The explicit solution of the drag and ablation equations of a single non-fragmenting meteoroid moving in any actual atmosphere was published several years ago (Pecina, Ceplecha 1983 and 1984). The solution yields theoretical relation of $l$, the distance flown by the meteoroid in its trajectory, as function of time, $t$, assuming that the height, $h$, is a known function of $l$. The photographic records of meteors and fireballs are coded by time marks, using a rotating shutter or a similar device to break the moving image. Time is thus the independent variable and for each time mark on a meteoroid trajectory, the observed distance along the trajectory, $l_{obs}$, as well as the observed height, $h_{obs}$, are values available from the geometry of double- or multiple-station photographs of the same meteoroid. Applying this solution to all available Prairie Network (PN) fireball-records (Ceplecha, McCrosky 1990), we recognized that majority of them gave good solutions with standard deviations somewhat bigger than the intrinsic geometrical precision of the data. We also noticed that, on an average, previous methods of evaluation of the meteoroid velocities (interpolation polynomials, numerical differentiation of the observed $l_{obs}$) used up to only several ten percent of the intrinsic precision of the PN observational data. When residuals of these solutions, i.e. $l_{obs} - l_{com}$, were represented as function of time, they proved to be random with time for about 75% of solutions. The remaining 25% of residuals showed systematic changes with time exceeding one standard deviation. We tried to explain these systematic time course of residuals by using different meteoroids first computed theoretically and then analysed by the same model as the natural PN fireballs were. The conclusion of these model computations: Systematic time changes of residuals in the non-fragmenting model exceeding one standard deviation are caused by sudden gross fragmentation at one or more trajectory points.

Thus we generalized the explicit solution of the drag and ablation equations of a single non-fragmenting meteoroid by allowing for one or more points, where sudden gross fragmentation can occur (Ceplecha 1992). Using this generalized solution, the distances along the meteoroid trajectory can be computed for any choice of input parameters and compared with the observed distances flown by the meteoroid. For the most precise and long fireball trajectories, the least-squares solution can thus yield the initial velocities, the ablation coefficients, the dynamical masses, the positions of gross-fragmentation points and the terminal mass. At a gross-fragmentation point, the ratio of the main mass to all the remaining fragments can be computed. The photometrically-determined meteoroid mass can be compared with the dynamic mass determined from our gross-fragmentation model and thus the meteoroid bulk density can be evaluated.

We applied the gross-fragmentation model to several PN
fireballs showing time changes of residuals and we recognized that, in all these cases, the new computed bulk densities of meteoroids resulted higher in comparison with the meteoroid densities determined with the no-gross-fragmentation model. Namely gross fragmentation early on the trajectory of a high velocity meteor makes quite a change in the computed meteoroid densities. As a part of the work we applied the gross-fragmentation model to Geminid meteoroids. From all the PN Geminids (McCrosky, Shao, 1990), only two (G15 and G54) have enough long and deep trajectories, enough observed change of velocity and enough precise heights and lengths measured for individual time-marks, that they allow the complete application of our gross-fragmentation model. If the previous non-gross-fragmentation model was used for G15 and G54, the time sequence of residuals of the solutions exhibited a prevailing systematic part and the bulk densities came out close to 1.0 g/cm³ (Tab.1). If the new gross-fragmentation model was applied to the same observational data, the systematic part of the time sequence of residuals was completely gone and the bulk densities resulted in 3 to 4 g/cm³. Thus the value of the bulk density of Geminids, 1 g/cm³, advocated for a long time and depending on decelerations determined as rough values by indirect methods, may have been caused by neglect of the gross-fragmentation effects on the meteoroid motion.

We applied the same gross-fragmentation model to a small-camera record of Geminid 0 32611 from the Ondřejov Observatory (Spurný, 1991) and to the best record on a Super Schmidt Geminid (No. 9725: Jacchia et al., 1965). Both these records have better precision than ±10 m in the distances along the trajectory. The small camera Geminid was the only one, which exhibited a mild gross-fragmentation. Again the bulk densities resulted between 3 to 4 g/cm³. To check the credibility of this concept, we used the same model for the Lost City fireball (McCrosky et al. 1971) and compared the results with reality known from the recovered meteorites. But in this case we should keep some precaution, because the velocities are substantially lower than for Geminids and the effect of gross fragmentation is not so severe as for the high velocity Geminids.

The complete statistical analysis including the uncertainty of the fragmentation point position was possible only for the PN Geminid G54 (Tab. 2). The standard deviation of the bulk density inside the gross-fragmentation model for G54 is ±0.4 (solution with 5 independent parameters). After adding the position of the fragmentation point as a sixth free parameter, the standard deviation is ±1.2 , i.e. three times bigger. In all other cases the positions of the fragmentation points cannot be kept as free parameter and they were chosen so that the sum of residuals was minimum from all solutions of different choices of fragmentation points (i.e. solutions with 5 parameters). These minima in respect to the position of the fragmentation point were quite shallow in all cases except G54 and did not allow to add the position of the fragmentation point as the sixth independent parameter. This is caused by a limited geometric precision of the data, especially when the fragmentation point lies close to the
beginning of the luminous trajectory. In case of the small-camera Geminid O 32611, the precision is high, but the gross-fragmentation effect did not strip so much mass from the main body as in the other cases.

The reason why G54 gave the most reliable results is caused by combination of several circumstances: precision of the data (±14 m in distances); big difference between the initial and terminal velocity; very low terminal height; and the most important effect: the gross-fragmentation point lies in the second half of the trajectory, where much lower precision of the distances along the trajectory still can give reliable data on the bulk density of the meteoroid (because of enough change in velocity).

After our partial experience with applying this model to the PN fireballs, we feel that gross fragmentation early on the luminous trajectory is a common phenomenon and may be responsible for the low bulk densities of meteoroids, when only simple models are used to compute the dynamic mass of the meteoroid. In the close future we intend to apply the gross-fragmentation model to all PN fireballs, which exhibit the time change of residua inside the no-gross-fragmentation solutions.

In case of the Lost City meteorite, the bulk density, the rough position of the fragmentation point, the shape coefficient and the terminal mass are known and since their values computed from our gross-fragmentation model came out quite close to this reality, it is highly probable that the bulk densities of Geminids in Table 1 are about 3 or 4 times greater than densities of the Geminid meteoroids postulated so far. This may also hold for at least a part of all the Geminids with gross fragmentation early on their trajectories.

References.


Jacchia L.G., Verniani F., Briggs R.E. (1965) Smithsonian Contr. Astrophys., 10, 2. (Data on individual time-marks taken from the original archive at the SAO)


Table 1

<table>
<thead>
<tr>
<th>Meteor No.</th>
<th>Fragmentation</th>
<th>TA 1.0</th>
<th>TA 1.1</th>
<th>TA 1.2</th>
<th>( \delta ) g/cm(^3)</th>
<th>( \delta ) g/cm(^3)</th>
<th>( \delta ) g/cm(^3)</th>
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<tbody>
<tr>
<td>SAO Geminid</td>
<td>no</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.0150 ± 0.0002</td>
<td>3.1 ± 1.1 ± 1.5</td>
<td>0.0127 ± 0.0002</td>
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<tr>
<td>SAO Geminid</td>
<td>gross</td>
<td>±1.1</td>
<td>±1.1</td>
<td>±1.1</td>
<td>±0.0003 ± 0.0002</td>
<td>±1.1 ± 1.1 ± 1.5</td>
<td>±0.0003 ± 0.0002</td>
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<td>SAO Geminid</td>
<td>no</td>
<td>0.9</td>
<td>1.1</td>
<td>1.2</td>
<td>0.0080 ± 0.0003</td>
<td>3.0 ± 1.1 ± 1.5</td>
<td>0.003 ± 0.0003</td>
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<td>±1.4</td>
<td>±1.4</td>
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<td>2.7</td>
<td>3.1</td>
<td>0.0072 ± 0.0002</td>
<td>2.5 ± 1.1 ± 1.5</td>
<td>0.001 ± 0.0003</td>
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<tr>
<td>Small camera</td>
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<td>±2.1</td>
<td>±2.1</td>
<td>±0.0002 ± 0.0002</td>
<td>±1.1 ± 1.1 ± 1.5</td>
<td>±0.0003 ± 0.0003</td>
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<tr>
<td>Super-Schmidt</td>
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<td>1.1</td>
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<td>0.0241 ± 0.0001</td>
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<td>0.015 ± 0.0003</td>
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<tr>
<td>SAO Geminid 9725</td>
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<td>±2.1</td>
<td>±2.1</td>
<td>±0.0001 ± 0.0001</td>
<td>±1.1 ± 1.1 ± 1.5</td>
<td>±0.0001 ± 0.0001</td>
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<tr>
<td>SAO Lost City</td>
<td>no</td>
<td>1.9</td>
<td>2.2</td>
<td>2.5</td>
<td>0.0185 ± 0.0006</td>
<td>2.8 ± 1.1 ± 1.5</td>
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<tr>
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<td>±1.2</td>
<td>±1.2</td>
<td>±0.0006 ± 0.0006</td>
<td>±1.1 ± 1.1 ± 1.5</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Fireball</td>
<td>gross</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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</table>

Standard deviations inside the models; errors in photometric mass not accounted for.

Table 2

<table>
<thead>
<tr>
<th>Systematic part of residuals</th>
<th>Fragmentation at</th>
<th>t=1.15 s</th>
<th>t=1.10 s</th>
<th>t=1.05 s</th>
<th>t=1.00 s</th>
<th>t=0.95 s</th>
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</thead>
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<tr>
<td>no fragmentation</td>
<td>solution</td>
<td>0.35</td>
<td>0.00</td>
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<td>0.54</td>
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<tr>
<td>bulk density ( \delta ) g/cm(^3)</td>
<td>1.2 ± 0.4</td>
<td>3.0 ± 1.3</td>
<td>3.9 ± 1.4</td>
<td>5.7 ± 1.9</td>
<td>10. ± 1.3</td>
<td></td>
</tr>
</tbody>
</table>

Resulting fragmentation point and density: \( t = (1.05 ± 0.06) \) s, \( \delta = (3.9 ± 1.2) \) g/cm\(^3\)

Fig. 1.  

Fig. 2.