

484672 IN-39

136215  
P.32

# Closed-Form Expressions for Crack-Mouth Displacements and Stress Intensity Factors for Chevron-Notched Short Bar and Short Rod Specimens Based on Experimental Compliance Measurements

R.T. Bubsey, T.W. Orange, W.S. Pierce, J.L. Shannon, Jr.  
*Lewis Research Center  
Cleveland, Ohio*

October 1992



(NASA-TM-83796) CLOSED FORM  
EXPRESSIONS FOR CRACK MOUTH  
DISPLACEMENTS AND STRESS INTENSITY  
FACTORS FOR CHEVRON NOTCHED SHORT  
BAR AND SHORT ROD SPECIMENS BASED  
ON EXPERIMENTAL COMPLIANCE  
MEASUREMENTS (NASA) 32 p

N93-15369

Unclass

G3/39 0136215

Closed-Form Expressions for Crack-Mouth Displacements and Stress Intensity Factors  
for Chevron-Notched Short Bar and Short Rod Specimens  
Based on Experimental Compliance Measurements

by R. T. Bubsey, T. W. Orange, W. S. Pierce, and J. L. Shannon, Jr.  
Lewis Research Center

### Summary

This report presents a set of equations describing certain fracture mechanics parameters for chevron-notch bar and rod specimens. They are developed by fitting compliance calibration data reported earlier. The equations present the various parameters in their most useful forms. The data encompass the entire range of the specimen geometries most commonly used. Their use will facilitate the testing and analysis of brittle metals, ceramics, and glasses.

### Introduction

This report presents a set of equations describing certain fracture mechanics parameters (mouth displacement and stress intensity factors) for chevron-notch rod and bar specimens.

First the background of the chevron-notch specimens and the experimental data to be used are discussed briefly. Then some particular characteristics of the chevron-notch specimens and their practical application are discussed more extensively. Finally, the fitted equations are presented and their fitting accuracies are discussed.

Measured displacement values and calculated stress intensity factors for each specimen at each crack length, along with differences from the fitted equations, are presented for archival purposes in an appendix.

### Symbols

(see fig. 1)

$a$	Crack length (measured from load line)
$a_0$	Distance from load line to tip of chevron
$a_m$	Crack length at which $Y^*$ is minimum
$B$	Thickness (bar specimen)
$C$	Specimen compliance, $C = EBV/P$
$D$	Diameter (rod specimen)
$E$	Elastic (Young's) modulus
$H$	Half-height (bar specimen)

$K_I$	Opening-mode stress intensity factor
$K_{Iv}$	Plane-strain (chevron-notch) fracture toughness, ASTM E 1304
$P$	Applied load
$V$	Crack mouth opening displacement
$W$	Width
$Y^*$	Dimensionless stress intensity factor for a trapezoidal crack, $K_I BW^{1/2}/P$
$Y_m^*$	Minimum value of $Y^*$ as a function of $\alpha$
$\alpha$	$a/W$
$\alpha_0$	$a_0/W$
$\alpha_m$	$a_m/W$
$\beta$	$W/B$

### Background

Fracture toughness tests of metals are conducted using specimens which are fatigue precracked to simulate a naturally-occurring flaw. Ceramic materials, however, are difficult to fatigue crack due to their brittle nature. Chevron-notch specimens are particularly useful for tests of ceramics since, unlike other specimens, they do not need to be precracked.

The chevron-notch specimens are fairly recent additions to the field of fracture mechanics. Consequently they do not have the same historical background of extensive stress and displacement analysis as do the more common specimen types. But, like the earliest specimen types, useful expressions can be developed using experimental compliance data.

Compliance data for the chevron-notch bar (ref. 1) and rod (ref. 2) specimens were previously reported. In each paper, one fitted equation was presented relating the minimum stress intensity factor to the initial notch length and to the specimen dimensions. A later paper (ref. 3) reported additional data for specimens having smaller initial notch lengths and also revised the previous equations to cover the wider range of crack lengths. As will be discussed later, these equations alone are not sufficient for analyses and tests involving less-brittle materials.

To make these data more complete and useful, a new set of equations are presented in the present report. These equations are developed by fitting curves to the existing data. They are presented in forms suitable for determining fracture toughness from maximum load, for determining the crack-extension resistance curve (R-curve), and for setting instrument sensitivities. They are usable over a wide range of specimen dimensions. This report is a summary and extension of refs. 1 to 3.

### Characteristics of Chevron-Notch Specimens

For most common fracture test specimens, the dimensionless stress intensity factor ( $Y$ ) increases continually with increasing relative crack length ( $a/W$ ). But due to the wedge

shape of the chevron-notch specimen, the corresponding factor ( $Y^*$ ) reaches a minimum, denoted  $Y_m^*$ , as the crack length reaches a value denoted  $a_m$ . The values of  $Y_m^*$  and  $\alpha_m$  are functions of specimen geometry only and are independent of material properties.

If the material being tested has a crack growth resistance curve which exhibits a relatively constant plateau (known as a "flat" R-curve), instability will occur at  $a=a_m$  and  $P=P_{max}$ . Then the fracture toughness ( $K_{Iv}$ ) can be calculated from

$$K_{Iv} = Y_m^* \frac{P_{max}}{BW^{1/2}} \quad (1)$$

and no other test measurements are necessary.

For some materials (even some ceramics), however, the R-curve increases continually with increasing crack extension (a "rising" R-curve). In ductile metals the rising R-curve is attributed to the formation of a plastically deformed zone ahead of the advancing crack. Green et. al. (ref. 4) and Hübner et. al. (refs. 5 and 6) have suggested that a rising R-curve in a ceramic may result from crack branching and microcracking ahead of the crack. In a previous study (ref. 7), bar specimens of hot-pressed silicon nitride and sintered aluminum oxide were tested. For the silicon nitride specimens, the fracture toughness was essentially constant over a range of specimen sizes. This is consistent with a flat R-curve. For the aluminum oxide, however, fracture toughness varied markedly with specimen size and geometry. This, in turn, is consistent with a rising R-curve. Shannon et. al. (ref. 8) have shown that eqn. (1) results in low values of apparent fracture toughness when the material tested has a rising R-curve.

For such materials eqn. (1) does not apply and it may be desirable to determine the complete R-curve. In this case ASTM Test Method E 561 (ref. 9) may be used for guidance. If crack mouth opening displacement (CMOD) is measured during the test (as in E 561) and appropriate compliance relations are available, one can calculate the instantaneous crack length. From crack length and load, one can calculate the crack extension resistance as

$$K_R = K_I = Y^* \frac{P}{BW^{1/2}} \quad (2)$$

A plot of crack extension resistance against crack advance is the R-curve.

### Experimental Procedure

The experimental procedure is described in detail in refs. 1 and 2 but will be summarized here. All specimens (fig. 1) were machined from a single 52-mm (2 in.) thick plate of aluminum alloy 7075-T651. Elastic modulus was determined (in the longitudinal direction) using 13mm (0.5 in.) diameter specimens. For the rod specimens the nominal diameter ( $D$ ) was 48 mm (1.9 in.) and widths ( $W$ ) were 1.5, 1.75, and 2.0 times the diameter ( $\beta = 1.5, 1.75,$  and 2.0). For the bar specimens the thickness ( $B$ ) and the full height ( $2H$ ) were 51 mm (2

in.) and the widths ( $W$ ) were 1.5 and 2.0 times the thickness ( $\beta = 1.5$  and 2.0).

Chevron notches were oriented in the L-T direction and prepared using a 1-mm (0.04 in.) thick slitting saw. The chevron was full-length ( $\alpha_1 = 1.0$ ) for all specimens except those listed in Table A5 of the Appendix. To simulate incremental crack advance, a 0.6-mm (0.025 in.) thick saw was used to cut straight across as shown in fig. 1. A few specimens had straight-thru notches for comparison with approximate solutions (refs. 1 and 2).

The clip gage was as shown in ASTM Test Method E 1304 (ref. 10). The instrumented specimens were loaded in a screw-powered testing machine under displacement control. Loads were limited so that the maximum stress intensity factor was less than about 11 MPa  $m^{1/2}$  (10 ksi  $in^{1/2}$ ). At least three replicate tests for each notch length were averaged to obtain the data reported here. Then the notch was extended (using the 0.6-mm saw) and the procedure repeated. For each specimen, 7 to 15 notch lengths (depending on the initial notch length) were tested.

### Analytical Procedure

Analysis of the data is based on the following equation (ref. 1)

$$Y^* = \left[ \frac{1}{2} \frac{\alpha_1 - \alpha_0}{\alpha - \alpha_0} \frac{d}{d\alpha} \frac{EBV}{P} \right]^{1/2} \quad (3)$$

and its derivative with respect to  $\alpha$ . Logarithms of the basic compliance data ( $C = EBV/P$ ) are fitted with a fourth degree polynomial in  $\alpha$ . The fitted curve is differentiated and values of  $Y^*$  calculated from eq 3. These are presented in Tables A1 and A2 of the Appendix to this report.

The values of  $Y_m^*$  and  $\alpha_m$  are determined using a two-step process. First, values of  $Y^*$  are fitted with a fourth degree polynomial in  $\alpha$  and the minimum of that curve is determined. Then seven points symmetrical about the tentative value of  $\alpha_m$  are selected. A fourth degree polynomial is then fit to the logarithms of the compliance derivatives for those seven points. The final values of  $Y_m^*$  and  $\alpha_m$  are determined from the minimum of the second polynomial. These are presented in Table A3 of the Appendix.

### Development of generalized equations

The following expressions are useful for computing the plane strain fracture toughness  $K_{Iv}$  when the material has a relatively "flat" R-curve.

$$\alpha_m = A_0 + A_1 \alpha_0 + A_2 \alpha_0^2 + A_3 \alpha_0^3 \quad (4)$$

and

$$Y_m^* = B_0 + B_1 \alpha_0 + B_2 \alpha_0^2 + B_3 \alpha_0^3 \quad (5)$$

These were developed by first fitting third-degree polynomials in  $\alpha_0$  for each specimen type (bar or rod) and each value of  $\beta$ . Then the coefficients of the intermediate polynomials were in turn fit to a second-degree polynomial in  $\beta$  to produce the final forms of eqs. (4) and (5). Values of the coefficients for eqs. (4) and (5) are given in Tables 1 and 2, respectively.

An expression for determining the relative crack length  $\alpha$  as a function of measured displacements is

$$\alpha = C_0 + C_1 U + C_2 U^2 + C_3 U^3 + C_4 U^4 \quad (6)$$

where  $U$  is the Saxena and Hudak form (ref. 11)

$$U = \frac{1}{\left(\frac{EBV}{P}\right)^{1/2} + 1}$$

The coefficients for eq. (6) are given in Table 3. This equation lends itself to computer-controlled fracture toughness testing since the subcritical crack growth can be determined from automated load and displacement data acquisition.

When the relative crack length  $\alpha$  is known, the stress intensity factor  $Y^*$  and the dimensionless compliance  $EBV/P$  can be computed from the following expressions:

$$Y^* = e^{D_0 + D_1 \alpha + D_2 \alpha^2 + D_3 \alpha^3 + D_4 \alpha^4} \quad (7)$$

and

$$\frac{EBV}{P} = e^{E_0 + E_1 \alpha + E_2 \alpha^2 + E_3 \alpha^3 + E_4 \alpha^4} \quad (8)$$

The coefficients for eqs. (7) and (8) are given in Tables 4 and 5, respectively.

### Discussion

Determining the parameters  $Y_m^*$  and  $\alpha_m$  from experimental compliance measurements presents a problem which is unique to the chevron-notch specimens. To illustrate, assume that we have developed a function  $f$  such that the experimental compliance values are approximated by

$$EBV/P = f(\alpha)$$

where  $f$  includes the data transform (if any) and a fitting function. To calculate  $Y^*$  we need calculate only the first derivative of the function  $f$ , as in eq. (3). But to determine  $Y_m^*$  and  $\alpha_m$  we must determine the *second* derivative as well. Substituting the function  $f$  into eq. (3), taking the derivative and setting it equal to zero, and eliminating non-zero terms we have

$$0 = \frac{1}{\alpha_m - \alpha_0} f'(\alpha_m) - f''(\alpha_m) \quad (9)$$

where  $f'$  and  $f''$  are the first and second derivatives and  $\alpha_m$  is the root of the equation (which can be found by standard numerical methods). But the higher the order of the derivative taken, the less reliable the value is likely to be. Furthermore, the results may be sensitive to the form chosen for the function  $f$  and the interval over which the data are fitted.

Figure 2, from ref. (1), shows the typical variation of  $Y^*$  with  $\alpha$  for different values of  $\alpha_0$ . Values derived from experimental compliance data would be expected to scatter about these lines. It is apparent from this figure that for a short initial notch (say,  $\alpha_0=0.2$ ),  $Y_m^*$  will be relatively insensitive to the method of curve fitting but  $\alpha_m$  will be very sensitive. However, for a long initial notch (say,  $\alpha_0=0.5$ ) the opposite will be true.

Equation (4) fits the calculated values of  $\alpha_m$  within  $\pm 0.013W$  for all bar specimens and within  $\pm 0.006W$  for all rod specimens. For two of the standard specimen geometries ( $\alpha_0=0.2$ ,  $\beta=2$ ; ref. 10), the calculated values of  $\alpha_m$  are within  $\pm 1.8\%$  of the finite-element results of Raju and Newman (ref. 12). Results for these geometries are shown in figure 3.

Equation (5) fits the calculated values of  $Y_m^*$  within  $\pm 1.0\%$  for all bar specimens and within  $\pm 2.7\%$  for all rod specimens. For the same standard geometries, the calculated values of  $Y_m^*$  are within  $+0.6\%$  of the finite-element results. Results for these geometries are shown in figure 4.

Table 3 of ref. 7 gives values of  $Y_m^*$  and a critical slope ratio  $r_c$ . That ratio is the ratio of the compliances corresponding to  $\alpha_m$  and  $\alpha_0$ . For specimens with  $\beta=2.0$ , the values of  $Y_m^*$  computed from Eq (5) for both the bar and rod specimens are within  $\pm 0.6\%$  of those in ref. 7. The critical slope ratio computed from eq. (8) is within  $+1\%$  for the bar specimen but is  $7.8\%$  low for the rod specimen.

Although determining the minimum values presents a special problem, the basic compliance technique is still valid. It is still possible to determine the stress intensity factor  $Y^*$  with reasonable accuracy. Fisher and Buzzard (ref. 13) used compliance measurements to determine stress intensity factors for the compact specimen. Their results were within  $\pm 2.7\%$  of accepted analytical values. There are no wide-range analytical results for the chevron-notch specimens for comparison, but it is reasonable to expect that the results presented here would be about as accurate as those of reference 13.

Within the ranges of  $\alpha$  and  $\alpha_0$  specified in Tables 3 to 5, eq. (6) fits the measured values of  $\alpha$  within  $\pm 0.003W$  for the bar specimen and within  $\pm 0.002W$  for the rod specimen; eq. (7) fits the calculated values of  $Y^*$  within  $\pm 2.9\%$  for the bar specimen and within  $\pm 2.1\%$  for the rod specimen. These are shown for the same standard geometries in figs. 5 and 6,

respectively.

Equation (8) fits the measured values of  $EBV/P$  within  $\pm 1.4\%$  for all bar and rod specimens, and fits the finite-element results (ref. 12) within  $+6.2\%$  for the same standard geometries. This is shown in figure 7.

### Conclusions

The equations presented here are in forms suitable for several purposes in fracture testing with chevron-notch specimens. They encompass the range of specimen geometries most commonly used. They provide a good fit with the basic compliance data and with available finite-element results.

Table 1 - Coefficients for Eq 4

Specimen	Coeff.	Expression
Bar	A <sub>0</sub>	-0.110 +0.354β -0.088β <sup>2</sup>
	A <sub>1</sub>	0.268 +1.628β -0.400β <sup>2</sup>
	A <sub>2</sub>	1.637 -6.358β +1.872β <sup>2</sup>
	A <sub>3</sub>	0.075 +4.462β -1.508β <sup>2</sup>
Rod	A <sub>0</sub>	0.147 +0.089β -0.026β <sup>2</sup>
	A <sub>1</sub>	0.358 +1.150β -0.096β <sup>2</sup>
	A <sub>2</sub>	2.860 -5.190β +0.770β <sup>2</sup>
	A <sub>3</sub>	-3.610 +5.100β -0.800β <sup>2</sup>

Range:  $1.5 \leq \beta \leq 2.0$ ,  $0 \leq \alpha_0 \leq 0.5$ ,  $\alpha_1 = 1$

Table 2 - Coefficients for Eq 5

Specimen	Coeff.	Expression
Bar	B <sub>0</sub>	-17.03 +29.94β -5.0β <sup>2</sup>
	B <sub>1</sub>	-116.00 +141.60β -29.6β <sup>2</sup>
	B <sub>2</sub>	1131.00 -1304.00β +342.0β <sup>2</sup>
	B <sub>3</sub>	-1351.00 +1654.00β -443.2β <sup>2</sup>
Rod	B <sub>0</sub>	5.47 +6.29β +2.46β <sup>2</sup>
	B <sub>1</sub>	-65.93 +72.62β -5.62β <sup>2</sup>
	B <sub>2</sub>	622.00 -659.80β +146.10β <sup>2</sup>
	B <sub>3</sub>	-541.40 +629.10β -135.20β <sup>2</sup>

Range:  $1.5 \leq \beta \leq 2.0$ ,  $0 \leq \alpha_0 \leq 0.5$ ,  $\alpha_1 = 1$

Table 3 - Coefficients for Eq 6

Specimen	Coeff	Expression		
Bar $\beta=1.5$	C <sub>0</sub>	3.09	-24.12 $\alpha_0$	+57.12 $\alpha_0^2$
	C <sub>1</sub>	-109.30	+1227.00 $\alpha_0$	-2876.00 $\alpha_0^2$
	C <sub>2</sub>	1908.00	-22216.00 $\alpha_0$	+51286.00 $\alpha_0^2$
	C <sub>3</sub>	-14900.00	+168580.00 $\alpha_0$	-381240.00 $\alpha_0^2$
	C <sub>4</sub>	41390.00	-451059.00 $\alpha_0$	+987080.00 $\alpha_0^2$
Bar $\beta=2.0$	C <sub>0</sub>	2.08	-8.74 $\alpha_0$	+16.93 $\alpha_0^2$
	C <sub>1</sub>	-63.31	+540.00 $\alpha_0$	-1019.00 $\alpha_0^2$
	C <sub>2</sub>	1086.00	-11296.00 $\alpha_0$	+20043.00 $\alpha_0^2$
	C <sub>3</sub>	-9327.00	+98493.00 $\alpha_0$	-158690.00 $\alpha_0^2$
	C <sub>4</sub>	28430.00	-284970.00 $\alpha_0$	+366330.00 $\alpha_0^2$
Rod $\beta=1.5$	C <sub>0</sub>	0.672	+4.85 $\alpha_0$	-23.93 $\alpha_0^2$
	C <sub>1</sub>	25.670	-361.90 $\alpha_0$	+1624.00 $\alpha_0^2$
	C <sub>2</sub>	-858.000	+9512.00 $\alpha_0$	-39580.00 $\alpha_0^2$
	C <sub>3</sub>	9219.000	-105260.00 $\alpha_0$	+411440.00 $\alpha_0^2$
	C <sub>4</sub>	-35145.000	+417050.00 $\alpha_0$	-1550300.00 $\alpha_0^2$
Rod $\beta=2.0$	C <sub>0</sub>	0.896	+7.24 $\alpha_0$	-26.5 $\alpha_0^2$
	C <sub>1</sub>	21.800	-590.40 $\alpha_0$	+2087.0 $\alpha_0^2$
	C <sub>2</sub>	-1192.000	+17166.00 $\alpha_0$	-58980.0 $\alpha_0^2$
	C <sub>3</sub>	16772.000	-213330.00 $\alpha_0$	+713640.0 $\alpha_0^2$
	C <sub>4</sub>	-78837.000	+961870.00 $\alpha_0$	-3146400.0 $\alpha_0^2$

Range:  $0.18 \leq \alpha_0 \leq 0.22$ ,  $\alpha_0 \leq \alpha \leq 0.8$ ,  $\alpha_1 = 1$

Table 4 - Coefficients for Eq 7

Specimen	Coeff.	Expression
Bar $\beta=1.5$	D <sub>0</sub>	3.329 +1.026 $\alpha_0$ +78.21 $\alpha_0^2$
	D <sub>1</sub>	-0.812 -58.080 $\alpha_0$ -334.40 $\alpha_0^2$
	D <sub>2</sub>	-2.061 +265.260 $\alpha_0$ +461.40 $\alpha_0^2$
	D <sub>3</sub>	4.350 -417.120 $\alpha_0$ -156.10 $\alpha_0^2$
	D <sub>4</sub>	0.349 +219.800 $\alpha_0$ -65.55 $\alpha_0^2$
Bar $\beta=2.0$	D <sub>0</sub>	4.308 -4.757 $\alpha_0$ +83.77 $\alpha_0^2$
	D <sub>1</sub>	-6.529 -19.190 $\alpha_0$ -358.70 $\alpha_0^2$
	D <sub>2</sub>	16.630 +172.000 $\alpha_0$ +483.10 $\alpha_0^2$
	D <sub>3</sub>	-22.170 -313.000 $\alpha_0$ -151.10 $\alpha_0^2$
	D <sub>4</sub>	13.220 +173.700 $\alpha_0$ -72.71 $\alpha_0^2$
Rod $\beta=1.5$	D <sub>0</sub>	-2.28 +106.3 $\alpha_0$ -567.0 $\alpha_0^2$ +1062 $\alpha_0^3$
	D <sub>1</sub>	29.61 -582.5 $\alpha_0$ +3100.0 $\alpha_0^2$ -5830 $\alpha_0^3$
	D <sub>2</sub>	-60.17 +1167.0 $\alpha_0$ -6070.0 $\alpha_0^2$ +11589 $\alpha_0^3$
	D <sub>3</sub>	52.60 -1022.0 $\alpha_0$ +5051.0 $\alpha_0^2$ -9869 $\alpha_0^3$
	D <sub>4</sub>	-15.00 +337.8 $\alpha_0$ -1517.0 $\alpha_0^2$ +3042 $\alpha_0^3$
Rod $\beta=2.0$	D <sub>0</sub>	2.19 +41.95 $\alpha_0$ -263.7 $\alpha_0^2$ +749.0 $\alpha_0^3$
	D <sub>1</sub>	-1.41 -128.20 $\alpha_0$ +945.6 $\alpha_0^2$ -3532.0 $\alpha_0^3$
	D <sub>2</sub>	22.30 +38.71 $\alpha_0$ -691.5 $\alpha_0^2$ +5784.0 $\alpha_0^3$
	D <sub>3</sub>	-40.95 +179.10 $\alpha_0$ -703.0 $\alpha_0^2$ -3646.0 $\alpha_0^3$
	D <sub>4</sub>	22.92 -131.30 $\alpha_0$ +741.2 $\alpha_0^2$ +607.4 $\alpha_0^3$

Range:  $0.1 \leq \alpha_0 \leq 0.35$  (bar),  $0.1 \leq \alpha_0 \leq 0.40$  (rod),  $\alpha_0 \leq \alpha \leq 0.8$ ,  $\alpha_1=1$

Table 5 - Coefficients for Eq 8

Specimen	Coeff.	Expression			
Bar $\beta=1.5$	E <sub>0</sub>	2.850	-6.48 $\alpha_0$	+61.56 $\alpha_0^2$	
	E <sub>1</sub>	1.177	+26.59 $\alpha_0$	-349.30 $\alpha_0^2$	
	E <sub>2</sub>	9.650	-8.37 $\alpha_0$	+708.90 $\alpha_0^2$	
	E <sub>3</sub>	-16.240	-62.60 $\alpha_0$	-597.00 $\alpha_0^2$	
	E <sub>4</sub>	10.450	+56.82 $\alpha_0$	+167.90 $\alpha_0^2$	
Bar $\beta=2.0$	E <sub>0</sub>	3.885	-17.75 $\alpha_0$	+94.97 $\alpha_0^2$	
	E <sub>1</sub>	-5.160	+123.20 $\alpha_0$	-624.20 $\alpha_0^2$	
	E <sub>2</sub>	34.270	-324.50 $\alpha_0$	+1562.00 $\alpha_0^2$	
	E <sub>3</sub>	-52.330	+386.80 $\alpha_0$	-1756.00 $\alpha_0^2$	
	E <sub>4</sub>	27.950	-173.20 $\alpha_0$	+741.40 $\alpha_0^2$	
Rod $\beta=1.5$	E <sub>0</sub>	3.91	-23.18 $\alpha_0$	+138.4 $\alpha_0^2$	-91.84 $\alpha_0^3$
	E <sub>1</sub>	-10.01	+237.70 $\alpha_0$	-1356.0 $\alpha_0^2$	+1325.00 $\alpha_0^3$
	E <sub>2</sub>	51.60	-758.80 $\alpha_0$	+4284.0 $\alpha_0^2$	-4777.00 $\alpha_0^3$
	E <sub>3</sub>	-74.66	+969.60 $\alpha_0$	-5480.0 $\alpha_0^2$	+6516.60 $\alpha_0^3$
	E <sub>4</sub>	37.83	-433.00 $\alpha_0$	+2464.5 $\alpha_0^2$	-3043.00 $\alpha_0^3$
Rod $\beta=2.0$	E <sub>0</sub>	2.92	+0.28 $\alpha_0$	+26.67 $\alpha_0^2$	+33.27 $\alpha_0^3$
	E <sub>1</sub>	1.68	+28.52 $\alpha_0$	-336.00 $\alpha_0^2$	+111.40 $\alpha_0^3$
	E <sub>2</sub>	20.59	-135.84 $\alpha_0$	+1157.00 $\alpha_0^2$	-918.40 $\alpha_0^3$
	E <sub>3</sub>	-39.16	+218.10 $\alpha_0$	-1581.00 $\alpha_0^2$	+1581.00 $\alpha_0^3$
	E <sub>4</sub>	22.64	-115.78 $\alpha_0$	+756.60 $\alpha_0^2$	-837.20 $\alpha_0^3$

Range:  $0.1 \leq \alpha_0 \leq 0.35$  (bar),  $0.1 \leq \alpha_0 \leq 0.40$  (rod),  $\alpha_0 \leq \alpha \leq 0.8$ ,  $\alpha_1=1$

## References

1. Munz, D., Bubsey, R. T., and Srawley, J. E., "Compliance and Stress Intensity Coefficients for Short Bar Specimens with Chevron Notches," *International Journal of Fracture*, Vol. 16, No. 4, August 1980, pp. 359-374.
2. Bubsey, R. T., Munz, D., Pierce, W. S., and Shannon, J. L., Jr, "Compliance Calibration of the Short Rod Chevron-Notch Specimen for Fracture Toughness Testing of Brittle Materials," *International Journal of Fracture*, Vol. 18, No. 2, February 1982, pp. 125-133.
3. Shannon, J. L., Jr., Bubsey, R. T., Pierce, W. S., and Munz, D., "Extended Range Stress Intensity Factor Expressions for Chevron-Notched Short Bar and Short Rod Fracture Toughness Specimens," *International Journal of Fracture*, Vol. 19, No. 3, July 1982, pp. R55-R58.
4. Green, D. J., Nicholson, P. S., and Embury, J. D., "Fracture Toughness of Partially Stabilized  $ZrO_2$  in the System  $CaO-ZrO_2$ ," *Journal of the American Ceramic Society*, Vol. 56, 1973, pp. 619-623.
5. Hübner, H., and Jillek, W., "Sub-Critical Crack Extension and Crack Resistance in Polycrystalline Alumina," *Journal of Material Science*, Vol. 12, 1977, pp. 117-125.
6. Kleinlein, F. W., and Hübner, H., "The Evaluation of Crack Resistance and Crack Velocity from Controlled Fracture Experiments of Ceramic Bend Specimens," in *Fracture 1977*, Proceedings of the 4th International Conference on Fracture, Waterloo, Ont., Canada, Vol. 3B, 1977, pp: 883-891.
7. Shannon, J. L., Jr., Bubsey, R. T., Munz, D., and Pierce, W. S., "Fracture Toughness of Brittle Materials Determined with Chevron Notch Specimens," in *Advances in Fracture Research (ICF5)*, Francois, D., ed., Vol. 2, 1982, pp. 1127-1144. Also available as NASA TM 81607, 1981.
8. Shannon, J. L., Jr., and Munz, D. G., "Specimen Size and Geometry Effects on Fracture Toughness of  $Al_2O_3$  Measured with Short Rod and Short Bar Chevron-Notch Specimens," in *Chevron-Notched Specimens: Testing and Stress Analysis, ASTM STP 855*, American Society for Testing and Materials, Philadelphia, 1984, pp. 97-121.
9. Standard Practice for R-Curve Determination, ASTM E 561-86, *Annual Book of ASTM Standards*, Vol. 03.01, 1991.
10. Standard Test Method for Plane-Strain (Chevron-Notch) Fracture Toughness of Metallic Materials, ASTM E 1304-89, *Annual Book of ASTM Standards*, Vol. 03.01, 1991.
11. Saxena, A., and Hudak, S. J., Jr., "Review and Extension of Compliance Information for Common Crack Growth Specimens," *International Journal of Fracture*, Vol. 14, No. 5, October 1978, pp. 453-468.
12. Raju, I. S., and Newman, J. C., Jr., "Three-Dimensional Finite-Element Analysis of Chevron-Notched Fracture Specimens, in *Chevron-Notched Specimens: Testing and Stress Analysis, ASTM STP 855*, American Society for Testing and Materials, Philadelphia, 1984, pp. 32-48.
13. Fisher, D. M., and Buzzard, R. J., "Experimental Compliance Calibration of the Compact Fracture Toughness Specimen." NASA TM 81665, 1980.

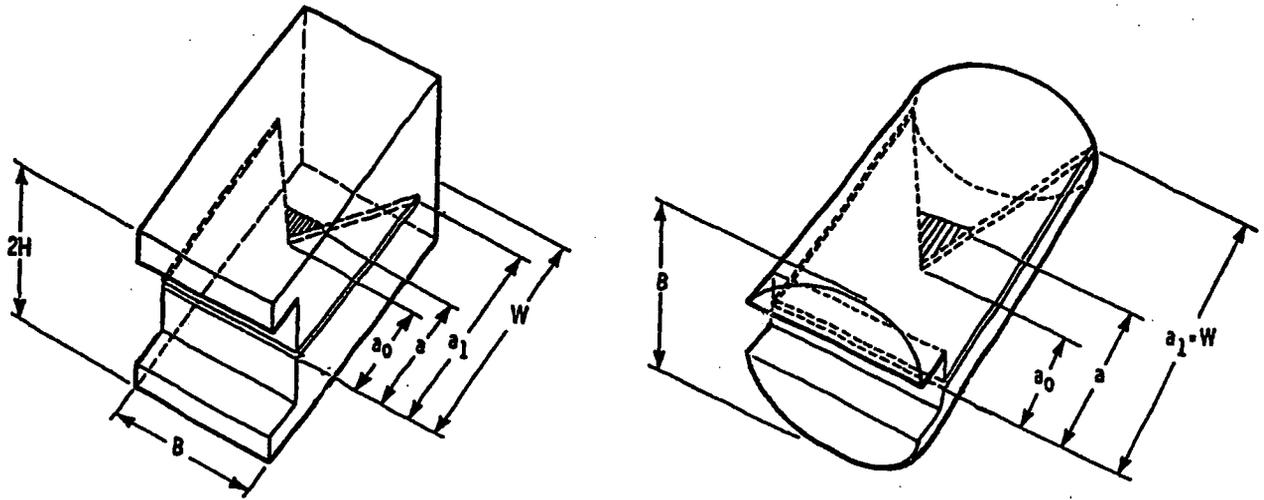


Figure 1.—Chevron-notch bar and rod specimens.

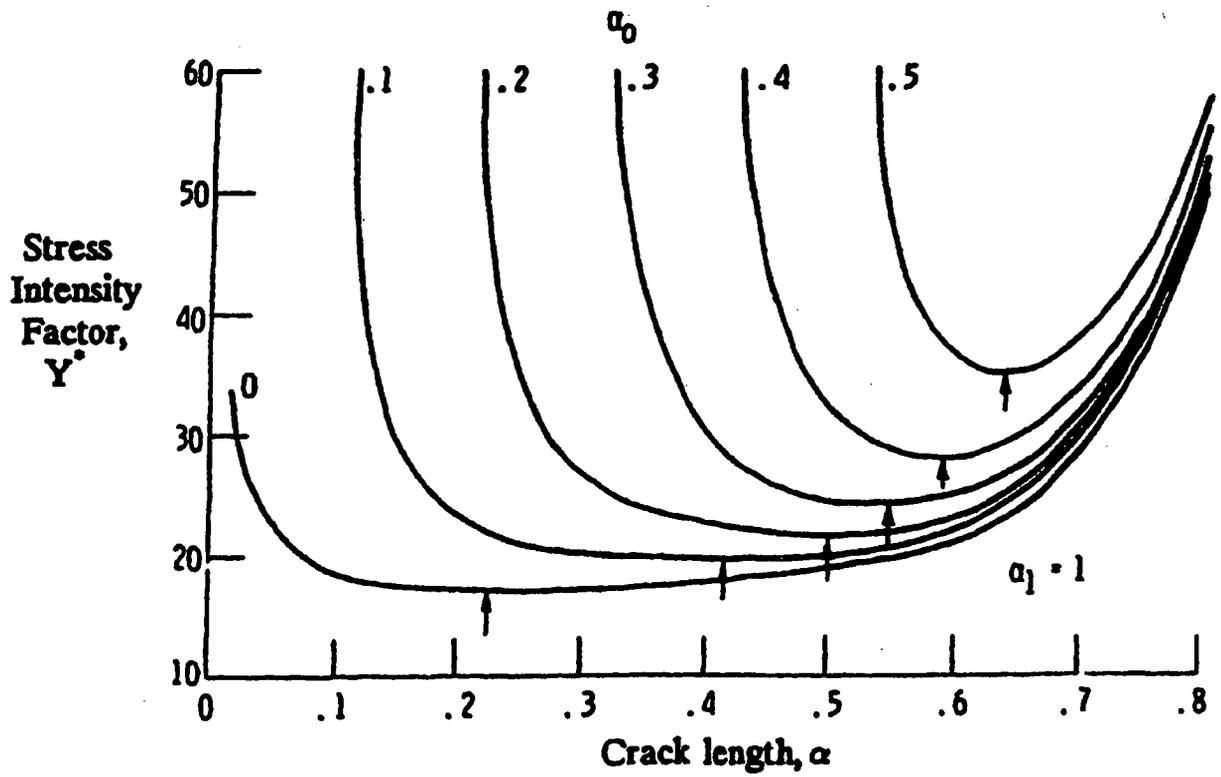


Figure 2.—Typical variation of stress intensity factor with crack length for chevron-notch specimens. [1]. Arrows denote minima.

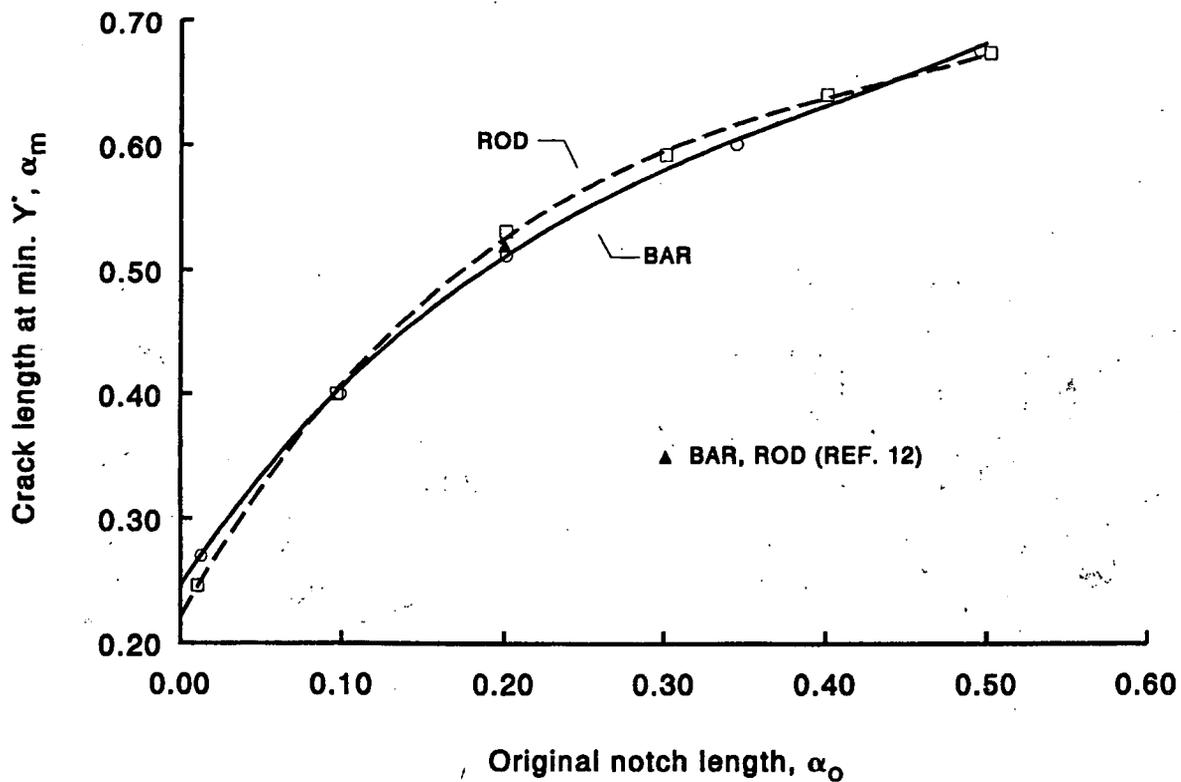


Figure 3.—Location of minimum stress intensity factor; equation (4) fit to data ( $\alpha_0 = 0.2$ ,  $\alpha_1 = 1.0$ ,  $\beta = 2$ ). Note that ref. 12 values for bar and rod coincide.

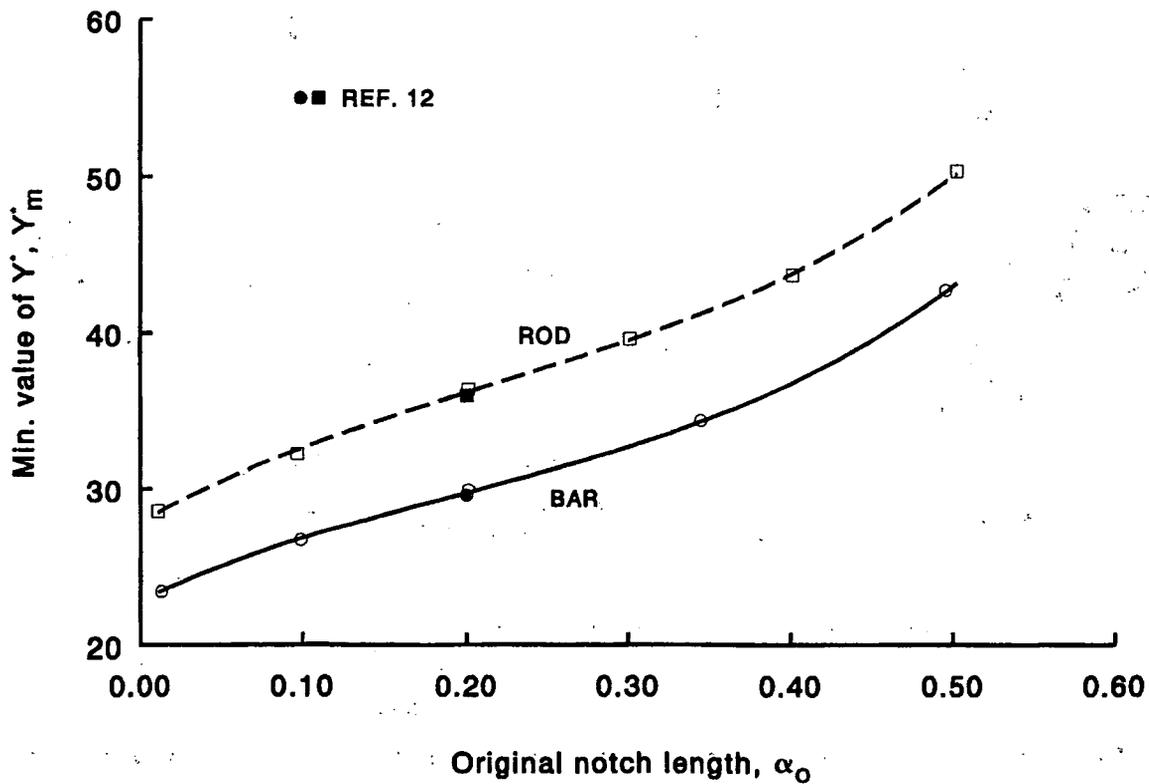


Figure 4.—Minimum value of stress intensity factor; equation (5) fit to data ( $\alpha_0 = 0.2$ ,  $\alpha_1 = 1.0$ ,  $\beta = 2$ ).

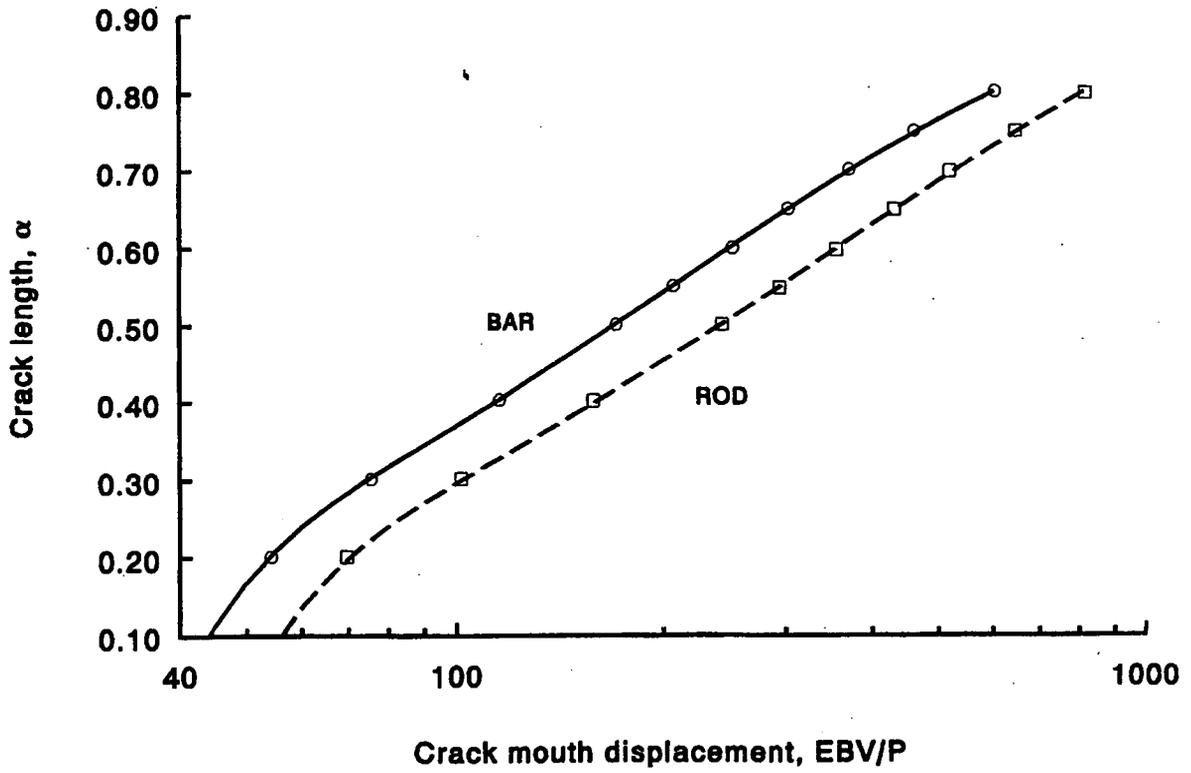


Figure 5.—Crack length as a function of mouth displacement; equation (6) fit to data ( $\alpha_0 = 0.2, \alpha_1 = 1.0, \beta = 2$ ).

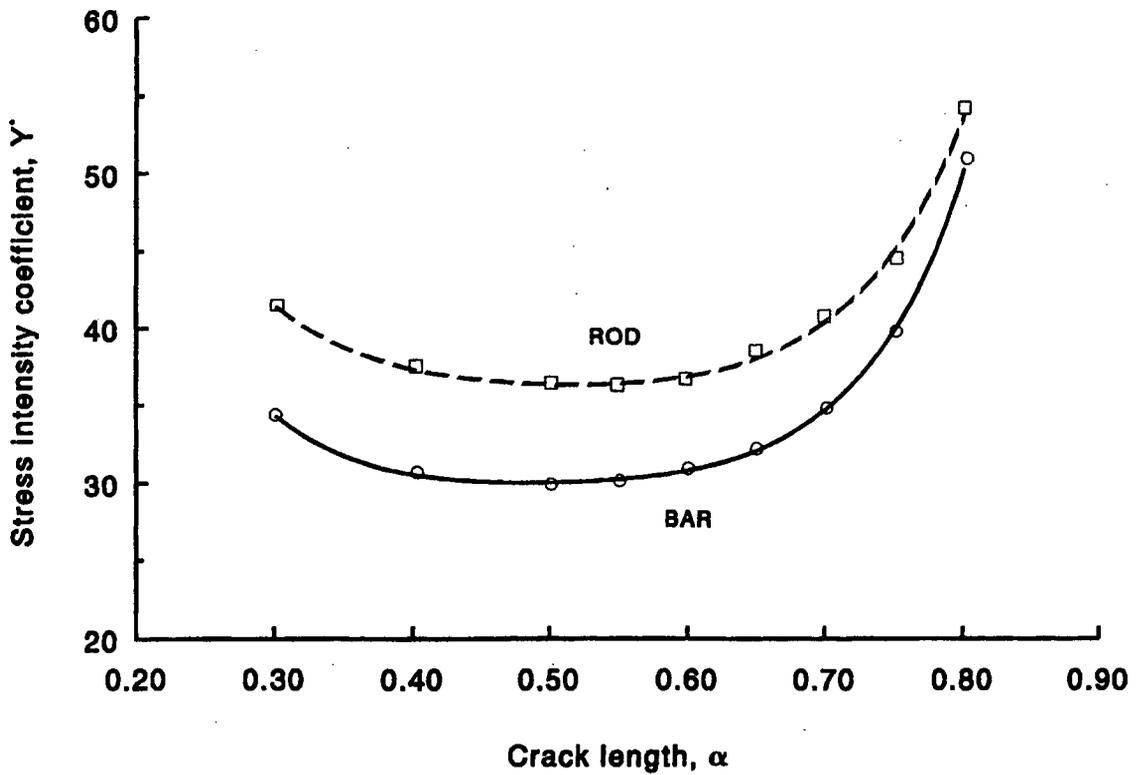


Figure 6.—Stress intensity factor as a function of crack length; equation (7) fit to data ( $\alpha_0 = 0.2, \alpha_1 = 1.0, \beta = 2$ ).

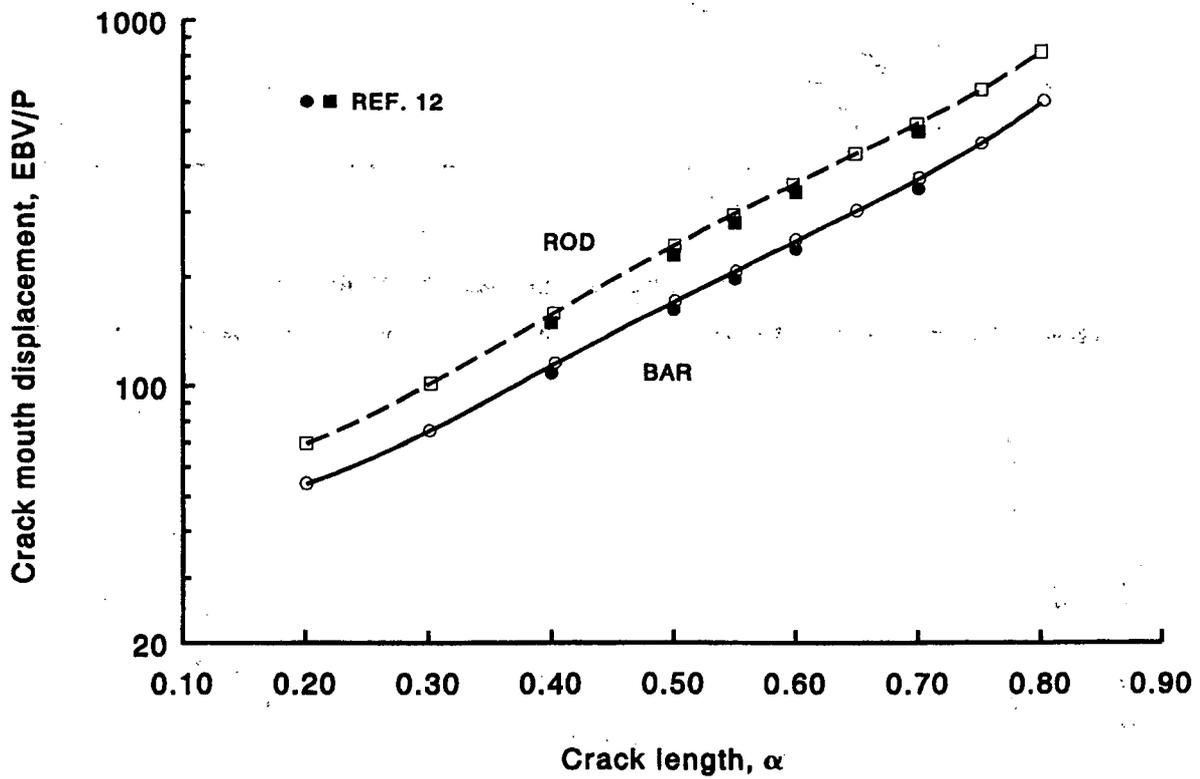


Figure 7.—Crack mouth displacement as a function of crack length; equation (8) fit to data ( $\alpha_0 = 0.2$ ,  $\alpha_1 = 1.0$ ,  $\beta = 2$ ).

## APPENDIX

### Test Data and Calculated Values

This appendix contains the specimen dimensional measurements and the calculated values of normalized crack mouth opening displacement (EBV/P) and stress intensity factor coefficient ( $Y^*$ ) that are used in this report. Similar information for specimens having straight-through cracks are included (these are not used in this report but were used in Refs. 1 and 2 and not fully documented there). Finally, previously unpublished data from bar specimens with  $\alpha_1 < 1$  are presented but not analyzed.

Table A1 - Test data and calculated values for bar specimens,  $\alpha_1 = 1$

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
1	2.000	0.013	0.013	(b)	---	13.67	(b)	---	$\infty$		
			0.044			15.27			31.50	(b)	---
			0.101			20.10			25.39		
			0.152			27.56			24.14		
			0.200			36.33			23.47		
			0.251			48.24			23.48		
			0.300			63.24			23.95		
			0.350			81.61			24.55		
			0.399			103.6			25.07		
			0.451			131.8			25.59		
			0.500			162.3			26.12		
			0.600			240.0			28.06		
			0.699			360.1			33.60		
			0.798			575.1			43.97		
9	2.000	0.099	0.099	(b)	---	27.02	26.90	-0.46	$\infty$		
			0.146			31.89	32.32	1.35	35.92	35.73	-0.52
			0.199			40.95	41.05	0.24	30.17	30.49	1.08
			0.250			52.87	52.54	-0.63	28.11	28.05	-0.23
			0.299			67.21	66.88	-0.49	27.12	27.00	-0.43
			0.349			85.54	85.20	-0.40	26.73	26.68	-0.19
			0.402			108.3	108.9	0.53	26.81	26.76	-0.17
			0.450			134.2	134.3	0.05	27.04	27.03	-0.05
			0.501			165.4	165.5	0.08	27.36	27.42	0.21
			0.551			201.0	200.8	-0.08	27.96	27.96	-0.01
			0.599			243.1	240.3	-1.16	28.70	28.79	0.30
			0.700			356.3	356.0	-0.10	33.31	33.13	-0.53
			0.800			585.3	582.2	-0.52	47.70	47.75	0.10

Table A1 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
15	2.000	0.201	0.201	0.202	0.001	54.20	54.19	-0.02	$\infty$		
			0.301	0.303	0.002	75.51	75.56	0.07	34.38	34.45	0.20
			0.403	0.403	0.000	115.4	114.7	-0.62	30.70	30.56	-0.45
			0.501	0.502	0.001	171.0	170.8	-0.10	29.92	30.07	0.50
			0.551	0.552	0.001	206.8	207.2	0.21	30.16	30.29	0.42
			0.600	0.603	0.003	251.3	249.2	-0.83	30.97	30.86	-0.36
			0.650	0.650	-0.000	302.1	301.0	-0.35	32.24	32.14	-0.32
			0.701	0.699	-0.002	370.5	368.8	-0.47	34.84	34.84	-0.01
			0.751	0.748	-0.003	460.4	460.3	-0.01	39.83	40.11	0.71
			0.802	0.803	0.001	603.3	600.9	-0.40	50.99	50.92	-0.13
			0.849	(c)	---	890.5	(c)	---	(d)	(c)	---
			0.901			1706					
			0.953			6153					
12	2.000	0.345	0.345	(b)	---	115.6	115.3	-0.28	$\infty$		
			0.403			130.4	130.3	-0.06	45.27	45.36	0.20
			0.501			181.1	179.9	-0.64	36.38	36.43	0.15
			0.551			215.1	215.5	0.19	34.74	34.90	0.47
			0.600			256.2	256.9	0.28	34.39	34.50	0.31
			0.650			306.6	307.5	0.30	35.03	35.16	0.38
			0.699			371.9	370.4	-0.40	37.14	37.29	0.39
			0.752			461.8	467.3	1.20	42.16	42.43	0.65
			0.802			612.2	616.6	0.72	52.32	52.63	0.59
11	2.000	0.495	0.495	(b)	---	216.8	(b)	---	$\infty$		
			0.550			240.1			51.10	(b)	---
			0.601			275.4			44.93		
			0.652			323.6			43.12		
			0.702			394.6			43.12		
			0.754			485.5			46.69		
			0.800			619.1			56.15		
			0.850			911.1			(c)		
0.901			1719								

Table A1 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
16	1.500	0.017	0.017	(b)	---	12.56	(b)	---	$\infty$		
			0.049			13.07			21.07	(b)	---
			0.099			15.30			19.83		
			0.149			19.65			18.63		
			0.201			24.81			17.50		
			0.248			30.79			17.20		
			0.300			38.95			17.35		
			0.350			48.70			17.46		
			0.400			59.49			17.71		
			0.450			73.34			18.34		
			0.499			89.34			19.18		
			0.548			108.7			20.14		
			0.599			133.3			20.86		
			0.699			208.3			26.65		
0.800			396.5			42.50					
24	1.500	0.098	0.131	(b)	---	21.73	21.60	-0.60	28.25	27.54	-2.53
			0.149			22.75	22.94	0.83	25.02	25.73	2.82
			0.196			27.09	27.25	0.61	22.29	22.45	0.71
			0.252			34.21	34.07	-0.41	20.44	20.31	-0.66
			0.299			41.40	41.34	-0.15	19.46	19.40	-0.32
			0.348			50.56	50.55	-0.02	19.00	18.99	-0.03
			0.400			61.91	62.30	0.63	19.07	18.98	-0.45
			0.449			75.19	75.44	0.33	19.40	19.28	-0.63
			0.500			91.55	91.66	0.13	19.86	19.88	0.11
			0.551			111.3	111.4	0.09	20.67	20.86	0.90
			0.602			135.9	136.2	0.22	22.18	22.37	0.87
			0.698			210.0	210.6	0.33	28.55	28.05	-1.76
			0.800			396.8	398.3	0.37	44.39	44.73	0.77
			30	1.500	0.196	0.196	0.196	0.000	35.60	35.69	0.25
0.307	0.307	-0.000				48.06	47.88	-0.37	23.51	23.89	1.61
0.401	0.398	-0.003				66.23	66.72	0.75	21.70	21.15	-2.52
0.501	0.497	-0.004				95.39	96.15	0.79	21.73	21.32	-1.89
0.548	0.547	-0.001				114.9	114.2	-0.58	22.20	22.04	-0.71
0.601	0.597	-0.004				139.7	139.7	0.02	23.26	23.43	0.72
0.650	0.646	-0.004				170.3	171.1	0.46	25.64	25.54	-0.39
0.699	0.697	-0.002				215.3	215.7	0.18	29.73	29.05	-2.30
0.751	0.750	-0.001				287.6	290.2	0.91	36.49	35.63	-2.35
0.799	0.796	-0.003				408.3	408.5	0.06	47.23	46.94	-0.61
0.851	(b)	---				687.8	(c)	---	(d)	(c)	---
0.901						1503					
0.954						6874					

Table A1 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
27	1.500	0.342	0.342	(b)	---	69.64	69.72	0.11	$\infty$		
			0.403			77.08	77.39	0.40	31.36	31.16	-0.63
			0.500			102.7	102.3	-0.38	26.13	26.06	-0.28
			0.552			121.2	122.1	0.74	25.72	25.78	0.22
			0.600			144.9	145.4	0.33	26.61	26.47	-0.53
			0.653			179.2	179.5	0.18	28.51	28.39	-0.44
			0.701			223.7	223.6	-0.05	31.81	31.68	-0.42
			0.753			296.9	298.6	0.58	38.61	38.32	-0.75
			0.803			425.8	426.8	0.23	51.41	50.92	-0.95
			0.852			727.4	(c)	---	(d)	(c)	---
			0.898			1452					
			0.952			6496					
25	1.500	0.481	0.481	(b)	---	125.3	(b)	---	$\infty$		
			0.551			142.2			35.21	(b)	---
			0.602			162.5			32.88		
			0.650			192.2			33.47		
			0.699			235.0			36.10		
			0.751			308.6			41.96		
			0.801			434.3			53.44		
			0.853			737.8			(d)		
			0.902			1580					
			0.950			5983					

- a) Calculated from compliance measurements (see text).
- b)  $\alpha_0$  is outside the range of validity for the equation.
- c)  $\alpha$  is outside the range of validity for the equation.
- d) Not calculated.

Table A2 - Test data and calculated values for rod specimens,  $\alpha_1 = 1$

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$					
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff			
61	2.000	0.011	0.011	(b)	---	16.74	(b)	---	$\infty$					
			0.045			17.99			33.86	(b)	---			
			0.098			24.67			29.85					
			0.148			34.83			29.22					
			0.197			48.45			29.09					
			0.246			65.79			28.91					
			0.298			88.03			29.09					
			0.348			115.9			29.62					
			0.399			149.3			30.20					
			0.451			189.2			30.95					
			0.500			235.2			31.79					
			0.550			289.4			33.06					
			0.599			353.3			34.75					
			0.700			525.8			39.29					
			0.800			825.0			47.83					
40	2.000	0.097	0.097	(b)	---	33.61	(b)	---	$\infty$					
			0.148			40.13			40.64	(b)	---			
			0.200			53.24			35.59					
			0.250			71.05			33.64					
			0.301			91.80			32.72					
			0.349			117.9			32.42					
			0.400			150.5			32.52					
			0.451			191.2			32.78					
			0.500			233.6			33.12					
			0.551			289.5			33.77					
			0.601			351.9			34.78					
			0.700			512.3			39.56					
			0.800			800.3			49.80					
			35	2.000	0.201	0.201	-0.103	-0.304	69.64	69.72	0.11	$\infty$		
						0.302	0.245	-0.057	101.6	101.7	0.15	41.51	41.45	-0.14
0.402	0.386	-0.016				158.6	158.7	0.07	37.52	37.31	-0.56			
0.501	0.491	-0.010				242.9	243.2	0.12	36.43	36.35	-0.23			
0.549	0.542	-0.007				293.9	294.7	0.27	36.32	36.40	0.22			
0.598	0.595	-0.003				355.4	355.4	0.02	36.73	36.86	0.34			
0.649	0.648	-0.001				430.3	430.0	-0.07	38.59	38.04	-1.43			
0.699	0.697	-0.002				518.6	520.5	0.36	40.83	40.43	-0.98			
0.751	0.750	-0.001				646.0	646.0	-0.01	44.61	45.26	1.45			
0.800	0.800	-0.000				816.2	817.4	0.14	54.23	53.81	-0.77			
0.852	(c)	---				1151	(c)	---	(d)	(c)	---			
0.900						1943								
0.951						6769								

Table A2 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
34	2.000	0.301	0.301	(b)	---	124.5	124.6	0.10	$\infty$		
			0.399			169.0	169.1	0.10	46.97	46.92	-0.10
			0.501			250.6	250.2	-0.18	41.08	40.88	-0.48
			0.549			300.9	300.5	-0.12	39.91	39.95	0.10
			0.600			361.9	363.1	0.33	39.49	39.62	0.33
			0.650			439.1	435.8	-0.75	40.13	40.08	-0.12
			0.701			527.1	527.5	0.07	42.37	41.95	-1.00
			0.752			652.5	650.2	-0.36	46.07	46.56	1.06
			0.801			824.7	822.9	-0.22	56.30	56.23	-0.13
			0.852			1162	(c)	---	(d)	(c)	---
			0.900			1975					
			0.952			6952					
			33	2.000	0.401	0.401	(b)	---	203.8	203.6	-0.10
0.500						266.7	265.9	-0.30	50.66	50.69	0.06
0.551						315.0	314.3	-0.24	46.14	46.10	-0.09
0.599						372.0	369.9	-0.56	44.10	44.18	0.18
0.651						445.2	442.9	-0.54	43.51	43.67	0.38
0.700						533.0	528.8	-0.79	44.86	44.79	-0.16
0.750						650.9	645.5	-0.82	48.42	48.72	0.62
0.800						825.7	817.0	-1.05	58.36	58.61	0.44
0.849						1147	(c)	---	(d)	(c)	---
0.899						1947					
0.951						6732					
31	2.000	0.502	0.502	(b)	---	309.8	(b)	---	$\infty$		
			0.551			340.7			64.82	(b)	---
			0.601			392.2			54.08		
			0.649			456.0			50.57		
			0.699			539.6			50.17		
			0.750			664.3			53.02		
			0.800			835.1			61.09		
			0.851			1170			(d)		
			0.901			2026					
			0.952			6900					

Table A2 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
55	1.750	0.200	0.200	(e)	---	56.91	(e)	---	$\infty$		
			0.298			78.93			35.38	(e)	---
			0.397			118.2			31.65		
			0.497			177.9			30.71		
			0.549			216.9			30.82		
			0.602			265.5			31.50		
			0.651			319.6			32.89		
			0.699			385.4			36.41		
			0.750			489.5			40.86		
			0.801			639.8			51.70		
			0.850			936.1			(d)		
			0.903			1837					
			0.951			6552					
54	1.750	0.301	0.301	(e)	---	98.80	(e)	---	$\infty$		
			0.401			131.1			39.29	(e)	---
			0.497			184.8			35.01		
			0.546			220.6			34.03		
			0.599			268.7			33.62		
			0.651			323.8			34.53		
			0.700			390.7			37.47		
			0.749			487.6			42.19		
			0.801			646.0			53.42		
			0.849			934.2			(d)		
			0.901			1794					
			0.952			6842					
			53	1.750	0.398	0.398	(e)	---	155.6	(e)	---
0.502						201.6			43.21	(e)	---
0.549						236.1			39.55		
0.598						277.7			37.49		
0.651						334.7			37.86		
0.700						401.7			40.43		
0.752						507.8			44.52		
0.802						662.9			54.84		
0.849						935.0			(d)		
0.900						1763					
0.953						6954					

Table A2 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
51	1.750	0.497	0.497	(e)	---	236.4	(e)	---	$\infty$		
			0.549			256.1			51.59	(e)	---
			0.600			294.3			43.43		
			0.652			343.6			41.61		
			0.698			387.6			43.24		
			0.751			506.5			48.25		
			0.803			670.1			59.01		
			0.851			957.4			(d)		
			0.901			1823					
			0.952			6746					
			50	1.500	0.018	0.018	(b)	---	14.97	(b)	---
0.046						15.55			25.98	(b)	---
0.099						18.94			22.21		
0.150						24.13			21.39		
0.199						30.89			21.27		
0.248						40.79			21.22		
0.297						51.47			21.02		
0.348						65.83			21.13		
0.401						83.20			21.47		
0.448						101.6			21.89		
0.498						124.2			22.61		
0.550						153.5			23.76		
0.598						185.3			25.10		
0.700						285.0			32.88		
0.797						491.2			43.05		
49	1.500	0.094	0.141	(b)	---	27.22	(b)	---	27.19	(b)	---
			0.150			27.87			27.27		
			0.197			33.85			26.38		
			0.247			43.47			24.70		
			0.301			55.28			23.11		
			0.353			69.29			22.77		
			0.402			85.60			23.20		
			0.450			105.0			23.43		
			0.502			129.1			23.57		
			0.549			153.9			24.28		
			0.600			188.9			25.76		
			0.697			284.2			31.78		
			0.799			502.2			46.23		

Table A2 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
45	1.500	0.201	0.201	0.200	-0.001	46.42	46.39	-0.06	$\infty$		
			0.301	0.301	-0.000	62.21	62.26	0.07	29.50	29.25	-0.83
			0.400	0.400	-0.000	90.26	90.16	-0.12	26.36	26.20	-0.61
			0.501	0.501	-0.000	132.1	132.4	0.28	25.73	25.59	-0.54
			0.554	0.555	0.001	161.4	161.4	-0.01	26.13	26.19	0.24
			0.599	0.600	0.001	191.6	191.3	-0.14	27.09	27.34	0.92
			0.649	0.648	-0.001	232.7	233.3	0.26	29.69	29.58	-0.37
			0.705	0.704	-0.001	297.5	299.1	0.56	33.90	33.95	0.15
			0.752	0.754	0.002	384.0	382.5	-0.40	39.33	40.14	2.06
			0.798	0.798	-0.000	509.5	511.2	0.33	49.63	50.09	0.92
			0.850	(c)	---	808.1	(c)	---	(d)	(c)	---
			0.900			1607					
			0.949			5931					
44	1.500	0.305	0.305	(b)	---	79.13	79.17	0.05	$\infty$		
			0.399			98.44	98.31	-0.14	33.29	33.10	-0.57
			0.498			137.0	137.7	0.50	29.65	29.48	-0.57
			0.550			165.7	165.9	0.12	29.05	29.14	0.31
			0.599			197.6	198.1	0.25	29.37	29.70	1.11
			0.650			239.8	240.1	0.14	31.45	31.40	-0.16
			0.701			298.1	297.6	-0.17	34.72	34.83	0.30
			0.750			377.8	381.2	0.89	40.09	40.85	1.89
			0.800			526.1	525.9	-0.03	51.60	52.12	1.00
			0.850			807.5	(c)	---	(d)	(c)	---
			0.899			1606					
			0.951			6415					
43	1.500	0.400	0.400	(b)	---	118.5	119.0	0.34	$\infty$		
			0.500			148.2	148.4	0.11	35.26	35.27	0.03
			0.550			171.1	172.9	1.05	32.94	32.93	-0.03
			0.599			203.0	203.5	0.23	32.47	32.59	0.3
			0.649			242.0	243.6	0.67	33.53	33.82	0.8
			0.702			302.0	302.8	0.24	37.21	37.10	-0.2
			0.750			382.2	385.1	0.74	42.36	42.76	0.9
			0.801			534.9	536.3	0.25	53.90	54.03	0.2
			0.852			832.2	(c)	---	(d)	(c)	---
			0.901			1651					
			0.950			6233					

Table A2 - continued

Spec. No.	$\beta$	$\alpha_0$	Relative Crack Length, $\alpha$			Normalized Crack Mouth Displacement, EBV/P			Stress Intensity Coefficient, $Y^*$		
			Test	Eq (6)	Diff.	Test	Eq (8)	% Diff	(a)	Eq (7)	% Diff
41	1.500	0.499	0.499	(b)	---	177.9	(b)	---	$\infty$		
			0.547			190.5			46.10	(b)	---
			0.598			218.4			40.23		
			0.653			258.4			38.27		
			0.702			313.6			39.94		
			0.749			388.9			45.64		
			0.800			543.1			57.03		
			0.848			814.1			(d)		
			0.899			1611					
			0.949			5993					

- a) Calculated from compliance measurements (see text).
- b)  $\alpha_0$  is outside the range of validity for the equation.
- c)  $\alpha$  is outside the range of validity for the equation.
- d) Not calculated.
- e) No equation was fitted for  $\beta=1.75$ .

Table A3 - Calculated values of  $Y_m^*$  and  $\alpha_m$ ,  $\alpha_1=1$

Spec. Type	$\beta$	No.	$\alpha_0$	Values at Minima <sup>(a)</sup>	
				$Y_m^*$	$\alpha_m$
Bar	2.00	1	0.013	23.42	0.270
		9	0.099	26.70	0.400
		15	0.201	29.91	0.512
		12	0.345	34.34	0.600
		11	0.495	42.72	0.675
	1.50	16	0.017	17.15	0.252
		24	0.098	18.96	0.370
		30	0.196	21.59	0.459
		27	0.342	25.63	0.546
		25	0.481	32.96	0.615
Rod	2.00	61	0.011	28.50	0.246
		40	0.097	32.25	0.400
		35	0.201	36.36	0.531
		34	0.301	39.61	0.592
		33	0.401	43.73	0.640
		31	0.502	50.30	0.673
	1.75	55	0.200	30.65	0.512
		54	0.301	33.67	0.576
		53	0.398	37.58	0.617
		51	0.497	41.57	0.647
	1.50	50	0.018	21.02	0.255
		49	0.094	22.90	0.370
		45	0.201	25.80	0.488
		44	0.305	29.01	0.558
		43	0.400	32.49	0.602
		41	0.499	38.17	0.638

a) Minimum stress intensity factor,  $Y_m^*$ , and relative crack length,  $\alpha_m$ , at which it occurs

Table A4 - Test data and calculated values for straight-through-crack specimens,  $\alpha_1 = 1$

Type	Spec. No.	$\beta$	(a) $\alpha$	(b) EBV/P	(c) Y			
Bar	5	2.00	0.099	9.16	7.12			
			0.201	24.49	10.17			
			0.298	50.87	12.92			
			0.398	91.51	15.66			
			0.500	151.4	18.89			
			0.548	189.8	20.45			
			0.601	236.7	22.00			
			0.650	287.9	23.82			
			0.703	355.5	27.44			
			0.751	438.8	32.93			
			0.800	581.0	42.58			
			0.852	868.0	65.85			
			0.901	1663	124.9			
	0.951	6189	328.3					
	Bar	18	1.50	0.132	9.20	6.20		
				0.198	15.50	7.50		
				0.300	29.93	9.40		
				0.395	50.01	11.13		
				0.493	79.39	13.03		
				0.548	98.30	14.60		
0.601				126.6	16.88			
0.652				158.4	19.76			
0.700				205.1	23.66			
0.753				277.6	30.36			
0.797				385.1	39.70			
0.853				681.6	64.49			
0.907				1602	136.1			
0.950				5831	328.2			
Rod				37	2.00	0.106	13.10	7.36
						0.208	36.20	11.98
						0.300	72.20	16.74
	0.400	133.0	20.72					
	0.499	217.5	21.92					
	0.551	273.1	21.94					
	0.601	340.0	22.53					
	0.651	414.5	24.82					
	0.700	504.8	29.78					
	0.749	621.3	38.61					
	0.800	796.2	54.13					
	0.850	1110	80.59					
	0.900	1916	130.3					
0.950	6354	234.3						

Table A4 - continued

Type	Spec. No.	$\beta$	(a) $\alpha$	(b) EBV/P	(c) Y
Rod	57	1.75	0.121	13.00	6.59
			0.197	26.00	9.63
			0.298	54.70	14.12
			0.398	97.70	17.61
			0.499	158.8	18.65
			0.549	197.4	18.64
			0.599	244.2	19.18
			0.648	300.7	21.28
			0.701	373.5	26.45
			0.750	468.8	35.15
			0.799	618.7	49.94
			0.848	890.5	76.12
			0.899	1678	130.0
			0.952	6690	260.2
	47	1.50	0.141	13.10	5.81
			0.192	20.00	7.70
			0.297	40.80	11.72
			0.403	73.50	14.97
			0.500	116.7	16.01
			0.549	142.8	16.14
			0.600	179.1	16.81
			0.651	221.4	19.01
			0.700	280.5	23.55
			0.749	360.1	31.65
0.800	499.4	46.39			
0.851	781.0	73.96			
0.899	1547	126.8			
0.949	5782	254.5			

- a) Relative crack length
- b) Normalized crack mouth displacement
- c) Stress intensity factor coefficient

Table A5 - Test data and calculated values for  $\alpha_1$  study  
(bar specimens,  $\beta=2$ )

Spec. No.	$\alpha_1$	$\alpha_0$	$\alpha$	EBV/P	$Y^*$	$\alpha_m$	$Y_m^*$
3	0.403	0.199	0.199	38.40	$\infty$	0.403	15.94
			0.223	40.31	20.32		
			0.249	43.38	17.57		
			0.275	48.04	16.80		
			0.301	54.42	16.29		
			0.325	61.27	15.98		
			0.351	70.08	15.98		
			0.375	80.50	15.97		
			0.402	92.73	15.98		
			0.451	120.4	17.29		
			0.500	150.2	17.43		
			2	0.600	0.198		
0.247	52.61	27.26					
0.300	64.94	23.06					
0.350	80.22	21.54					
0.400	99.94	20.91					
0.449	124.0	20.67					
0.500	153.7	20.68					
0.550	188.9	20.93					
0.600	231.3	21.53					
0.648	280.2	23.82					
0.697	342.9	26.78					
4	0.800	0.198				0.198	49.52
			0.298	69.44	29.02		
			0.399	106.8	25.93		
			0.448	130.5	25.37		
			0.500	160.7	25.25		
			0.550	196.2	25.33		
			0.599	235.3	25.70		
			0.649	284.1	26.98		
			0.700	346.9	29.51		
			0.750	436.5	33.77		
0.802	576.8	43.17					
15	1.000	0.201	See Table A1			0.512	29.91

# REPORT DOCUMENTATION PAGE

*Form Approved*  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

<b>1. AGENCY USE ONLY</b> <i>(Leave blank)</i>	<b>2. REPORT DATE</b> October 1992	<b>3. REPORT TYPE AND DATES COVERED</b> Technical Memorandum	
<b>4. TITLE AND SUBTITLE</b> Closed-Form Expressions for Crack-Mouth Displacements and Stress Intensity Factors for Chevron-Notched Short Bar and Short Rod Specimens Based on Experimental Compliance Measurements		<b>5. FUNDING NUMBERS</b>  WU-505-63-5B	
<b>6. AUTHOR(S)</b>  R.T. Bubsey, T.W. Orange, W.S. Pierce, and J.L. Shannon, Jr.		<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  E-2293	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>  NASA TM-83796	
<b>9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Washington, D.C. 20546-0001		<b>11. SUPPLEMENTARY NOTES</b>  Responsible person, T.W. Orange, (216) 433-3301.	
<b>12a. DISTRIBUTION/AVAILABILITY STATEMENT</b>  Unclassified - Unlimited Subject Category 39		<b>12b. DISTRIBUTION CODE</b>	
<b>13. ABSTRACT</b> <i>(Maximum 200 words)</i>  This report presents a set of equations describing certain fracture mechanics parameters for chevron-notch bar and rod specimens. They are developed by fitting compliance calibration data reported earlier. The equations present the various parameters in their most useful forms. The data encompass the entire range of the specimen geometries most commonly used. Their use will facilitate the testing and analysis of brittle metals, ceramics, and glasses.			
<b>14. SUBJECT TERMS</b> Specimen geometry; Compliance; Stress intensity factors; Fracture mechanics; Crack open displacement		<b>15. NUMBER OF PAGES</b> 32	
		<b>16. PRICE CODE</b> A03	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> Unclassified	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> Unclassified	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> Unclassified	<b>20. LIMITATION OF ABSTRACT</b>