On LAMs and SAMs for Halley's Rotation

S. J. Peale
Dept. of Physics
University of California
Santa Barbara, CA

Abstract

Non principal axis rotation for comet Halley is inferred from dual periodicities evident in the observations. The modes where the spin axis precesses around the axis of minimum moment of inertia (long axis mode or LAM) and where it precesses around the axis of maximum moment of inertia (short axis mode or SAM) are described from an inertial point of view. The currently favored LAM model for Halley's rotation state satisfies observational and dynamical constraints that apparently no SAM can satisfy. But it cannot reproduce the observed post perihelion brightening through seasonal illumination of localized sources on the nucleus, whereas a SAM can easily produce post or pre perihelion brightening by this mechanism. However, the likelihood of a LAM rotation for elongated nuclei of periodic comets such as Halley together with Halley's extreme post perihelion behavior far from the Sun suggest that Halley's post perihelion brightening may be due to effects other than seasonal illumination of localized sources, and therefore such brightening may not constrain its rotation state.

INTRODUCTION

The properties of cometary comas which vary with time include the overall brightness, the intensities of emission lines from constituent molecules and the corresponding abundances of these molecules, the overall abundance of dust particles, and both molecular and dust jets that spray like garden hoses from the rotating nucleus. Superposed on the long term variation in comet activity due to the changing heliocentric distance are quasiperiodic variations of relatively short time scale. Both the long term and short period changes will concern us here. The extreme localization of the gas and dust emitting regions on the cometary nucleus was revealed in the images of comet Halley's nucleus obtained at the time of the encounter by the spacecraft Giotto (Keller et al. 1987). It is then natural to adopt a model of a cometary nucleus like the Halley prototype, where the surface is an insulating crust pierced in only a few places to allow volatiles to escape from the localized spots when warmed by the Sun. Much of the short term variability in the coma properties and the garden hose jets are then understood in terms of the distribution of localized sources being periodically warmed with resulting periodic gas and dust emission as the comet rotates under the Sun. As other physical processes also contribute to comet variability, it is important to understand the rotation state and its evolution in order to isolate for study the many processes affecting comet activity.

Toward that end, an enormous effort has been made to constrain the rotation state of Comet Halley. References to the many observational contraints are contained in papers in which models of Halley's rotation are constructed (e.g., Julian, 1987, Peale and Lissauer, 1989, Belton, 1990, Belton et al. 1991, Samarasinha and A'Hearn, 1991). Although there is some range in the precise values of the periods of variation deduced by the observers (Belton, 1990), the case for the existence of two dominant periods, one near 2.5 days and the other near 7.4 days, is compelling. The dual periodicity seems to require that Halley's nucleus be in a state of non principal axis rotation, where precession of the instantaneous spin axis in a frame of reference fixed in the body leads to a periodic variation in the geometry of the solar illumination of the several localized sources that is superposed on a higher frequency variation caused by the rotation itself.
Non principal axis rotation is a common theme for all recently published models for Halley's rotation. In this state, the changing position of the spin axis relative to a frame of reference fixed in the nucleus leads to a corresponding change in the instantaneous equator. The centrifugal distortion of the nucleus due to the rotation thereby also periodically changes, and this flexing leads to dissipation of rotational kinetic energy. A completely isolated body would then eventually assume a minimum energy state of rotation about the axis of maximum moment of inertia consistent with the conserved angular momentum. For Halley, the time constant for an exponential decay to this state is about $10^Q$ years (Peale and Lissauer, 1989), where $1/Q$ is the specific dissipation function with $Q$ having values near 100 for rock and probably smaller values for ice. Even with $Q = 1$, it is clear that non principal axis rotation, excited by either a piece breaking off the nucleus and suddenly changing the inertia tensor or by torques from the reaction to the localized jet emissions, would persist for many apparitions, and we may neglect the effects of damping.

The spin axis can precess stably about either the axis of maximum moment of inertia (Short Axis Mode or SAM) or that of minimum moment of inertia (Long Axis Mode or LAM). If $E$ is the rotational kinetic energy and $M$ is the spin angular momentum, the two modes can be distinguished conveniently with

$$A \leq \frac{M^2}{2E} < B,$$

for a LAM,

$$B < \frac{M^2}{2E} \leq C,$$

for a SAM, \hspace{1cm} (1)

where $A < B < C$ are the principal moments of inertia. Although both SAMs and LAMs have been proposed for Halley's nucleus, Belton et al. (1991) and Samarasinha and A'Hearn (1991) independently find the same LAM model for Halley as the only one satisfying a long list of compelling observational constraints, where no SAM will do. However, one constraint not satisfied by this LAM model is the extreme post perihelion brightening and increased amplitude of short term variation in the light curve observed in both the 1910 and 1986 apparitions of comet Halley (Newburn, 1981; Schleicher, et al. 1990).

In the following we shall describe LAM's and SAM's as viewed from inertial space, construct light curves for particular published Halley models of source distribution and strength and rotation state, demonstrate why LAM's cannot yield the post perihelion brightening shown by Halley's light curve without unlikely high energy states or additional degrees of freedom beyond the simplest assumptions, show that such asymmetry is easy to produce with a SAM model by demonstrating a light curve that is asymmetric about perihelion, state the effects of torques on two published models where LAMs gain support for Halley's rotation state, describe a conjecture about the expectation of LAMs, and end with a discussion softening the remaining objection to a Halley LAM.

LAM

Halley's nucleus is about twice as long as it is thick, so we shall always approximate the nucleus with smooth ellipsoids of roughly this shape in describing the two rotational modes. If the equivalent ellipsoid is axially symmetric about the long axis ($B=C$), the only possible rotational mode is a LAM. Halley's long axis is easy to define, so we follow its motion as viewed from inertial space. Figure 1 shows a LAM which is characterized by the long axis and the angular velocity $\varpi$ rotating around the fixed angular momentum with average period $P_\varpi$ while the nucleus rotates around the long axis with period $P_L$. The angular velocities $\omega_L$ and $\omega_\varpi$ corresponding to $P_L$ and $P_\varpi$ are shown as projections of $\varpi$ onto M and the long axis respectively. If $B = C$, the rates are constant, $P_\varpi$ is constant, the path of $\varpi$ on the
surface of the ellipsoid is a circle (indicated by the dashed dashed curve in Figure 1) and the long axis traces out a small circle on the sky. If $B < C$, the time $P_w$ between successive passes of the long axis through the same longitude is no longer constant, and the long axis now nods about a small circle on the sky with a nodding period $P_w/2$. The amplitude of the nodding depends on $C - B$. $P_w$ is just the period of precession of the spin axis in the frame of reference fixed in the body. The important characteristics of a LAM are that the nucleus rotates through 360° about the long axis as the long axis nods about a complete small circle on the sky whose center is pierced by an extension of the angular momentum.

\[ \text{Figure 1. LAM} \]

\[ \text{Figure 2. SAM} \]

**SAM**

In Figure 2 we have shown a representation of a SAM for a shape similar to Halley’s, where the long axis rotates about the angular momentum $M$ as before, but now it nods about a great circle in the plane perpendicular to $M$ with period $P_w$ instead of about a small circle. The nucleus no longer rotates completely around the long axis but oscillates with an angular amplitude less than 90° with period $P_w$. This latter period is just the period of precession of the angular velocity in the body frame of reference. Again $P_w$, the time between successive passes of the long axis through the same longitude, is not constant. The trace of spin vector $\omega$ on the ellipsoid surface is the curve shown dashed in Figure 2. The important characteristics of the SAM which distinguish it from a LAM are that the nucleus now oscillates about the long axis through a limited range of angles while the long axis spins about the angular momentum while nodding about a great circle on the sky. In both LAMs and SAMs the ratio of the periods is determined within limited ranges by the initial conditions. (See Samarasinha and A’Hearn (1991) for a complete exposition of rigid body motion for a Halley–like shape from the inertial point of view.)

**LIGHT CURVES**

What are the consequences for light curves for LAMs and SAMs? To construct a light curve from a given rotational model, we distribute a finite number of localized sources of gas and dust over the nucleus and assume that the amount of light emitted by the coma is proportional to the currently visible mass. The rate of mass ejection is represented by

\[
\frac{dm}{dt} = \sum_i \frac{dm^{(i)}}{dt_{\text{max}}} g(r) \cos \theta^{(i)}; \quad \theta^{(i)} < 90^\circ.
\] (2)
The superscript $i$ indicates the $i$'th source, $\theta_{0}^{(i)}$ is the angle between the normal to the surface at the source position and the direction to the Sun. The contribution to $dm/dt$ from the $i$'th source is zero if $\theta_{0}^{(i)} > 90^\circ$, i.e., if the source is in shadow. The function
\[
g_p(r) = \alpha \left(\frac{r}{r_0}\right)^{-\beta} \left[1 + \left(\frac{r}{r_0}\right)^{\gamma}\right]^{\delta} \tag{3}\]
is a measure of the dependence of the evaporation of water ice on heliocentric distance $r$ (Marsden et al. 1973), where $\beta$, $\gamma$, $\delta = 2.15$, $5.093$, $4.6142$, $r_0 = 2.808$ AU, and where the subscript $p$ indicates that $\alpha$ is chosen so $g_p(r) = 1$ at the perihelion distance. The strength of a source is indicated by $dm/dt_{max}^{(i)}$, which is that value of the mass flux at perihelion with the Sun directly overhead. A point on the light curve is given by
\[
\ell(t) = K \int_{t \pm \Delta t} f(t') \frac{dm}{dt}(t') dt', \tag{4}\]
where $f(t')$ can assume one of three forms: a) a linear function which is unity at $t' = t$ and zero at $t' = t - \Delta t$, b) a difference of two exponentials modeling the decay of parent and daughter molecules, c) the same as b) but multiplied by an aperture function for narrow angle viewing. See Peale and Lissauer (1989) for examples of $f(t')$. The coefficient $K$ is chosen to normalize $\ell(t)$ to unity at its maximum value.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perihelion passage</td>
<td>$T = Feb. 9.45894$, 1986</td>
</tr>
<tr>
<td>Perihelion distance</td>
<td>$r_p = 0.5871029$ AU</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>$e = 0.9672755$</td>
</tr>
<tr>
<td>Period</td>
<td>$P = 75.99094$ years</td>
</tr>
<tr>
<td>Semimajor axis</td>
<td>$a = 17.94077$ AU</td>
</tr>
</tbody>
</table>

**Table 1: Parameters for Halley's orbit**

Figure 3 shows the light curve generated by the LAM model of Belton et al. (1991), where the distribution of the sources on the nucleus can be found in their paper and the remaining parameters are given in the figure. The linear weighting function is used to generate the curves in both Figures 3 and 4 with $\Delta t = 2$ days. The parameters used for Halley's orbit are given in Table 1 (Yeomans, Private communication, 1989). The curve was generated by integrating in both directions from an initial orientation of the nucleus at the Giotto encounter. This model is an axially symmetric LAM with the ratios of the lengths of the axes of the equivalent ellipsoid being indicated by $a : b : c$. Note that this model produces a light curve that is more or less symmetric about perihelion with similar amplitudes of short period variation pre and post perihelion. The favored axially symmetric LAM model of Samarasinha and A'Hearn (1991) with the source distribution of Belton et al. yields a similar light curve, except there is greater similarity in the shape of the short period variations from period to period on one side of perihelion. This increased similarity results from the adjustment of the parameters to make $P_{+}$ and $P_{-}$ exactly commensurate. Schleicher et al. (1990) advocate this commensurability to generate nearly the same orientation of the nucleus with respect to the Sun about every 7.4 days after perihelion. This in turn leads to the observed repeatability of the shape of the short period variation in the light from the molecules $C_2$ and $C_3$. For comparison the light curve for the least objectionable SAM model of Samarasinha and A'Hearn is shown in Figure 4. The distribution and relative strengths of the sources from Belton et al. and the nucleus orientation at encounter are used again, but the integration proceeds in one direction starting from 172 days before perihelion. The axis
ratios are somewhat arbitrarily chosen to make it easy to get the desired ratio of the periods with reasonable initial conditions. This light curve is also symmetric about perihelion passage with similar amplitudes of variation pre and post perihelion. The commensurate periods lead to the striking similarity in the shape of the light curve from period to period with, however, a phase shift as the comet passes perihelion.

-5 jets (3.1,1.3.1)  
-abc=1.0;0.53:0.53  
P_1=3.89d, P_2=7.1d  
-Obliquity = 17°  
-Vernal equinox 68° before perihelion

Days from Perihelion

Figure 3. Light curve for LAM model of Belton et al. (1991).

Days from Perihelion

Figure 4. Light curve for SAM model of Samarasinha and A’Hearn (1991).

But Halley is considerably more active after perihelion than before. This is indicated in Figure 5, where estimates of the \( H_2O \) production rate from several sets of observations ranging from the ultraviolet to the radio regions of the spectrum are shown. Although there is a large scatter among the various estimates, all except those from the radio observations of \( OH \) by Gérard et al. show a marked increase in production after perihelion. From the localized nature of the sources of coma material on Halley’s nucleus, Weissman (1987) has proposed that this asymmetry results from a seasonal effect where some major localized sources remain in shadow pre perihelion but become illuminated post perihelion in the springtime of the appropriate hemisphere. For Halley’s shape and likely value of \( M^2/2E \) relatively far from its minimum value of \( A \), this requires a SAM model for the rotation state. Recall that rotation about the long axis in the SAM state was limited to an oscillation, which restricted, say, the positive axis of maximum moment of inertia to always make an acute angle with the angular momentum vector \( M \). If \( M \) has a non negligible inclination to the orbit normal (obliquity) and the equinox is near the perihelion, a localized source near the axis of maximum moment would receive little illumination either pre or post perihelion depending on which hemisphere the source was located. The ease with which asymmetric light curves are generated with a SAM model is illustrated by the solid curve in Figure 5, where the axis ratios and initial energy state of Figure 4 are used. But the obliquity of \( M \) is 30°, the equinox is 30° before perihelion, and two sources having ordinary spherical coordinates in the body frame (\( z \) and \( x \) axes along axes of maximum and minimum moments respectively) of \( \phi, \theta = -90°, 20° \) and \( 144°, 109° \) are assumed with the source near the pole having 6 times the strength of that near the equator. Pre perihelion dominance of the light curve can be generated by leaving \( M \) where it is and reflecting the sources through the origin of the body frame, or by leaving the sources as they were and interchanging the positions of the vernal and autumnal equinoxes.

An asymmetric light curve can also be generated with a LAM provided \( M^2/2E \) is sufficiently close to \( A \). In the latter case the angle between the axis of minimum moment (\( z \) axis) and \( M \) is relatively small and the cone swept out by \( z \) axis as it rotates around \( M \)
with period $P$, has a small angle. In this case, one could hide a source from the Sun for part of the orbit if it is placed close to the tip of the long axis of the nucleus and $M$ is given a significant obliquity. However, as the cone angle (angle between the $x$ axis and $M$) increases for lower energy states, the fact that the nucleus rotates completely around the $x$ axis for a LAM as the $x$ axis rotates completely around $M$ makes it more and more difficult to keep a source, even near the tip, from receiving full illumination sometime during the cycle at any part of the orbit. This is verified either by comparing the maximum values of $\cos \theta_o$ for a source near the long axis tip in different parts of the orbit or by actually constructing the light curves. In the latter case, a source with spherical coordinates $(\phi, \theta) = (0^\circ, 70^\circ)$ on a Belton et al. nucleus yielded an essentially symmetric light curve when the angle between the $x$ axis and $M$ was greater than about $45^\circ$. That angle is near $65^\circ$ for the best LAM model satisfying other observational constraints. For this latter LAM model there can clearly be no "Springtime for Halley" and the light curve variation depending on geometry would be symmetric about perihelion for any distribution of sources.

![Figure 5. Estimates of $H_2O$ production rates from observations and a sample asymmetric light curve for a SAM model.](image1)

![Figure 6. Mean light curve of Comet Halley from Green and Morris (1987).](image2)

**OTHER CONSIDERATIONS**

Non gravitational forces lead to an increasing orbital period for Comet Halley of 4 days per apparition, which has apparently remained unchanged for 2000 years (Yeomans and Kiang, 1981). These non gravitational forces are interpreted as a reaction to the jet emission of material from the nucleus coupled with a phase lag in the response of a source to the periodic illumination and/or the asymmetry in $\frac{dm}{dt}$ about perihelion. Since the jets are extremely localized, torques on the nucleus must accompany the secular acceleration. But the rotational state of Halley's comet seems to be stable over the past 20 centuries, in spite of the fact that torques from the jets are capable of significant changes over a single apparition (Peale and Lissauer, 1989; Julian, 1990). This stability favors the LAM model for Halley's rotation state (Julian, 1990). If we distribute $2 \times 10^7$ g/sec (e.g. Krankowski, et al., 1986) over the 5 Belton et al. sources at the time of encounter, extrapolate back to the perihelion with Eq. (3) to obtain $\frac{dm}{dt}_{\text{max}}$, assume all the jets are perpendicular to the surface of the equivalent ellipsoid, and give the ejected mass a velocity of 0.3 km/sec (e.g., Peale, 1989), the LAM rotation state of Belton et al. is essentially unchanged from before to after an apparition, whereas the (least objectionable) SAM of Samarasinha and A'Hearn had $P_*$ decreased from 3.65 to 3.24 days and $P_o$ decreased from 7.3 to 6.44 days.
The reason for this contrasting behavior is clear from the description of the two modes of rotation above. Recall that for the LAM, the nucleus rotates repeatedly through 360° about the long axis as the long axis rotates around M. A source displaced from the central part of the nucleus could at some point be illuminated as that part of the nucleus was receding from the Sun in the φ motion. As such the reaction would contribute a torque in the direction of the angular momentum and $P_φ$ would be decreasing. However, after the nucleus has rotated 180° about the long axis, that same source will be illuminated as that part of the nucleus is approaching the Sun. The reaction then gives a torque opposite M and $P_φ$ is increased by the reaction. As the rotation periods are relatively short compared to the time Halley is close to the Sun, the gains in angular momentum are almost balanced by the losses. The imbalance that occurs because Halley is closer to the Sun (stronger jets) during last half of the period $P_φ$ on the inbound leg does change the rotation state, but this change is essentially erased on the outbound leg where Halley is further from the Sun during the last half of the period $P_φ$. The result is little net change in the rotation state after the apparition. This behavior is consistent with the apparently stable rotation state for Halley's nucleus. For a SAM the nucleus only oscillates about the long axis as that axis spins around M. This means that most sources that are illuminated as their part of the nucleus is receding from the Sun will always be illuminated in such a receding geometry and the change in the angular momentum and hence the rotation state will be secular. The only way a SAM state could remain unchanged against the reaction torques would be if the sources were distributed in an unlikely special way to balance each other's effects.

But the contrasting response of LAMs and SAMs to the torques only results if we restrict the jets to be perpendicular to the equivalent ellipsoidal surface of the nucleus. For example, if we give the direction of the Belton et al. jet #4 a 10° rotation about the long axis in the LAM model, there is a secular torque about that axis and initial periods $P_φ$, $P_φ = 3.69, 7.38$ days change to 3.85, 8.38 days in a single apparition. This effect led Julian (1990) to reject significant inclination of the jets, because of Halley's apparent stability. But the images of Halley reveal a rugged topography, and inclination of the jets to the equivalent ellipsoidal surface should be the rule rather than the exception. The stability of Halley's rotation may mean that we are overestimating the torques from the observed mass flux.

Given that a LAM is the currently a favored rotation state for Halley, are LAMs to be expected on other grounds? For a nucleus shaped like Halley's, the answer is a qualified yes. First of all, only a LAM is possible for a symmetric rotator ($A < B = C$). Halley is probably not dynamically symmetric but may be nearly so, such that $B < M^2/2E < C$ for a SAM is a small range for $B < C$. A reasonable conjecture is that random jets eventually drive rotation away from either extreme energy state in the absence of significant dissipation. For $B < C$ a relative small change in energy will change a SAM into a LAM such that LAMs may be the preferred state for long, thin nuclei of short period comets that have made many perihelion passes.

If LAMs are not unlikely, can the favored LAM for Halley be embellished by another degree of freedom such as activating additional sources after perihelion? Weissman (1987) finds that thermal inertia effects are totally inadequate to explain the post perihelion brightening for Halley, but comets are a varied lot showing all sorts of erratic behaviors (e.g., Jacchia, 1974), so almost anything else proposed may not be totally unreasonable. Samarasinha and A'Hearn (1991) offer the nuclear model of Brin and Mendis (1974) where more and more insulating crust is blown off the nucleus as it approaches the Sun. The nucleus departs with more exposed area than it had on approach and is therefore brighter post perihelion. The dust layer rebuilds itself as the comet wanes such that it can repeat the performance at the next apparition. Some periodic comets are brighter pre perihelion at each apparition (e.g. Encke, Yeomans, private communication, 1991), which could be the effect of geometry with stable sources, but it appears that effects other than geometry may be good candidates as
contributors to the asymmetry of Halley’s light curve.

This is especially true when one considers the complete light curve for Halley determined by Green and Morris (1987). Points from this light curve are shown in Figure 6. Like the estimates for the H$_2$O production rates in Figure 5, the light curve within ±100 days of perihelion shows an asymmetry of a factor of a few. However, on day 350 the estimated brightness is about 620 times that on day -350—a difference of about 7 magnitudes! At 350 days, Halley is about 4.8 AU from the Sun. The function $g_p(r)$ defined in Eq. (3) plummets drastically beyond 2 or 3 AU and our synthetic light curves generated using this function obviously cannot even be close to reality for Halley with any rotation state with fixed sources that we could choose. As $g_p$ is based on the evaporative properties of water ice, it appears that some other volatile must be dominating the activity during the distant post perihelion phase. There was even an outburst of dust from Halley when it was 14.3 AU from the Sun. Yeomans (private communication, 1991) notes that the CO outgassed from Halley is about 17% that of H$_2$O, and it is the likely volatile causing the distant activity. He also points out that if the nucleus is very inhomogeneous, a pocket of CO could emerge from shadow or have a protective covering removed as Halley retreats from the Sun. But this would require a relative low energy SAM! Weissman (1988) points out evidence that sizable pieces break off from and stay close to the nuclei of some comets as they pass perihelion. The increase in surface area would yield a post perihelion brightening. However, in Halley’s case the same behavior would have to occur at each apparition, and the increased surface area would not be expected to be so much more effective at 5 or 14 AU than it was at, say, 1 AU. Asymmetries due to seasonal effects on fixed sources whose output remains simply proportional to the instantaneous illumination tend to dominate close to perihelion, where the most rapid changes in solar aspect occur. The large distant asymmetries seem to be due to something else, and that something else may have also influenced the asymmetry near the perihelion. These considerations leave open the possibility that the asymmetry in Halley’s light curve may be found within the nuclear properties rather than in a rotational geometry change from a LAM to a SAM.

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REFERENCES


