TRASYS Form Factor Matrix Normalization

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SUMMARY

A method has been developed for adjusting a TRASYS enclosure form factor matrix to unity. This approach is not limited to closed geometries, and in fact, it is primarily intended for use with open geometries. The purpose of this approach is to prevent optimistic form factors to space. In this method, nodal form factor sums are calculated within 0.05 of unity using TRASYS, although deviations as large as 0.10 may be acceptable, and then, a process is employed to distribute the difference amongst the nodes. A specific example has been analyzed with this method, and a comparison was performed with a standard approach for calculating radiation conductors. In this comparison, hot and cold case temperatures were determined. Exterior nodes exhibited temperature differences as large as 7°C and 3°C for the hot and cold cases, respectively when compared with the standard approach, while interior nodes demonstrated temperature differences from 0°C to 5°C. These results indicate that temperature predictions can be artificially biased if the form factor computation error is lumped into the individual form factors to space.

NOMENCLATURE

\begin{itemize}
  \item $A_i$: area of $i$th node
  \item AU: astronomical units
  \item BCS: block coordinate system
  \item DDA: dual-drive actuator
  \item FFCAL: form factor calculation segment within TRASYS
  \item FFRATL: maximum internodal subelement distance to average internodal subelement distance ratio
  \item $F_{ij}$: form factor from node $i$ to node $j$
  \item GLL: Galileo Project
  \item GMM: geometric math model
\end{itemize}

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HGA  high gain antenna  
LGA  low gain antenna  
MLI  multilayer insulation  
NELCT  number of subelements used in Nusselt unit sphere method  
PWS  plasma wave science  
$S_i$  nodal form factor sum for node $i$  
S/C  spacecraft  
TMM  thermal math model  
TRASYS  Thermal Radiation Analyzer System  
$\alpha_s$  solar absorptivity  
$\Delta_i$  difference between nodal form factor sum for node $i$ and unity  
$\varepsilon$  hemispherical emissivity  

*Superscripts*

cal  calculated directly through TRASYS  
red  calculated through form factor reduction process  
uni  calculated from process to adjust form factor matrix to unity  

**STATEMENT OF THE PROBLEM**

TRASYS (Ref. 1) is a software system which is utilized for the determination of internodal form factors and environmental heating in primarily extraterrestrial thermal analyses. When GMMs are of moderate or large size, it becomes increasingly more difficult to verify their form factor calculations. Internodal shadowing and complex-shaped geometry are some reasons contributing to this obstacle. Thus, individual form factor verification is simply not practical for sizeable models. Of more pragmatic importance is the form factor from each node to space. TRASYS does not directly determine form factors to space in its standard operating mode. Instead, TRASYS implicitly uses the difference of the nodal form factor sum and unity. Therefore, any form factor computation error will be directly imbedded in the form factor to space. It should be noted that TRASYS possesses an option to enable direct calculation of the form factor to space. However, this option is computationally-intensive and has demonstrated computational errors (Refs. 2, 3, and 4). A more significant shortcoming with this approach is its inability to save form factor to space calculations on the restart file. Clearly, an approach that can address how the computational error is distributed over all nodes is required.

**FORM FACTOR MATRIX NORMALIZATION**

*Standard Form Factor Calculation Mode*

The FFCAL segment is responsible for form factor calculations within TRASYS. It is reliant upon a parameter known as FFRATL which represents the maximum internodal subelement distance to average internodal subelement distance ratio. The default value is 15.0, but it may
be respecified by the user. If the calculated FFRATL is less than the specified value, the double summation (numerical integration) technique is used for that particular \( F_g \). However, if the calculated FFRATL is greater than the specified value, the Nusselt unit sphere technique is employed. The Nusselt unit sphere technique is more accurate than the double summation method, but it is also more time-consuming as well. The default FFRATL value has been demonstrated to be an empirically optimal in terms of computation time and accuracy.

Creating Enclosures from Open Geometries

It has been indicated the individual form factors to space may be inherently erroneous if there is no provision to verify the calculation. A suggested approach is to construct an enclosure around the open geometry. This does not simply imply surrounding the geometry within a large sphere, but rather using appropriate-sized surfaces to complete closure. A simplistic example would be using a sufficiently-nodalized hemisphere to enclose a circular disk. The closing surfaces should be nodalized so that each enclosing nodal area is no more than one order of magnitude larger than the smallest node in the geometry, but ideally, it should be of the same magnitude. Such a constraint upon the enclosing area helps to ensure accurate form factor calculations to and from these nodes.

Optimizing Form Factor Calculations

In many instances, it is not tractable to determine the validity of every form factor calculation especially if the geometry does not constitute a complete enclosure. For enclosures, a more global but yet effective way of determining form factor calculation accuracy is the nodal form factor sum which must be unity. This idea may be extended to non-enclosures since it was previously explained how open geometries may be closed out. Usually, accuracy within \( \pm 0.05 \) of unity is acceptable, but there may be cases where accuracy within \( \pm 0.10 \) of unity is acceptable since temperature differences are expected to be small. Nodal form factor sums may not be acceptable even after the standard TRASYS form factor calculation procedure is implemented. Accuracy may be improved by recomputing individual form factors for those nodes whose form factor sums are unacceptable by forcing the Nusselt unit sphere technique and by using more nodal subelement resolution. In terms of application within TRASYS (see Fig. 1), the previous form factor calculation is restarted, recomputed nodes are identified through RECOMP option in the form factor data block, Nusselt unit sphere method is specified by setting FFRATL to -1.0, and higher nodal resolution is specified by setting NELCT equal to between 75 and 100 prior to the FFCAL call. With correctly-specified geometry, recomputation will usually bring form factor sums between 0.95 and 1.05.

Figure 1 - TRASYS run stream for form factor recomputation; italicized text indicates user input

HEADER OPTIONS DATA
RSI $ READ RESTART TAPE FROM INITIAL FF RUN
RSO $ WRITE RESTART OUTPUT TAPE
If form factor recomputation does not produce acceptable nodal form factor sums, it would be advisable to reexamine the geometry for potential geometry problems such as gaps between nodes, inactive side of a node being viewed, or a node lying directly upon or intersecting another node.

Reducing Form Factor Sums Greater than Unity

Even after form factor recomputation, there may be a number of nodes whose form factor sums are unacceptably greater than unity. A simple algorithm has been devised to reduce the individual form factors on a weighted basis so that the nodal form factor sum is consequently reduced to or below unity. For any of the nodes in question, the difference from unity is determined as,

\[ \Delta_i = \sum_j F_{ij}^{\text{cal}} - 1 \]  

Or,

\[ \Delta_i = S_i^{\text{cal}} - 1 \]  

It is assumed that \( \Delta_i \) represents the form factor computational error and furthermore, it is assumed that the error is proportional to the size of the nodal form factor. Hence, each nodal

\[ F_{ij}^{\text{red}} = F_{ij}^{\text{cal}} - \frac{F_{ij}^{\text{cal}}}{\sum_j F_{ij}^{\text{cal}}} \Delta_i \]
form factor may be reduced based upon its fractional make-up of the form factor sum, and this weighing is demonstrated as the second term in Eq. 3. Eq. 3 may be rewritten as:

\[ F_{ij}^{\text{red}} = \frac{F_{ij}^{\text{cal}}}{S_i^{\text{cal}}} \]  

(4)

When the reduction process is complete, Eq. 4 indicates that the summation of the reduced nodal form factors should total unity. It should be noted that although the ith nodal form factor sum has been set to unity, the reduction process implicitly affects the jth nodal form factor sum due to form factor reciprocity. Consequently, there may be instances where the jth nodal form factor sum is perilously close to 0.95, and the reduction process will lead to an unacceptable form factor sum for the jth node. In these cases, this jth node should be excluded from the reduction process, and the weighing should be based on the remaining nodal form factors.

Adjusting Form Factor Matrix to Unity

Following the reduction process, the nodal form factor sums should not be greater than unity. It is possible to devise a process to increase form factor sums to unity at this point. However, the application of this process to every node would be difficult, because of the interdependency of the form factors through reciprocity. Instead, the main objective is to prevent the difference between the nodal form factor sum and unity from erroneously being added to the form factor to space. Therefore, the nodal form factor deviation from unity is assumed to be added to the form factor to itself (Eq. 5). Here, the implicit assumption is that there is virtually no temperature differences between the nodes. Once Eq. 5 has been performed for all nodes, the form factor matrix should be entirely adjusted to unity. However, if this approach results in non-conservative modeling, an analogous form factor weighted process to increase form factor sums to unity may be applied to particular nodes of interest. Eq. 4 with a sign change would be applicable for this process.

\[ F_{ui}^{\text{adj}} = F_{ui}^{\text{red}} + (1 - \sum_i F_{ij}^{\text{red}}) \]  

(5)

It should be kept in mind that the enclosing nodes represent space. These enclosing surfaces may be removed from the GMM, and the form factors to space have been adjusted so that they are more rigorous in a global sense. In the form factor matrix normalization process, the computational error has been distributed throughout the GMM nodes. Therefore, the individual form factors to space do not have all the computational error imbedded in them.

Implementing the Normalized Form Factor Matrix

The GMM can be modified to remove the enclosing nodes. In order to facilitate removal, the
enclosing nodes should be specified in a separate BCS. Additionally, the form factor matrix must also be modified so that all form factors to or from the enclosing nodes are removed. The remaining form factors may be input through the form factor data block. This TRASYS run stream in depicted in Fig. 2. Note that an input restart file is not required since an entire form factor matrix is entered in the form factor data block. Also, note that the option to initially zero the entire form factor matrix is utilized since only non-zero form factors are input. This prevents TRASYS from calculating form factors that were known to be zero.

Figure 2 - Implementation of normalized form factors; italicized text indicates user input

HEADER OPTIONS DATA
.
RSO $ WRITE AN OUTPUT RESTART FILE
.
HEADER SURFACE DATA
.
geometry without enclosing surfaces
.
HEADER FORM FACTOR DATA
FIG model configuration name
node array
ZERO $ INITIALLY SETS ENTIRE FA MATRIX TO ZERO
normalized form factors without enclosing surfaces
.
HEADER OPERATIONS DATA
.
L FFCAL $ CALL TO FFDATA NOT NEED SINCE HEADER FORM FACTOR DATA USED
.
END OF DATA

Available Computer Codes for Normalization

A FORTRAN program known as PL-PULL (Ref. 5) has been developed by Rockwell International with the capability to normalize a form factor matrix as described above.

A SAMPLE APPLICATION

Form factor matrix normalization has been applied in the case of the GLL HGA GMM (Ref. 6). The hardware configuration is shown in Fig. 3, along with the GMM nodalization. The intent of this model is to be able to predict primarily exterior surface temperatures during its Venus flyby while in the stowed configuration, but internal components of interest such as the DDA and the S-band antenna feed have been also modeled. The TMM generally shares a one-to-one correspondence with the GMM with the exception of the ribs which are individually distinct and then collapsed into one bulk representation. This type of modeling is valid since the S/C is expected to be spinning about the axis of antenna symmetry when the HGA is stowed.
Figure 3 - GLL HGA hardware description (left) and model nodalization (right)
The external node descriptions are given in Table 1. The antenna is radiatively isolated from the rest of the S/C with an MLI blanket known as the bus shade. The lower tower is covered with MLI blankets as well as the stowed ribs and upper support structure. The radome and PWS support structure are covered with a single layer of black Kapton. The LGA is painted with white paint. The tip shade is carbon-filled Kapton and is used to provide protection from high solar irradiances. It should be noted that the DDA has a significant conductive tie with the S/C main body, and the main body is treated as a 25°C boundary temperature. For this sample problem, two extreme cases were investigated: 1) a hot case at 0.72 AU (near-Venus), and 2) a cold case at 5.0 AU (near-Jupiter). Fig. 3 indicates the direction of the solar flux. The central tower region was of great interest thermally, and therefore, an enclosure around this area was constructed in the GMM so that a global verification of the form factor calculation could be obtained (see Fig. 4). Initially, form factors were computed by using the standard TRASYS values in the FFCAL segment. The nodal form factor sums for some of the central tower nodes are summarized in Table 2, along with the corresponding form factors to space and absorbed solar heating at 1 AU. Afterward, the form factor matrix was normalized. The enclosing the open geometry resulted in 20 GMM nodes outside of the acceptable form factor sum range between 0.95 and 1.05. These nodes were recomputed using the Nusselt unit sphere technique,

<table>
<thead>
<tr>
<th>Node Number(s)</th>
<th>Description</th>
<th>Exterior Surface</th>
<th>$\alpha / \varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 4</td>
<td>Bus shade, HGA side</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>5 - 11</td>
<td>Lower Tower</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>12</td>
<td>Radome</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>68, 69</td>
<td>Upper Support Structure</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>71 - 75</td>
<td>PWS Support Structure</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>76, 77</td>
<td>Tip Shade Support Structure</td>
<td>Black Paint</td>
<td>0.93 /0.87</td>
</tr>
<tr>
<td>78, 79</td>
<td>Tip Shade</td>
<td>Carbon-Filled Kapton</td>
<td>0.90/0.81</td>
</tr>
<tr>
<td>80</td>
<td>LGA</td>
<td>White Paint</td>
<td>0.30/0.85</td>
</tr>
<tr>
<td>81, 82</td>
<td>Tip Shield MLI</td>
<td>ITO-Coated Carbon-Filled Kapton</td>
<td>0.50/0.71</td>
</tr>
<tr>
<td>113 - 118</td>
<td>Rib MLI</td>
<td>Black Kapton</td>
<td>0.85/0.75</td>
</tr>
<tr>
<td>150 -153</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2 - Summary of Normalization Process

<table>
<thead>
<tr>
<th>Node No.</th>
<th>OPEN GEOMETRY</th>
<th>FORM FACTOR MATRIX NORMALIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Form Factor Sum</td>
<td>Form Factor to Space</td>
</tr>
<tr>
<td>1</td>
<td>0.1274</td>
<td>0.8726</td>
</tr>
<tr>
<td>4</td>
<td>0.3257</td>
<td>0.6743</td>
</tr>
<tr>
<td>7</td>
<td>0.4514</td>
<td>0.5486</td>
</tr>
<tr>
<td>10</td>
<td>0.5779</td>
<td>0.4231</td>
</tr>
<tr>
<td>12</td>
<td>0.6009</td>
<td>0.3991</td>
</tr>
<tr>
<td>69</td>
<td>0.7797</td>
<td>0.2203</td>
</tr>
<tr>
<td>71</td>
<td>0.7869</td>
<td>0.2131</td>
</tr>
<tr>
<td>72</td>
<td>0.7273</td>
<td>0.2727</td>
</tr>
<tr>
<td>76</td>
<td>0.6272</td>
<td>0.3728</td>
</tr>
</tbody>
</table>

Note:

- "≤" indicates the value is less than open geometry form factor to space, and "≥" indicates the value is greater than the open geometry form factor to space.
and following this, all nodal form factor sums were within an acceptable range. Next, the form factor sum that exceed unity are reduced and then, all the form factor sums are adjusted to unity. Lastly, the enclosing nodes are removed, and the adjusted form factor matrix for the open geometry remains. Radiation conductors and absorbed heating were calculated. Table 2 summarizes the normalization process. Once radiation conductors and absorbed heating were determined, temperature estimates were determined at 0.72 AU and 5.0 AU using the thermal model from Ref. 6, and the results are given in Table 3.

Discussion of Results

A quick glance at the temperature results indicates that the difference between the standard form factor calculation and form factor matrix normalization may be as larger as 7°C in the hot case and 3°C in the cold case. For the hot case, notice that the temperature of node 7 is warmer for form factor normalization when compared with the standard calculation. However, it should be also indicated that the temperature of node 72 is cooler when the same comparison is made. There is appears to be no apparent trend when comparing temperature differences. However, when Table 2 is reviewed for the comparison between the form factor to space, a pattern develops. In general, when the form factor to space using form factor normalization is less than that of the standard calculation the temperature using the normalization method is greater than the corresponding temperature using the standard technique. In addition, the converse appears to be generally true. A reduced form factor to space usually implies a warmer nodal temperature. However, node 10 is an exception to this generalization, and it seems more influenced by the part of the normalization process where form factors are recomputed to obtain a nodal form factor sum between 0.95 and 1.05. The initial form factor sum within the enclosure was 0.9007 and after recomputation, it was increased to 1.0093. Consequently, this may have changed not only the form factor to space, but also other internodal form factors may have increased or decreased. The normalization process does not always reduce the form factor to space, but rather, it attempts to distribute the form factor computational error over all the nodes. In the process, the analyst strives to verify and revise the form factor calculation in a global way.

The temperature differences in the cold case are less marked than the hot case. At 5.0 AU, the environmental heat load is much smaller than at 0.72 AU, and the temperature distribution should be driven by the radiation coupling to space. For the most part, the form factor to space between the two methods are small, thus leading to only small temperature differences.
Table 3 - Hot and Cold Case Temperature Estimates in °C

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Open Geometry</th>
<th>Form Factor Matrix Normalization</th>
<th>( \Delta T = T_{\text{norm}} - T_{\text{open}}, ^\circ \text{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hot</td>
<td>Cold</td>
<td>Hot</td>
</tr>
<tr>
<td>1</td>
<td>-3.2</td>
<td>-168.4</td>
<td>-2.2</td>
</tr>
<tr>
<td>4</td>
<td>-3.2</td>
<td>-168.4</td>
<td>-2.2</td>
</tr>
<tr>
<td>7</td>
<td>6.0</td>
<td>-152.2</td>
<td>10.1</td>
</tr>
<tr>
<td>10</td>
<td>68.1</td>
<td>-137.7</td>
<td>65.9</td>
</tr>
<tr>
<td>12</td>
<td>70.0</td>
<td>-141.6</td>
<td>634.1</td>
</tr>
<tr>
<td>69</td>
<td>64.0</td>
<td>-143.2</td>
<td>63.7</td>
</tr>
<tr>
<td>71</td>
<td>70.3</td>
<td>-141.4</td>
<td>72.2</td>
</tr>
<tr>
<td>72</td>
<td>69.0</td>
<td>-99.6</td>
<td>62.2</td>
</tr>
<tr>
<td>76</td>
<td>72.2</td>
<td>-133.2</td>
<td>68.9</td>
</tr>
<tr>
<td>33\textbf{b}</td>
<td>35.7</td>
<td>-38.8</td>
<td>35.8</td>
</tr>
<tr>
<td>65\textbf{c}</td>
<td>63.9</td>
<td>-145.0</td>
<td>59.4</td>
</tr>
</tbody>
</table>

Notes:

\( ^a \) Temperature difference between form factor normalization and standard (open geometry) approaches
\( ^b \) Internal node - DDA
\( ^c \) Internal node - S-band antenna feed

Two internal thermal model nodes have been included in Table 3. The DDA (node 33) is coupled to a 25°C boundary, and is largely unaffected by normalization. However, the S-band antenna feed (node 65) is more responsive to the external radiative environment, and this environment can be characterized by node 12 (see Fig. 3). Since the temperature of node 12 for normalization is cooler than the standard method, the S-band antenna feed has a similar character.

When dealing with thermal models, the question of uncertainty arises frequently. As inferred from the results of this sample case, unverified form factor calculations may cause an uncertainty of approximately ±5°C. Unless the thermal design is very forgiving, unverified form factors could result in optimistic thermal performance. Therefore, some method of form factor validation should be performed, and form factor normalization provides such an avenue.
CONCLUSIONS

A method that may globally verify and revise TRASYS form factor calculations has been presented. The primary features of this approach are reducing form factors on a weighed form factor basis and adding a self-viewing form factor to adjust nodal form factor sums to unity. In comparison to the standard method of determining form factors, this process may result in temperatures that may differ by ±5°C. It is recommended that this approach be utilized so that form factor computational error would be distributed over the entire geometric model rather than any one node.

ACKNOWLEDGEMENTS

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REFERENCES


