Cometary orbital evolution in the outer planetary region

A. Manara\textsuperscript{1} and G.B. Valsecchi\textsuperscript{2}

\textsuperscript{1}Osservatorio Astronomico di Brera, via Brera 28, 20121 Milano, Italy
\textsuperscript{2}IAS — Planetologia, viale dell'Università 11, 00185 Roma, Italy

Abstract

Numerical integrations of fictitious objects are carried out in order to elucidate the dynamical behaviour of potential short-period comets when they move in orbits at distances from the Sun comparable to those of Uranus and Neptune. As in the case of observed short-period comets, close encounters with the planets play a major role for the orbital evolution, and this is especially true for encounters with initial orbits nearly tangent to that of the planet. A comparison with integrations in which the planetary masses are larger by a factor 10 shows that, in the latter case, the orbital evolution is greatly accelerated, but the dynamical paths in phase space followed by the comets are altered.

Introduction

A key question about the origin of Jupiter-family comets (those of orbital period \( P < 20 \) yr) is whether they come from a flattened trans-neptunian disk or from an isotropic inner core of the Oort cloud (Quinn et al., 1990). To model the orbital evolution of potential short-period comets from the situation in which they are under the dynamical control of Neptune to the orbits in which they can be observed requires a formidable amount of computer time, and in fact the only attempt done so far has used a model of the solar system in which the masses of the four outer planets are enhanced by a factor \( \mu = 10 \) to shorten the process in terms of number of orbits (and thus of CPU time) required (Quinn et al., 1990).

Researches done in the past on the dynamical evolution of short-period comets have led to a rather clear overall picture of the dynamics in the region of Jupiter and Saturn, and an equally good overall picture of the dynamics of comets also in the Uranus and Neptune region would be desirable. We try to accomplish this by examining in some detail the orbital evolutions of fictitious comets started from low-eccentricity, low-inclination orbits not far from those of the planets.
The numerical integrations

The model of the solar system used in this work consists of the Sun and the four outer planets Jupiter, Saturn, Uranus and Neptune, with the inner planets added to the Sun; the four outer planets are on elliptical unperturbed orbits (further details are given in Manara and Valsecchi, 1991).

We have integrated the motion of 100 fictitious comets with Everhart's routine RADAU (Everhart, 1985); the objects are divided in four groups of 25 each, whose initial conditions are set up choosing at random:

- the reciprocal semimajor axis $1/a$, such that: $0.9 \times 1/a_p < 1/a < 1.1 \times 1/a_p$, where $a_p$ is, in turn, the semimajor axis of each of the four outer planets; thus, for each group of 25 objects we have $0.173 \ AU^{-1} < 1/a < 0.211 \ AU^{-1}$, $0.094 \ AU^{-1} < 1/a < 0.115 \ AU^{-1}$, $0.047 \ AU^{-1} < 1/a < 0.057 \ AU^{-1}$, and $0.030 \ AU^{-1} < 1/a < 0.037 \ AU^{-1}$ respectively;

- the eccentricity $e$ and the inclination $i$, such that: $0 < e < 0.1$, $0 < \sin i < 0.1$;

- the argument of perihelion $\omega$, the longitude of node $\Omega$, and the mean anomaly $M$, such that: $0^\circ < \omega < 360^\circ$, $0^\circ < \Omega < 360^\circ$, $0^\circ < M < 360^\circ$;

- the mean anomalies of the outer planets $M_J$, $M_S$, $M_U$, and $M_N$, such that: $0^\circ < M_J < 360^\circ$, $0^\circ < M_S < 360^\circ$, $0^\circ < M_U < 360^\circ$, $0^\circ < M_N < 360^\circ$,

for each comet to be started.

Each object is followed for 1000 revolutions about the Sun, keeping track of its close planetary encounters.

The role of close encounters

Close planetary encounters, i.e. encounters within the planetary Hill radius

$$r_H = a_p \sqrt[3]{\frac{m_p}{3m_{\text{Sun}}}}$$

where $m_p$ and $m_{\text{Sun}}$ are respectively the masses of the planet and the Sun, are found to govern the evolution of the majority of the comets that show substantial orbital changes at the end of the simulation (Manara and Valsecchi, 1991). This is true mostly for objects interacting with Jupiter and Saturn; comets started close to Uranus and Neptune have a more limited orbital evolution, and in fact none of them encounters closely either Saturn or Jupiter before the end of our integrations.

In their case, close encounters with Uranus and Neptune do influence the evolution, but are not able to spread the orbital elements all over the available phase space, as it happens with Jupiter and Saturn.
The distribution of perturbations at close encounters shows distinct tail asymmetries that are related to the positions of the pre-encounter orbits in the phase space of orbital elements \( a-e-i \). Moreover, the majority of the strongest perturbations, i.e. of those contained in the asymmetric tails of the distribution, are experienced by comets in orbits nearly tangent to that of the planet encountered (Manara and Valsecchi, 1991). This suggests that the regions of phase space corresponding to orbits nearly tangent to those of the planets constitute a preferential path followed by comets on their way towards short-period orbits. In fact, those objects started from the vicinity (in \( a-e-i \) space) of Neptune, that tend to pass from the dynamical control of the latter planet to that of Uranus, do so moving along the "nearly-tangent-orbits" path; they first have the perihelion distance \( q \) lowered at constant aphelion distance \( Q \approx 30 \) AU, and then, when \( q \approx 20 \) AU, have \( Q \) lowered due to encounters with Uranus, while \( q \) remains nearly constant.

### What happens for larger planetary masses

As seen in the previous section, the orbital evolution in the Uranus-Neptune region appears to be less dominated by planetary close encounters than it is in the Jupiter-Saturn region. This may seem to support the plausibility of a treatment of the multi-stage process of capture of comets into short-period orbits as a diffusion process, as done by Quinn et al. (1990), thus justifying the enhancement of planetary masses used in their integrations.

However, Valsecchi (1991) finds, using Ōpik's theory of close encounters (Ōpik, 1976; Carusi et al., 1990) — which is valid in the case of encounters in crossing orbits (Greenberg et al., 1988) — that changing the planetary masses by a factor \( \mu \) as large as 10 changes also significantly the distributions of energy perturbations at close encounters.

We have recomputed the first 25 objects, those started close to the orbit of Neptune, from the same initial conditions, with \( \mu = 10 \), as done by Quinn et al. (1990). The aim of this computation is a comparison with that done with the realistic masses, in order to check if the effect of the increased masses is only that of shortening the multi-stage capture process, in terms of number of revolutions of the comets integrated, or if the alteration of the energy perturbation distribution also affects the paths followed in phase space by the comets.

A preliminary analysis of the output of this additional integration shows that the orbital evolutions are not only (and simply) accelerated, but that they take place along different routes in phase space, at larger eccentricities and inclinations than in the case of the actual masses. This happens essentially because planetary encounters displace the orbits in \( a-e-i \) space in such a way as to approximately preserve the value of the Tisserand invariant \( T_p \) relative to the planet encountered

\[
T_p = \frac{a_p}{a} + 2\sqrt{\frac{a(1-e^2)}{a_p}} \cos i
\]
(\(T_p\) would be more precisely conserved in a restricted circular 3-body problem), so that to larger \(1/a\) perturbations generally correspond larger perturbations in \(e\) and \(i\). As a consequence of this, the objects started in Neptune's zone, that for \(\mu = 1\) had shown a smaller sensitivity to the effects of planetary close encounters, for \(\mu = 10\) are scattered all over the available phase space, and the nearly-tangent-orbits path for passing under the control of Uranus is practically not recognizable any more in our data. The overall behaviour is similar to that of the fictitious comets started close to Jupiter and Saturn in the \(\mu = 1\) integration, for which close encounters are evidently governing the evolution. The detailed analysis of the outcomes of this integration will appear in Valsecchi and Manara (1991).

Summarizing, it appears that, for low-eccentricity and low-inclination initial conditions, the orbital evolution for \(\mu = 1\) is dominated by close planetary encounters in the region of Jupiter and Saturn, but not so much so in that of Uranus and Neptune; for \(\mu = 10\), however, also the evolution in the Uranus-Neptune region appears to be dominated by close encounters. This finding casts doubts on the plausibility of treating the multi-stage capture of comets into short-period orbits as a diffusion process, given the stochastic nature of a scattering process essentially dominated by close planetary encounters. The question of the necessity of postulating the Kuiper belt to explain the observed population of Jupiter-family comets, as done by Quinn et al. (1990), basing on computations performed with \(\mu = 10\), should therefore be considered as still open.

References


