Application of Artificial Neural Networks in Nonlinear Analysis of Trusses

J. Alam
Youngstown State University
Youngstown, Ohio

and

L. Berke
Lewis Research Center
Cleveland, Ohio

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NONLINEAR ANALYSIS OF TRUSSES

J. Alam
Youngstown State University
Civil Engineering Department
Youngstown, Ohio 44555

and

L. Berke
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

In the present study, a method is developed to incorporate neural network models for material response into nonlinear elastic truss analysis. Different feedforward network configurations are developed to assess the accuracy of neural network modeling of nonlinear material response. In addition to this, a scheme based upon linear interpolation for material data, is also implemented for comparison purposes.

It is found that the neural network approach can yield very accurate results if used with care. For the type of problems under consideration, it offers a viable alternative to other material modeling methods.

INTRODUCTION

It has been an ongoing effort to create machines which are capable of exhibiting intelligent behavior. The research in the Artificial Intelligence (AI) area is directed to achieve this goal. The traditional AI based approaches lead to the development of Expert Systems. They are useful in cases where a given problem can be specified in the form of well defined rules. By using symbolic logic useful inferences can be drawn from the rules data base by following standard search techniques. However, this approach was not very successful in pattern recognition tasks where the available information is noisy or incomplete.

Neural networks (NN), inspired by the research in cognitive and biological sciences in the functioning of the brain have provided an alternative approach to the solution of pattern recognition tasks, such as text to speech synthesis (ref. 1), image compression and processing (ref. 2), and nonlinear signal processing (ref. 3). Several neural network models have been suggested. Hopfield (ref. 4) was successful in applying network approach for a class of optimization problems. One of the most popular neural network models is based upon the studies of Rumelhart, Hinton, and Williams (ref. 5). It is a multi-layered feedforward network, in which learning is accomplished by backpropagation algorithm. It has been used extensively in solving a wide variety of problems. The computational NN models resemble the biological model of neurons. However, this resemblance is not very close and to avoid misunderstandings, they are called Artificial Neural Nets (ANN).
The application of feedforward networks based upon backpropagation algorithm in Computational Structural Technology (CST) is relatively new in its origin. Rehak et al. (ref. 6) developed NN models for simulating the dynamic behavior of structures. Troudet and Merrill (ref. 7) adopted network methodology for estimating fatigue life of structural components. Berke and Hajela (ref. 8) used ANN for structural analysis and optimization of trusses. Their use in plate and shell analysis is reported in reference 9. They have shown considerable promise in material modeling. Jain et al. (ref. 10) used it to model the tri-axial behavior of soils. Ghaboussi et al. (ref. 11) have reported the modeling of nonlinear behavior of concrete.

It was suggested in references 8 and 11 that the trained neural networks can be incorporated into existing structural analysis programs. The use of nonlinear analysis programs require an accurate description of material behavior. With the increased complexity in modeling material behavior, it is becoming necessary to look for new approaches to represent it in a form which is computationally efficient and can easily be added to the existing stress analysis programs. The use of ANN seems to be particularly appealing for this type of nonlinear materials modeling. It allows to capture the material response by training an appropriately configured network for given material data. Once the network is trained for the desired accuracy a small data file containing the weights of connections and network biases is saved. It can be used later on for eliciting proper material response such as stresses for known strains or stress increments for given strain increments. The next step would be to incorporate this trained NN into a stress analysis program.

OBJECTIVE OF THE STUDY

The present study is conducted to explore the possibility of training feedforward network for known material data and then incorporate it into a structural analysis program considering material nonlinearity. For this purpose, nonlinear elastic analysis of trusses is chosen. The material behavior is assumed to be known in the form of equations so that comparison between the actual material behavior and the one predicted by neural nets is possible for the evaluation of the effectiveness of the neural network approach. Different trusses are used for analysis to allow comparison between the different approaches.

NONLINEAR ELASTIC ANALYSIS OF TRUSSES

A stiffness based matrix formulation (ref. 12) is used for the analysis of trusses. The joint equilibrium equations for a truss can be written as:

\[
[K] \{u\} = \{f\}
\]

(1)

where \([K]\) is overall assembled stiffness matrix of truss, \(\{u\}\) and \(\{f\}\) represent the joint displacement vector and joint force vector, respectively. The material behavior is given by the following equation (ref. 13):
\[ \sigma = E_0(\varepsilon - 5\varepsilon^2) \quad \text{for } \varepsilon \geq 0 \]
\[ \sigma = E_0(\varepsilon + 5\varepsilon^2) \quad \text{for } \varepsilon < 0 \]

where \( E_0 = 200 \) units

For the equations dimensionally consistent units can be chosen. Due to nonlinear material behavior in equation (2) the stiffness matrix in equation (1) is a function of displacements \( \{u\} \), and has to be solved in an iterative manner. The initial stiffness method as suggested in reference 13 was used. This method saves the updating of the stiffness matrix at each iteration. The stiffness matrix formulated for the linear elastic case can be used for each iteration. This method is more stable in comparison to the tangent stiffness method which requires updating of the stiffness matrix at the beginning of each iteration. However, it suffers from the slow convergence rate at loads near maximum loads.

The total load is applied in \( n \) prescribed steps. The equations for the \( p^{th} \) load step and the \( i^{th} \) iteration are:

\[ [K] \left\{ \Delta u_i^P \right\} = \{R_i\} \]

where

\[ \{R_i\} = \{f^P\} - \{f_{i-1}\} \]

The vector \( \{f^P\} \) represents the applied joint forces for the \( p^{th} \) load step. \( \{\Delta u_i^P\} \) is the change in the joint displacement vector for the \( i^{th} \) iteration for the load step \( p \). \( \{R_i\} \) is a residual force vector showing the error in the joint equilibrium due to the force vector \( \{f_{i-1}\} \) computed from the forces in the members of the truss. The displacements for the \( i+1 \) iteration are updated by the following equation:

\[ \{u_{i+1}^P\} = \{u_i^P\} + \{\Delta u_i^P\} \]

For the \( k^{th} \) member of the truss the elongations are calculated as follows:

\[ e_k = (u_1 - u_m) \cos v - (v_1 - v_m) \sin v \]

where \( u_1 \) and \( u_m \) are the end joints of the member \( k \). The \( u_1 \) and \( u_m \) are the x-displacements of joints 1 and m, respectively. Similarly \( v \)'s are the joint displacements along the y-direction. \( v \) is the slope of the \( k^{th} \) member with respect to the positive x-axis. The strains in each member of the truss are computed by:
\[ \varepsilon_k = \frac{e_k}{l_k} \quad k = 1, 2, \ldots, s \] (7)

Here \( l_k \) is the length of the \( k \)th member and \( s \) is the total number of members in the truss. The stresses in each member of the truss can now be computed by using equation (2) or by the trained neural network. The forces in each member are obtained from these stresses. The member forces are transformed to obtain the equivalent joint forces to assemble the new \( \{f_i\} \) vector. The residual force vector for the \( i+1 \) iteration will be:

\[ \{R_{i+1}\} = \{f^p\} - \{f_i\} \] (8)

The new joint equilibrium equations are:

\[ |K| \{\Delta u^p_{i+1}\} = \{R_{i+1}\} \] (9)

By using Gauss-Jordan method the incremental displacements are calculated and the procedure can be repeated as described earlier. To stop the iterative process an error norm given by the following equation is calculated:

\[ \delta = \sqrt{\frac{\sum_{q=1}^{r} (R_{i,q})^2}{r}} \] (10)

where \( \delta \) is the rms error, and \( r \) is the total number of nonzero joint displacements in the truss. When the calculated value of rms error \( \delta \) is less than the prescribed tolerance the iteration process is stopped and the values of the joint displacements, member forces, stresses and strains are printed. The applied joint loads are incremented for the next load step \( p+1 \). This procedure is repeated until all the load steps are completed.

**ARTIFICIAL NEURAL NETWORK MATERIAL MODELING**

Figure 1 shows the neural network configuration. It has one processing element (PE) in the input layer with strain as the input. The output layer consists of one PE providing the value of stress. The processing elements in the middle layer also, known as hidden layer, are varied from 5 to 15 to find an appropriate network configuration which predicts stresses for a given strain with the smallest error. The computer program NETS (ref. 14) was used for all the network training. In the program the backpropagation algorithm was implemented at Lyndon B. Johnson Space Flight Center of NASA.

For incorporating the neural network model of material behavior in the nonlinear truss analysis program, two small functions, "init" and "propagate," were written in C language. The
first one initializes the net while the second one propagates the given input strain to get the corresponding stress. The output $o_i$ of an $i^{th}$ PE can be obtained as follows:

$$o_i = \frac{1}{1 + e^{-a_i}} \quad (11)$$

where $w_{ij}$ is the weight of the connection between PE $i$ and PE $j$. $I_j$ is the input from the $j^{th}$ PE and $b_i$ is the bias value in

$$a_i = \sum_{j=1}^{n_i} w_{ij} I_j + b_i \quad (12)$$

for the $i^{th}$ PE.

RESULTS AND DISCUSSIONS

To assess the accuracy and to choose an appropriate network configuration for the nonlinear truss analysis several cases listed below were selected for numerical experimentation.

<table>
<thead>
<tr>
<th>Case</th>
<th>Network Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-5-1.13 network</td>
</tr>
<tr>
<td>2</td>
<td>1-10-1.13 network</td>
</tr>
<tr>
<td>3</td>
<td>1-15-1.13 network</td>
</tr>
<tr>
<td>4</td>
<td>1-5-1.19 network</td>
</tr>
<tr>
<td>5</td>
<td>1-10-1.19 network</td>
</tr>
</tbody>
</table>

In these cases the first number denotes the number of input units, which is always one. The second number represents the number of hidden units and it varies from 5 to 15. The third number which is one in all the cases, is the number of output units. The last number after the period is the total number of input-output pairs used for network training. These pairs are obtained from equation (2). All the training data was scaled between 0 and 1 due to the restriction placed by the backpropagation algorithm implemented in the NETS program. Scaling between 0.1 and 0.9 is also customary. All the networks are trained with a maximum error not exceeding 1.8 percent and rms error less than 1 percent. After the training the files containing weights and biases are saved for each of the network to assess the accuracy of all the neural networks.

The input strains used for training are augmented by additional values of strains from the stress-strain equation (2) to propagate the data. The predicted values for stresses from neural nets for these strains are plotted along with the actual stress-strain curves obtained from equation (2). Figure 2 contains the plots for cases 1 to 3. It shows good prediction capability of all the cases with case 3 being closest to the actual stress strain curve. Similarly in figure 3, data from cases 4 and 5 is plotted. Both cases are in good agreement with the known results. Figure 4 shows the comparison between cases 3 and 4. These two models are very close to the chosen stress-strain curve for the study. It is difficult to select the best case from these plots. Therefore, for a closer look at the accuracy of the results, percent error in neural network interpolations versus strains are plotted on figures 5 to 7. It can be seen from these figures that the percent error is within $\pm 3$ percent when the strains used for training are also used for the prediction of stresses. For other value of strains, these errors could be significant. Especially at
the two end points of the stress-strain curve where strain values are nearly \( \pm 0.2 \). However, in nonlinear analysis, truss members do not attain these high values of strains. The other location where errors in predictions are significant, is near the strain level zero. It should be noted that at these strain levels the actual stress is approaching zero. Any small variation in the prediction by Neural net causes a large relative percent error because in calculating the percent error, the difference between the actual and the predicted stress is divided by a stress value which is small in magnitude. This artificially magnifies the magnitude of the error. This shows that neural network predictions although very accurate, could be in error at few points and a careful checking is necessary before selecting an appropriate network configuration for materials modeling.

Figure 8 shows four truss geometries with dimensions and applied loading used in this study. For all these trusses a uniform cross section area of 1 unit magnitude is assumed. Manual calculations are performed for Truss1 and Truss2. The initial stiffness method is implemented in a computer program written in C language to perform the nonlinear elastic truss analysis. For both cases the nonlinear stress-strain function given by equation (2) is used. These results for load and load point displacements are plotted on figures 9 and 10. For both of the trusses the results from the computer program and manual calculations are in close agreement. It shows that the initial stiffness method with incremental solution strategy can be used for nonlinear analysis purposes. In addition to these calculations, further calculations are done by dropping the nonlinear term from the equation (2). This results in a linear elastic material. The incremental solution leads to a straight line response for both of the trusses. This serves as a further verification of the computer program. In addition to this, it can also be seen from figures 9 and 10 that there is significant nonlinearity in the responses of the trusses. The initial stiffness method has been able to capture this nonlinearity without any instability and errors.

The computer program was slightly modified to include functions “init” and “propagate.” This allows to obtain the nonlinear stress-strain material data from the trained neural networks. The Truss1 is analyzed by this program for cases 1 to 5. For all calculations 10 load steps and a tolerance of 0.005 are used. The results for applied load and load point displacements for all these five case are shown on figures 11 and 12. Except for case 1 all other cases show fairly good agreement with the manual calculations. It can be concluded from these figures that 1-5-1.19 net produces the best results. It also leads to the conclusion that use of more input-output pairs for training the network provide better accuracy. For this problem due to availability of assumed stress-strain function any number of input-output pairs can be generated for network training. However, most of the time the material data are available only at discrete points. Therefore, computer program was slightly changed so that it can use material data supplied in form of a table. This table was created using the function given by the equation (2). For intermediate points, a linear interpolation scheme was implemented in the computer program. This case is indicated as linear interpolation on the plots. It more accurately emulates the existing nonlinear analysis programs in which material data is provided at discrete points. The results for cases 3 and 4 and from the linear interpolation scheme are shown on figure 13 for Truss1. The results from case 4 and linear interpolation scheme are very close to exact results. Case 3 was chosen because it had the smallest maximum error of 0.0123 and rms error of 0.0057 among all other cases after training the network. However these plots show that this network does not produce the best results for Truss1. This particular network was trained for 23 604 cycles. It is known that overtraining usually reduces the generalization capability of network models. Overall, all the models used, in the study provide adequate approximate solutions. It is encouraging that the errors noted earlier seems to average out when neural nets are used for material modeling. In situations where material data is available only at few points from experiments, neural net approximations will be able to provide results within reasonable accuracy.
For Truss1 a load of 9.6 units is applied in 10 increments with a tolerance of 0.005. A time function is used to evaluate the total execution time for all the five cases. These results are shown on figure 14 along with the case when the exact stress-strain function is used for material modeling in the computer program. It can be seen that cases 4 and 5 take more time when compared with the similar networks of cases 2 and 3, respectively, that have identical number of hidden units. This difference is due to a higher number of iterations needed for convergence. This also points to a situation in which the best neural net model may not be fastest although it has the smallest number of hidden units.

Similar calculations are performed for Truss2 and shown on figure 15. Only cases 3 and 4 are used for neural net modeling. Once again the best results for applied load and load point displacements are produced by case 4 when compared with the exact results from manual calculations. For this truss the neural net model predicts better results than the linear interpolation scheme. Figures 16 and 17 show the results for applied load and load point displacements for Truss3 and Truss4. For both cases it was not possible to perform a manual computation. Therefore, the results from the initial stiffness program using the stress-strain function given by equation (2), are considered as the exact solutions. They are also plotted on these figures for comparison purposes. For both cases 1-5-1.19 net produces extremely accurate results. They are very close to the results from initial stiffness method. The linear interpolation scheme in both cases fails to provide accurate solutions. In this study the stress-strain function used for material response is a second order parabola. The linear interpolation scheme was not able to approximate it closely. For material behavior with sharper gradients than a parabola this approximation scheme will further deteriorate. However, as mentioned earlier, this is the most commonly used scheme in the existing general purpose nonlinear finite element analysis programs. These results show that it is prone to errors. It also shows that if neural network modeling is used carefully extremely accurate results can be obtained. Even in cases where the model is not chosen carefully, results of reasonable accuracy are attainable. It is anticipated that for materials with responses more complicated than a parabolic model, linear interpolation scheme should lead to larger errors. In such cases neural network material modeling can effectively be used in stress analysis problems.

To further investigate the convergence characteristics of all these schemes, maximum load calculations are done for all four trusses. For manual calculations maximum load is obtained by choosing the peak points of the load-displacement plots. For initial stiffness method the peak load is obtained by incrementing the applied loads till the solutions fail to converge. The unloading part of the load displacement curve can not be calculated from this implementation of initial stiffness method due to incremental loading control. Table I shows that for first three trusses, neural network is very close to exact maximum loads. The same can be interpreted from figure 18. This is an indication that neural net material modeling does not appreciably change the convergence characteristics of initial stiffness method.

The maximum displacements were calculated for applied load levels of 9.6, 20.5, 13.5, and 12.8 for the trusses by all the suggested schemes. The numerical values are shown in table II and the results are on chart 19. Neural network models compare favorably with exact solutions.

For the previous cases the total execution times are also obtained. They are listed in table III. They are also shown on figure 20. For the first three trusses the time taken by the two neural network models and the linear interpolation method are comparable. However, this trend breaks down for the Truss4. In that case neural net provides accurate solutions but takes more
time to converge to the results as compared to the linear interpolation scheme. These are only a few cases, and it is difficult to conclude on this basis that neural network material modeling will take less computer time compared to other approaches. The time taken by the initial stiffness method using the stress-strain function from equation (2) is the smallest. In this case the function evaluation is extremely fast because it is in a form of a simple equation. This indicates that in nonlinear analysis most of the time is taken in obtaining the material response. For large size nonlinear analysis problems, the choice of appropriate material modeling is very essential to keep the analysis time within reasonable limits.

To further investigate the accuracy of neural net modeling with the other techniques, the forces in the members of the Truss3 are plotted on figures 21 to 24. The member numbering is shown on figure 8. The applied load was increased in five steps to the maximum value of 13.5 units. It can be seen that all the techniques produce results very close to the exact solution with NN models having a slight edge over the linear interpolation scheme.

CONCLUSIONS

From the results and the discussion the following conclusions can be drawn.

1. Neural network material modeling can successfully be implemented in a general purpose analysis program.

2. The neural network models provide reasonably accurate solutions to the nonlinear elastic analysis of trusses. If the network configuration is chosen with care extremely accurate solutions are possible.

3. The network trained with larger input-output pairs reproduce the material behavior more accurately.

4. The linear interpolation scheme for handling the material data is most commonly used in general purpose nonlinear finite element analysis programs. The neural networks are able to provide more accurate solutions in comparison to this scheme.

5. The incorporation of neural network material modeling in an analysis program does not appreciably change the convergence characteristics of the initial stiffness method.

6. It is not possible to conclude from this study that neural net material modeling will result in savings in computer time. It is comparable to the linear interpolation scheme.

7. It is anticipated that in case of more complicated material models neural network approach will be able to provide more accurate results as compared to other schemes.

8. Neural network offers a viable alternative for material modeling. In this approach material information can be captured in a small file containing weights and biases leading to significant data base compression.

9. The network trained with least errors may not provide the most accurate solution to the analysis problems.
10. More work is needed to establish guidelines for configuring an appropriate network model. Especially in selecting the number of hidden processing elements. At this time it is primarily a trial and error process.

REFERENCES


TABLE I.—PREDICTED MAXIMUM LOAD CAPACITY OF TRUSSES

<table>
<thead>
<tr>
<th>Truss number</th>
<th>Truss1</th>
<th>Truss2</th>
<th>Truss3</th>
<th>Truss4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manual calculations</td>
<td>10.0</td>
<td>21.478</td>
<td>-----</td>
<td>-----</td>
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<tr>
<td>Stress-strain function</td>
<td>10.0</td>
<td>21.478</td>
<td>13.9</td>
<td>13.3</td>
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<tr>
<td>Neural network</td>
<td>1-10-1.13</td>
<td>9.8</td>
<td>21.478</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>1-5-1.19</td>
<td>9.6</td>
<td>21.478</td>
<td>13.5</td>
</tr>
<tr>
<td>Linear interpolation</td>
<td>10.0</td>
<td>20.5</td>
<td>13.8</td>
<td>13.3</td>
</tr>
</tbody>
</table>

TABLE II.—MAXIMUM DISPLACEMENTS FOR TRUSSES

<table>
<thead>
<tr>
<th>Truss number</th>
<th>Truss1</th>
<th>Truss2</th>
<th>Truss3</th>
<th>Truss4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied load</td>
<td>9.6</td>
<td>20.5</td>
<td>13.5</td>
<td>12.8</td>
</tr>
<tr>
<td>Stress-strain function</td>
<td>0.799</td>
<td>0.985</td>
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<td></td>
<td>1-5-1.19</td>
<td>0.928</td>
<td>1.104</td>
<td>2.444</td>
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<tr>
<td>Linear interpolation</td>
<td>0.799</td>
<td>1.085</td>
<td>2.335</td>
<td>1.244</td>
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</table>

TABLE III.—COMPUTER TIME FOR THE ANALYSIS OF DIFFERENT TRUSSES

<table>
<thead>
<tr>
<th>Truss number</th>
<th>Truss1</th>
<th>Truss2</th>
<th>Truss3</th>
<th>Truss4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied load</td>
<td>9.6</td>
<td>20.5</td>
<td>13.5</td>
<td>12.8</td>
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<tr>
<td>Stress-strain function</td>
<td>8.07</td>
<td>15.1</td>
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<td>33.67</td>
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<td></td>
<td>1-5-1.19</td>
<td>25.58</td>
<td>42.18</td>
<td>35.32</td>
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<tr>
<td>Linear interpolation</td>
<td>12.52</td>
<td>37.56</td>
<td>13.13</td>
<td>55.75</td>
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</tbody>
</table>
Fig. 1 The Configuration of a Neural Network
Fig. 2 Neural Network predictions for stresses
Fig. 3 Neural Network predictions for stresses
Fig. 4 Neural Network predictions for stresses
Fig. 6 Errors in Neural Net interpolations
Fig. 8 Different Configuration of Trusses
Fig. 9 Applied load vs. maximum displacement for Truss1
Fig. 10 Applied load vs. maximum displacement for Truss2
Fig. 11 Applied load vs. maximum displacement for Truss 1
Fig. 12 Applied load vs. maximum displacement for Truss 1
Fig. 13 Applied load vs. maximum displacement for Truss1
Fig. 14 CPU times from different Neural Net models for Truss1
Fig. 15 Applied load vs. maximum displacement for Truss2
Fig. 16 Applied load vs. maximum displacement for Truss3
Fig. 17 Applied load vs. maximum displacement for Truss4
Fig. 18 Predicted maximum load capacity from different material models
Fig. 19 Maximum displacement for trusses from different material models

Fig. 20 CPU times for trusses from different material models

Fig. 21 Compressive force in member 1 and 2 of Truss3

Fig.22 Compressive force in member 3 of Truss3

Fig. 23: Tensile force in member 4 of Truss3

Fig. 24 Tensile force in member 5 of Truss3

In the present study, a method is developed to incorporate neural network model based upon the Backpropagation algorithm for material response into nonlinear elastic truss analysis using the initial stiffness method. Different network configurations are developed to assess the accuracy of neural network modeling of nonlinear material response. In addition to this, a scheme based upon linear interpolation for material data, is also implemented for comparison purposes. It is found that neural network approach can yield very accurate results if used with care. For the type of problems under consideration, it offers a viable alternative to other material modeling methods.