Minimizing Noise-Temperature Measurement Errors

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An analysis of noise-temperature measurement errors of low-noise amplifiers has been performed. Results of this analysis can be used to optimize measurement schemes for minimum errors. For the cases evaluated, the effective noise temperature of a Kaoband maser can be measured most accurately by switching between an ambient and a 2-K cooled load without an isolation attenuator. A measurement accuracy of 0.3 K was obtained for this example.

I. Introduction

An analysis of low-noise amplifier (LNA) noise-temperature measurement errors is performed based on the Y-factor (power ratio measurement) method [1]. The results provide a guide for selecting a measurement scheme based on minimizing errors. The individual error sources are identified.

Load (input source termination) high-frequency noise-temperature corrections [2,3] are not addressed in this article because the effect upon the error analysis is minimal. For example, at 32 GHz, the high-frequency noise-temperature correction effectively reduces the noise temperature of a load at 2-K physical temperature by 0.67 K. The effect of this on the noise-temperature measurement error for the optimum case analyzed in this article, case 4 in Table 1, with no attenuator loss, is 0.03 K. This is determined using the equations and computer program as described, with the load noise temperature reduced 0.67 K. This small error is acceptable for the purpose of this article.

The effects of microwave component mismatch errors and receiver nonlinearity are not analyzed in this article.

II. Analysis

Two noise-temperature measurement schemes, shown in Figs. 1 and 2, are analyzed. Figure 1 shows the Y factor measured with the noise source on and off. The scheme in Fig. 2 requires that the Y factor be measured while switching between two loads at different temperatures.

The following equations were used in a Supercalc 4 spreadsheet computer program to analyze the noise-temperature measurement errors as a function of attenuator setting $L$, in dB. The measurement errors are determined by differencing $T_e$ using nominal and perturbed parameter values. The amplifier noise-temperature differences (measurement errors) are shown as summed (worst-case) and rms.

Both the $Y$-factor measurement and the attenuator values are assigned errors of the form dB, A, and dB, B, where A represents a fixed error and B represents a fractional or percentage error per dB. The terms used in the program associated with Eqs. (1) and (2) are defined at the bottoms of Figs. 3, 5, 7, 9, and 11. (Figures 4, 6, 8, 10, and 12 plot the data presented in each preceding figure.) For Fig. 1, using the noise source,
\[
Y = \frac{(T/L) + (Tn/L) + TL + Te}{(T/L) + TL + Te} \tag{1}
\]

and

\[
Te = \frac{T}{(T/L) - Tn(1 - 1/L)} \tag{2}
\]

calibrated more accurately by a standards laboratory. The approximate value of \(L\), dB, resulting in the minimum \(Te\)-rms measurement error, is indicated for each case, except case 5.

A comparison of cases 1 and 2 with cases 3, 4, and 5 shows that with the accuracies assumed, two loads are
References

Table 1. Summary of Supercalc 4 computer programs NOISE1ND and NOISE1LD analysis of an LNA noise-temperature measurement delta or error (\(DTe\)).

<table>
<thead>
<tr>
<th>Case</th>
<th>Figure</th>
<th>Method</th>
<th>Configuration</th>
<th>(L) dB</th>
<th>(DTe) (rms), K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 4</td>
<td>1 input load and noise source</td>
<td>(T = 300) K, (Tn = 1000) K, (DYG = 0.01)</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>5, 6</td>
<td>1 input load and noise source</td>
<td>(T = 300) K, (Tn = 60,000) K, (DYG = 0.02)</td>
<td>20</td>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
<td>7, 8</td>
<td>2 input loads</td>
<td>(T1 = 80) K, (T2 = 300) K, (DYG = 0.01)</td>
<td>10</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>9, 10</td>
<td>2 input loads</td>
<td>(T1 = 2) K, (T2 = 300) K, (DYG = 0.02)</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>11, 12</td>
<td>2 input loads</td>
<td>(T1 = 2) K, (T2 = 300) K, (DYG = 0.01)</td>
<td>10</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Fig. 1. Low-noise amplifier measurement scheme using a load at temperature $T$, a noise source, and a fixed attenuator.

Fig. 2. Low-noise amplifier measurement scheme using two loads at temperatures $T_1$ and $T_2$ and a fixed attenuator.
### INPUT

<table>
<thead>
<tr>
<th>L, dB</th>
<th>T, SEC</th>
<th>0</th>
<th>3</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>23</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>10</td>
<td>31.62</td>
<td>100</td>
<td>199.53</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### RESULTS

<table>
<thead>
<tr>
<th>Te</th>
<th>ERROR, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.995</td>
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</tbody>
</table>

### NOMINAL CALC

<table>
<thead>
<tr>
<th>TL</th>
<th>0.9976</th>
<th>1.8</th>
<th>1.937</th>
<th>1.98</th>
<th>1.990</th>
<th>1.998</th>
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</thead>
<tbody>
<tr>
<td>Y</td>
<td>4.28947</td>
<td>4.226</td>
<td>3.793296</td>
<td>3.950</td>
<td>3.050</td>
<td>2.113586</td>
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<tr>
<td>YDB</td>
<td>6.32404</td>
<td>6.259</td>
<td>5.790167</td>
<td>4.843</td>
<td>3.250199</td>
<td>2.2241</td>
</tr>
<tr>
<td>Te</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

### ERROR CALC

| L  | n | 1 | 3 | 10 | 31.62 | 20.61 | 23.7 | 30.91 |

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**DEFINITIONS**

- L = ATTEN LOSS
- OL = DELTA L
- TL = TEMP CONTRI OF L
- T = INPUT LOAD TEMP
- DT = DELTA T
- Tp = PHY TEMP OF L
- DTp = DELTA Tp
- Y = Y FACTOR
- OYL = DELTA Y
- NON-LINEARITY
- OYN = OELTA Y, RADIOMETER
- DYG = DELTA Y, RADIOMETER
- Te = LNA NOISE TEMPERATURE
- Tn = NOISE SOURCE CONTRIBUTION

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**Fig. 3.** Supercalc 4 computer program NOISEIND printout of the analysis of Fig. 1, showing the measurement error as a function of attenuator IDea L, dB, and assumed Input parameter errors; noise source = 1000 K.
Fig. 4. Plot of the data in Fig. 3.
**Fig. 5.** Supercalc 4 computer program NOISE1ND printout of the analysis of Fig. 1, showing the measurement error as a function of attenuator loss $L$, dB, and assumed input parameter errors; noise source = 60,000 K. The higher gain change (DYG) than that in Fig. 3 is appropriate for $L$ = small attenuation.
Fig. 6. Plot of the data in Fig. 5.
Fig. 7. Supercalc 4 computer program NOISEILD printout of the analysis of Fig. 2, showing the measurement error as a function of attenuator loss L, dB, and assumed input parameter errors; T1 = 80 K and T2 = 300 K.
Fig. 8. Plot of the data in Fig. 7.
**INPUT**

\[
T_2 = 300 \quad T_1 = 2 \quad T_p = 2 \\
D_T_2 = .1 \quad D_T_1 = .01 \quad D_T_p = .01 \quad T_e = 4
\]

\[
D_YLDB, A = .01 \quad D_YLDB, B = .01 \quad B_{MHz} = 50 \quad D_YG = .02
\]

\[
L_{DB} | 0 \quad 3 \quad 10 \quad 15 \quad 20 \quad 23 \quad 30
\]

\[
L \quad 1.99526 \quad 10.316228 \quad 100.199.526 \quad 1000
\]

**RESULTS**

<table>
<thead>
<tr>
<th>(T_e)</th>
<th>(\text{ERROR, K})</th>
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<tr>
<td>.008800</td>
<td>1.008800</td>
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<td>.002013</td>
<td>1.002013</td>
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</tbody>
</table>

**NOMINAL**

<table>
<thead>
<tr>
<th>(L)</th>
<th>(T_L)</th>
<th>(Y)</th>
<th>(Y_{DB})</th>
<th>(T_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.99526</td>
<td>10.316228</td>
<td>100.199.526</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(T_p)</th>
<th>(D_T_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>300</td>
</tr>
</tbody>
</table>

**ERROR CALC**

\[
L+DL_{DB} = .01 \quad 3.1 \quad 10.31 \quad 15.46 \quad 20.61 \quad 23.7 \quad 30.91
\]

\[
L+DL = 1.00201 \quad 2.04174 \quad 10.73989 \quad 35.1560 \quad 115.0800 \quad 234.8723 \quad 463.8286
\]

\[
T_2+DT_2 = 300.1
\]

\[
T_1+DT_1 = 2.01
\]

\[
T_p+DT_p = 2.01
\]

**DEFINITIONS**

- \(L=\text{ATTEN LOSS}\)
- \(DL=\text{DELTA L}\)
- \(T_L=\text{TEMP CONTR OF L}\)
- \(T_e=\text{LNA NOISE TEMP}\)
- \(Y=\text{Y FACTOR}\)
- \(D_Y=\text{DELTA Y FACTOR NON-LINEARITY}\)
- \(D_YLDB, A = \text{DELTA Y, RADIOMETER NOISE (T,B)}\)
- \(D_YG=\text{DELTA Y, RADIOMETER GAIN DELTA G}\)

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Fig. 9. Supercalc 4 computer program NOISE1LD printout of the analysis of Fig. 2, showing the measurement error as a function of attenuator loss \(L, \text{dB}\), and assumed input parameter errors; \(T_1 = 2 \text{ K}\) and \(T_2 = 300 \text{ K}\).
Fig. 10. Plot of the data in Fig. 9.
Fig. 11. Supercalc 4 computer program NOISE1LD printout of the analysis of Fig. 2, showing the measurement error as a function of attenuator loss $L$, dB, and assumed input parameter errors; $T_1 = 2$ K and $T_2 = 300$ K. The lower gain change ($DYG$) than that in Fig. 9 is appropriate for case 5, $L = 10$ dB.
Fig. 12. Plot of the data in Fig. 11.