic field near its surface and rough estimates of key material properties is, of course, not quite so straightforward.

Both the rich variety of geomagnetically significant Earth properties and processes and the mathematical subtlety of the implied geophysics present formidable obstacles to core flow estimation. It has thus seemed appropriate to seek a qualitative understanding of geomagnetic secular variation in terms of a simple model, attempt quantitative explication of the observations, and to then systematically restore essential geophysical detail. In addition to using a simple magnetic earth model, I have further restricted my study to the geomagnetic effects of induction by hypothetically steady fluid motion (and steady magnetic diffusion) near the top of Earth's core.

The purpose of this series of papers is to share the findings of my decade-long investigation of steady induction effects in geomagnetism. These findings include qualitative geophysical arguments, rigorous mathematical proofs, technical development and application of new methods, and quantitative numerical results. Considered space is devoted to the technical introduction, background theory, mathematics, and physics needed to make these findings clear to the general reader. Indeed, some reviewers have demanded more detail precisely where others have demanded less. I hope that experts will share my belief that the findings transcend occasionally divergent stylistic preferences.

1.1 A Simple Magnetic Earth Model and Some Enabling Conditions

Following ROBERTS & SCOTT (1965), BACKUS (1968), BENTON (1979) and others (see, e.g., HIDE & MALIN, 1981; VOORHIES, 1984, 1986a, 1987a; BLOXHAM & JACKSON, 1991), attention is focused upon the fluid motion near the top of the core by adopting a simple model of the magnetic Earth: the source-free mantle/frozen-flux core model (or SFM/FFC model). In this model, a rigid, impenetrable, electrically insulating mantle of uniform magnetic permeability surrounds a spherical, inviscid, perfectly conducting outer core in anelastic flow. The collective use of these eight suppositions to account for recent broad-scale geomagnetic secular change is expected to yield errors of about 7%-as will be discussed in Part II (VOORHIES, 1986c, 1987c). In the SFM/FFC model, geomagnetic lines of force that thread the core-mantle boundary (CMB) are rooted in

the fluid at the top of the core; lateral motion of this fluid induces secular change by advecting the two footpoints of each such magnetic field line so as to reconfigure the scaloidal magnetic field outside the core. In and just above a SFM, the magnetic flux density vector \mathbf{B} is the negative gradient of the scalar potential V : $\mathbf{B} = -\nabla V$.

Granting these simplifying suppositions, even complete and perfect information about the geomagnetic field at Earth's surface does not allow unambiguous determination of the fluid motion at the top of the core (ROBERTS & SCOTT, 1965; BACKUS, 1968). The radial component of the induction equation at the top of a frozen-flux core attributes the time rate of change of the radial magnetic flux density component to the surface convergence of the product of this component with the lateral fluid velocity ($\partial_t B_r = -\nabla_s \cdot B_r \mathbf{v}_s$), so any cryptic flow which induces no secular change is not detected (any \mathbf{v}_s' such that $\nabla_s \cdot B_r \mathbf{v}_s' = 0$). Despite this toroidal ambiguity in $B_r \mathbf{v}_s$ (BACKUS, 1982), some components of this motion could be so determined at special locations (BENTON, 1981). Moreover, this motion could be uniquely determined from such information under certain conditions: e.g., if and when the motion is steady (VOORHIES & BACKUS, 1985) or, in broad areas, if and when it is tangentially geostrophic (HILLS, 1979; BACKUS & LEMOUEL, 1986).

Such enabling conditions, and the SFM/FFC model itself, can be viewed as approximations to a more realistic core geodynamo model: supposition of steady flow is accurate to lowest order in a temporal Taylor series expansion of the fluid motion; tangential geostrophy might hold to lowest order in core dynamics. The inaccuracy of these approximations might be demonstrated quantitatively by analysis of geomagnetic observations. If only to establish the importance of other effects, such conditions may then be treated as hypotheses to be tested against geomagnetic data (or models thereof) in the context of the SFM/FFC model (or refinements thereof).

Geomagnetic data will be neither complete nor perfect within the foreseeable future, so geomagnetic testing of such hypotheses is further embedded in at least one additional supposition regarding the geomagnetic field itself. For example, an initial condition is needed to test the steady motions hypothesis; therefore, the rigor of geomagnetic tests of this hypothesis is limited by the completeness and accuracy with which an initial geomagnetic field can be specified.

Magnetic field measurements do not isolate the slowly varying, broad-scale portion of the field—the "core field" of interest here. This isolation is imperfectly achieved with truncated spherical harmonic models derived to fit select, often low-pass space- and time- filtered, geomagnetic data (see, e.g., LANGEL, 1987); however, such models do represent the relevant portion of the observations. In recognition of the effects of external and non-core internal sources, the use of such models is here preferred to the use of raw data. Spherical harmonic models are convenient to work with; yet substitution of spatially complete, but spectrally incomplete, truncated spherical harmonic models for (in effect) spectrally complete, but spatially incomplete, measurements is not without pitfalls. For example, spherical harmonic (Gauss) coefficients of the scalar geomagnetic potential are uncertain and cross-correlated (LANGEL, ESTES & SABAKA, 1989; LANGEL, 1991); moreover, coefficients that are not estimated are not necessarily assumed to be zero. Spherical harmonic models may represent reduced, filtered, and analyzed data; but neither derived model parameters such

as Gauss coefficients nor annual averages of magnetic observatory data should be confused with the result of a physical measurement.

1.2 Some Motivation for Considering Steady Core Surface Flow

The supposition of statistically steady surficial core flow during intervals of a few decades or more is adopted for a variety of reasons. Theoretical arguments (invoked *a posteriori*) based upon ties between core surface flow and extremely slowly varying thermal, topographic, and compositional anomalies in the deep mantle support this supposition. For example, obstacles to a persistent westward flow posed by depressions of the CMB could result in an effectively steady, yet spatially complicated flow near the core surface. RUFF & ANDERSON (1980) and BLOXHAM & GUBBINS (1987) suggest "thermal core-mantle interactions," whereby laterally varying heat flow from the core to the thermally heterogeneous deep mantle establishes a steady, superficial core circulation. JAULT & LEMOUEL (1989) suggest "topographic locking," whereby the pattern of perturbation pressure driving core surface circulation becomes locked to the topography of the CMB. Another possibility is "Lorentz linkage," whereby strong Lorentz forces inhibit the motion of fluid parcels lying on field lines which thread regions of anomalously high conductivity in the deep mantle. Such patches of relatively stagnant fluid may be separated by mobile streams wherein fluid parcels are but loosely coupled to overlying areas of weak mantle conductivity. Such streams could slowly but steadily transport fluid laterally from large-scale source regions towards sinks. The sources could be buoyant FeO or FeS enriched reservoirs trapped near the CMB after columns or plumes from the inner core boundary have impinged upon the CMB (suggested by S. I. BRAGINSKY, pers. comm., 1989); the sinks might be compositionally depleted due to wüstite underplating of the initially low-conductivity region overlying the stream. One might also imagine steady drift of an otherwise standing wave field.

Facts, practical considerations, and empirical arguments may offer more compelling reasons. For example, supposition of surficially steady flow is simple, annihilates a key formal ambiguity (VOORHIES & BACKUS, 1985), and retains the lowest order term in a temporal Taylor (or Fourier) series expansion of the flow which, by definition, always merits consideration. BLOXHAM (1987a) suggested that all geomagnetic estimates of core surface flow suppose steadiness on some time scale. Those which do not (e.g., BENTON & CELAYA, 1991) still suppose some other form for the time dependence and make at least one quasi-steady approximation. Moreover, the primarily kinematical supposition of steady flow provides an alternative to specific dynamical assumptions and might thus shed some light on core dynamics—a view strongly advocated by WHALER (1991). It also brings all relevant data to bear on the problem of estimating a single core surface flow—a statistically interesting property in light of practical ambiguities resulting from the finitude of geomagnetic measurements, each of which is influenced by myriad geomagnetic phenomena. Perhaps most importantly, it turns out that such a single steady core surface flow explicates recent secular change to better than first-order accuracy.

Most of the recent secular variation (SV) indicated by broad-scale spherical harmonic models of the observed geomagnetic field can be explained quantitatively in the SFM/FFC model by a steady surficial core flow (VOORHIES & BENTON, 1983; VOORHIES, 1984, 1986a)—especially when the derived flow is not artificially constrained to induce minimal

narrow-scale SV (VOORHIES, 1986b). Published studies by K. Whaler, J. Bloxham, and others, and unpublished, if not wholly reported, studies by this author and others confirm this fact. In particular, this fact remains when the steady flow is constrained to be, in effect, superficially geostrophic (VOORHIES, 1986d; 1991); when the spatial complexity of the flow is damped (BLOXHAM, 1986, 1987a,b, 1988a, 1989; VOORHIES, 1987b, 1988a; WHALER & CLARKE, 1988); and when both geostrophic and smoothness constraints are enforced (VOORHIES, 1987c, 1988a, 1989, 1991; BLOXHAM, 1987c, 1988b). This fact supports the argument that time-varying flow near the core surface and departures from the SFM/FFC model in the real Earth are of secondary importance in the correct explication of recent SV. It also supports the argument that the flow near the top of the core is similar to the estimated flows—many estimates derived by different groups using diverse methods and models being positively correlated (WHALER, 1990).

These arguments seem provocative enough to motivate an in-depth exposition of both the physical foundation upon which, and the methods by which, this fact is established. Moreover, colleagues suggest that the approach, theory, method, techniques, and results of some recent attempts at steady superficial core flow estimation merit more than mention at meetings. There are, of course, other reasons to investigate steady induction effects in geomagnetism—one being to learn to distinguish such effects from other effects.

Limited quantitative success of a simple interpretation does not prove it correct. Indeed, interpretation of recent SV solely in terms of the SFM/FFC model and steady superficial core flow is in error due to the extreme simplicity of these seemingly relevant idealizations. Nonetheless, the demonstration that these idealizations provide a powerful tool for quantitative explication of SV reflects favorably upon the power of any more realistic core geodynamo model. It also shows that statistically steady superficial core flow can dominate geomagnetic field behavior over appreciable intervals, may thus be a component of a more realistic core geodynamo model, and might even be a minor feature of an accurate core geodynamo theory. The geomagnetic effects of steady motional induction thus merit study not only to establish interesting facts, estimate core surface flow, and provide a fascinating contrast to other possible sources of SV, but because an understanding of these effects is important, and perhaps essential, to the correct interpretation of SV—if not the phenomenon of geomagnetism itself.

1.3 Outline of the Series on Steady Induction Effects

To investigate steady induction effects in geomagnetism, and to help establish the importance of other effects, it is often useful to regard the SFM/FFC model as a first approximation and to treat the supposition of steady superficial core flow as a hypothesis. To test hypotheses against observations it seems appropriate to (a) understand both; (b) develop a satisfactory method for mimicking the relevant data in accord with the hypotheses; (c) apply the method to make quantitative predictions; and (d) subtract predicted from observed values and measure such residuals in units of the estimated uncertainty in the data. This series of papers describes an effort to meet these requirements using relevant geomagnetic field models to test the SFM/FFC earth model (and refinements thereof) and the hypothesis of (piecewise, statistically) steady superficial core flow.

In Part I, the steady surficial core flow estimation problem is examined and solved in the context of the simple SFM/FFC earth model. The present, introductory paper (IA) reviews the basic theory and develops some implications of the steady motions hypothesis. Paper IB develops a method for solving the non-linear inverse motional induction problem posed by the hypothesis of (piecewise, statistically) steady core surface flow and the adoption of an initial geomagnetic condition. This inverse problem is generally non-linear because of the nature of the solution to the forward problem and because neither the models nor the observations they represent are either complete or perfect. Paper IC describes application of this method to the Definitive Geomagnetic Reference Field (DGRF) models (IAGA, 1988). Paper ID presents numerical results of applying the method and conclusions drawn therefrom.

In Part II, the SFM/FFC model is reexamined; errors induced by oversimplifying suppositions are assessed and targeted for systematic elimination. In Part III, the supposition of perfect core conductivity is replaced with that of steady magnetic flux diffusion near the top of a resistive core and an effort to allow for, and indeed estimate, deep mantle electrical conductivity is described.

2. THEORY

Many geomagnetic estimates of core surface motions rely on the magnetic induction equation for a fluid medium of isotropic magnetic permeability μ and steady, isotropic electrical conductivity σ

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \sigma^{-1} [\nabla \times (\mu^{-1} \mathbf{B})] \quad (1a)$$

where \mathbf{B} represents the magnetic flux density vector, $\partial_t \mathbf{B}$ its partial derivative with respect to time t , and \mathbf{v} the fluid velocity vector. The first term on the right of (1a) describes motional induction; the second describes magnetic flux diffusion. The optional appendices offer a derivation of this equation for general readers and an analysis of conditions under which it is a useful summary of the Maxwell equations.

For homogeneous μ and uniform σ (1a) reduces to

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (1b)$$

where $\eta \equiv [\mu\sigma]^{-1}$ represents the magnetic diffusivity and the vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla \nabla \cdot \mathbf{A} - \nabla^2 \mathbf{A}$ has been applied to solenoidal \mathbf{B} .

In spherical polar coordinates (r, θ, ϕ) with orthonormal unit vectors $(\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}})$, let \mathbf{A} represent a vector field with radial component $A_r \equiv \hat{\mathbf{r}} \cdot \mathbf{A}$ and surficial component $\mathbf{A}_s \equiv \mathbf{A} - \hat{\mathbf{r}} A_r$. Let $\nabla_s \cdot$ denote the surface divergence operator on the sphere of radius r : $\nabla_s \cdot \mathbf{A} = \nabla \cdot \mathbf{A} - r^{-2} \partial_r r^2 A_r = [r \sin \theta]^{-1} [\partial_\theta A_\theta \sin \theta + \partial_\phi A_\phi]$. Because $\hat{\mathbf{r}} \cdot [\nabla \times (\mathbf{A} \times \mathbf{Z})] = \nabla_s \cdot [A_r \mathbf{Z}_s - Z_r \mathbf{A}_s]$, the radial component of (1b) is

$$\partial_t B_r + \nabla_s \cdot [B_r \mathbf{v}_s - u \mathbf{B}_s] = \eta r^{-1} \nabla_r^2 B_r \equiv d_r \quad (1c)$$

where B_r and $u \equiv v_r$ represent, respectively, the radial components of \mathbf{B} and \mathbf{v} , \mathbf{B}_s and \mathbf{v}_s the surficial portions of \mathbf{B} and \mathbf{v} , and d_r denotes the radial magnetic flux density diffusion. Equation (1c) is the radial induction equation for uniform, isotropic media. To restore effects of displacement currents, add $\tau_q \partial_t^2 B_r$ to the left of (1c) - where $\tau_q \equiv \epsilon/\sigma$

is the charge relaxation time and ϵ is the uniform, isotropic dielectric permittivity (Appendix A); omission of this term filters out electromagnetic radiation in accord with the quasi-steady approximation.

2.1 Radial Induction at the Top of a Spherical Core

The magnetic flux density vector $\mathbf{B}(\mathbf{r},t)$ and the fluid velocity vector $\mathbf{v}(\mathbf{r},t)$ within the Earth may vary with time t and position vector \mathbf{r} reckoned in geobarycentric spherical polar coordinates (r,θ,ϕ) fixed to the solid Earth; henceforth r is radius, θ is colatitude south from the reference pole of rotation, and ϕ is longitude east from the Greenwich meridian. The upper part of the outer core is treated as a low-viscosity liquid of uniform μ and uniform σ wherein (1) applies. The CMB is approximated by the sphere of radius $b = 3480$ km—the mean PREM core radius (DZIEWONSKI & ANDERSON, 1981). The surrounding mantle is treated as a perfectly rigid, impenetrable, electrical insulator of uniform magnetic permeability wherein, perforce, $\mathbf{v} = 0$ and scaloidal $\mathbf{B} = -\nabla V$. The viscous boundary layer just beneath the CMB is treated simply as a sheet vortex; points on the spherical surface just beneath it lie at the top of the free-streaming core and are assigned positions $\mathbf{r} = \mathbf{b}$. The kinematic boundary condition is applied at \mathbf{b} ; thus $\mathbf{v}(\mathbf{b},t) \cdot \hat{\mathbf{r}} \equiv u(\mathbf{b},t) = 0$, $\mathbf{v}(\mathbf{b},t) = \mathbf{v}_s(\mathbf{b},t)$ is tangent to \mathbf{b} , and (1c) at (\mathbf{b},t) is

$$\partial_t B_r(\mathbf{b},t) + \nabla_s \cdot [B_r(\mathbf{b},t) \mathbf{v}_s(\mathbf{b},t)] = \eta [r^{-1} \nabla^2 B_r^*(\mathbf{r},t)] \Big|_{\mathbf{b}} \equiv d_r(\mathbf{b},t) \quad (2)$$

The asterisk on B_r^* emphasizes that only portions of the field that are not purely scaloidal contribute to d_r .

Across a spherical boundary B_r is continuous, $\partial_r B_r$ can jump with B_s due to a sheet electric current, and $\partial_r^2 B_r$ will jump with the lateral electric current density \mathbf{J}_s (see Appendix A). The radial component of any part of the field which is not purely scaloidal, say B_r^* , vanishes at the spherical base of a source-free mantle, so it is negligible at \mathbf{b} by continuity; however, $\partial_r^2 B_r^*$ will typically be non-trivial at \mathbf{b} , as will $d_r(\mathbf{b},t)$. When the core is treated as a perfect conductor η vanishes and $d_r(\mathbf{b},t)$ is zero—unless a sheet current and jump in $\partial_r B_r^*$ dictate otherwise. Though radial flux diffusion at the top of the free-stream may well account for about 6% of broad-scale secular change (VOORHIES, 1986c; 1988 unpublished, 1989), $d_r(\mathbf{b},t)$ is omitted in part I. The result is the celebrated ROBERTS & SCOTT (1965) equation

$$\partial_t B_r(\mathbf{b},t) + \nabla_s \cdot [B_r(\mathbf{b},t) \mathbf{v}_s(\mathbf{b},t)] = 0 \quad (3)$$

which describes frozen-flux motional induction at the surface of a free-streaming spherical outer core.

2.2 Kinematics

Mass density $\rho(\mathbf{r},t)$ is treated as the sum of a mean state $\rho_0(\mathbf{r})$ a fluctuating perturbation $\rho'(\mathbf{r},t)$. Mass conservation at (\mathbf{r},t) implies

$$\partial_t \rho' + \nabla \cdot [\rho' \mathbf{v}] + \nabla \cdot [\rho_0 \mathbf{v}] = 0 \quad (4)$$

where $\nabla \cdot$ is the full divergence operator. The anelastic approximation

$$\nabla \cdot [\rho_0(\mathbf{r}) \mathbf{v}(\mathbf{r},t)] = 0 \quad (5)$$

filters out high-frequency acoustic radiation in much the same way that omission of displacement currents from the Ampere-Maxwell law filters

out high-frequency electromagnetic radiation. It does not require vertical advection of the mean stratification and is considered more appropriate to the slow outer core flow of interest here than is the supposition of solenoidal flow (VOORHIES, 1987c, 1988 unpublished ms.). Equation (5) differs from equation (1.36) of GUBBINS & ROBERTS (1987) in that advection and flow convergence are allowed to change $\rho'(\mathbf{r}, t)$.

In a spherical outer core, $\rho_o(\mathbf{r}) = \rho_o(r)$, so (5) and the kinematic boundary condition at \mathbf{b} imply incompressible flow at \mathbf{b} : $\nabla \cdot \mathbf{v}|_{\mathbf{b}} = 0$. Then $\nabla_s \cdot \mathbf{v}_s(\mathbf{b}, t) = -\partial_r u(\mathbf{b}, t)$, surficial convergence (or confluence $\nabla_s \cdot \mathbf{v}_s < 0$) implies downwelling ($\partial_r u > 0$), and (3) reduces to

$$\partial_t B_r(\mathbf{b}, t) + \mathbf{v}_s(\mathbf{b}, t) \cdot \nabla_s B_r(\mathbf{b}, t) = B_r(\mathbf{b}, t) \partial_r u(\mathbf{b}, t). \quad (6)$$

This is the usual frozen-flux radial induction equation at \mathbf{b} .

In this frozen-flux core (FFC) approximation, the mean square radial magnetic flux density linking the core changes only when fluid downwelling correlates with the squared radial magnetic flux density. To see this, let $\langle \psi(\mathbf{r}, t) \rangle$ represent the mean value of any scalar field $\psi(\mathbf{r}, t)$ averaged over the sphere of radius r

$$\langle \psi(\mathbf{r}, t) \rangle \equiv (4\pi)^{-1} \int_0^{2\pi} \int_0^\pi [\psi(\mathbf{r}, t)] \sin\theta d\theta d\phi \quad (7)$$

and let $\psi(r, t)^{\text{rms}}$ denote the root mean square value of $\psi(\mathbf{r}, t)$ averaged over the sphere of radius r , $\langle \psi(\mathbf{r}, t)^2 \rangle^{1/2}$. Note that $\langle \nabla_s \cdot \mathbf{A} \rangle = 0$ for any vector field $\mathbf{A}(\mathbf{r}, t)$ with single-valued differentiable components. Now consider the mean square radial field averaged over \mathbf{b}

$$(4\pi)^{-1} \int_0^{2\pi} \int_0^\pi [B_r(\mathbf{b}, t)]^2 \sin\theta d\theta d\phi \equiv \langle [B_r(\mathbf{b}, t)]^2 \rangle \equiv [B_r(\mathbf{b}, t)^{\text{rms}}]^2. \quad (8)$$

The value of (8) indicated by a finite set of Gauss coefficients is easily calculated in the SFM model (LOWES, 1966; VOORHIES, 1984). With (3) and $\nabla_s \cdot \mathbf{v}_s(\mathbf{b}, t) = -\partial_r u(\mathbf{b}, t)$ the time derivative of (8) is

$$\begin{aligned} \langle 2B_r(\mathbf{b}, t) \partial_t B_r(\mathbf{b}, t) \rangle &= \langle 2B_r(\mathbf{b}, t) [B_r(\mathbf{b}, t) \partial_r u(\mathbf{b}, t) - \mathbf{v}_s(\mathbf{b}, t) \cdot \nabla_s B_r(\mathbf{b}, t)] \rangle \\ &= \langle 2B_r^2 \partial_r u - \nabla_s \cdot [B_r^2 \mathbf{v}_s] + B_r^2 \nabla_s \cdot \mathbf{v}_s \rangle \\ &= \langle [B_r(\mathbf{b}, t)]^2 \partial_r u(\mathbf{b}, t) \rangle. \end{aligned} \quad (9)$$

Indeed, at (\mathbf{b}, t) we have $2\langle B_r \partial_t B_r \rangle = \langle B_r^2 \partial_r u \rangle = 2\langle B_r \mathbf{v}_s \cdot \nabla_s B_r \rangle$; yet the contribution to (9) from lateral advection is one half, but opposite, that from downwelling.

In the special case of purely toroidal flow at the surface of a FFC $\mathbf{v}_s(\mathbf{b}, t) = \mathbf{v}_T(\mathbf{b}, t) \equiv [\nabla_s T(\mathbf{b}, t)] \times \hat{\mathbf{r}}$, where $-T$ is the streamfunction; then $\nabla_s \cdot \mathbf{v}_T(\mathbf{b}, t) = 0$, there is no downwelling, and the right-hand side of (6) and (9) vanish with $\partial_r u(\mathbf{b}, t)$. Any purely toroidal flow at the surface of a FFC therefore conserves the mean square radial flux density averaged over the surface. This is obviously so for rigid rotations, but also holds for arbitrarily complicated toroidal flows—allowing a quick check on the compatibility of estimates of $B_r(\mathbf{b}, t)$ with the

hypotheses of purely toroidal FFC surface flow. Alternately, if the flow is toroidal, then there is a second way to locate the core magnetically: seek the sphere upon which $\langle B_r(\mathbf{r}, t)^2 \rangle$ is unchanged over some time interval. (The first way is described by HIDE & MALIN (1981), VOORHIES & BENTON (1982), VOORHIES (1984), BENTON & VOORHIES (1987)). Mean square radial field conservation is also easy to impose on models of the evolving geomagnetic field; such biased models might be useful for seeking purely toroidal flows. Of course, a geomagnetic field model constrained to be smooth in a well-defined sense that also yields small values of (9) will, perhaps unintentionally, be more compatible with the no-upwelling hypothesis than models that are not so constrained.

It is clear that supposition of sphericity is not needed (BACKUS, 1968) to equate the time rate of change of the mean square magnetic flux density normal to the closed surface of a perfect fluid conductor bounded by a rigid, impenetrable exterior with the mean value of the product of the squared normal flux density and the surface convergence of the surficial fluid velocity averaged over the surface.

2.3 Simplified Dynamics

The dynamics of the outer core is a subject of debate; however, a reasonable approximation to the equation of fluid motion therein is

$$\rho[\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \partial_t \Omega \times \mathbf{r} + 2\Omega \times \mathbf{v}] = \nabla \cdot \Pi + \rho \mathbf{g}_e + \mathbf{J} \times \mathbf{B} \quad (10)$$

where Ω is the bulk angular velocity of the solid earth; Π is the hydrodynamic stress tensor (including pressure and bulk and shear viscous stresses) and $\nabla \cdot \Pi$ is its divergence; $\mathbf{g}_e = -\nabla[\Phi_N - 0.5|\Omega \times \mathbf{r}|^2]$ is the effective gravitational acceleration for Newtonian potential Φ_N and centrifugal potential $-0.5|\Omega \times \mathbf{r}|^2$; and \mathbf{J} is the electric current density (from A3d). Physically, $\nabla \cdot \Pi$ is the macroscopic representation of net microscopic electromagnetic force densities and the Lorentz force density $\mathbf{J} \times \mathbf{B}$ represents the divergence of macroscopic magnetic stresses (see, e.g., VOORHIES, 1991).

The mean state of the outer core is taken to be the equilibrium between forces caused by hydrostatic pressure and by gravitational attraction and centrifugal effects: $\nabla p_0(\mathbf{r}) \equiv \rho_0(\mathbf{r}) \mathbf{g}_e(\mathbf{r})$, where ∇ is the gradient operator and p_0 is $-1/3$ the trace of diagonal Π for a Newtonian fluid at rest in the reference frame rotating at Ω_0 . This mean state is subtracted from (10). High-frequency fluctuations in Ω and tidal effects are omitted (in Part I). For a modest to weak magnetic field, scale analyses of the residual perturbation momentum equation in the upper part of the core suggest that, just beneath a thin (8 cm) viscous sub-layer, the relative, advective, and precessional pseudo-force densities on the left and viscous, centrifugal, and Lorentz effects on the right of (10) contribute little to the primarily geostrophic balance between Coriolis, perturbation pressure, and radial buoyancy forces (see, e.g., HILLS, 1979; LEMOUEL, 1984; BENTON, 1985; VOORHIES, 1991). Then with $\rho' \ll \rho_0$, omission of perturbations in \mathbf{g}_e , and $\mathbf{g}_e \approx -\hat{\mathbf{r}}g$,

$$2\rho_0 \Omega_0 \times \mathbf{v}' + \nabla p' + \rho' \hat{\mathbf{r}} g \approx 0 \quad (11)$$

where the prime indicates the (non-tidal) perturbation relative to the mean state. A modest magnetic field at the top of the free-stream is consistent with an effectively SFM, but need not imply a weak field at

depth (e.g., below a 90-km-thick magnetic boundary layer). With $\Omega(t) = \Omega_0 = \Omega_0(\hat{r}\cos\theta - \hat{\theta}\sin\theta)$, the radial component of the curl of (11) evaluated at \mathbf{b} reduces to the geostrophic radial vorticity balance

$$\partial_r u \cos\theta + \frac{v}{b} \sin\theta \approx 0 \quad (12)$$

which has long been used to constrain some numerical estimates of core surface flow. Physically, (12) requires downwelling to be accompanied by poleward flow except at the equator (where $v = 0$) and at the poles (where $\partial_r u = 0$) (VOORHIES, 1987d). Note that (12) approximates the full radial vorticity equation under somewhat less restrictive conditions than those under which (11) follows from (10) (BENTON, 1985; VOORHIES, 1991). Flows obeying (12) are "surficially geostrophic" and include tangentially geostrophic flows defined by BACKUS & LEMOUEL (1986).

3. STEADY MOTIONAL INDUCTION: THE FORWARD PROBLEM

For steady flow, (3) reduces to

$$\partial_t B_r(\mathbf{b}, t) + \nabla_s \cdot [B_r(\mathbf{b}, t) \mathbf{v}_s(\mathbf{b})] = 0 \quad (13a)$$

For steady anelastic flow, (6) reduces to

$$\partial_t B_r(\mathbf{b}, t) + \mathbf{v}_s(\mathbf{b}) \cdot \nabla_s B_r(\mathbf{b}, t) = B_r(\mathbf{b}, t) \partial_r u(\mathbf{b}). \quad (13b)$$

Equations (13) also describe frozen-flux motional induction during an interval $t_0 \leq t \leq t_f$ when the flow is steady in the statistical sense: when correlations between fluctuations about short time averages of \mathbf{v}_s and B_r contribute negligibly to short time averages of $\partial_t B_r$ and the short time averages of \mathbf{v}_s do not vary during the long time interval $[t_0, t_f]$ (VOORHIES, 1986a, section 2.2). A flow that is statistically steady during consecutive intervals is piecewise statistically steady; though \mathbf{v}_s may change between intervals, (13) holds within each interval. Effectively piecewise steady flow might arise physically when long intervals of statistically steady flow, hence statistically balanced forces, are punctuated by rapid shifts to a new flow configuration.

Given initial condition $B_r(\mathbf{b}, t_0)$ and $\mathbf{v}_s(\mathbf{b})$ the forward solution to (13) is of transcendental exponential operator form [Voorhies, 1986b]

$$B_r(\mathbf{b}, t) = \left\{ e^{-\frac{(t-t_0)}{b} \nabla_s \cdot \mathbf{v}_s(\mathbf{b})} \right\} \circ \{ B(\mathbf{b}, t_0) \} \quad (14a)$$

or, perhaps more clearly,

$$B_r(\mathbf{b}, t) = \left\{ e^{-\frac{(t-t_0)}{b} \nabla_s \cdot [\mathbf{v}(\mathbf{b})]} \right\} \circ B(\mathbf{b}, t_0). \quad (14b)$$

This is a special case of the general solution to forward steady motional induction problems presented by HOYNG (1985). To derive (14) time differentiate (13a) and back substitute $k-1$ times:

$$\partial_t^2 B_r + \nabla_s \cdot [\mathbf{v}_s \partial_t B_r] = \partial_t^2 B_r - \nabla_s \cdot [\mathbf{v}_s \{ \nabla_s \cdot [\mathbf{v}_s B_r] \}] = 0 \quad (15a)$$

$$\partial_t^3 B_r + \nabla_s \cdot [\mathbf{v}_s \partial_t^2 B_r] = \partial_t^3 B_r + \nabla_s \cdot [\mathbf{v}_s (\nabla_s \cdot [\mathbf{v}_s (\nabla_s \cdot [\mathbf{v}_s B_r])])] = 0 \quad (15b)$$

$$\partial_t^k B_r = (-1)^k (\nabla_s \cdot [\mathbf{v}_s])^k B_r \quad (15c)$$

where (15c) follows by logical induction. Equation (15c) at t_0 is substituted into the Taylor series expansion for $B_r(\mathbf{b}, t)$ about t_0

$$\begin{aligned} B_r(\mathbf{b}, t) &= \sum_{k=0}^{\infty} \frac{1}{k!} (t-t_0)^k \partial_t^k B_r(\mathbf{b}, t_0) \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (t-t_0)^k (\nabla_s \cdot [\mathbf{v}_s])^k B_r(\mathbf{b}, t_0) \\ &= \sum_{k=0}^{\infty} \frac{1}{k!} \{- (t-t_0) \nabla_s \cdot [\mathbf{v}_s]\}^k B_r(\mathbf{b}, t_0) \\ &= \{ e^{- (t-t_0) \nabla_s \cdot [\mathbf{v}_s(\mathbf{b})]} B_r(\mathbf{b}, t_0) \} \end{aligned}$$

which is (14b) or, with the transcendental exponential operator taking precedence over the surface divergence operator, (14a).

One feature of evolution equation (14) is that action of a given steady surface flow on different initial radial magnetic flux conditions induces predicted flux configurations which can diverge exponentially with time in some regions and converge exponentially with time in other regions. Consider the iterative map obtained by forward numerical solution of the steady motional induction problem (13b) with an arbitrarily small time step δt

$$B_r(\mathbf{b}, t+\delta t) = B_r(\mathbf{b}, t) + [\partial_t B_r(\mathbf{b}, t)] \delta t. \quad (16)$$

With time measured in units of δt

$$B_r(\mathbf{b}, t+1) = B_r(\mathbf{b}, t) + \partial_t B_r(\mathbf{b}, t) \quad (17a)$$

$$= B_r(\mathbf{b}, t) - \mathbf{v}(\mathbf{b}) \cdot \nabla_s B_r(\mathbf{b}, t) - B_r(\mathbf{b}, t) \nabla_s \cdot \mathbf{v}(\mathbf{b}) \quad (17b)$$

$$\frac{dB_r(\mathbf{b}, t+1)}{dB_r(\mathbf{b}, t)} = 1 - \nabla_s \cdot \mathbf{v}(\mathbf{b}) = 1 + \partial_r u \quad (17c)$$

and the Liapunov exponent is

$$L \equiv \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \ln \left| \frac{dB_r(\mathbf{b}, t+1)}{dB_r(\mathbf{b}, t)} \right| = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N \ln |1 + \partial_r u| = \ln |1 + \partial_r u|. \quad (18)$$

Restoration of the original measure of time yields

$$L = \ln |1 + \delta t \partial_r u|. \quad (19)$$

The sign of the Liapunov exponent is the sign of the downwelling!

For downwelling $\partial_r u > 0$, $L > 0$, and chaotic behavior is indicated. Downwelling draws in magnetic field-line footpoints; basins of downwelling are field-line footpoint attractors. For upwelling $\partial_r u < 0$, $L < 0$, and 'normal' behavior is indicated. Upwelling blows away field line footpoints; domes of upwelling are field-line footpoint repulsors (VOORHIES, 1987e). L is zero for purely toroidal flow.

Downwelling at a stagnation point ($\mathbf{v}_s = 0$) causes exponential growth of B_r with time constant $[\partial_r u]^{-1}$; a strongly magnetized unipolar region called a core spot (Benton & Voorhies, 1981 discussion) or a flux bundle or spot (BLOXHAM & GUBBINS, 1985) may form in an area of downwelling. Upwelling at a stagnation point causes exponential decay of B_r ; a weakly magnetized region may form in an area of upwelling. Elsewhere the evolution of B_r is complicated by lateral advection: laterally variable downwelling may strengthen horizontal gradients $\nabla_s B_r$ whose advection by \mathbf{v}_s can lead to chaotic field behavior.

If \mathbf{v}_s is a rigid rotation (e.g., bulk westward flow), then $B_r(\mathbf{b}, t)$ is periodic in time. Between extremes is the quasi-periodic regime characterized by weak meridional shear, non-zonal flow components, and regions of up- and downwelling embedded in a bulk flow. For example, an existing core spot advected through a mild upwelling spreads out and weakens, then intensifies upon encountering a mild downwelling and, if not absorbed or disrupted downstream, eventually circumnavigates the CMB to reencounter the same regions of up- and downwelling. Finally, advection of very steep lateral gradients in B_r produced by vigorous downwelling plumes can yield chaotic field behavior. The steady surficial motional induction problem is apparently a fine, easily visualized example of deterministic chaos which, depending on \mathbf{v}_s , exhibits: extreme sensitivity to initial magnetic conditions; periodic, quasi-periodic, and/or non-periodic behavior; fixed attractive or repulsive stagnation points with associated basins of attraction or repulsion; and regionally chaotic behavior (VOORHIES, 1988b).

4. SOME EFFECTS OF PERSISTENT, SURFICIALLY GEOSTROPHIC FLOW

An interesting conceptual view of secular change results from supposing core surface flow is both steady and surficially geostrophic. With $\partial_r u = -v \tan \theta / b$, chaos may rule in regions of poleward flow. Because downwelling, which may form core spots, is accompanied by poleward flow, I expect poleward drifting core spots. Conversely, regions of low radial flux density formed by upwelling shift towards lower latitudes. The implied flux partitioning mechanism can create an axial dipole moment from a non-dipole field with a non-axisymmetric component; moreover, persistent, surficially geostrophic motional induction can cause this axial dipole to grow and fluctuate.

To see how persistent, surficially geostrophic flow can create and fortify an axial dipole moment, note that the time rate of change of the axial dipole coefficient for a conventional Schmidt-normalized spherical harmonic expansion of the scalar geomagnetic potential near Earth's surface ($r = a = 6.3712$ Mm) is

$$\partial_t g_1^0 = \frac{3}{2} \langle \partial_t B_r(\mathbf{a}, t) \cos \theta \rangle. \quad (20a)$$

With either (6) or (13b) just beneath a SFM,

$$\begin{aligned}\partial_t g_1^0 &= \frac{3b^3}{2a^3} \langle \partial_t B_r(\mathbf{b}, t) \cos\theta \rangle = \frac{-3b^3}{2a^3} \langle \nabla_s \cdot (B_r \mathbf{v}_s \cos\theta) + B_r v \sin\theta / b \rangle \\ &= \frac{-3b^3}{2a^3} \langle B_r v \sin\theta / b \rangle.\end{aligned}\quad (20b)$$

Frozen-flux variations in the axial dipole require net transport of radial field towards the rotation axis, hence poleward flow. Surficially geostrophic poleward flow requires downwelling (12), so

$$\partial_t g_1^0 = \frac{3b^3}{2a^3} \langle B_r \cos\theta \partial_r u \rangle.\quad (20c)$$

The time rate of change of the axial dipole is proportional to the mean product of the downwelling and the axial projection of the radial field.

Pressure perturbations are single valued, so the horizontal components of (11) imply that $\partial_r u$ has no axisymmetric component; therefore, only the non-axisymmetric part of the radial field B_r^{na} contributes to (20c) for tangentially geostrophic flow. Tangentially geostrophic downwelling where $B_r^{na} \cos\theta \leq 0$ (or ≥ 0) decreases (or increases) g_1^0 . This can create a normal (or reversed) polarity axial dipole from a purely non-dipolar field with a non-axisymmetric component. This axial dipole will grow until the correlation between $B_r^{na} \cos\theta$ and the downwelling changes sign. For steady flow this could be quite some time—roughly half the time required for B_r^{na} to change sign. Indeed, then $\partial_t^2 g_1^0 = 3b^3 \langle B_r \mathbf{v}_s \cdot \nabla_s \partial_r u \cos\theta \rangle / 2a^3$.

It is suggested that surficially geostrophic downwelling in a like polarity core spot intensifies the spot and shifts it poleward, strengthening the axial dipole; upwelling in a like polarity core spot disperses its flux and shifts it equatorward, weakening the axial dipole. Conversely, downwelling in a reversed flux patch intensifies it and shifts it poleward, weakening the axial dipole; upwelling in a reversed flux patch disperses it and shifts it equatorward, strengthening the axial dipole (VOORHIES, 1987d). If such spots or patches drift westward, alternately encountering regions of steady (or merely persistent) upwelling and downwelling, then quasi-periodic dipole oscillations are expected.

Because there is no downwelling at the poles, a core spot should neither form at nor reach the highest latitudes—which should then be regions of weak radial field. Because downwelling implies poleward flow, there can be no downwelling at a stagnation point—except at the equator. Runaway intensification of a core spot is therefore limited by the tendency for the flux to shift poleward (towards regions of weaker downwelling). Downwelling at an equatorial stagnation point appears problematic: though flux advection from adjacent latitudes vanishes with v , an equatorial 'dot' might develop from weak equatorial flux density after a few times $[\partial_r u]^{-1}$. Yet such a point feature can develop no absolute flux linkage as it has no area, its source region (the equator) has no area, and downwelling just off the equator is accompanied by the

usual poleward flow. Moreover, Lorentz forces and magnetic diffusion likely become important near such a dot in violation of (12) and (3).

Suppose a basically non-axisymmetric configuration of radial field, perhaps established by a previous episode of convective/diffusive flux expulsion, is subjected to a persistent, surficially geostrophic flow which is partly non-zonal. The expected value of (20c) may be zero, but the chance of (20c) being exactly zero (of there being no correlation between axial field $B_r \cos\theta$ of one polarity and the downwelling) is nil. So suppose areas of downwelling correlate initially with areas of normal polarity. Then normal polarity flux is attracted by basins of downwelling, tending to form core spots, but is also shifted poleward, forming high-latitude core spots. The poleward shift inhibits runaway intensification of core spots and thus runaway growth of the mean square radial field. It seems unlikely that all the flux would end up in an axisymmetric configuration; indeed, as normal polarity field-line footpoints become stranded poleward of the downwelling foci, dipole growth should slow. It seems more likely that flux partitioning would continue until some maximum eligible fraction of the normal polarity flux has been shifted poleward. Roughly equal partitioning of the mean square radial field between dipole and non-dipole configurations might be a preferred, possibly metastable state; however, this has not yet been demonstrated and continued downwelling might transport reversed flux poleward. Perhaps more importantly, nearly zonal advection of high-latitude core spots through the high-latitude fringes of alternating regions of up- or downwelling would induce quasi-periodic dipole oscillations. With a few core spots and a few foci of up- and downwelling, local effects may dominate dipole oscillations at the CMB. Yet the global dipole oscillations would appear relatively more prominent at Earth's surface.

This simple picture can be enriched by magnetic flux expulsion: the entrainment of toroidally magnetized fluid into the magnetic boundary layer, toroidal to poloidal field conversion by the implied laterally heterogeneous vertical motion, and diffusion across the CMB. Weak field flux expulsion might occur at the low latitudes favored by surficially geostrophic upwelling (estimates mapped and posted by VOORHIES (1988a) show foci of steady surficially geostrophic up- and downwelling to be confined to within about $\pm 30^\circ$ of the equator) and perhaps at the high latitudes seemingly favored by density and angular momentum poor plumes from the inner core boundary. So the simple scenario should be modified to include radial flux diffusion $d_r(\mathbf{b}, t)$ (which may also help regulate core spot amplitudes by eliminating strong curvature $r^{-1}\nabla^2 r B_r$) and effects of the strong field with appreciable curl expected at depth. Sufficiently vigorous vertical advection of such fields may generate Lorentz forces that violate even surficial geostrophy and may thus change such a flow. Yet such changes may but punctuate intervals of ordinary convective vigor when the Lorentz forces implied by warping a strong toroidal field tend to oppose the driving vertical motion. Then the simple scenario may commonly hold to a fair approximation.

To help complete the simple picture, suppose that despite such opposing forces, vertical motion at depth grows locally strong enough to not only thin the magnetic boundary layer, but to bring toroidally magnetized fluid close to the CMB at some intermediate latitude; then toroidal to poloidal field conversion by vertical motion and flux diffusion can form core spot pairs. If, upon restoration of a

surficially geostrophic balance, the spot foci remain in the region of upwelling, then such pairs should weaken and drift equatorward. Though such spot pairs are distinguished from the unipolar core spots discussed above, the distinction may be artificial. Low-latitude flux expulsion and ageostrophic effects may form a spot pair that splits into different hemispheres; such spots should be repelled by the upwelling. Subsequent restoration of a surficially geostrophic balance and interactions with regions of surficially geostrophic downwelling may lead to further separation of the originally paired spots and the formation of seemingly unrelated unipolar regions.

In the long view, a statistically steady, surficially geostrophic flow regime characterized by quasi-periodic dipole oscillations could be punctuated by vigorous convection (overstability being preferred in fluids with small Prandtl and magnetic Prandtl numbers (CHANDRASEKHAR, 1961) and emergence of one or more bipolar regions. If these spot pairs emerge at low latitude and drift into hemispheres of (either like or) opposing polarity, subsequent flux partitioning upon restoration of a new, persistent, surficially geostrophic flow may dramatically alter the dipole moment. Indeed, if enough flux of opposing polarity were involved, this could appear as an excursion, or even a reversal, of the geomagnetic axial dipole.

The simple scenario of approximately steady, frozen-flux, surficially geostrophic core flow predicts: (1) low-latitude regions of low flux (and occasional spot pairs); (2) high-latitude unipolar core spots; (3) polar regions devoid of spots; and (4) a strong axial dipole. Many maps of the broad-scale radial field at the CMB (constructed by many workers since BOOKER [1969]) reveal such features. It follows that this scenario deserves closer scrutiny. To this end, and in search of a quantitatively acceptable explication of recent SV, subsequent papers document an effort to describe broad-scale models of the slowly varying observed geomagnetic field in terms of (piecewise, statistically) steady, optionally surficially geostrophic, core surface motions.

5. SUMMARY

The simple source-free mantle/frozen-flux core (SFM/FFC) earth model has been used to review and extend the kinematic and elementary dynamic theory of geomagnetic secular change. The ROBERTS & SCOTT [1965] equation was used to prove that the mean square radial magnetic flux density averaged over a FFC, $\langle B_r(\mathbf{b}, t)^2 \rangle$, changes if and only if fluid downwelling, $-\nabla_s \cdot \mathbf{v}_s(\mathbf{b}, t) = \partial_r u(\mathbf{b}, t)$, correlates with squared radial magnetic flux density, and is thus conserved by any purely toroidal FFC surface flow. For a SFM, the contribution from the broad-scale portion of the radial magnetic flux density to $\langle B_r(\mathbf{b}, t)^2 \rangle$ can be easily estimated from broad-scale models of the evolving geomagnetic field and used to check the compatibility of such models with the no-upwelling hypothesis.

The kinematical forward problem posed by steady motional induction at the top of a FFC was solved analytically (albeit not in closed form). The sign of the Liapunov exponent describing evolution of the radial magnetic flux density $B_r(\mathbf{b}, t)$ was shown to be the sign of the downwelling $\partial_r u(\mathbf{b})$ —raising the possibility of steady motional induction of geomagnetic chaos. Steady motional induction at the top of a FFC was found to be a fine, easily visualized example of deterministic chaos

which, depending on $\mathbf{v}_s(\mathbf{b})$, exhibits extreme sensitivity to initial geomagnetic conditions; periodic, quasi-periodic, and non-periodic field behavior; fixed basins of magnetic field-line footpoint attraction or repulsion surrounding fixed stagnation points; and regionally chaotic field behavior.

Downwelling implies poleward flow for surficially geostrophic core flow. Some aspects of persistent, surficially geostrophic core motions were described, including the flux partitioning mechanism whereby the formation and poleward drift of core spots in regions of fluid downwelling can create and fortify an axial dipole from a field that is at least partly, and could be entirely, non-axisymmetric. Features expected to result from persistent, surficially geostrophic flow are evident in many mapped estimates of the broad-scale radial field near the top of the core, and may be present in records of paleomagnetic secular variation. However, magnetic flux diffusion, Lorentz forces, and time-dependent flow are expected to play key roles in the correct interpretation of very long term SV and geomagnetic reversals.

In one long-term scenario, intervals of persistent, partly non-axisymmetric, surficially geostrophic core flow characterized by quasi-periodic global dipole oscillations and regionally chaotic field behavior are intermittently punctuated by major changes of the flow in the outer core. The instability envisioned is of internal origin—a natural consequence of thermo-compositional convection within a rapidly rotating, roughly spherical annulus of strongly magnetized liquid metal-alloy subject to very slowly changing, inhomogeneous cooling from above (due to thermal instability of the deep mantle) and a very slowly changing, heterogeneous flux of buoyant material from below (due to condensation of the inner core). It is characterized by uncommonly strong poloidal flow within the upper core, entrainment of toroidally magnetized fluid into the magnetic boundary layer (say the upper 90 km of the core), conversion of toroidal to poloidal field by the vertical motion, flux diffusion across the CMB, and formation of core spot pairs—particularly, but by no means entirely, at the low latitudes favored by surficially geostrophic flow. There may be many such pairs which split ageostrophically; if enough spots of sufficient flux happen to drift towards, if not into, hemispheres of opposing polarity, then the process appears to be either an excursion or a reversal of the axial dipole. Relaxation to a new pattern of persistent, partly non-axisymmetric, surficially geostrophic core flow leads to intensification and poleward drift of such spots, formation of unipolar core spots at high latitudes and reestablishment of an axial dipole. The polarity of the new dipole is the polarity of the spots which happen to correlate with the new, surficially geostrophic pattern of downwelling.

This is of course but one of many conceivable scenarios in need of development and, more importantly, testing. Many paleomagnetic data and geomagnetic data can be brought to bear upon speculations, properly framed hypotheses about the core, and geodynamo theory in general if the physical suppositions, mathematical means, and numerical methods needed to connect hypotheses and observations are developed. In order to test simple hypotheses such as (statistically) steady, piecewise steady, or persistent surficially geostrophic flow and, perhaps more importantly, in search of a quantitatively acceptable explication of recent SV, a method has been developed to connect the SFM/FFC kinematics of secular

change to broad-scale models of the evolving geomagnetic field. This method is described in the next paper.

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APPENDIX A

The magnetic induction equation, though familiar to some, is not as widely known as can be hoped. Moreover, some textbook derivations of it (e.g., ROBERTS, 1967; JACKSON, 1975; GUBBINS & ROBERTS, 1987) skip a few points of interest in geomagnetism; in particular, a common rationale for using Ampere's law is inadequate for some geomagnetic purposes. So we return to the differential form of the macroscopic Maxwell equations.

Let q represent the electric charge density scalar (with dimensions of C/m^3), \mathbf{E} the electric field vector (V/m), \mathbf{D} the electric displacement vector (C/m^2), \mathbf{J} the electric current density vector (A/m^2), \mathbf{B} the magnetic flux density pseudo-vector (T), and \mathbf{H} the magnetic field strength pseudo-vector (A/m). The Maxwell equations describing the position \mathbf{r} and time t -dependent electromagnetic field are then

$$\nabla \cdot \mathbf{D} = q \quad (\text{A1a}) \qquad \qquad \qquad \nabla \cdot \mathbf{B} = 0 \quad (\text{A1b})$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad (\text{A1c}) \qquad \qquad \qquad \nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D} \quad (\text{A1d})$$

where ∂ denotes partial differentiation with respect to the ensuing subscripted variable, $\nabla \cdot$ the divergence operator, and $\nabla \times$ the curl operator. The divergence of the Ampere-Maxwell law (A1d) is charge conservation

$$\partial_t q + \nabla \cdot \mathbf{J} = 0. \quad (\text{A1e})$$

In the reference frame K' moving with a physical medium the macroscopic constitutive relations are

$$\mathbf{B}' = \underline{\underline{\mu}} \cdot \mathbf{H}' \quad (\text{A2a}) \qquad \qquad \mathbf{D}' = \underline{\underline{\epsilon}} \cdot \mathbf{E}' \quad (\text{A2b}) \qquad \qquad \mathbf{J}' = \underline{\underline{\sigma}} \cdot \mathbf{E}' \quad (\text{A2c})$$

where $\underline{\underline{\mu}}(\mathbf{r}', t')$ represents the second-rank magnetic permeability tensor (H/m), $\underline{\underline{\epsilon}}(\mathbf{r}', t')$ the dielectric permittivity tensor (F/m), and $\underline{\underline{\sigma}}(\mathbf{r}', t')$ the electrical conductivity tensor (S/m) of the medium. In (A2) the dot product \cdot indicates the first-rank result of a second-rank tensor operating on a first rank vector (or pseudo-vector). For non-linear media $\underline{\underline{\mu}}$, $\underline{\underline{\epsilon}}$, and $\underline{\underline{\sigma}}$ depend implicitly on \mathbf{H}' and \mathbf{E}' . For steady media $\underline{\underline{\mu}}$, $\underline{\underline{\epsilon}}$, and $\underline{\underline{\sigma}}$ are independent of t' . For homogeneous media $\underline{\underline{\mu}}$, $\underline{\underline{\epsilon}}$, and $\underline{\underline{\sigma}}$ are independent of \mathbf{r}' . Because \mathbf{H}' and \mathbf{E}' are generally functions of both \mathbf{r}' and t' a medium which is steady, or homogeneous, or both (i.e., uniform) will generally be linear. For isotropic media $\underline{\underline{\mu}}$, $\underline{\underline{\epsilon}}$, and $\underline{\underline{\sigma}}$ are diagonal matrices, each with three identical elements, and are described by the scalars μ , ϵ , and σ . In an isotropic medium, the speed of light is $c = (\mu\epsilon)^{-1/2}$, the charge relaxation time is $\tau_q = \epsilon/\sigma$, and the magnetic diffusivity is $\eta = (\mu\sigma)^{-1}$.

Jump conditions across the interface between two different media are derived from the integral forms of (A1) (see, e.g., JACKSON, 1975). Let $\hat{\mathbf{n}}$ be the unit vector normal to the interface, $\mathbf{A}_n \equiv \mathbf{A} \cdot \hat{\mathbf{n}}$, and $\mathbf{A}_s \equiv \mathbf{A} - \mathbf{A}_n \hat{\mathbf{n}}$. Then these conditions are continuity of B_n and \mathbf{E}_s and jumps $\{D_n\} = \Sigma$ and $\{H_s\} = \mathbf{C} \times \hat{\mathbf{n}}$, where Σ is the idealized surface charge at the interface and \mathbf{C} is the idealized surface current at the interface. Given $\underline{\underline{\mu}}$, $\underline{\underline{\epsilon}}$, $\underline{\underline{\sigma}}$, and time-dependent boundary conditions, suitable initial conditions at $t' = 0$ for solving (A1, A2) are $\mathbf{E}'(\mathbf{r}', 0)$ and $\mathbf{B}'(\mathbf{r}', 0)$.

For a medium moving with uniform velocity \mathbf{u} relative to an inertial reference frame, the comoving frame K' is inertial and equations (A1) and (A2) are implicitly covariant under the special relativistic Lorentz transformation (see, e.g., JACKSON, 1975). For the outer core, the fluid velocity \mathbf{v} is not uniform so K' is not inertial. Moreover, \mathbf{v} is defined relative to the non-inertial reference frame K fixed to the imperfectly rigid, solid Earth which is in non-uniform rotational and orbital motion relative to an inertial reference frame.

The portions of the electromagnetic field of geophysical interest are described qualitatively by their length scale of spatial variation λ , time scale of temporal variation τ , and typical speed $U \equiv \lambda/\tau \sim |\mathbf{v}|$. Slow changes of the geomagnetic field attributed to core sources are described by $\lambda \leq 2\pi b = 22 \text{ Mm}$ and $\tau \geq 1 \text{ year} = 32 \text{ Ms}$, so $U \leq 0.69 \text{ m/s}$. Typically $\lambda \geq \lambda_0 \approx 2\pi b/14 = 1.6 \text{ Mm}$, $\tau \geq \tau_0 \approx 10^2 \text{ years}$, and $U \approx 5 \times 10^{-4} \text{ m/s}$. Clearly U is much less than the speed of sound $c_s \approx 10^4 \text{ m/s}$ within the Earth and the vacuum speed of light $c_0 = 3.0 \times 10^8 \text{ m/s}$; however, Earth is not vacuum, so geophysical scale analyses comparing U with c_0 may be irrelevant. If $\epsilon_0 \leq \epsilon \leq 10^{-2} \text{ F/m}$ and if $\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$, then $c \geq 9 \times 10^3 \text{ m/s} \gg U$; but do we know ϵ ? Because hydrodynamic stress and key bulk material properties (elastic moduli) used to describe sound are macroscopic representations of microscopic electromagnetic interactions between particles, sound is an electromagnetic phenomenon. So the top speed at which information can propagate electromagnetically within the Earth is not less than the sound speed. Then $c \geq c_s \gg U$ and progress can be made even in the absence of measured ultra-low frequency values for ϵ and μ of deep-Earth materials.

With $U \ll c \leq c_0$ the general transformation from K' to K should approach the Lorentz transformation which, in turn, approaches the Galilean transformation $\mathbf{B}' = \mathbf{B}$ and $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$; moreover, $\mathbf{J}' = \mathbf{J} - q\mathbf{v}$ and charge invariance imply $q' = q$. Scale analysis of Faraday's law (A1c) yields $|\mathbf{E}'| \sim \lambda |\mathbf{B}'|/\tau = U |\mathbf{B}'|$, so the Ampere-Maxwell law (A1d) is approximated by Ampere's law ($\nabla \times \mathbf{H} = \mathbf{J}$) because the displacement current is allegedly small compared with the curl of the field strength: $|\partial_t \mathbf{D}'|/|\nabla \times \mathbf{H}'| \sim \epsilon |\mathbf{E}'| \mu \lambda / |\mathbf{B}'| \tau \sim (U/c)^2 \ll 1$ (ROBERTS, 1967). This should hold even with $c \ll c_0$; yet $|\partial_t \mathbf{D}'|/|\nabla \times \mathbf{H}'| = 1$ in regions where $\mathbf{J} = \mathbf{0}$ and the geomagnetic field is supposedly scaloidal. Conditions under which Ampere's law is useful are offered in Appendix B: for steady ϵ and σ it is shown that τ must be very much greater than τ_q for this ultra-low frequency, electromagnetically quasi-steady approximation to be useful.

For electrically neutral media $q = 0$ and $\mathbf{J}' = \mathbf{J} - q\mathbf{v} = \mathbf{J}$. More generally, $|q\mathbf{v}| = |(\nabla \cdot \epsilon \mathbf{E})\mathbf{v}| \sim \epsilon |\mathbf{E}| U/\lambda \sim U^2 |\mathbf{B}| \epsilon/\lambda$ and scale analysis of Ampere's law yields $|\mathbf{B}'|/\mu \lambda \sim |\mathbf{J}'|$; therefore, $|q\mathbf{v}|/|\mathbf{J}'| \sim U^2/c^2$ and \mathbf{J}' is approximated by \mathbf{J} (ROBERTS, 1967). This holds even with $c \ll c_0$.

These approximations yield the pre-Maxwell equations appropriate to the magnetohydrodynamics of an isotropic medium

$$\nabla \cdot \epsilon \mathbf{E} = q \quad (\text{A3a})$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{A3b})$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad (\text{A3c})$$

$$\nabla \times (\mathbf{B}/\mu) = \mathbf{J} \quad (\text{A3d})$$

and Ohm's law

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (\text{A3e})$$

The $\mathbf{v} \times \mathbf{B}$ term is an order-zero relativistic effect—much like classical kinetic energy $m_0 \mathbf{v}^2/2$. (Recall that for invariant rest mass m_0 , $E = m_0 c^2 (1 - \mathbf{v}^2/c^2)^{-1/2} = m_0 c^2 + m_0 \mathbf{v}^2/2 + \text{terms of order } \mathbf{v}^2/c^2$ (OKUN, 1989)).

Jump conditions across the fixed interface between two different media in which (A3a-d) hold are derived from (A3b), (A3c), the divergence of (A3d), and (A3d). These conditions are continuity of B_n , E_s , J_n , and jump $\{\mathbf{B}_s/\mu\} = \mathbf{C} \times \hat{\mathbf{n}}$ across the interface. If \mathbf{J} is finite, then $\{\mathbf{B}_s/\mu\} = 0$. If σ is finite at the interface, then finite \mathbf{E} , \mathbf{v} , and \mathbf{B} in (A3e) imply finite \mathbf{J} ; then it seems $\mathbf{C} = 0$ and \mathbf{B}_s/μ is continuous. If the finitude of σ is omitted (as in frozen-flux), then $\mathbf{C} \neq 0$ and discontinuous \mathbf{B}_s/μ are allowed; then \mathbf{B}_s may jump even if μ does not. The appropriate initial condition at time $t = 0$ is $\mathbf{B}(\mathbf{r}, 0)$. The initial condition on \mathbf{E} is not important in the quasi-steady approximation and must be sacrificed because (A3d) is a singular perturbation of (A1d). Given solenoidal \mathbf{B} (A3b), Ampere's law (A3d) determines solenoidal \mathbf{J} ; further given \mathbf{v} , Ohm's law (A3e) determines \mathbf{E} . Then Gauss's law (A3a) determines q and $\{\epsilon E_n\} = \Sigma$. Comparison of $\mathbf{V} \cdot \mathbf{J} = 0$ with the true charge conservation law (A1e) confirms that sources of displacement current: $\partial_t q$ (along with $q\mathbf{v}$, $\partial_t \Sigma$, and $\Sigma\mathbf{v}$) are omitted in the quasi-steady approximation.

Evaluation of $\partial_t q$ from (A3a-e) yields

$$\begin{aligned} \partial_t q &= \partial_t \nabla \cdot (\epsilon \mathbf{E}) = \partial_t \nabla \cdot (\tau_q \mathbf{J} - \epsilon \mathbf{v} \times \mathbf{B}) = \partial_t (\nabla \tau_q \cdot \mathbf{J}) - \nabla \cdot \partial_t (\epsilon \mathbf{v} \times \mathbf{B}) \\ &= \nabla \cdot \partial_t (\tau_q \nabla \times \mathbf{H} - c^{-2} \mathbf{v} \times \mathbf{H}) \end{aligned} \quad (\text{A3f})$$

which must be negligible compared with q/τ_q , $|\mathbf{J}|/\lambda$, and $|\mathbf{B}|/\mu\lambda^2$ for Ampere's law to be useful in conjunction with (A3e). If the quasi-steady approximation is to be strictly compatible with true charge conservation (A1e), then (A3f) must be identically zero. For uniform media this would require $\epsilon \nabla \cdot \partial_t (\mathbf{v} \times \mathbf{B}) = \epsilon \partial_t (\mathbf{B} \cdot \boldsymbol{\omega} - \mu \mathbf{v} \cdot \mathbf{J}) = 0$, where $\boldsymbol{\omega} \equiv \nabla \times \mathbf{v}$ is the vorticity. This requirement is always met in the comoving frame ($\mathbf{v} = 0$); in the limit as ϵ , and thus τ_q and c^{-2} , approach zero; in electrically neutral media; and when $(\mathbf{B} \cdot \boldsymbol{\omega} - \mu \mathbf{v} \cdot \mathbf{J})$ is steady (e.g., irrotational sub-relativistic motion of an electrical insulator). The unimportance of (A3f) is necessary, but not sufficient, for use of the quasi-steady approximation.

Elimination of \mathbf{E} from (A3c) using (A3e) yields

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\mathbf{J}/\sigma). \quad (\text{A4})$$

Substitution of Ampere's law (A3d) into (A4) yields the magnetic induction equation (1a).

Omission of $q\mathbf{v}$ does not imply $\mathbf{J} = 0$. Charge density q is a macroscopic average over the microscopic charge carriers of their individual charge alone; charge current density \mathbf{J} is an average over the charge carriers of their individual charge multiplied by their individual velocity. Similarly, mass density ρ is a macroscopic average over the microscopic mass carriers of their individual mass alone; mass current density $\rho\mathbf{v}$ is an average over the mass carriers of their individual mass multiplied by their individual velocity. Because $\rho\mathbf{v}$ is dominated by ponderous mass carriers (neutral molecules, 'holes', and ions) while \mathbf{J} is dominated by mobile charge carriers (conduction and free electrons), $\rho\mathbf{v}$ need not be obviously related to \mathbf{J} . The electro-

magnetic quasi-steady approximation ($\nabla \cdot \mathbf{J} = 0$) does not imply the acoustic quasi-steady approximation ($\nabla \cdot \rho \mathbf{v} \approx \nabla \cdot \rho_0 \mathbf{v} = 0$, where $\rho_0(\mathbf{r})$ is the mean density stratification $\rho_0(\mathbf{r}) \gg \rho(\mathbf{r}, t) - \rho_0(\mathbf{r}) = \rho'(\mathbf{r}, t)$); moreover, a steady mass current density ($\partial_t[\rho \mathbf{v}] \approx \partial_t[\rho_0 \mathbf{v}] = \rho_0 \partial_t \mathbf{v} = 0$) does not imply a steady charge current density ($\partial_t \mathbf{J} = 0$).

APPENDIX B

For media in which ϵ and σ are non-trivial, isotropic, and uniform equations (A1a), (A1e), (A2b), and (A2c) in comoving frame K' require

$$\epsilon \nabla' \cdot \partial_t \mathbf{E}' + \sigma \nabla' \cdot \mathbf{E}' = \partial_t q' + \tau_q^{-1} q' = 0 \quad (\text{B1})$$

so

$$q'(\mathbf{r}', t') = q'(\mathbf{r}', 0) e^{-t'/\tau_q} = \epsilon \nabla' \cdot \mathbf{E}'(\mathbf{r}', t') = \epsilon \nabla' \cdot \mathbf{E}'(\mathbf{r}', 0) e^{-t'/\tau_q}. \quad (\text{B2})$$

Provided $\tau_q > 0$, both q' and $\nabla' \cdot \mathbf{E}'$ decay exponentially towards zero with time constant $\tau_q = \epsilon/\sigma$. (Note ϵ , μ , and σ are regarded as purely real for simplicity).

If ϵ and σ are non-trivial, isotropic, and steady, but possibly inhomogeneous, then in K' , equations (A1d), (A2b), and (A2c) require

$$\partial_t \mathbf{E}' + \tau_q^{-1} \mathbf{E}' = \epsilon^{-1} \nabla' \times \mathbf{H}'. \quad (\text{B3})$$

Because $\tau_q(\mathbf{r}') = \epsilon(\mathbf{r}')/\sigma(\mathbf{r}')$ is also isotropic and steady, the solution of this first-order equation is

$$\begin{aligned} \mathbf{E}'(\mathbf{r}', t') = & \mathbf{E}'(\mathbf{r}', 0) e^{-t'/\tau_q} + \\ & \epsilon(\mathbf{r}')^{-1} \int_0^{t'} \nabla' \times \mathbf{H}'(\mathbf{r}', \tau) [e^{-(t' - \tau)/\tau_q}] dt. \end{aligned} \quad (\text{B4})$$

Suppose τ_q is positive definite—which excludes 'anti-ferroelectrics' or 'anti-conductors.' Then \mathbf{E}' forgets its initial condition at $t'=0$ after a few τ_q and is effectively the exponentially weighted running average of $\epsilon^{-1} \nabla' \times \mathbf{H}'$ thereafter (BACKUS, 1982). Time integration of (A1c) after using (B4) to eliminate $\mathbf{E}'(\mathbf{r}', t' > 0)$ yields

$$\begin{aligned} \mathbf{B}'(\mathbf{r}', t') = & \mathbf{B}(\mathbf{r}', 0) - \nabla' \times \mathbf{E}'(\mathbf{r}', 0) \tau_q [1 - e^{-t'/\tau_q}] - \\ & \nabla' \times \epsilon(\mathbf{r}')^{-1} \int_0^T \left\{ \int_0^T \nabla' \times \mathbf{H}'(\mathbf{r}', \tau) [e^{-(T - \tau)/\tau_q}] d\tau \right\} dT. \end{aligned} \quad (\text{B5})$$

If part of \mathbf{H}' does not vary exponentially on time scales near τ_q when $t' \gg \tau_q > 0$, then only values of this part of $\epsilon^{-1} \nabla' \times \mathbf{H}'$ within a few τ_q of t' contribute much to \mathbf{E}' because the weight factor in (B4) definitely decays exponentially. Slow or secular variations in \mathbf{E}' and \mathbf{H}' on time scales long compared with τ_q may then be considered quasi-steady: $\mathbf{E}' \approx \tau_q \epsilon^{-1} \nabla' \times \mathbf{H}'$ or, by (A2c),

$$\nabla \times \mathbf{H}'(\mathbf{r}', t') \approx \mathbf{J}'(\mathbf{r}', t') \quad (\text{B6})$$

which is Ampere's law. Physically, the Maxwell displacement current in (A1d) is omitted in (B6) because fast changes in \mathbf{D}' , hence in \mathbf{q}' , \mathbf{E}' , and thus \mathbf{B}' by (B5), contribute negligibly to the slowly varying portions of interest. A more rigorous derivation of (B6) follows.

We are concerned with the comoving frame K' but omit the prime notation for now. Again suppose ϵ and σ are non-trivial, isotropic, and steady; suppose $\mathbf{H}(\mathbf{r}, t)$ is also real. During the time interval from 0 to t , $\mathbf{H}(\mathbf{r}, \tau)$ is the sum of a constant $\mathbf{H}_0(\mathbf{r})$, a linear trend $\dot{\mathbf{H}}_0(\mathbf{r})[\tau - t_0]$, and a fluctuating portion $\mathbf{h}(\mathbf{r}, \tau) = \sum_k \mathbf{h}_k(\mathbf{r}) \sin(\omega_k \tau + \phi_k)$:

$$\mathbf{H}(\mathbf{r}, \tau) = \mathbf{H}_0(\mathbf{r}) + \dot{\mathbf{H}}_0(\mathbf{r})[\tau - t_0] + \sum_{k=1}^{\infty} \mathbf{h}_k(\mathbf{r}) \sin(\omega_k \tau + \phi_k). \quad (\text{B7})$$

If ω_k were $2\pi k/t$, then the sum would be the Fourier representation of the zero mean detrended magnetic field strength during the interval. We do not insist on this choice so as to allow flexibility in defining the mean, the trend, and any fundamental period; indeed, a transform over continuous ω may replace the discrete series. With (B7) the three contributions to the integral on the right of (B4) are:

$$\int_0^t \nabla \times \mathbf{H}_0(\mathbf{r}) e^{-(t-\tau)/\tau_q} d\tau = \nabla \times \mathbf{H}_0(\mathbf{r}) \tau_q [1 - e^{-t/\tau_q}] \quad (\text{B8a})$$

$$\int_0^t \nabla \times \dot{\mathbf{H}}_0(\mathbf{r}) [\tau - t_0] e^{-(t-\tau)/\tau_q} d\tau = \nabla \times \dot{\mathbf{H}}_0(\mathbf{r}) \tau_q (t - [t_0 + \tau_q]) [1 - e^{-t/\tau_q}] \quad (\text{B8b})$$

$$\begin{aligned} \int_0^t \nabla \times \mathbf{h}_k(\mathbf{r}) \sin(\omega_k \tau + \phi_k) e^{-(t-\tau)/\tau_q} d\tau \\ = \nabla \times \mathbf{h}_k(\mathbf{r}) \tau_q [1 + \omega_k^2 \tau_q^2]^{-1} (\sin(\omega_k t + \phi_k) - \omega_k \tau_q \cos(\omega_k t + \phi_k) \\ - [\sin \phi_k - \omega_k \tau_q \cos \phi_k] e^{-t/\tau_q}) \end{aligned} \quad (\text{B8c})$$

$$= \nabla \times \mathbf{h}_k(\mathbf{r}) \tau_q [1 + \omega_k^2 \tau_q^2]^{-1} [\sin \phi_k - \omega_k \tau_q \cos \phi_k] [1 - e^{-t/\tau_q}]$$

where the last step would follow from (B8c) only if the ω_k were $2\pi k/t$. With (B8a-c), multiplication of (B4) by $\sigma(\mathbf{r})$ yields

$$\begin{aligned} \mathbf{J}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, 0) e^{-t/\tau_q} + \nabla \times \mathbf{H}_0(\mathbf{r}) [1 - e^{-t/\tau_q}] \\ + \nabla \times \dot{\mathbf{H}}_0(\mathbf{r}) (t - [t_0 + \tau_q]) [1 - e^{-t/\tau_q}] \end{aligned}$$

$$\begin{aligned}
& + \sum_{k=1}^{\infty} \nabla \times \mathbf{h}_k(\mathbf{r}) [1 + \omega_k^2 \tau_q^2]^{-1} \{ \sin(\omega_k t + \phi_k) - \omega_k \tau_q (\cos \omega_k t + \phi_k) \\
& \quad - [\sin \phi_k - \omega_k \tau_q \cos \phi_k] e^{-t/\tau_q} \}. \quad (B9)
\end{aligned}$$

When $t \gg \tau_q$, $\exp(-t/\tau_q) \approx 0$ and

$$\begin{aligned}
\mathbf{J}(\mathbf{r}, t) &= \nabla \times \mathbf{H}_0(\mathbf{r}) + \nabla \times \mathbf{H}_0(\mathbf{r}) [t - t_0 - \tau_q] + \\
& \quad \sum_{k=1}^{\infty} \nabla \times \mathbf{h}_k(\mathbf{r}) [1 + \omega_k^2 \tau_q^2]^{-1} [\sin(\omega_k t + \phi_k) - \omega_k \tau_q \cos(\omega_k t + \phi_k)]. \quad (B10a)
\end{aligned}$$

For $t \gg \tau_q$, one may choose t_0 : $\tau_q \ll |t - t_0|$. So choosing implies $[t - t_0 - \tau_q] \approx [t - t_0]$. If $|\omega_k| \tau_q \ll 1$ for all non-trivial $\nabla \times \mathbf{h}_k$, then to order zero in the small quantity $\omega_k \tau_q [1 + \omega_k^2 \tau_q^2]^{-1} \approx \omega_k \tau_q \approx 0$,

$$\begin{aligned}
\mathbf{J}(\mathbf{r}, t) &= \nabla \times \mathbf{H}_0(\mathbf{r}) + \nabla \times \mathbf{H}_0(\mathbf{r}) [t - t_0] + \sum_{k=1}^{\infty} \nabla \times \mathbf{h}_k(\mathbf{r}) \sin(\omega_k t + \phi_k) \\
&= \nabla \times \mathbf{H}(\mathbf{r}, t). \quad (B10b)
\end{aligned}$$

Restoring the primes converts (B10b) into Ampere's law in the comoving frame (B6). Ampere's law holds only for frequencies $|\omega_k|$ small compared with τ_q^{-1} , so it is strictly an ultra-low frequency approximation.

If ω_k were $2\pi k/t$, then $|\omega_k| \tau_q \ll 1$ would imply $\tau_q \ll 2\pi t/k$. Ampere's law is thus broken by any non-trivial $\nabla \times \mathbf{h}_k$ with $k \geq 2\pi t/\tau_q$. If, however, the $\nabla \times \mathbf{h}_k(\mathbf{r})$ are negligible for $k \geq K$, then the sum can be truncated at $k = K$; then Ampere's law will be useful provided $\omega_K \tau_q = K \tau_q / 2\pi t \ll 1$ (i.e., $t \gg K \tau_q / 2\pi$). More generally, ω_k (or continuous ω) can be very high; yet Ampere's law will hold if the $\nabla \times \mathbf{h}_k$ are negligible for $|\omega_k| \gg \tau_q^{-1}$. Clearly the use of Ampere's law is tantamount to truncating the sum over frequencies; higher frequencies require (A1d).

When the total electromagnetic field has finite energy in finite volume, it has finite energy density at all but a finite number of singular point sources with finite charge and finite magnetic moment. Then we may insist that the macroscopic averaging procedure give $\mathbf{h}_k^2(\mathbf{r})$ which approach zero as $|\omega_k|$ approaches infinity. Unfortunately, it is not clear that the cutoff frequency above which the \mathbf{h}_k are negligible will be small compared with τ_q . On Earth's surface, high-frequency electromagnetic oscillations comprising the solar and geothermal fluxes break Ampere's law, but have far less energy density than the main geomagnetic field; yet within Earth's core, the electromagnetic energy density associated with high-frequency inter-molecular collisions (hence hydrostatic pressure) vastly exceeds that of the main field. Nevertheless, when ϵ and σ (and μ) are steady, the temporal linearity of the Maxwell equations in the comoving frame ensures that different frequencies are linearly independent. Then it is useful geomagnetically to filter out high-frequency electromagnetic oscillations by truncating the sum in (B10b). This is accomplished experimentally either by using low-pass magnetometers or by averaging the results in time.

Because Ampere's law holds to order zero in the relaxation time τ_q , it is tempting to think of it as the limit as ϵ and thus c^{-2} approach zero; then (A3f) would approach zero. Although the quasi-steady approximation (A3d) and (A3e) follows from treating c^{-2} and ϵ (but not μ) as if they were zero, it does not require ϵ to be treated as if it were zero in (A1a) or (A2b). Use of Ampere's law merely requires $|\partial_t \mathbf{D}|/|\mathbf{J}| = |\partial_t(\epsilon \mathbf{E})|/|\sigma \mathbf{E}| \sim |\lambda \nabla \cdot \partial_t(\epsilon \mathbf{E})|/|\sigma \mathbf{E}| \sim \tau_q/\tau \ll 1$. Galilean invariance, Ohm's law, and the omission of $q\mathbf{v}$, hence the quasi-steady approximation, further require terms of order U^2/c^2 to be negligible.

In the special case when σ is identically zero in some region, then \mathbf{J} is zero, q is constant, and τ_q is infinite. Then t cannot be greater than τ_q and Ampere's law holds only if $\partial_t \mathbf{D} = 0$. Yet Ampere's law can still be used if displacement current sources outside the region, which give rise to solenoidal $\partial_t \mathbf{D} = \nabla \times \mathbf{H}$ within the region, contribute negligibly to the portion of \mathbf{H} of interest. Then this portion of \mathbf{H} is irrotational and originates outside the region of interest.

Curiously, one rationale for the omission of displacement currents ($|\partial_t \mathbf{D}|/|\nabla \times \mathbf{H}| \sim (\lambda/\tau c)^2 = U^2/c^2 \ll 1$) typically fails in regions where \mathbf{H} is irrotational (where μ is homogeneous and \mathbf{B} is scaloidal). Moreover, this rationale is often used to justify the use of Ampere's law in conductors, where it is not sufficient. Fortunately, conditions under which $U^2/c^2 \ll 1$ but $\tau \approx \tau_q$ appear to be but rarely encountered in solid Earth geomagnetism; however, they appear to be common where fluctuations in the conductivity and dielectric properties of sub-relativistic plasmas are of considerable interest. Such non-linear media are considered briefly in Appendix C—as are steady, anisotropic media.

APPENDIX C

Real media are at best statistically steady and statistically isotropic, so it seems worth considering the case of inhomogeneous, anisotropic, and unsteady media. This includes non-linear media in which the dependence of $\underline{\epsilon}$, $\underline{\sigma}$, and $\underline{\mu}$ on electromagnetic fields is implicit in the dependence of their elements on position \mathbf{r} and time t . The Ampere-Maxwell law (A1d) and the anisotropic, possibly non-linear, constitutive relations in the comoving frame require

$$\underline{\epsilon} \cdot \partial_t \underline{\mathbf{E}} + (\underline{\sigma} + \partial_t \underline{\epsilon}) \cdot \underline{\mathbf{E}} = \nabla \times \underline{\mathbf{H}} = \nabla \times [\underline{\mu}^{-1} \cdot \underline{\mathbf{B}}] \quad (C1)$$

where the prime notation is again suppressed and only media in which possibly non-linear second-rank operators $\underline{\epsilon}^{-1}$, $\underline{\sigma}^{-1}$ and $\underline{\mu}^{-1}$ exist are considered. Define the tensor relaxation time operator $\underline{\tau}_q(\mathbf{r}, t)$ via

$$\int [\underline{\epsilon}^{-1} \cdot (\underline{\sigma} + \partial_t \underline{\epsilon})] dt \equiv \underline{\tau}_q^{-1} \quad (C2)$$

where the integrand is the second-rank result of matrix multiplication. With (C2) and (A2c), the solution of the first-order equation (C1) is written

$$\underline{\mathbf{E}}(\mathbf{r}, t) = \{ e^{-\underline{\tau}_q^{-1} t} \} \cdot \underline{\mathbf{E}}(\mathbf{r}, 0) + \int_0^t \{ e^{-(t-\tau)\underline{\tau}_q^{-1}} \} \cdot \{ \underline{\epsilon}(\mathbf{r}, \tau)^{-1} \cdot \nabla \times [\underline{\mu}(\mathbf{r}, \tau)^{-1} \cdot \underline{\mathbf{B}}(\mathbf{r}, \tau)] \} dt \quad (C3)$$

where $\exp(-t\underline{\tau}_q^{-1})$ is itself a matrix operator that varies with (\mathbf{r}, t) .

If it is possible to represent the elements of $\underline{\tau}_q^{-1}$, $\underline{\epsilon}^{-1}$, $\underline{\mu}^{-1}$, and \mathbf{B} in terms of their mean values, trends, and fluctuating portions during the interval from 0 to t , then one may proceed by analogy with Appendix B. Substitution of these expansions into (C3) reveals that the curly bracketed term on the far right of (C3)

$$\mathbf{U}(\mathbf{r}, \tau) = (\underline{\epsilon}(\mathbf{r}, \tau)^{-1} \cdot \nabla \times [\underline{\mu}(\mathbf{r}, \tau)^{-1} \cdot \mathbf{B}(\mathbf{r}, \tau)]) \quad (C4)$$

will include terms proportional to: (a) $(\tau - t_0)^n$ for $n = 0, 1, 2$, or 3 ; (b) the products of single oscillations and $(\tau - t_0)^m$ with $m = 0, 1$, or 2 ; (c) the products of two oscillations and $(\tau - t_0)^i$ with $i = 0$ or 1 ; and (d) the products of three oscillations. The interactions of $\underline{\epsilon}$, $\underline{\mu}$, and \mathbf{B} generally will contribute mean, secular, and oscillatory terms to the expansion for \mathbf{U} . For example, under resonant conditions in which the two oscillations in the $i = 0$ terms of case (c) have the same frequency and phase, there generally will be a non-zero contribution to the mean value of \mathbf{U} . Therefore the mean value of \mathbf{U} will generally not be the same as obtained by combining the mean values of $\underline{\epsilon}$, $\underline{\mu}$, and \mathbf{B} according to (C4). This can be the case even if $\underline{\epsilon}^{-1}$ is steady, provided $\underline{\mu}$ and \mathbf{B} are not. And the exponential operator in (C3) must contain the trend and oscillations in $\underline{\tau}_q$ as well as its mean value.

It therefore seems extremely unlikely that Ampere's law will hold for unsteady media; however, the problem does not arise for steady media even if they are anisotropic. And from the macroscopic perspective, unsteady media often appear to be near a statistically steady state. If this is the case, then the macroscopic constitutive relations (A2) can be considered definitions of the statistically steady, macroscopic properties of the medium.

Of particular interest in solid-Earth geomagnetism are cases in which $\underline{\epsilon}$ and \underline{g} in (A2b) and (A2c) are taken to be (statistically) steady, albeit possible anisotropic and inhomogeneous. To the extent that $\underline{\epsilon}$ and \underline{g} are steady, $\underline{\tau}_q^{-1} \equiv \underline{\epsilon}^{-1} \cdot \underline{g}$ is steady. If \mathbf{H} is used instead of $\underline{\mu} \cdot \mathbf{B}$, then one can proceed by analogy with Appendix B, albeit in matrix notation, without placing restrictions on $\underline{\mu}$. Indeed, with possibly complex $\underline{\epsilon}(\mathbf{r})$ and $\underline{g}(\mathbf{r})$, and with

$$\mathbf{H}(\mathbf{r}, \tau) = \mathbf{H}_0(\mathbf{r}) + \mathbf{H}_0(\mathbf{r})[\tau - t_0] + \sum_k h_k(\mathbf{r}) e^{i\omega_k \tau}$$

equation (C3) becomes

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) = & (e^{-t\underline{\tau}_q^{-1}}) \cdot \mathbf{E}(\mathbf{r}, 0) + [\mathbf{I} - e^{-t\underline{\tau}_q^{-1}}] \cdot \underline{\sigma}^{-1} \cdot \nabla \times \mathbf{H}_0 \\ & + \{[t - t_0] \underline{\mathbf{I}} - \underline{\tau}_q + [e^{-t\underline{\tau}_q^{-1}}] \cdot [t_0 \underline{\mathbf{I}} - \underline{\tau}_q]\} \cdot \underline{\sigma}^{-1} \cdot \nabla \times \mathbf{H}_0 \\ & + [\mathbf{I} - e^{-t\underline{\tau}_q^{-1}}] \cdot [\underline{\tau}_q^{-1} + i\omega_k \underline{\mathbf{I}}] \cdot \underline{\epsilon}^{-1} \cdot \nabla \times [\sum_k h_k e^{i\omega_k t}] \end{aligned} \quad (C5)$$

where $\underline{\mathbf{I}}$ is the identity matrix. If, for large t , $\exp(-t\underline{\tau}_q^{-1}) \approx \underline{\mathbf{0}}$, then (C5) simplifies to

$$\mathbf{E}(\mathbf{r}, t) \approx \sigma^{-1} \cdot \nabla \times \mathbf{H}_0 + \{ [t - t_0] \underline{\underline{\mathbf{I}}} - \underline{\underline{\tau}}_q \} \cdot \sigma^{-1} \nabla \times \mathbf{H}_0 \\ + [\underline{\underline{\tau}}_q^{-1} + i\omega_k \underline{\underline{\mathbf{I}}}]^{-1} \cdot \underline{\underline{\epsilon}}^{-1} \cdot \sum_k \nabla \times \mathbf{h}_k e^{i\omega_k t}. \quad (\text{C6})$$

If the frequencies of interest are so low that $[\underline{\underline{\tau}}_q^{-1} + i\omega_k \underline{\underline{\mathbf{I}}}]^{-1} \approx \underline{\underline{\tau}}_q$, and if the elements of possibly complex $\underline{\underline{\tau}}_q^{-1}$ are negligible compared with $[t - t_0] \underline{\underline{\mathbf{I}}}$, then (C6) reduces to Ampere's law. In such cases, Ampere's law is a justifiable ultra-low frequency approximation to the Ampere-Maxwell law for inhomogeneous, anisotropic media with unsteady magnetic permeability.

The possibility of unsteady $\underline{\underline{\mu}}$ yields a flexibility which may be useful when dealing with time-dependent crustal rock magnetization $\mathbf{M}(\mathbf{r}, t)$, where $\mathbf{M}(\mathbf{r}, t) \equiv [\underline{\underline{\mu}}_0^{-1} \underline{\underline{\mu}}(\mathbf{r}, t) - \underline{\underline{\mathbf{I}}}] \cdot \mathbf{H}(\mathbf{r}, t)$. Note that $\underline{\underline{\mu}}_i(\mathbf{r}, t) = \sum_j \underline{\underline{\mu}}_{ij}(\mathbf{r}, t)$ can be a sum of matrices. One such portion could be the product of a matrix $\underline{\underline{\mathcal{Q}}}(\mathbf{r}, t)$ and the diagonal matrix with elements inversely proportional to $\mathbf{H}(\mathbf{r}, t)$. This would yield a hard magnetization which is independent of the 'inducing' \mathbf{H} , but, depending on the form of $\underline{\underline{\mathcal{Q}}}$, might depend on other macroscopic parameters such as temperature $T(\mathbf{r}, t)$. Other portions of $\underline{\underline{\mu}}(\mathbf{r}, t)$ can be constructed to mimic other sorts of highly non-linear behavior, including thermo-viscous remanent magnetization.

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13. ABSTRACT (Maximum 200 words) Geomagnetic effects of magnetic induction by hypothetically steady fluid motion and steady magnetic flux diffusion near the top of Earth's core are investigated using electromagnetic theory, simple magnetic earth models, and numerical experiments with geomagnetic field models. The problem of estimating a steady fluid velocity field near the top of Earth's core which induces the secular variation indicated by broad-scale models of the observed geomagnetic field is examined and solved. In Part I, the steady surficial core flow estimation problem is solved in the context of the source-free mantle/frozen-flux core model. In the first paper (IA), the theory underlying such estimates is reviewed and some consequences of various kinematic and dynamic flow hypotheses are derived. For a frozen-flux core, fluid downwelling is required to change the mean square normal magnetic flux density averaged over the core-mantle boundary. For surficially geostrophic flow, downwelling implies poleward flow. The solution of the forward steady motional induction problem at the surface of a frozen-flux core is derived and found to be a fine, easily visualized example of deterministic chaos. Geomagnetic effects of statistically steady core surface flow may well dominate secular variation over several decades. Indeed, effects of persistent, if not steady, surficially geostrophic core flow are described which may help explain certain features of the present broad-scale geomagnetic field and perhaps paleomagnetic secular variation.				
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