SUBLIMATION RATES OF CARBON MONOXIDE AND CARBON DIOXIDE FROM COMETS AT LARGE HELIOCENTRIC DISTANCES

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Using a simple model for outgassing from a small flat surface area, the sublimation rates of carbon monoxide and carbon dioxide, two species more volatile than water ice that are known to be present in comets, are calculated for a suddenly activated discrete source on the rotating nucleus. The instantaneous sublimation rate depends upon the comet's heliocentric distance and the Sun's zenith angle at the location of the source. The values are derived for the constants of CO and CO₂ in an expression that yields the local rotation-averaged sublimation rate as a function of the comet's spin parameters and the source's cometocentric latitude.

Numerous examples demonstrate that comets can flare up unpredictably at virtually any point of the orbit, including large heliocentric distances. An outburst of major proportions was recently experienced by Halley's comet at 14 AU from the Sun (West et al. 1991). While the actual cause of these unexpected events cannot be stated with certainty, one of the more attractive amongst the plausible scenarios is sudden activation of a discrete source of dust ejecta (that makes up the observed halo) driven by expanding gases much more volatile than water (that have no transitions in the observed optical region of the spectrum and thus remain undetected).

Judging from information based in part upon the results of experiments on board the spacecraft that intercepted Halley's comet, in part upon remote observations, carbon monoxide (e.g., Eberhardt et al. 1986, 1987, Woods et al. 1986, Krankowsky and Eberhardt 1990, Krankowsky 1991) and carbon dioxide (Krankowsky et al. 1986, Feldman et al. 1986, Combes et al. 1988, Krankowsky and Eberhardt 1990, Krankowsky 1991) are the most likely species potentially capable of instigating such eruptive events.

To facilitate studies of the production of these volatiles from an isolated source on the nucleus surface, it is necessary to determine the diurnal and seasonal variations in the sublimation rate, that is, its dependence upon the Sun's local zenith angle \( z \) and the comet's heliocentric distance \( r \). A simple model for outgassing from a flat surface element, based upon the assumption that the absorbed solar energy is spent on sublimation and thermal reradiation (but not on heat conduction into the nucleus) and already employed in studies of the water production (Sekanina 1988), has now been applied to carbon monoxide and carbon dioxide. The dependence of the saturated vapor pressure \( p \) (in dyn/cm²) upon the temperature \( T \) (in °K) under equilibrium conditions has been expressed by

\[
\log p = A - \frac{B}{T} + CT, \tag{1}
\]

where \( A = 10.11, B = 334, \) and \( C = 0 \) have been adopted for CO to fit the data listed by Egerton and Edmondson (1928), while \( A = 11.4320, B = 1275.62, C = 0.006833 \) for CO₂ have been taken from the same reference. The vapor pressures listed by Stull (1972) are generally in fair agreement with the adopted values of the coefficients, even though more elaborate formulas could be developed to improve the match. As in the case of water ice, solutions for the sublimation rate \( Z(z, r) \) (measured in molecules/cm²/s) that was calculated from the equation of energy balance have been sought to satisfy a law

\[
Z(z, r) = Z_0(r) \cdot \zeta(z, r), \tag{2}
\]
where \( Z_0 \) is the sublimation rate at the subsolar point and \( \zeta(z, r) \leq 1 \) is the dimensionless relative sublimation rate at the Sun's zenith angle \( z \), plotted for CO in Fig. 1 and for CO\(_2\) in Fig. 2 at four different heliocentric distances. When almost all of the absorbed solar energy is spent on sublimation, the rate \( \zeta \) varies essentially as \( \cos z \). This is true up to a certain zenith distance of the Sun, beyond which \( \zeta \) begins to drop more rapidly. The approximation introduced in Sekanina (1988) for the relative sublimation rate of water ice has also proven useful for CO and CO\(_2\). Thus,

\[
\zeta(z, r) = \begin{cases} 
\cos z - f(r) \cdot \sin^2 z & \text{for } 0 \leq z \leq z_c, \\
0 & \text{for } z > z_c.
\end{cases}
\]  

(3)

Here \( z_c = \arccos\{[1 + (2f)^{-2}]^{1/2} - (2f)^{-1}\} \) is the Sun's critical zenith angle, beyond which the sublimation rate is negligibly low compared with that at the subsolar point and \( f(r) \) is a function of heliocentric distance that is discussed below. The optimum representations of the relative sublimation rates that were achieved by applying the approximation (3) are shown in Figs. 1 and 2 by dashed curves. This approach allows one to write the rotation-averaged sublimation rate \( \langle Z \rangle \) at a given distance \( r \) (assumed constant during rotation) as

\[
\langle Z \rangle = \frac{1}{2\pi} \int_{-\Theta_c}^{\Theta_c} Z(z) d\Theta = \left(\frac{Z_0}{\pi}\right) \cdot \left( a \Theta_c + b \sin \Theta_c + c \sin 2\Theta_c \right),
\]  

(4)

where \( \Theta_c(r) \) is a critical hour angle of the Sun that depends upon \( z_c \) and the cometocentric latitudes of the active source, \( \phi_{\text{act}} \), and the subsolar point, \( \phi_{\text{ss}} \),

\[
\cos \Theta_c = \cos z_c \sec \phi_{\text{act}} \sec \phi_{\text{ss}} - \tan \phi_{\text{act}} \tan \phi_{\text{ss}}.
\]  

(5)

FIG. 1. Relative sublimation rate \( \zeta(z, r) \) of carbon monoxide vs. the Sun's zenith angle at the location of the source as a function of heliocentric distance. The solid curves are the rates derived from the energy-balance equation. The broken curves are the least-squares solutions that accommodate the approximation (3). The curves have been shifted vertically for the sake of clarity.

FIG. 2. Relative sublimation rate \( \zeta(z, r) \) of carbon dioxide vs. the Sun's zenith angle at the location of the source as a function of heliocentric distance. The solid curves are the rates derived from the energy-balance equation. The broken curves are the least-squares solutions that accommodate the approximation (3). The curves have been shifted vertically for the sake of clarity.
and the coefficients $a$, $b$, and $c$ are functions of $\phi_{act}$, $\phi_{ss}$, and $f(r)$:

$$
a = \sin \phi_{act} \sin \phi_{ss} - \frac{1}{2} f(1 + \sin^2 \phi_{act} \cos^2 \phi_{ss} + \cos 2\phi_{act} \sin^2 \phi_{ss}),
$$

$$
b = \cos \phi_{act} \cos \phi_{ss}(1 + 2f \sin \phi_{act} \sin \phi_{ss}),
$$

$$
c = \frac{1}{4} f \cos^2 \phi_{act} \cos^2 \phi_{ss}.
$$

The peak (subsolar-point) sublimation rate $Z_0(r)$ is expressed by a slightly generalized version of the formula used in Sekanina (1988) for water ice:

$$
\log Z_0(r) = A + B \log(r/r_0) + C (r/r_0)^{\beta} + D \log[1 + (r/r_0)^{\gamma}],
$$

where $A$, $B$, $C$, $D$, $\beta$, and $\gamma$ are constants for the given ice and $r_0$ is a "normalizing" heliocentric distance that is related to the heat of sublimation $L$ (crudely, $r_0 \propto 1/L^2$). Figure 3 compares the heliocentric variations in the sublimation rates of CO and CO$_2$ at the subsolar point with that of water ice. For CO the function $f(r)$ is fitted empirically by

$$
f(r) = E(r/r_0)^2 \left[1 + F(r/r_0) + G(r/r_0)^3\right].
$$

For CO$_2$ this expression yields a fit that is somewhat inferior to that offered by a law

$$
f(r) = \tilde{E}(r/r_0)^{7/4} \exp \left[\tilde{F}(r/r_0)^{7/4}\right].
$$

When $r$ is in AU and $Z_0(r)$ in molecules/cm$^2$/s, the coefficients have the numerical values listed in Table I for an assumed unit bolometric emissivity and an albedo of 4 percent. The coefficients $A$ through $\gamma$ provide an excellent fit up to $r \approx 500$ AU for CO and up to $r \approx 40$ AU for CO$_2$. The coefficients $E$ through $G$ offer a very good fit up to $r \approx 300$ AU for CO and the coefficients $\tilde{E}$ through $\tilde{G}$ are applicable up to $r \approx 20$ AU for CO$_2$.

### TABLE I. Sublimation constants for carbon monoxide and carbon dioxide.

<table>
<thead>
<tr>
<th>Species</th>
<th>$r_0$ (AU)</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$E$</th>
<th>$F$</th>
<th>$G$</th>
<th>$\tilde{E}$</th>
<th>$\tilde{F}$</th>
<th>$\tilde{G}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>285</td>
<td>14.184</td>
<td>-2.00</td>
<td>-0.74</td>
<td>-0.63</td>
<td>2.09</td>
<td>14.95</td>
<td>1.386</td>
<td>-1.337</td>
<td>1.343</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>20.2</td>
<td>15.961</td>
<td>-1.95</td>
<td>-0.75</td>
<td>-1.74</td>
<td>1.50</td>
<td>8.55</td>
<td>1.882</td>
<td>-1.775</td>
<td>1.779</td>
<td>0.78</td>
<td>0.90</td>
<td>2.70</td>
</tr>
</tbody>
</table>
Once the nucleus rotation parameters (determining the values of $\Theta_c$ and $\phi_{wa}$) are available, Eqs. (2) through (9) describe fully the sublimation rate for any cometocentric latitude $\phi_{act}$, subject to the assumptions of the model employed.

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REFERENCES


