SIMULATED FAMILIES: A TEST FOR DIFFERENT METHODS OF FAMILY IDENTIFICATION

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ABSTRACT

A set of families generated in fictitious impact events (leading to a wide range of "structures" in the orbital element space have been superimposed to various backgrounds of different densities in order to investigate the efficiency and the limitations of the methods used by Zappalá et al. (1990) and by Bendjoya et al. (1990) for identifying asteroid families. In addition an evaluation of the expected interlopers at different significance levels and the possibility of improving the definition of the level of maximum significance of a given family were analyzed.

INTRODUCTION

The asteroid family determination has appeared for a longtime as an analysis method dependent problem. For the first time two different methods have given a good agreement between the families defined by each method. These two methods are the hierarchical single linkage clustering method used by Zappala et al. (1990) and the wavelet transform cluster analysis used by Bendjoya et al. (1990).

The purpose of this paper is to show how the efficiency of both methods has been tested in a situation which is 'a priori' known. The idea is to "create" in the osculating element space a family the members of which are known. Both methods have been tested with respect to some physical parameters of the families and with respect to the background.

Part two will introduce the way the artificial families have been generated and the chosen parameters will be presented. Part three will briefly remind both clustering methods and the fourth part will give comparative results in which the number of interlopers has been computed for each cases.

SIMULATED FAMILIES

The origin of asteroid families is commonly admitted to be due to a collisional event. Dynamical families have been artificially generated by giving to the family members a typical ejection velocity following a power law. Since the definition of families is based on purely dynamical arguments, the power law for the ejection velocity can be justified by laboratory fragmentation experiments.

The number of fragments \( dN \) with a mean ejection velocity between \( V \) and \( V + dV \) is given by:

\[
\begin{align*}
    dN &= 0 \text{ if } V < V_{\text{min}} \\
    dN &= C V^\alpha \, dV \text{ if } V \geq V_{\text{min}}
\end{align*}
\]

\( V_{\text{min}} \) is the minimum ejection velocity; \( C \) is a constant. The histogram for the ejection velocity at infinity is given fig 1 with the power law fitting it. From laboratory fragmentation experiments \( \alpha \) has been taken equal to 3.25. The diameter of the parent body has been chosen equal to 100 km.

A family of 50 members has been generated for the following cases: \( V_{\text{min}} = 50 \text{ m/s}, 100 \text{ m/s}, 150 \text{ m/s and 200 m/s} \). Each family has been respectively plunged in a background (BG) of 300, 600, 900 asteroids. Fig. 2 shows three examples of simulated families in different backgrounds in the \((a, e)\) and \((a, i)\) planes where \( a, e, i \) are respectively the semi major axis, the eccentricity and the inclination of the asteroids.
In order to test the reliability of such artificial families, the density of the simulations has been compared to the family densities in the 6 studied zones (see Zappala et al., 1990) of the asteroid main belt. It appears that, except for zone 2 which is the Flora region, most of the real families have to be compared to the simulation with 50 members in a background of 300 asteroids.

THE TWO CLUSTERING METHODS

(i). The single linkage hierarchical clustering method is a grouping method that consists in agglomerating successively the closest clusters. Being given a distance the dimension of which is a velocity one, for the problem we are interested on, the two closest objects are identified and agglomerated in order to be replaced by a unique object. All the distances are then updated and the same rule is played again as long as two objects survive. It is then possible to build a dendrogram giving for any threshold of the distance the number of clusters existing at this value of the distance. Families are then defined by comparing this dendrogram with a similar one derived from a quasi random distribution of elements matching the large scale structures of the real distribution (see Zappala et al., 1990).

(ii). The wavelet transform cluster analysis is a density method based on the computation of a wavelet coefficient on each node of a network superimposed to the set of data. The wavelet coefficient can be seen as an indicator of proximity. The wavelet transform acts as a zoom and point out structures which have the same size as the studied scale. By applying the wavelet transform on a quasi random distribution, defined as above, it is possible to define a threshold that quantify the risk that a detected structure is due to chance. The analysis is performed for a set of scale and structures appear to fit into each other from small scales to large ones. A criterion based on the philosophy of the wavelet transform is then apply in order to cut the hierarchy and to define families (see Bendjoya et al. 1990 and Bendjoya et al. in this proceedings).
Table I presents the results obtained by the two methods. In each case the number of interlopers, i.e. the asteroids which are not original members but the relative velocity of which includes them in a computed family. In this table N represents the number of asteroids originally belonging to the family that are identified by the methods as belonging to a family, I represents the number of interlopers.

CONCLUSION

This work allows to believe 'a posteriori' in the reliability of the previous analysis made on the real families of the asteroid main belt since the density of most of these families is comparable to the easier cases that is 50 objects in a background of 300 asteroids. For the more difficult cases (such as for the Flora region) this analysis on the simulated families permits to point out the need for making a distinction between clusters and families and for having a new nomenclature. Finally the cluster analysis on simulated families could appear as a tool for testing very rapidly a part of the reliability of a new method of proper element computation in the sense that the cluster analysis on an artificial family made both in the osculating element space and in the proper element space must give very close results.
### Table I

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<th>BG</th>
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<td>N.</td>
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**single linkage method**

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**wavelet transform method**

### References
