MODELING HUMAN RESPONSE ERRORS IN SYNTHETIC FLIGHT SIMULATOR DOMAIN

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ABSTRACT

This paper presents a control theoretic approach to modeling human response errors (HRE) in the flight simulation domain. The human pilot is modeled as a supervisor of a highly automated system. The synthesis uses the theory of optimal control pilot modeling for integrating the pilot's observation error and the error due to the simulation model (experimental error). Methods for solving the HRE problem are suggested. Experimental verification of the models will be tested in a flight quality handling simulation.

I. INTRODUCTION

The use of flight simulator in pilot training is as old as flying itself. However, it was not until the late part of 1940s that the human pilot was considered as a part of the simulation model (ref. 3). In this respect, the human pilot is considered to be a complex servo-mechanical system whose position in the simulation loop represents that of a sub-optimal controller (ref. 1, 4, 5, 16).

Control theoretic models have been shown to be very robust mathematical tools for modeling servo systems (ref. 5, 7, 10, 17). Whether the human is modeled as an observer (ref. 5, 6, 7), a controller (ref. 6, 8, 14, 21), a supervisor (ref. 9, 13), or a problem solver in fault diagnosis domain (ref. 12, 13, 17), the most important goal is to predict the human performance and behavior in a human-machine interaction system (ref. 12, 15, 20).

The application of control model in the human pilot training simulation have been
promising and accepted as the conventional approach to modeling the pilot handling quality fidelity (ref. 6, 11). There is one important drawback in the current control models for flight simulators. That is, the performance of the model is based solely on the knowledge of the plant response before control is applied. In this sense, the human response error is a simplistic assumption of a Gaussian white noise with zero mean and variance which depends on the plant dynamics.

In this paper consideration is given to human response error (HRE) models which are additive components of both the model representation error and the experiment error respectively. The HRE models are conceptualized with generality in mind thereby allowing the simulationist the flexibility to experiment on a variety of flight handling quality (FHQ) tests. Methods for solving the HRE problem are suggested.

**Symbols**

\[ a_i(i=0,1;j \in \{m, o\}) \] characteristic gain function of the term in second-order error dynamic equation

- \( B_m \) control vector for simulated model
- \( B_r \) control vector for reference model
- \( C_r \) coefficient matrix for the reference system output
- \( D_r \) coefficient of error matrix for the reference system output
- \( d \) cardinality index
- \( E(\cdot) \) expected value operator
- \( e \) additive human response error term
- \( e_m \) simulation model error
- \( e_o \) observed model error
- \( e_m \) expected latent error of expected input and conditional control
- \( F \) cost functional of least-square equation for \( e_o \)
- \( \text{HRE} \) Human Response Error
- \( i \) index \((i \rightarrow 0, 1)\)
- \( J_o \) HRE cost functional
- \( j \) index operator, \( j \in \{m, o\} \)
- \( K \) scaling factor
- \( L_m(\cdot) \) a function describing unknown dynamic input response
- \( M \) expected value of \( v^2 \)
- \( m \) index for "model"
- \( N_m(\cdot) \) known part of system dynamics for the simulation model
THE HRE MODELING PROBLEM

A. The optimal control model for HRE problem (OCM/HRE)

The OCM/HRE system of interest are derived by the following dynamic equations,

\[ \dot{X}_m = N_m(X, t) + L_m(X, u, t) + \eta(t) \ldots (1) \]

where \( X \in \mathbb{R}^n \) is plant state vector whose components may represent aircraft dynamics such as velocity, flight path angle, and altitude; \( U \in \mathbb{R}^n \) is a control vector whose components may represent flap deflection, pitch roll angle, and elevator deflection; \( N_m(x, t) \) is a known part of the system dynamics in the model before control is applied (i.e., the initial system state);

\( L_m(x, u, t) \) represent (the unknown response plant dynamics when the control vector \( u \) is applied, and \( \eta(t) \) is \( \in \mathbb{R}^n \) is an unknown disturbance vector or the neuromotor noise of the human pilot.

The time variable \( t \) represents time. Equation (1) represents the OCM/HRE model.

B. The classical OCM

Following the classical optimal control model (OCM); see, e.g; Ref 5. Let us define a linear quadratic time invariant reference model which generates a desired trajectory (see ref. 2)
\[ \dot{X}_r = A_r X_r + B_r U_r, \] ................. (2)

and the measurement

\[ Y_r = C_r X_r + D_r U_r, \] .............................. (3)

is observed in the reference model \( r \), where \( X_r \) is the reference plant state vector, \( U_r \) is the reference pilot control vector, \( Y_r \) is the observed system output from the reference model (i.e; output vector utilized by the pilot in performing the control task). \( A_r \in \mathbb{R}^{n \times n} \) is a constant stable system matrix, \( B_r \in \mathbb{R}^{n \times d} \) is a constant control vector and \( C_r \in \mathbb{R}^{p \times n} \) and \( D_r \in \mathbb{R}^{p \times d}; Y_r \in \mathbb{R}^p \). Note that \( C_r \) is a known matrix;

Let \( v = D_r U_r \) ................................ (4)

with \( E(v) = 0 \) ................................. (5)

\[ E(vv^T) = M \] .................................. (6)

where \( M \) is a known \( p \times p \) positive matrix.

C. The HRE Model

We are interested in modeling the response errors in the system. Starting from the reference model; let us suppose that we had an estimate of the state before the simulation (measurements) are made; which we will call \( \hat{X}_r \), where

\[ E \left[ (X_r - \hat{X}_r) (X_r - \hat{X}_r)^T \right] = J \] ................................. (7)

where \( J \) is a known \( n \times n \) positive matrix. Observing equation (3) shows that \( Y_r \) is a weighted-least-square of the estimate vector \( X \). The usual criterion (ref. 2, 7) is a minimization of a quadratic form
\[ F = \frac{1}{2}(X_r - \overline{X}_r)^T J^{-1}_r (X_r - \overline{X}_r) + (Y_r - C_r X_r)^T M^{-1}_r (Y_r - C_r X_r) \] \hspace{1cm} (8)

To determine \( X_r \), consider the differential of equation (8):

\[ dF = dx_r^T \left[ J^{-1}_r (X_r - \overline{X}_r) - C_r^T M^{-1}_r (Y_r - C_r X_r) \right] \hspace{1cm} (9) \]

In order that \( dJ = 0 \) for arbitrary \( dx^T \), the coefficient of \( dx^T \), in equation (9) must vanish:

\[ (J^{-1}_r + C_r^T M^{-1}_r C_r) X_r = J^{-1}_r \overline{X}_r + C_r^T M^{-1}_r Y_r = (J^{-1}_r + C_r^T M^{-1}_r C_r) \overline{X}_r + C_r^T M^{-1}_r (Y_r - C_r \overline{X}_r) \]

\[ \hat{X}_r = \overline{X}_r + P C_r^T M^{-1}_r (Y_r - C_r \overline{X}_r) \] \hspace{1cm} (10)

Where \( P^{-1} = J^{-1}_r + C_r^T M^{-1}_r C_r \). \hspace{1cm} (11)

\( P \) is the covariance matrix of the error in the estimate \( \overline{X}_r \), that we have

\[ P = E \left[ (\hat{X}_r - X_r) (\hat{X}_r - X_r)^T \right] \] \hspace{1cm} (11b)

**Theorem 1:** The observation error estimate \( e_o = \overline{X}_r - X_r \) (see ref. 2).

**Proof:**

By adding and subtracting \( X_r \) in the \( e_o \) term we have

\[ e_o = \overline{X}_r - X_r + \hat{X}_r - \overline{X}_r \] \hspace{1cm} (12a)

\[ = \overline{X}_r - X_r + PC_r^T M^{-1}_r [D_r U_r - C_r (\overline{X}_r - X_r)] \hspace{1cm} (12b) \]

Since \( \overline{X}_r - X_r \) and \( D_r U_r \) are independent, it follows equation (12b) that

\[ E(e_o e_o^T) = (I - KC_r) J (I - KC_r)^T + KMK^T \] \hspace{1cm} (13)

and \( I \) is a unit matrix.
where \( K = PC, M^{-1} \) .................................................. (14)

Premultiplying equation (11) by \( P \) and postmultiplying by \( J \), we have

\[
J = P + PC^T M^{-1} C, J .................................................. (15)
\]

or \( P + (I - KC) J .................................................. (16) \)

By using equation (16) in equation (13):

\[
E(e_o e_o^T) = P - PC^T K + KM^T = P - PC^T M^{-1} C, P + PC^T M^{-1} C, P = P \ldots (17)
\]

Thus, we have established a model for observation error, \( e_o \) in equation (12b and their computing properties in equations (13)-(17).

We are now interested in establishing the existence of model error \( e_m \). To do this, we can introduce the command vector \( U(t) \) into equation (1). By rewriting equation (1) with the \( B_m U_m(t) \) component we have:

\[
\dot{X}_m = N_m(x, t) + L_m(x, u, t) - B_m U_m(t) + \eta(t) + B_m U_m(t) \ldots (18)
\]

\[
\dot{X}_m = N_m(x, t) + Z_m(x, u, t) + B_m U_m(t) + \eta_m(t) \ldots (19)
\]

where the term \( Z_m(x, u, t) \) is defined by

\[
Z_m(x, u, t) = L_m(x, u, t) - B_m U_m(t) \ldots \ldots \ldots (20)
\]

\( B_m \in \mathbb{R}^{zd} \) is a known constant matrix of rank \( d \) selected from the experimental model. Next, we define a model error \( e_m \) to be the difference between the plant state vector and the reference vector,
\[ e_m = X_m - X_r \] \hfill (21)

Therefore the total human response error (HRE) comprises of the model error and reference error vectors respectively. That is

\[ HRE = e_m + e_o \] \hfill (22)

D. Properties of HRE

There various properties of HRE that need to be investigated experimentally.

Case 1: If the model state vector \( X_m \) is absent, then \( e_m = 0 \) thus, \( HRE = e_o \) which is the classical method of state estimation. Thus HRE has all the properties discussed under section C above.

Case 2: If the reference state vector \( X_r \) is absent, then \( X_m \) describes the synthetic simulation model whose validity is by experimental observation only. In this case \( HRE = e_m \). However, there is an error or experimental bias introduced by the difference between unknown (latent) response \( L_m(x,u,t) \) and the input control \( B_m U_m(t) \) as defined by \( Z_m(x,u,t) \) in equation (20). Let \( \dot{e}_m \) define this error such that

\[ \dot{e}_m = E \left[ (Z_m(x,u,t)) \right] \] \hfill (23)

Then, \( HRE = e_m + \dot{e}_m \) \hfill (24)

Case 3: If \( e_m + e_o = 0 \), then, we say that the simulation model described by \( X_m \) has a high fidelity. This is never attained in reality.

Case 4: The order of the system.

From equation (21): \( e_m = X_m - X_r \), and the time rate of change of the error \( e_m \), is

\[ \dot{e}_m = \dot{X}_m - \dot{X}_r \] \hfill (25)

\( X_r(t_o), X_m(t_o) \) given. Similarly;

\[ \dot{e} = \dot{e}_m + \dot{e}_o \] \hfill (26)

is the time rate of change of HRE. Clearly, HRE can be modeled as a second-order system with
the minimization criterion defined by

\[ J_0 = \frac{1}{2} \|e\|^2 \] .......................... (27)

where \( e = \text{HRE} \).

Case 5: HRE is a second-order error dynamic system. This property follows directly from case 4 above. Since \( e_0 \) and \( e_m \) are independent, we can define the error dynamic equations by:

\[ \dot{e}_m = a_i^m \dot{e}_m + a_o^m e_m = 0 \] ........................ (28)

\[ \dot{e}_o + a_i^o \dot{e}_o = a_o^o e_o = 0 \] .......................... (29)

Where \( a_i^1 (i = o, 1; j \in (m, o)) \) is the characteristic gain vector associated with each system of equation.

Case 6: \( L_m(x,u,t) \) can be determined experimentally as follows: using the second-order gradient method, we guess a control parameter \( u(t=0) \) and determine \( X(t=0) \) from \( N_m(x(t=0), u(t=0) = 0, \) and then \( L_m(X(0), u(0)) \). We can then determine the first and second derivatives of \( L_m(x,u,t) \) with respect to \( u \). Thus, we can approximate the \((L_m \text{ vs } u)\)-curve by a quadratic curve:

\[ L_m^* = L_m(x_o, u_o, 0) + \left( \frac{\partial L_m}{\partial u} \right) (u-u_o) + \left( \frac{\partial^2 L_m}{\partial u^2} \right) (u-u_o)^2 \ldots (30) \]

Case 7: Time frequency property of HRE

Previous human response models in the aircraft simulation domain have been described by Taylor (ref. 18, 19) in terms of time frequency and power spectrum density functions. In a particular case in which \( e_m = e_o \), the autocorrelation function describing HRE is found by

\[ \varphi_{oo}(\tau) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f_o(t) f_o(t + \tau) \, dt \] ........................ (31)

where \( f_o(t) \) is fitted distribution describing the observation error, \( e_o \). In this case the power
The spectrum is

\[ \phi_{\infty}(\omega) = \frac{1}{T} \int_{-\pi T}^{\pi T} \varphi_{\infty}(\tau) e^{i \omega \tau} d\tau \] .............. (32)

If \( e_o \neq e_m \) during the period of observation \( T \);

\[ \varphi_{mo}(\tau) = \frac{1}{T} \int_{-\pi T}^{\pi T} f_o(t) f_m(t + \tau) d\tau \] .............. (33)

defines the crosscorrelation function of \( f_o(t) \) and \( f_m(t) \); and

\[ \phi_{mo}(j \omega) = \overline{F_m(n)} F_o(n) \] .............. (34)

where

\[ \overline{F(n)} F_o(n) = \frac{1}{T} \int_{-\pi T}^{\pi T} \text{Exp}(-j n \omega_m) \varphi_{mo}(\tau) d\tau \] .............. (35)

**CONCLUDING REMARKS**

The discussion in this paper is geared towards modeling human response errors in a synthetic simulation domain in which flight handling qualities are the main tasks. The following conceptual contributions are prevalent to this paper.

1. We model the HRE as a component of two types of errors: the model error constructed around the simulation domain; and the reference error which is the theoretical state space model commonly used. In addition, we introduce the concept of experimental latent error which is the disparity between the theoretical input vector and the human input response at a given state space.
2. We discuss the various properties of HRE and their implications.

3. We formulate a cost minimization model of a simulation environment in terms of the HRE function.

4. We demonstrate how the HRE model can be used in both the time and frequency domains.

   It should be noted here that the discussions in this paper need further theoretical proofs as well as actual experimentation to warrant their applications in flight handling quality characterization.

REFERENCES


