Optimum Design of High Speed Prop Rotor Including the Coupling of Performance, Aeroelastic Stability and Structures

by

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The primary design objectives of the 450 knot high speed civil prop-rotor are improved cruise propulsive efficiency and acceptable hover performance [1-5]. The rotor system should also be reasonably low weight and easy to operate. The vehicle design process is multidisciplinary in nature and involves a merging of several technical disciplines, such as aerodynamics, aeroelastic stability analysis, dynamics, structures and acoustics. For example, as speed increases, with the increase in rotor aerodynamic forces, the coupled motion of the prop-rotor and the wing elastic modes become unstable causing whirl flutter instability. Individual rotor blade stability is also important. Rotor aerodynamic performance requires ideal combination of planform, twist, airfoils and sweep to achieve maximum cruise propulsive efficiency while maintaining sufficient lifting capability and control properties in hover and in low-speeds. The complex rotor flow field associated with high tip velocities further complicates the aerodynamic analysis. In the helicopter mode, vibration can be a major source of problem and its alleviation will play an important role in the rotor blade design process. In the area of structures, the objective is to achieve maximum structural performance with minimum weight and cost requirements. Composite structures, combining the stiffness flexibility and bending-torsion coupling, are ideal for providing superior light-weight structure and Advanced Technology composite Blades (ATB) are currently being used in the XV-15 tiltrotor aircraft. Acoustics also plays an important role, as both external and internal noise generated by the vehicle are important design issues.

There also exists several conflicting design requirements. For example, cruise condition demands thin blade airfoils for high drag divergence Mach number (M_{dd}), but thin airfoils have
lower values of $C_{l\text{max}}$ ($C_l$ is the lift coefficient). This reduces the rotor figure of merit (FOM) by reducing $C_T/\sigma$ ($C_T$ is the thrust coefficient and $\sigma$ is the thrust-weighted solidity). Therefore, to satisfy hover requirements, the rotor solidity has to increase. This can be eliminated by selecting airfoils with high $C_{l\text{max}}$ and high $L/D$ ratios at relatively low Mach numbers and low drag but high $M_{\text{dd}}$ at high Mach numbers and low lift. Similarly, although airframe drag can be alleviated by sweeping the outboard sections, that operate at high Mach numbers, a swept rotor blade must be aeroelastically tailored to minimize the blade torsional and bending loads. The tradeoff between airfoil thickness and blade sweep angle therefore requires an indepth-study since they both influence aerodynamic and aeroelastic performance in hover and cruise.

The application of formal optimization procedures to tiltrotor design, therefore, seems appropriate as it can provide design trends while reducing the "man-in-the-loop" type of iterations. However, it is essential to integrate the necessary disciplinary couplings within the optimization process. Some initial investigations are due to Chattopadhayay and Narayan [7]. The purpose of the proposed paper is to enhance the state of the art by developing a multidisciplinary optimization procedure for high speed civil prop-rotors, including the couplings of aerodynamic performance, aeroelastic stability and structures in both high speed cruise and hover flight conditions.

Problem Statement

An optimization procedure is developed for the design of high speed prop-rotors to be used in civil tiltrotor applications. The goal is to couple aerodynamic performance, aeroelastic stability and structural design requirements inside a closed-loop optimization procedure. The objective is to minimize the gross weight and maximize the propulsive efficiency in high speed cruise. Constraints are imposed on the rotor aeroelastic stability in both hover and cruise and rotor figure of merit in hover. Both structural and aerodynamic design variables are used.

Multiobjective Optimization Formulation

Due to the fact that the optimization problem involves more than one design objective, the objective function formulation is complicated. In most of the existing work, the individual objective functions are combined using weight factors in a linear fashion [8]. Such methods are judgmental as the answer depends upon the weight factors which are often hard to justify. Also, in a rotary wing design environment, where complex nonlinear functions are involved, such methods are not well posed. Therefore an investigation is currently under way to evaluate the best formulation procedure appropriate for such applications. A Kreisselmeier-Steinhauser (K-S) function approach is used in this paper and a brief description of the approach follows.
Kreisselmeier-Steinhauser (K-S) function approach: The optimization problem can be stated as follows.

Minimize \( F_k(\phi_n) \) \( k = 1, 2, \ldots, \text{NOBJ} \) (objective functions)
subject to \( g_j(\phi_n) \leq 0 \) \( j = 1, 2, \ldots, \text{NCON} \) (inequality constraints)
\( \varphi_{nL} \leq \varphi_n \leq \varphi_{nU} \) (side constraints)

where NOBJ denotes the number of objective functions, NDV is the number of design variables and NCON is the total number of constraints. The subscripts L and U represent lower and upper bounds, respectively, on the design variable \( \varphi \). Using the K-S function approach, the first step in formulating the objective function in this approach involves transformation of the original objective functions into reduced objective functions [9]. These reduced objective functions are of the form

\[
F_k^*(\varphi) = \frac{F_k(\varphi)}{F_k(\varphi_0)} - 1 - g_{\text{max}} \leq 0 \quad k = 1, 2, \ldots, \text{NOBJ}
\]

(1)

where \( F_k(\varphi_0) \) are the values of \( F_k \) calculated at the beginning of each iteration. The quantity \( g_{\text{max}} \) is the value of the largest constraint corresponding to the design variable vector \( \varphi \) and assumed to be constant for this iteration. Because these reduced objective functions are analogous to the previous constraints, a new constraint vector \( g_j(\varphi_0) \), \( j = 1, 2, \ldots, M \), is introduced, where \( M = \text{NCON} + \text{NOBJ} \). The new objective function to be minimized is then defined, using the K-S function as follows.

\[
\bar{F}(\varphi) = g_{\text{max}} + \frac{1}{\rho} \log_e \sum_{m=1}^{M} e^{\rho(g_m(\varphi) - g_{\text{max}})}
\]

(2)

where the multiplier \( \rho \) is analogous to a draw-down factor controlling the distance from the surface of the K-S objective function to the surface of the maximum function value. The design variable vector \( \varphi \) is identical to that used in the Global Criteria approach. The method was found to provide faster convergence by Chattopadhyay and McCarthy [10] in a rotor blade optimization problem.
Optimization Implementation

The basic algorithm used is the method of feasible directions as implemented in the optimization program CONMIN [11]. The optimization is to be coupled with a comprehensive analysis code. The blade is to be trimmed at each step of design optimization such that each feasible design (i.e., a design that satisfies all constraints) produced by the optimizer represents a trimmed configuration. Since the optimization process requires many evaluations of the objective function and constraints before an optimum design is obtained, the process can be very expensive if full analyses are made for each function evaluation. The objective function and constraints are therefore approximated by an approximation technique which is described below.

Approximation Technique

A two-point exponential approximation [12] is proposed for approximation of the objective functions and the constraints. This technique takes its name from the fact that the exponent used in the expansion is based upon gradient information from the previous design point. This technique is formulated as follows.

\[
\hat{F}(\Phi) = F(\Phi_0) + \sum_{n=1}^{NDV} \left[ \frac{\phi_n}{\phi_{0n}} \right]^{P_n} - 1.0 \left[ \frac{\phi_{0n}}{p_n} \frac{\partial F(\Phi_0)}{\partial \phi_n} \right]
\]

where

\[
p_n = \frac{\log \left( \frac{\partial F(\Phi_1)}{\partial \phi_n} \right)}{\log \left( \frac{\partial F(\Phi_0)}{\partial \phi_{0n}} \right)} + 1.0
\]

The quantity \(\Phi_1\) refers to the design variable vector from the previous iteration and the quantity \(\Phi_0\) denotes the current design vector. A similar expression is obtained for the constraint vector. The exponent \(p_n\) can be considered as a "goodness of fit" parameter, which explicitly determines the trade-offs between traditional and reciprocal Taylor series based expansions (also known as a
hybrid approximation technique). It can be seen from Eqn. 4 that in the limiting case of \( p_n = 1 \), the expansion is identical to the traditional first order Taylor series, and when \( p_n = -1 \), the two-point exponential approximation equates to the reciprocal expansion form. The exponent is then defined to lie within this interval, such that if \( p_n > 1 \), it is set identically equal to one, and if \( p_n < -1 \), it is set equal to -1.

**Analysis**

The aerodynamic, dynamic and aeroelastic analysis is performed using the comprehensive analysis code CAMRAD/JA [13]. The structural analysis is performed using a code developed in-house.

**Blade Modeling**

The formulation and modeling assumptions used in the integrated optimization problem are described in this section.

**Aerodynamic Model**

The aerodynamic lifting line offset, from reference axis, is based on a cubic model (Figure 1) with in-plane curvature as follows.

\[
x_{ac}(\hat{y}) = \varepsilon_0 + \varepsilon_1 \hat{y} + \varepsilon_2 \hat{y}^2 + \varepsilon_3 \hat{y}^3
\]

(5)

where \( \varepsilon_0, \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) are constants that determine the curvature and \( \hat{y} \) is the nondimensional radial location. These curvature parameters, \( \varepsilon_i \) (\( i = 0 - 3 \)), are defined such that

\[
|\varepsilon_i| \leq \zeta_i
\]

(6)

where \( \zeta_i \) are prescribed bounds for the curvature parameters. These bounds allow for either forward or backward in-plane curvature. When these parameters are equal to zero, the lifting line will be a straight line. Based upon this quadratic lifting line, the sweep variation, in degrees, is calculated as follows.

\[
\Lambda(\hat{y}) = 180 \left( \varepsilon_1 + 2\varepsilon_2 \hat{y} + 3\varepsilon_3 \hat{y}^2 \right) / \pi
\]

(7)
The twist angle of attack, $\theta(\hat{y})$, is defined to have the following spanwise variation.

$$\theta(\hat{y}) = \theta_{\text{ideal}} + \theta_{\text{perturbation}}$$  \hspace{1cm} (8)

where

$$\theta_{\text{ideal}} = \tan^{-1}\left(\frac{V_\infty}{\Omega \hat{y}}\right)$$  \hspace{1cm} (9)

$$\theta_{\text{perturbation}}(\hat{y}) = \theta_1 (\hat{y} - 0.75) + \theta_2 (\hat{y} - 0.75)^2 + \theta_3 (\hat{y} - 0.75)^3$$  \hspace{1cm} (10)

and $V_\infty$ represent the free stream velocity. It is important to note that twist plays an important role in both cruise and hover flight conditions and at high Mach numbers the twist has a significant effect on the cruise efficiency. Therefore, the above cubic distribution is chosen to provide the optimizer with more flexibility by using the coefficients $\theta_1 - \theta_3$ as design variables.

**Structural Model**

The load carrying structural member is a single cell composite box beam (Figure 2).
The beam is modeled with unequal vertical and horizontal wall thicknesses and the beam cross section is described by stretching, bending, twisting, shearing and torsion related warping. The box beam walls are made of layers of laminated orthotropic composite plies. A symmetric ply arrangement of (90°/45°/0°/-45°)ₙ is used for the horizontal walls and a symmetric arrangement of (0°/±45°/0°)ₙ is used for the vertical walls. The spanwise thickness distribution, \( t(y) \), for each ply is based on the following spanwise distribution.

\[
t(y) = t_0 + t_1 y + t_2 \frac{1}{y} + t_3 \frac{1}{y^2}
\]  

where \( t_0 \) - \( t_3 \) are coefficients that determine the thickness distribution. The total thickness of an individual ply with orientation \( \psi \) is calculated as follows

\[
t_{\psi}(\tilde{y}) = n_\psi t(\tilde{y})
\]  

where \( t_{\psi} \) is the total thickness and \( n_\psi \) is the total number of plies with orientation \( \psi \). The stiffnesses, as required by CAMRAD/JA, are calculated based on the formulation developed by Smith and Chopra [14].

Non structural tuning weights, \( w(\tilde{y}) \), are placed at the leading edge and the following cubic spanwise distribution is assumed.

\[
w(\tilde{y}) = w_0 + w_1 \tilde{y} + w_2 \tilde{y}^2 + w_3 \tilde{y}^3
\]
where \( w_0 - w_3 \) are the coefficients describing the weight distribution. The total blade weight comprises the weight of the box beam, the tuning weights, the skin and the honeycomb weight (used in the trailing edge section).

**Objective function, design variables and constraints:** The objective function to be minimized is the total weight which comprises the drive system weight and the total weight of the three blades as shown below.

\[
W = W_{\text{drive}} + W_{\text{blade}} \tag{14}
\]

where

\[
W_{\text{drive}} = W_{\text{engine}} + W_{\text{trans}} + W_{\text{fuel}} \tag{15}
\]

and the individual weights are based upon the following empirical formulae.

\[
W_{\text{engine}} = \eta_e \Omega (SHP)(0.12) + 175 \tag{16}
\]

\[
W_{\text{trans}} = 300 \left( \frac{1.1}{\text{RPM}} \right)^{0.8} \tag{17}
\]

\[
W_{\text{fuel}} = 1.5 (SFC)(SHP) \tag{18}
\]

In the above equations, SFC is the specific fuel consumption, \( \eta_e \Omega \) is an empirical efficiency parameter, RPM is the rotational velocity of the rotor and SHP is the shaft horse power. The blade weight, \( W_{\text{blade}} \), includes the box beam, the skin, the honeycomb and the non structural or tuning weights, placed at the leading edge. Both structural and aerodynamic design variables are used during the optimization. Following is a summary of the design variables.

(i) thickness distribution coefficients, \( t_1 - t_3 \).
(ii) number of plies with orientation \( \phi \), \( n_{\phi i} \) (\( i = 1-5 \)).
(iii) non structural weight distribution coefficients, \( w_0 - w_3 \).
(iv) coefficients of sweep distribution, \( \varepsilon_0 - \varepsilon_3 \).
(v) coefficients of twist distribution, \( \theta_1 - \theta_3 \).
To ensure aeroelastic stability in both hover and high speed cruise, constraints are imposed on the real part of the stability root as follows.

\[ \alpha_{ki} \leq 0 \quad k = 1, 2, \ldots, \text{NMOD} \]
\[ i = 1, 2 \quad (1 - \text{hover}, 2 - \text{axial}) \]  

where NMOD represents the total number of modes constrained. Also a lower bound constraint is imposed on the hover figure of merit FM as follows.

\[ \text{FM} \geq \text{FM}_{\text{allow}} \]  

where FM\(_{\text{allow}}\) is the required figure of merit in hover.

**Results**

Some preliminary results obtained are presented in this section. The objective function used is minimizing the blade weight increment from reference to the composite model. Constraints are imposed on the rotor aeroelastic stability in hover and cruise. The rotor used, as a reference, is the XV-15 Advanced Technology Blade which is three bladed rotor with a rigid hub and zero sweep. The blade radius is 12.5 feet. The rotor was designed for a cruise velocity of 300 knots. The optimization is performed with both multi and single objective function formulation techniques. The program CAMRAD/JA is used for the aerodynamic, dynamic and aeroelastic analysis. The blade is discretized into 51 segments structural stations and 21 aerodynamic stations. Using a wind tunnel trim option, in each case, the optimum rotor is trimmed to the same CT as the baseline or the reference rotor such that the same lifting capability is maintained after optimization. The optimization is performed at a cruise velocity of 400 knots at an altitude of 25,000 feet above sea level. In hover, a rotational velocity of \( \Omega = 570 \ \text{RPM} \) (tip speed of 746 ft/s) is used at sea-level conditions. An uniform inflow model is used for both hover and axial analysis. In hover the rotor is trimmed to a design \( C_T/\sigma = 0.13 \) and an L/D value of 5.3 is used in cruise. The material used for the composite box beam is T300/5208 Graphite Epoxy with a volume fraction of 70 percent. The aeroelastic stability of the reference blade is significantly improved as indicated in Figs. 4 and 5 for hover and cruise, respectively.

The final paper will contain results of the comprehensive optimization problem using total vehicle weight and high speed cruise propulsive efficiency as objective functions. The results of
the optimization, performed at a cruise velocity of 400 knots, will be compared against the baseline XV-15 ATB blade which has never been tested at such speeds.

Figure 3 Stability roots in hover
Figure 4 Stability roots in cruise

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References


