THE DAMPER PLACEMENT PROBLEM FOR LARGE FLEXIBLE SPACE STRUCTURES

by

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ABSTRACT

The damper placement problem for large flexible space structures is formulated as a combinatorial optimization problem. The objective is to determine the \( p \) truss members of the structure to replace with active (or passive) dampers so that the modal damping ratio is as large as possible for all significant modes of vibration. Equivalently, given a strain energy matrix with rows indexed on the modes and the columns indexed on the truss members we seek to find the set of \( p \) columns such that the smallest row sum, over the \( p \) columns, is maximized. We develop a tabu search heuristic for the damper placement problems on the CSI Phase I Evolutionary Model (10 modes and 1507 truss members). The resulting solutions are shown to be of high quality.

1. INTRODUCTION

The demand for larger sized space structures with lower mass has led to the development of highly flexible structures where, in effect, every point can move relative to the next. Traditionally, structural motion is viewed more simply in terms of a sum of several dozen or more independent motions called natural motions. The problem of controlling the motion of a flexible structure is then reduced to controlling the natural motions. Associated with each natural motion are three parameters: a mode which is a natural spatial shape, a natural frequency which expresses the rate of oscillation, and a natural decay rate which is a measure of the time required for the motion to decay. The contribution of each natural motion to the overall motion depends on the degree to which it is excited by external forces.

The overall structural motion of a flexible truss structure can be reduced by the use of structural dampers that both sense and dissipate vibrations. We focus on where to locate these dampers so that vibrations arising from the control or operation of the structure and its payloads or by cyclic thermal expansion and contraction of the space structure can be damped as effectively as possible. There are several mechanisms available for vibrational damping. We consider the replacement of some of the truss members by active dampers which sense axial displacement (strain) and induce a compensating displacement. (A related option is to replace some of the truss members with passive dampers which dissipate strain energy due to their material properties.) Each of these techniques for damping increases the weight and cost of the truss structure. Hence, structural designers are required to locate as few dampers as possible and still maintain an appropriate level of vibrational damping.

2. FORMULATION

The CSI Phase I Evolutionary Design (see Figure 1) is an example of a large flexible space truss structure. A normal modes analysis of a finite element model of this structure yielded a 10 (\( n_{\text{modes}} \)) by 1507 (\( n_{\text{membs}} \)) modal strain energy matrix. Let \( D^T \) denote this matrix with row index set \( I \) and column index set \( J \). The entries in the matrix have been normalized so that each \( d_{ij} \) denotes the percentage of the total modal strain energy imparted in mode \( i \) to truss member \( j \).

The goal of the damper placement problem is to select \( p \) truss members to be replaced by active (passive) dampers so that the modal damping ratio is maximized for all significant modes. Maximizing the modal damping ratio is a widely accepted goal in damper placement problems (see Anderson et al. 1991). However, the modal damping ratio is difficult to determine explicitly and, consequently, the placement of active (or passive) dampers has proved difficult (cf. Padula and Sandridge 1992 and Preumont et al. 1991). Both active and passive dampers dissipate forces which are internal to the structure and are most effective replacing truss...
members with maximum extension or compression. The truss elements with maximum internal displacement are those with the largest strain energy over all modes. Given a finite element model and the results of a normal modes analysis the modal strain energy in each candidate location (truss member) for each significant normal vibration mode can be estimated quite accurately. The damping achieved with active dampers depends on the properties of the damper and the control law that is implemented. Following Padula and Sandridge (1992) we use a force-feedback control law (cf. Preumont et al. (1991)) yielding damping ratios that are directly proportional to the fraction of modal strain energy. Hence, the maximization of the modal damping ratio for all modes can be accomplished by selecting the damper locations that maximize the minimum sum of modal strain energy over the p chosen locations. Padula and Sandridge (1992) formulate this problem as a mixed 0/1 integer linear program (MILP).

Alternatively, the damper placement problem may be formulated as a combinatorial optimization problem. That is, given \( D_N \) we seek to find the \( n\) modes by \( p \) submatrix whose smallest row sum is as large as possible. Let \( Z(X) = \min_{i \in I} \sum_{j \in X} d_{ij} \). Then the damper placement problem becomes

\[
\max_{X \subseteq I} Z(X) \quad \text{subject to } |X| = p.
\]

3. COMPUTATIONAL RESULTS

There are several ways in which tabu search (and many other heuristic search strategies) can be of use. First, it can simply be used to generate solutions to the damper placement problem. However, tabu search by itself provides no information about the quality of the solution found. Solving the linear programming (LP) relaxation of the MILP mentioned above is one way to get a good upper bound. Solving the MILP with a branch and bound code will provide even better upper bounds as well as a lower bound (the MILP solution). Table 1 compares the quality of solutions generated by the MILP formulation (solved by LINDO with a limit of 10,000 iterations) and tabu search. Secondly, tabu search can be used to try and improve upon the MILP solution or the LP relaxation of MILP. In the latter case fractional solutions will be present and a mechanism for choosing a subset of the optimal decision variables must be found. We picked the \( p \) (where \( p = 8, 16, \) or 32) decision variables with largest value (closest to one). For example, when \( p = 8 \) the LP solution had 12 non-zero decision variables in the optimal solution. Of these 12 five had a value of one. When \( p = 32 \) there are even fewer choices to be made. The LP optimal solution had only 35 non-zero decision variables of which 29 had a value of one. Table 2 summarizes the performance of tabu search under three different initial solutions—random, MILP solution, and LP relaxation. Reported timings are for a 16 MHz 386-class micro-computer. The solutions generated by LINDO for the MILP formulation were computed on a CONVEX computer in about 4 minutes, this corresponds to approximately 200 hours of computational effort on the 386 micro-computer.

REFERENCES


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Table 1. Best objective function value comparisons

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Table 2. Tabu Search results from different initial solutions

Figure 1. CSI Phase I Evolutionary Design