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OPTIMIZATION FOR EFFICIENT STRUCTURE-CONTROL SYSTEMS

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INTRODUCTION

The efficiency of a structure-control system is a nondimensional parameter which indicates the fraction of the total control power expended usefully in controlling a finite-dimensional system. The balance of control power is wasted on the truncated dynamics serving no useful purpose towards the control objectives. Recently, it has been demonstrated that the concept of efficiency can be used to address a number of control issues encountered in the control of dynamic systems such as the spillover effects, selection of a good input configuration and obtaining reduced order control models. Reference (1) introduced the concept and presented analyses of several Linear Quadratic Regulator designs on the basis of their efficiencies. Encouraged by the results of Ref. (1), Ref. (2) introduces an efficiency modal analysis of a structure-control system which gives an internal characterization of the controller design and establishes the link between the control design and the initial disturbances to affect efficient structure-control system designs. The efficiency modal analysis leads to identification of principal controller directions (or controller modes) distinct from the structural natural modes. Thus ultimately, many issues of the structure-control system revolve around the idea of insuring compatability of the structural modes and the controller modes with each other, the better the match the higher the efficiency. A key feature in controlling a reduced order model of a high dimensional (or ∞-dimensional distributed parameter system) structural dynamic system must be to achieve high efficiency of the control system while satisfying the control objectives and/or constraints. Formally, this can be achieved by designing the control system and structural parameters simultaneously within an optimization framework. The subject of this paper is to present such a design procedure.

An important aspect of the efficiency approach to structure-control system is that the behavior of the full-order system can be ascertained based on the reduced-order design model without any knowledge of the truncated system dynamics. In case of finite element models (FEM) of structural systems the full order system is the high-dimensional first-cut model of the system known as the N^{th}

order evaluation model, where N is the total finite-element model structural degrees of freedom. In the case of distributed parameter partial differential equation formulation, the full-order model is the ∞-dimensional system.

Two types of efficiency are defined for structure-control systems in Ref. 1. The first is the global efficiency ϵ^* which compares the total control power expended on the full-order system by a spatially discrete finite number of point inputs, to the control power that would have been expended to control the full-order system by a spatially continuous input field. Thus, the global efficiency is a predominant indicator of the effect of nature of input configuration on utilizing the available control power. The second efficiency e compares the control power lost to the truncated dynamics thereby not serving the purpose of control to the total control power expended on the full-order physical system via the reduced-order control design model. In the case of global efficiency there is an interest in the performance of a spatially distributed control design which is dynamically similar (Ref. 3) to the point-input control design. The performance of the distributed input design constitutes a globally optimal performance. In the case of relative model efficiency e, both control powers compared pertain to the same control design model employing point-inputs, hence e constitutes a relative measure of power performance. In this paper, the focus will be on the relative model efficiency. References 1, 2, and 4 include more details on the efficiency approach to structure-control systems.

The subject of structure-control systems is inherently multidisciplinary. A variety of objectives and constraints can be proposed both at the system-level and subsystem level (structure or control subsystems) to bring about an interdisciplinary study of the problem. For space-structures an ultimate objective is to have a minimum mass structure subject to structural and/or control system constraints. References 5-8 include a variety of optimization formulations of the problem. One aspect of structural-control system optimization seems to be the variety of objective and constraint function formulations that are proposed. While abundance of various formulations is desirable at one hand, many different formulations also point out the need for being able to pose objective and constraint functions that are truly multidisciplinary and therefore can address a variety of design issues for the structure-control system. It is here that the power efficiency of the structure-control system as a non-dimensional indicator of the merit of the system design seems to offer a unique potential.

In our recent work in Ref. 9, as a further enhancement of the optimization formulation presented in Ref. 10, we included a lower bound on the minimum efficiency achievable under all possible initial disturbances as a system-level constraint. Other constraints included in Refs. 9, 10 were on closed-loop damped frequencies and damping ratios. Furthermore, the question of a reduced-order design model was not addressed in Ref. 10. The inclusion of the efficiency constraint in Ref. 9, on the other hand, brought the controller reduction problem into the picture which is implicit in the definition of the relative model efficiency. The feasibility of the optimization procedure for minimum mass including an efficiency constraint was clearly demonstrated in Ref. 9, by several examples using the ACOSS-FOUR structure (Fig. 1). The evaluation model had 12 degrees of freedom and the reduced order control design model included the lowest 8 structural modes. The control law was designed via the linear quadratic regulator theory (LQR) for apriori assumed unit weighting parameters for the states and the control inputs. The design variables were the 12 cross-sectional areas of the members of the structure.

From a broader perspective for the structure-control system, the design variables can and should include control system design parameters as well as structural system design variables. To this end, if the control law is designed via the LQR theory, the state and control weighting parameters can be considered as additional design variables. Recently, Ref. 11 included the control and state weighting parameters as design variables along with the member cross-sectional areas of the ACOSS-FOUR structure for an optimization problem with robustness constraints. However, Ref. 11 used the full-order (12 modes, 24 states) system model in its illustrations.

In view of the illustrations given in Refs. 9 and 11, the next evolution in optimization of the structure-control system with a focus on the efficiency of the design with a reduced-order model is to include the control weighting parameters as design variables along with the structural design variables. This paper represents this next step in the system optimization. Thus an optimization problem that is not only of more practical interest but also of a more genuine interdisciplinary character is presented in this paper.

EFFICIENCY ANALYSIS FOR A STRUCTURE-CONTROL SYSTEM

Consider an N^{th} order FEM evaluation model of the structural system

$$\mathbf{M}\ddot{\underset{\sim}{q}} + \mathbf{E}\dot{\underset{\sim}{q}} + \mathbf{K}\underset{\sim}{q} = \mathbf{D}\underset{\sim}{F}(t) \tag{1}$$

where M, K and D are the mass, stiffness and input influence matrices. $q(t)$ is the N-vector of nodal displacements and $\underset{\sim}{F}_{(t)}$ is the m-vector of point inputs. To control the structure described by (1), reduced-order modal state-space equations are considered

$$\dot{\underset{\sim}{x}} = \mathbf{A}\underset{\sim}{x} + \mathbf{B}\underset{\sim}{F}(t) \tag{2}$$

$$\underset{\sim}{x} = [\,\eta_c^T \quad \dot{\eta}_c\,]^T$$

where η_c^T are the $n < N$ structural modes controlled. Hence, considering the structural modal problem associated with (1) and denoting the orthonormalized modal matrix Φ of the full order evaluation model we have

$$\underset{\sim}{q} = \Phi\eta = [\, \Phi_c \quad \Phi_R \,] \begin{bmatrix} \eta_c \\ \eta_R \end{bmatrix} \tag{3}$$

where R denotes truncated structural modes. The modal-state space system of (2) is the reduced $2n^{th}$ order control design-space model. The A and B matrices have the form

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\omega^2 & \mathbf{0} \end{array}\right], \qquad \mathbf{B} = \left[\begin{array}{c} \mathbf{0} \\ \hline \Phi_c^T \mathbf{D} \end{array}\right] \tag{4,5}$$

where $\omega^2 = \text{diag}\,[\omega_1 ....\omega_n]$ with ω_r a natural frequency and I is the n^{th} order identity matrix.

Due to any arbitrary input $F(t)$ the control power associated with the input on the actual full-order evaluation system (1) is given by the integral

$$S^R = \int \underset{\sim}{F}^T \mathbf{D}^T \mathbf{M}^{-1} \mathbf{D} \underset{\sim}{F} dt \tag{6a}$$

The portion of this total expended power on the actual physical system that is projected onto a reduced-order dynamic system represented by (2) is

$$S_C^M = \int \underset{\sim}{F}^T \mathbf{B}^T \mathbf{B} \underset{\sim}{F} dt \tag{6b}$$

We refer to S^R as the real (total) control power expended and S_C^M as the modal control power expended on the modal control design model. One has (Ref. 1)

$$S^R \geq S_C^M \tag{7}$$

and the control power wasted to the truncated dynamics is

$$S_U^M = S^R - S_C^M \tag{8}$$

The relative model input power efficiency is defined as

$$e\% = \frac{S_C^M}{S^R} \times 100 \tag{9}$$

with a maximum possible efficiency of 100%.

Associated with e, a power spillover quotient can be defined as

$$Sq\% = \frac{S_U^M}{S^R} \times 100 = (1 - e) \times 100 \tag{10}$$

We note that while S_C^M is indicative of a quantity for the reduced control design model through the appearance of the B matrix, S^R is a quantity for the evaluation model through the appearance of the evaluation model mass matrix M. This observation establishes that the model efficiency relates the power performance of the full-order evaluation model of the actual physical system. Most importantly the definition of model efficiency is valid regardless of the specific functional dependence of the input field $F(t)$ which is the physical input to the real system. For example, it does not matter from the point of definition whether $F(t)$ is a control input or not.

Specifically, however, if the input $F(t)$ on the physical system has the functional form of the state-feedback of the reduced-control design model (2) as:

$$\underset{\sim}{F}(t) = -\mathbf{G}\underset{\sim}{x} \tag{11}$$

where G is a stabilizing constant control feedback gain matrix of dimension $m \times 2n$, then it can be shown that S_C^M, S^R become:

$$S^R = \underset{\sim}{x}_o^T \mathbf{P}^R \underset{\sim}{x}_o, \qquad S_C^M = \underset{\sim}{x}_o^T \mathbf{P}_C^M \underset{\sim}{x}_o, \qquad \underset{\sim}{x}_o = \underset{\sim}{x}(t_o) \tag{12}$$

P^R and P_C^M are symmetric positive definite matrices referred to as real and modal control power matrices, respectively. They are the solutions of the Lyapunov equations associated with the closed-loop control system

$$\mathbf{A}_{cl}^T \mathbf{P}^R + \mathbf{P}^R \mathbf{A}_{cl} + \mathbf{G}\mathbf{D}^T \mathbf{M}^{-1} \mathbf{D}\mathbf{G} = 0 \tag{13}$$

$$\mathbf{A}_{cl}^T \mathbf{P}_C^M + \mathbf{P}_C^M \mathbf{A}_{cl} + \mathbf{G}\mathbf{B}^T \mathbf{B}\mathbf{G} = 0 \tag{14}$$

$$\mathbf{A}_{cl} = \mathbf{A} + \mathbf{B}\mathbf{G} \tag{15}$$

Both power matrices are $2n^{th}$ order; they are computed based on the reduced control design model. However, note that the real power matrix P^R still inherently involves the evaluation model. It follows that, for a stable structure-control system, the model efficiency becomes

$$e = \frac{\underset{\sim}{x}_o^T \mathbf{P}_C^M \underset{\sim}{x}_o}{\underset{\sim}{x}_o^T \mathbf{P}^R \underset{\sim}{x}_o} \tag{16}$$

Hence, the efficiency of the system in general depends on the initial disturbance state and the structure and control system parameters carried into the power matrices via the Lyapunov Equations (13, 14). As simple as definition (16) of efficiency of the system appears, it does hold a host of internal information about the working of the structure and control system thereby characterizing the control/structure interactions uniquely as we outline below.

Since the control power matrices are Hermitian matrices, the efficiency quotient (16) essentially represents a Rayleigh's quotient. Consider the eigenvalue problem associated with the power matrices (Ref. 2)

$$\mathbf{P}_C^M t_i = \lambda_i^e \mathbf{P}^R t_i \qquad i = 1, 2, \ldots, 2n \tag{17}$$

where λ_i^e and t_i are defined as the i^{th} characteristic efficiency and the i^{th} controller efficiency mode, respectively. The eigenvector t_i is also referred to as the principal controller direction. Introducing the efficiency modal matrix T:

$$\mathbf{T} = [t_1 \quad t_2 \quad \ldots \quad t_{2n}] \tag{18}$$

the following orthonomality relations can be stated

$$\mathbf{T}^T \mathbf{P}^R \mathbf{T} = \mathbf{I}_{2n\times 2n}, \qquad \mathbf{T}^T \mathbf{P}_C^M \mathbf{T} = \Lambda^e \tag{19}$$

where

$$\Lambda^e = \text{diag}\,[\lambda_1^e \quad \lambda_2^e \quad \ldots \quad \lambda_{2n}^e], \qquad \lambda_1^e \leq \ldots \leq \lambda_{2n}^e \leq 1 \tag{20}$$

From the properties of a Rayleigh's quotient, for any arbitrary vector (initial disturbance state) x_o, the value of the quotient (16) is bracketed by

$$\lambda_1^e \leq e \leq \lambda_{2n}^e \leq 1 \tag{21}$$

where the upper bound of 1 follows from the property (7). We shall refer to λ_1^e as the fundamental efficiency. It is the minimum efficiency achievable by the structure-control system regardless of the initial state x_o.

Again, since the Rayleigh's quotient is stationary around an eigenvalue λ_i^e it follows that if the initial disturbance $x_o = t_i$, that is if it matches the ith controller efficiency mode exactly, the efficiency will be exactly λ_i^e. Next, defining an efficiency modal transformation

$$\underset{\sim}{x} = \mathbf{T}\underset{\sim}{\epsilon}, \qquad \underset{\sim}{x}_o = \mathbf{T}\underset{\sim}{\epsilon}_o \tag{22}$$

an efficiency expansion expression can be written as

$$e = \sum_{i=1}^{2n} e_i, \qquad e_i = c_i^2 \lambda_i^e, \qquad c_i^2 = \frac{\epsilon_{io}^2}{\|\epsilon_o\|^2} \tag{23}$$

where ϵ_i and e_i represent the ith efficiency state and the i^{th} efficiency component, respectively. From the above analysis, we note that the controller efficiency modal matrix T and the characteristic efficiencies λ_i^e are uniquely determined for a particular structure-control system design. For different initial disturbances x_o, the resulting efficiency e can readily be computed via the efficiency expansion of Eqs. (23). There will be no need for reanalysis of the system when the disturbance changes. The controller efficiency modal analysis presented above and characterized by $\{\Lambda^e, T\}$ is unique for the structure-control system and is in addition to the purely structural modal properties characterized by $[\omega^2, \Phi]$. For the sake of brevity and without elaborating further, the structure and control system analysis revolves around how compatible the modal properties Φ and T are. The modal matrix T

gives an internal characterization of the structure-control system and should prove to be a valuable analysis/design tool (Ref. 4).

OPTIMIZATION PROBLEM FORMULATION

In the design of structural-control systems it is natural to strive for a high model efficiency ϵ regardless of initial state disturbances. The consequence is that a high efficiency of any given reduced-order control design model will imply that there is low control power spillover to the truncated dynamics and hence minimized residual interaction with the design model. Furthermore, by definition, a high efficiency simply means a more efficient use of resources available, which is a common sense engineering design principle. We can then pose a structure-control optimization problem which incorporates the system efficiency.

Optimization Problem

Objective:

$$Minimize\ the\ total\ structural\ weight \tag{24}$$

subject to

Constraints on the reduced-order control design model:

$$\xi_{i\ \min} \geq \xi_i^* \tag{25a}$$

$$\omega_i \geq \omega_i^* \qquad i\epsilon\{1,2,\ldots,n\} \tag{25b}$$

$$e\% \geq e^*\% \tag{25c}$$

Control System Design Performance Index (CDPI):

$$\text{CDPI} = \text{Minimize} : \frac{1}{2}\int_{O}^{\infty}(\underset{\sim}{x}^T\overline{Q}\underset{\sim}{x} + \underset{\sim}{f}^T\overline{R}\underset{\sim}{f})dt \tag{26}$$

where $\overline{Q} \geq O$, $\overline{R} > O$ are weighting matrices defined by

$$\overline{Q} = \delta Q, \quad \overline{R} = \gamma R \tag{27a, b}$$

where Q and R are specified constant matrices and δ and γ are the control system design variables.

Design Variables: {Structural design variables, γ, δ}

In (25), i denotes a chosen set of modes from a set of n modes in the design space. ζ_i is the damping ratio of the controlled system and ω_i is the closed-loop frequency. An * denotes minimum desirable constraint values. The minimum weight optimization problem using only the first two types of constraints has been studied in Ref. (10). The novel feature of the problem posed here is the inclusion of the nondimensional structure control system parameter, the efficiency e in addition to the already too familiar other nondimensional parameter, the damping ratio ζ. The constraint on ζ reflects a concern on the quality of response, whereas the constraint on e reflects a concern on the use of the control power. An equally important feature of this optimization formulation is that the goodness of the reduced order design model relative to the full order system is explicitly but intricately incorporated to the design via the introduction of the efficiency constraint.

Returning to the efficiency constraint (25c) and the definition of efficiency (16) it is certain that the solution of the problem will also be sensitive to the initial modal state disturbance which is affected by the structural design variables. To circumvent this dependence of the problem solution on x_o we invoke a feature noted in the previous section that the minimum efficiency achievable is the fundamental efficiency λ_1^e regardless of the initial disturbance. Hence, the efficiency constraint (25c) can be substituted by a constraint on the fundamental efficiency

$$\lambda_1^e > e^* \tag{25d}$$

guaranteeing a lower bound on the model efficiency regardless of initial disturbances where sensitivity of λ_1^e depends only on the system matrices via the efficiency eigenvalue problem (17). Hence, we solve the optimization problem subject to the constraints (25a), (25b) and (25d).

The sensitivity expressions for the objective function and the damping ratio ζ and the closed-loop frequencies ω_i are exactly the same as given in Ref. (10) where it is assumed that the control gain matrix G is the steady-state solution of the $2n^{th}$ order matrix Riccati equation associated with the minimization of the Control Design Performance Index (CDPI). The sensitivity expression of efficiency λ_1^e is given in Ref. 9. The sensitivities with respect to the control design variables are given in Ref. (11).

ILLUSTRATIVE EXAMPLES

The ACOSS-FOUR structure shown in Fig. 1 was used to design a minimum weight structure with constraints on the closed-loop eigenvalues and the fundamental efficiency. This structure has twelve degrees of freedom ($N = 12$) and four masses of two units each attached at nodes 1 through 4. The dimensions and the elastic properties of the structure are specified in consistent nondimensional units in Ref. (7). Six colocated actuators and sensors are in six bipods. The control approach used is the linear quadratic regulator with steady-state gain feedback via minimizing the control design performance index, (Eqs. 26, 27). In Eqs. 27, the weighting matrices Q and R for the state and control variables were assumed to be equal to the identity matrices and the parameters δ and γ were used as design variables along with the 12 structural member cross-sectional areas.

The nominal initial design is denoted by Design A with cross-sectional areas of the members equal to those given in Table 1. This initial design weighs 43.69 units. The initial values for the control parameters δ and γ were chosen as unity.

The constraints imposed on the optimum designs were as follows:

$$\omega_1 \geq 1.425 \tag{28a}$$

$$\omega_2 \geq 1.757 \tag{28b}$$

$$\zeta_1 \geq 1.5\ \zeta_1(\text{initial}) \tag{28c}$$

$$\lambda_1 = e_{min} \geq 1.75\ \lambda_1(\text{initial}) = 1.75\ e_{min}(\text{initial}) \tag{28d}$$

The first two constraints on the closed-loop damped frequencies correspond to a 10% increase over the initial closed-loop damped frequencies which were practically equal to the corresponding structural natural frequencies. The damping constraint demands a 50% increase in the damping of the first mode over that of the initial design. The fundamental efficiency constraint, which is the minimum possible efficiency (the worst case) for all conceivable initial state disturbances $\underline{x}_o$, requires a 75% increase over the minimum efficiency of the initial design. We should note that the designation with subscript "1" in this constraint has no connotation with the first structural mode, quite differently it refers to the first efficiency mode or first principal controller mode, the significance of which is brought about through the definition of concept of efficiency of the structure-control system.

The NEWSUMT-A software based on the extended interior penalty function method with Newton's method of unconstrained minimization (Ref. 12) was used to obtain optimum designs. Two optimization problems were solved each with a different reduced-order control design model and a different input configuration. These designs were denoted as Design B and Design C. Design B used the first eight natural structural modes in the reduced-order control design model ($n = 8$) with 6 inputs ($m = 6$) located on the six bipods of the structure. Design C used the first six natural structural modes in the reduced order control design model ($n = 6$) with 2 inputs ($m = 2$) located on the two bipods attached to node 2.

The results of optimizations are given in Table 2 which includes values obtained for the constrained quantities $\omega_1, \omega_2, \lambda_1^e$ and ζ_1 and the objective functions, weights of the structures. In addition, the resulting real control powers expended, S^R and the amount of this power that was absorbed by the reduced-order design models, S_C^M and the respective model efficiencies, $e\%$ are listed in Table 2. As the initial disturbance state x_o, a unit displacement in the x-direction at node 2 was assumed. Note that the initial disturbance affects only the value of model efficiency e, but not the value of the fundamental efficiency λ_1^e.

The design variables, cross-sectional areas of elements and the control weighting parameters are listed in Table 1 for all Designs A-C. The structural frequencies, the characteristic controller power efficiency spectrum and the damping ratios of the closed-loop designs are also listed in Tables 3-5, respectively.

It is observed from Table 2 that all optimum designs result in considerable weight reduction in comparison to the initial weight and the constraints are satisfied. Particularly, fundamental efficiencies of optimum designs have been increased resulting in significant improvements also in the model efficiencies as intended. From Table 2 we note that the total control powers S^R expended on the 12^{th} order evaluation models have been affected with larger percentages of them absorbed by the 8^{th} and 6^{th}-order control design models of the optimum designs. While the control design models of the optimum designs have higher structural frequencies than the initial design, the truncated frequencies have been lowered, thus making the response of the truncated dynamics more susceptible to excitation by the control powers spilled over inefficiently in the optimum design. Thus it becomes even of more concern that the optimum designs have higher efficiencies than the initial design. For both control design models this has been achieved.

The control power S_C^M absorbed by a design model increases with the cube of the structural frequencies and may increase or decrease with the damping ratios depending on the separation between the closed-loop natural frequencies (moduli of the closed-loop eigenvalues) and the open-loop structural natural frequencies (Ref. 13). Therefore, the increases in the control powers S_C^M absorbed by both of the optimum designs are expected. From an alternate perspective, the initial strain energies in the design models of the optimum designs B and C are higher than the initial design A for the assumed unit initial displacement at node 2. However, much higher damping ratios realized in the optimum designs as listed in Table 5 by virtue of the required increase in the damping ratio of the fundamental structural mode, result in considerable decrease in the settling time of the closed-loop system. Thus, from this perspective also, power absorbed by the optimum designs S_C^M must increase to remove higher levels of initial strain energy in a much shorter time. Again note that the optimum designs have higher levels of efficiencies in using the control powers.

The line-of-sight error responses at node 1 of both optimum designs for the evaluation models and the control design models are shown in Figures 2 and 3 for ($n = 6, m = 2$) and ($n = 8, m = 6$), respectively. Figure 2a shows the responses of the 12-mode evaluation models of the initial design (Design A - solid curve) and the optimum design (Design C - dashed curve) for the control design model of 6 lowest natural modes and 2 inputs. Design C has an efficiency of 95.1% versus the 53.6%

efficiency of the initial Design A. Figure 2b shows the responses of the 12-mode evaluation model (solid curve) and the 6-mode control design model (dashed curve) of optimum Design C. Similarly, for the control design model of 8 lowest natural modes and 6 inputs, Figure 3a shows the responses of the 12-mode evaluation models of the initial design (Design A - solid curve) and the optimum design (Design B - dashed curve) with respective efficiencies of 61.5% and 88.6%. Figure 3b shows the responses of the 12-mode evaluation model (solid curve) and the 8-mode control design model (dashed curve) of optimum Design B.

The relative model efficiency e is a figure of merit which can also be used to ascertain the quality of response of the evaluation model of a controlled structure based on the study and simulation of a reduced-order control design model without any need for simulation of the evaluation model which can be very taxing on computational resources. It is shown in Ref. 4 that the mean square response of truncated dynamics is inversely proportional to the fourth power of the truncated natural frequencies and directly proportional to the time-weighted control power spilled over to the truncated dynamics which is quantified by the spillover quotient-inefficiency defined by Eq. (10). Certainly, if the controlled frequencies and the truncated frequencies are well-separated, specifically, if the truncated frequencies are high frequencies and a high system efficiency is realized, then one would hardly expect any degradation of the response of the reduced-order control design model due to excitation of truncated dynamics. In other words, in such cases, the inefficiency figure would further be attenuated when it is translated to its effect on the system response. In contrast, if the truncated frequencies are not well-separated from the controlled frequencies and they are of low natural frequencies, then the inefficiency figure will further be magnified when it is correlated to the system response. In case of such low frequencies in the truncated dynamics it becomes even of more concern to obtain very high system efficiencies. With efficiency of the system obtained based on the reduced-order design model and its implications on the evaluation model response known apriori through such observations, the designer will not have to simulate the evaluation model. Due to such aspects of the structure-control system, consideration of the efficiency of the system becomes essential for the designer. Furthermore, even if the effect of truncated dynamics on the response is ascertained to be insignificant, still striving for higher efficiency to conserve control power makes sound design engineering.

As for the optimum designs B and C illustrated in this paper, from Table 3 it is noted that the first truncated frequencies, mode 9 for Design B and mode 7 for Design C are almost coincident with the highest controlled frequencies, modes 8 and 6, respectively. Thus although the highest controlled frequencies and the first truncated frequencies are clearly separated in the initial Designs A, in the optimum designs, this feature is lost. In spite of the higher efficiencies obtained for the optimum Designs B and C one may expect that the near resonance excitation of the truncated mode 7 for Design C and the truncated mode 9 for Design B by the 6^{th} and 8^{th} modes, respectively, will be discernible in the evaluation model responses over the responses of the reduced control design models. This is clearly verified in the evaluation model responses, especially in Fig. 2b, in spite of the 95% efficiency obtained for the optimum design. One may seek to improve the situation by either attempting a higher efficiency design or by putting a frequency separation constraint between the control design model and the truncated frequencies.

Finally, some remarks are in order as to the choice of different input configurations for the two optimum designs B and C. As discussed in Ref. 1, efficiency is a genuine parameter that reflects the interdisciplinary nature of the structure-control system design. As such it is also an indicator of the effects of changes in the input configuration and the design model order as well as of the compatability of the particular input configuration with the reduced-order design model. Indeed, it is illustrated in Ref. 1 that for the initial Design A with the 6-mode design model inclusion of inputs 3-6 degrades the efficiency of the system, whereas their inclusion improves the efficiency of the system for the 8-mode design model. Thus, for the optimization problems formulated and illustrated in this paper with the objective of improving the efficiencies of the 8-mode and 6-mode reduced-order designs, from the study of efficiencies of the initial design A, the input configurations were chosen with 6 inputs and the first 2 inputs, respectively, culminating in satisfaction of our objectives for both designs.

CONCLUSIONS

Incorporation of the efficiency concept as a norm of the structure-control system design and analysis enhances the overall quality of the system. Structure-Control system efficiency is a physically based nondimensional parameter indicating the degree of usefulness of a fundamental quantity in the design and analysis of many engineering disciplines, namely, the power. Our work heretofore demonstrates that a focus on the system efficiency does not curtail the designer's ability in monitoring other important quantities of the overall design; on the contrary, it brings in an added, but necessary, dimension to the structure-control system which is a time-tested proven concept in engineering design. The improvement of efficiency, in the least, simply makes better use of available control power since it results in reduced power spillover to the unmodelled dynamics. Furthermore, this reduction is not merely qualitative but it is quantified via efficiency. Consequently, monitoring of efficiency of the system is tantamount to gauging the goodness of any reduced-order control design model relative to the full-order physical system, which is characterized typically by a higher-order evaluation model in the case of FEM models or the ∞-dimensional model in the case of distributed parameter systems. More importantly, all control design computations only involve the reduced-order control design model while extracting information about the behavior of the full-order system which makes efficiency a practical design tool for the structure-control engineer. Our work demonstrates that efficiency is an essential feature that must be addressed in the design of structure-control systems for flexible systems.

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Table 1: Design Variables

Element Number	Design A	Design B $n = 8, \quad m = 6$	Design C $n = 6, \quad m = 2$
1	1000.0	246.9	126.5
2	1000.0	403.6	280.7
3	100.0	175.4	575.5
4	100.0	257.2	576.2
5	1000.0	228.2	407.9
6	1000.0	253.9	271.1
7	100.0	54.2	84.7
8	100.0	226.1	68.2
9	100.0	224.6	547.8
10	100.0	528.2	171.8
11	100.0	604.9	416.2
12	100.0	597.4	270.8
Weight	43.69	21.97	26.79
δ	1.00	4.24	2.45
γ	1.00	0.24	0.41

Table 2: Optimum Designs

Constraints	Design A Initial Design n = 8, 6	Design B n = 8 modes m = 6 inputs	Design C n = 6 modes m = 2 inputs
ω_1	1.295	1.425	1.425
ω_2	1.596	1.757	1.757
ζ_1	0.056, 0.031	0.290	0.130
$\lambda_1 \%$	40.7, 52.5	71.2	92.1
S^R	4.88, 17.88	89.00	26.02
S_C^M	3.00, 9.59	78.91	24.74
e%	61.5, 53.6	88.6	95.1
Weight	43.69	21.97	26.79

Table 3: Structural Frequencies ω_s^2

Structural Mode	Design A	Design B n = 8, m = 6	Design C n = 6, m = 2
1	1.68	2.11	2.06
2	2.55	3.27	3.15
3	7.31	7.83	8.43
4	7.52	11.17	13.85
5	9.98	17.34	19.27
6	16.06	22.80	24.17
7	20.01	44.61	24.43
8	20.17	50.40	43.32
9	66.24	50.52	55.84
10	77.46	96.96	70.42
11	97.42	107.40	92.49
12	151.30	110.70	112.86

Table 4: Characteristic Efficiency Spectrum $\{\lambda\}$ %

Controller Efficiency Mode	Design A n = 8, 6	Design B n = 8, m = 6	Design C n = 6, m = 2
1	99.98, 59.95	99.89	97.85
2	99.98, 59.95	99.82	97.85
3	99.62, 59.95	99.22	97.83
4	99.57, 59.95	98.56	97.83
5	98.32, 59.95	94.01	97.77
6	98.10, 59.95	93.78	97.73
7	76.90, 52.65	92.88	92.24
8	76.21, 52.65	91.99	92.23
9	62.92, 52.65	89.57	92.16
10	61.98, 52.65	88.51	92.14
11	56.92, 52.65	81.21	92.14
12	53.29, 52.65	78.98	92.14
13	42.87, -	76.88	-
14	42.81, -	74.14	-
15	42.77, -	72.63	-
16	40.77, -	71.24	-

Table 5: Closed-Loop Damping Ratios ζ

Mode	Design A n = 8, 6	Design B n = 8, m = 6	Design C n = 6, m = 2
1	0.056, 0.031	0.290	0.130
2	0.067, 0.034	0.107	0.171
3	0.074, 0.009	0.335	0.121
4	0.081, 0.063	0.106	0.120
5	0.085, 0.077	0.100	0.160
6	0.087, 0.049	0.189	0.124
7	0.076, -	0.205	-
8	0.072, -	0.196	-

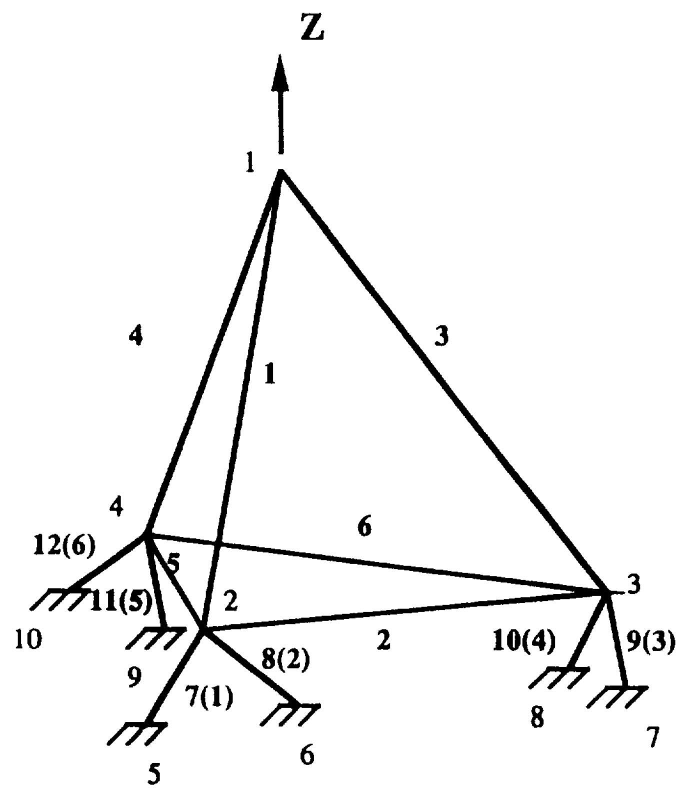

Figure 1: Tetrahedral ACOSS-FOUR structure
(actuator numbers are in parentheses)

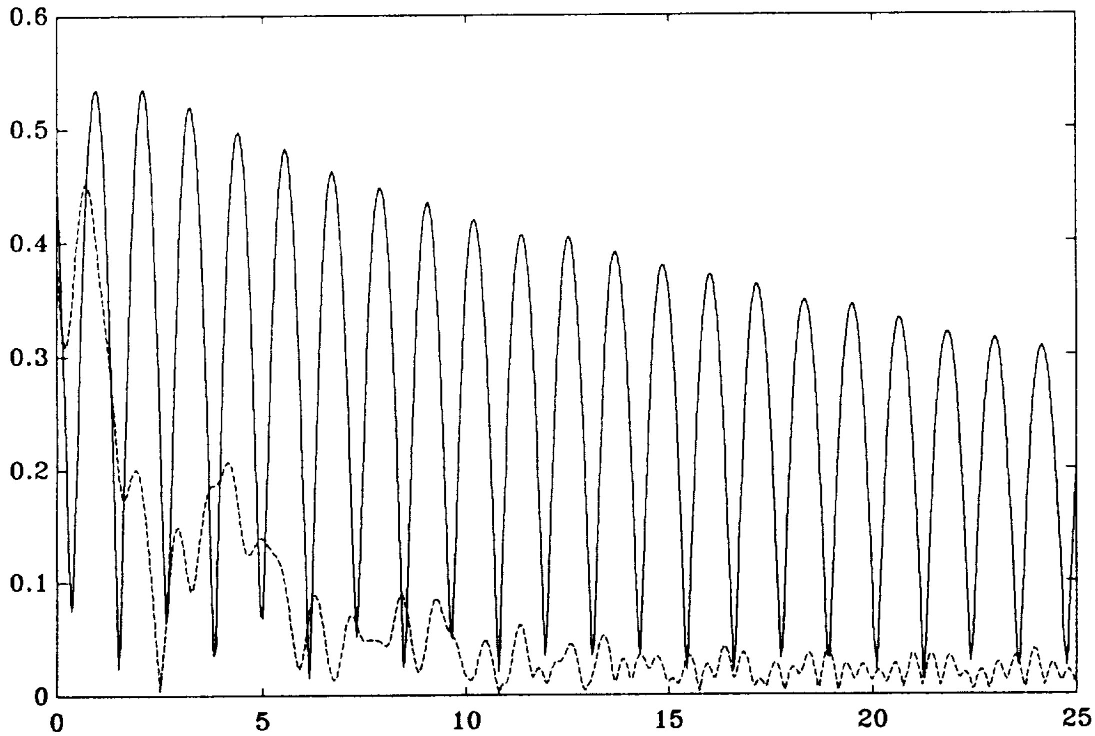

Figure 2a: Line-of-sight error evaluation model responses for the 6^{th} order control design model; initial design A (solid) and optimum design C (dashed)

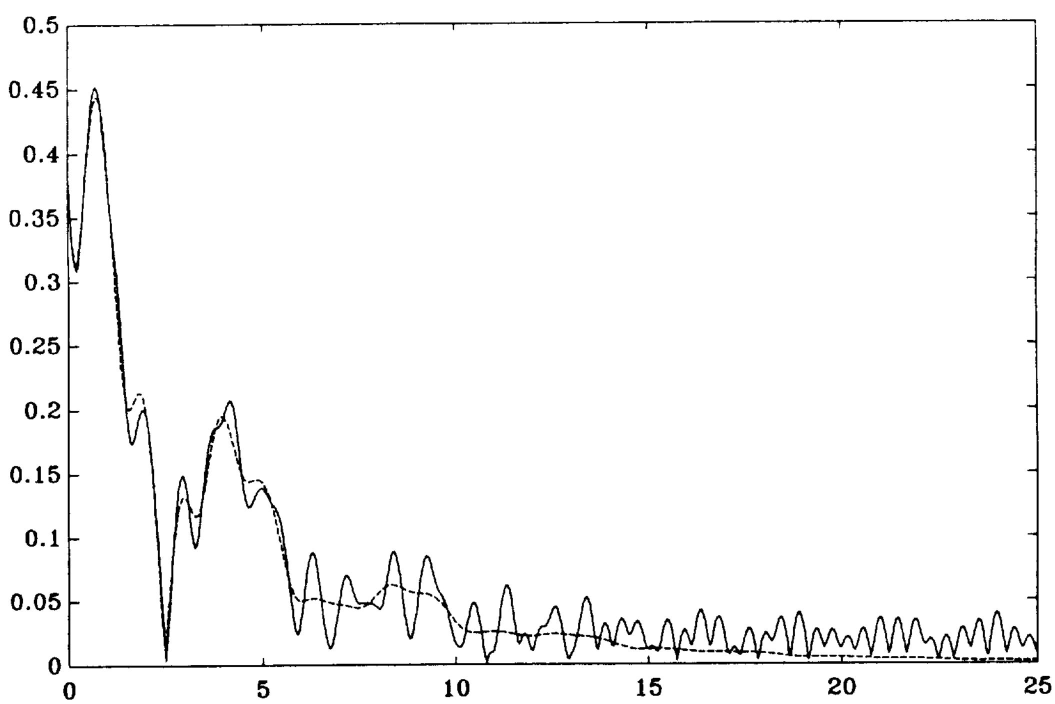

Figure 2b: Line-of-sight error responses of the evaluation model (solid) and 6^{th} order control design model (dashed) for optimum design C

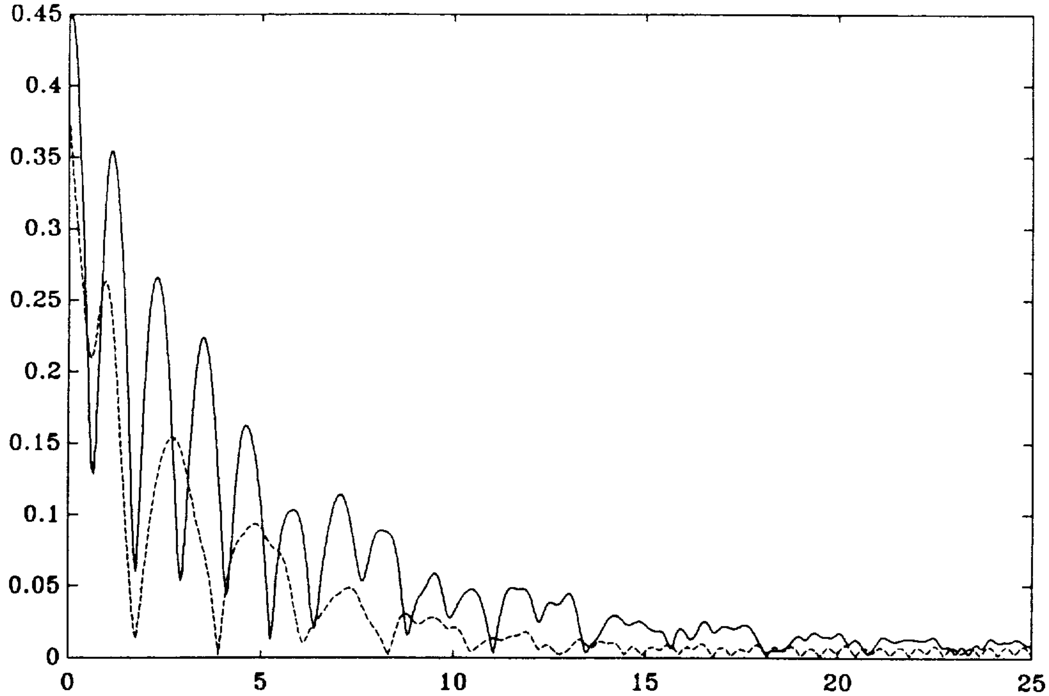

Figure 3a: Line-of-sight error evaluation model responses for the 8^{th} order control design model; initial design A (solid) and optimum design B (dashed)

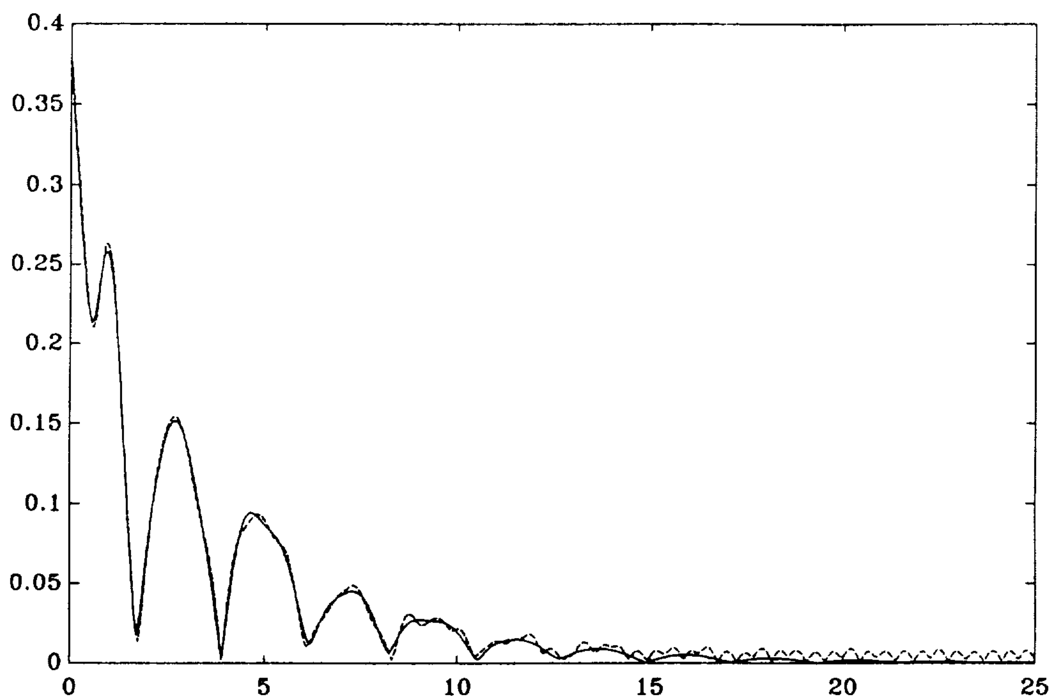

Figure 3b: Line-of-sight error responses of the evaluation model (dashed) and 8^{th} order control design model (solid) for optimum design B