DAMPING CHARACTERIZATION IN LARGE STRUCTURES

NASA/JPL Grant No.: NAGW-1599

FINAL TECHNICAL REPORT

Performance Period: January 1, 1989 - December 31, 1990

Submitted by: School of Engineering and Architecture
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January 24, 1991
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1.0 EXECUTIVE SUMMARY

This research project has as its main goal the development of method(s) for selecting the damping characteristics of components of a large structure or multibody system, in such a way as to produce some desired system damping characteristics. The main need for such an analytical device is in the simulation of the dynamics of multibody systems consisting, at least partially, of flexible components. The reason for this need is that all existing simulation codes for multibody systems require component-by-component characterization of complex systems, whereas requirements (including damping) often appear at the overall system level.

The main goal was met in large part by the development of a method that will in fact synthesize component damping matrices from a given system damping matrix. The restrictions to the method are that the desired system damping matrix must be diagonal (which is almost always the case) and that interbody connections must be by simple hinges.

In addition to the technical outcome, this project contributed positively to the educational and research infrastructure of Tuskegee University - a Historically Black Institution. All the students supported under this grant completed their degrees, and the pieces of equipment purchased via this project are being used to expand research efforts in System and Structural Dynamics.
2.0 INTRODUCTION

Many engineering systems comprise several bodies connected together, with active control between bodies. Specific examples of such systems include robots and manipulators, space vehicles, missiles, and precision pointing systems. Because of the increasing tendency towards lightweight components, many such systems are partially or totally composed of flexible bodies. The dynamics of such systems can be studied by experimentation or analysis, or, preferably, both. When an analytical approach is used, modeling is usually one of the first issues to be addressed. In the study of a complex structure or a system of interconnected flexible bodies, most modeling strategies rely on a finite dimensional representation of each flexible component; and the smaller the dimension, the more tractable the analysis. Structural damping is one of the most poorly understood parameters of a structure. Very often it is simply ignored. When this is not possible, such as when one is interested in stability issues for an actively controlled structure, damping is introduced in an ad hoc fashion, usually in the form of a system damping matrix, which is assumed to be diagonal. A rule of thumb is then used to assign values to the diagonal elements, which generally represent the damping ratio corresponding to each retained mode of the structure.

There are situations where one is compelled to work with components of a structure. Such a situation may arise in the analysis of a large structure such as an aircraft or a space station; here, it is common practice to assign different components of the structure to different analysts. And, if modal viewpoint is adopted, modal information, including damping information is needed at the component level. A similar situation arises when it is desired to simulate the motions of a system of interconnected, actively controlled flexible bodies, using a simulation package such as DISCOS[1] or TREETOPS[2]. These programs require that each body in a given system be characterized
separately. That is, mass, stiffness, and damping matrices of each component of the system must be supplied separately to the program. Generally, it is desired to have a diagonal damping matrix for the whole system with each element having a specified value (usually 1%). Knowing what is desired for the system damping matrix, there still remains a major task of determining the values that must be assigned to the component damping matrices such that when they are assembled, they yield the desired system damping matrix. Experience with structural analysis and simulation of the Galileo spacecraft[3,4] has shown that using component damping matrices that are diagonal leads to a system damping matrix that is far from being sparse.

3.0 OBJECTIVE AND PROBLEM STATEMENT

The principal objective of this research project is to search for a reliable, systematic, and efficient procedure for generating the damping matrices that must be assigned to the components of a given large structure so that the damping matrix of the structure as a whole (system damping matrix) has any desired form and content. The secondary objective is to initiate a fundamental re-evaluation of current methods of representing damping in structures, and indicate a path for future research in this area.

The problem to be solved is really an offshoot of a bigger problem. The big problem is that of simulating the dynamics of a system of coupled rigid/flexible bodies. This simulation problem can be solved with the aid of one of the existing multibody simulation codes such as DISCOS or TREETOPS. In order to use these codes, the system is usually modeled in a NASTRAN-like environment, so that mass, stiffness, and modal matrices (among other quantities) are available for the free-free vibration modes of each flexible body in the system. Additionally, these codes require that a damping matrix be available for each flexible body in the system. Since NASTRAN does not produce damping matrices,
these component damping matrices must be supplied by the analyst. In general, it is desired that the damping matrix for the whole system viewed as one, be a diagonal matrix whose elements represent the damping ratios (usually 1%) for the retained modes. To achieve this goal, the damping matrices for the flexible components in the system must be selected judiciously. These matrices cannot be arbitrary; they cannot even be diagonal.

What is attempted here, therefore, is to find a scientifically sound method or methods for selecting the elements of the component damping matrices so that the requirements on the system damping matrix are met.

4.0 ANALYSIS

Consider a system $S$ consisting of $n$ subsystems $S_i$ ($i=1,2,...,n$) connected together as shown in Fig. 1. For each of the subsystems $S_i$, it is possible to write:

$$M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i = F_i \quad (1)$$

And if body $i$ has $n_i$ degrees of freedom, then $M_i$, $C_i$, and $K_i$ have dimension $n_i$ by $n_i$, and are the mass, damping and stiffness matrices respectively. $F_i$ and $x_i$ are row vectors of dimension $n_i$ by 1 and represent the forcing function and displacement vector respectively.

It is also possible to view the whole system as one structure, and write
\[ \mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \mathbf{F} \quad (2) \]

Suppose that modal analyses was performed for equation (2) to produce the system modal matrix \( u \). This implies the coordinate transformation

\[ x = u\dot{q} \quad (3) \]

Equation (3) can now be used to transform (2) into

\[ I\dot{q} + c\dot{q} + k\dot{q} = u^T\mathbf{F} \quad (4) \]

where \( I \) is an identity matrix, \( k \) is a diagonal matrix with

\[ k_j = \omega_j^2 \quad (5) \]

and

\[ c = u^T\mathbf{C}u \quad (6) \]

Normally, \( c \) is not diagonal; but it is common practice to assume that it is, with

\[ c_j = 2\zeta_j\omega_j \quad (7) \]

where \( \zeta_j \) is the damping ratio corresponding to the \( j \)th mode of the system. If it should become necessary to reconstruct the \( \mathbf{C} \) matrix from \( c \), this can be done by pre- and post-multiplying equation (6) by \( u \) and \( u^T \) respectively:

\[ \mathbf{C} = ucu^T \quad (8) \]

Similarly, a modal matrix \( u_i \) can be found for each subsystem, so
that equation (1) can also be transformed into

\[ I_iq_i + c_iq_i + k_iq_i = u_iF_i \]  

(9)

where, as usual, \( I_i \) is an identity matrix, \( k_i \) is a diagonal matrix and

\[ c_i = u_i^T C_i u_i \]  

(10)

System NASTRAN models can be used to generate \( k_i, u_i \), as well as \( k \) and \( u \). To characterize the system, multibody simulation codes can only accept subsystem information as input. So that a given simulation problem will require that \( k_i, u_i \) and \( c_i \) (or \( M_i, K_i \) and \( C_i \)) be available. \( k_i \) and \( u_i \) are readily obtainable from NASTRAN output, but \( c_i \) will have to be determined by the analyst. In general, the goal is to pick the elements of \( c_i \) in such a way that the system damping matrix \( c \) is diagonal, with the damping ratio for each mode having the constant value of about 1%. In other words, it is desired to influence the elements of \( c \) through those of the matrices \( c_i \). To do this effectively, it is important to understand the relationship between the \( c_i \)'s an \( c \). That is, it is necessary to examine the mathematics of the process by which a given multibody code assembles its subsystems into the full system.

4.1 **Subsystem Assemblage Process**

The analytical basis of the component assemblage process leading to the construction of the system damping matrix from component damping matrices is illustrated below with simple examples.
Consider the two planar systems A and B shown in Figure 2. Each of the systems consists of two rigid rods connected together by a one degree of freedom hinge; and motion about the hinge is restricted by a torsional spring and damper system. All motions of the systems are restricted to a plane. Each such system can be viewed as a simple flexible body. Equations of motion for A and B can be written in matrix form as

\[
\begin{bmatrix}
I_{A1} & 0 \\
0 & I_{A2}
\end{bmatrix}
\begin{bmatrix}
\ddot{\alpha}_1 \\
\ddot{\alpha}_2
\end{bmatrix}
+ \begin{bmatrix}
b_A & -b_A \\
-b_A & b_A
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha}_1 \\
\dot{\alpha}_2
\end{bmatrix}
+ \begin{bmatrix}
k_A & -k_A \\
-k_A & k_A
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix}
= \begin{bmatrix}
T_{A1} \\
T_{A2}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
I_{B1} & 0 \\
0 & I_{B2}
\end{bmatrix}
\begin{bmatrix}
\ddot{\beta}_1 \\
\ddot{\beta}_2
\end{bmatrix}
+ \begin{bmatrix}
b_B & -b_B \\
-b_B & b_B
\end{bmatrix}
\begin{bmatrix}
\dot{\beta}_1 \\
\dot{\beta}_2
\end{bmatrix}
+ \begin{bmatrix}
k_B & -k_B \\
-k_B & k_B
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix}
= \begin{bmatrix}
T_{B1} \\
T_{B2}
\end{bmatrix}
\]

respectively.
Now, consider A and B connected together at point P to form one system S shown in Figure 3 below. There are at least three ways in which the connection at P can be implemented:

- rigid connection
- frictionless hinge
- hinge connection with spring and dashpot.

The first option is of no interest here. Assuming a frictionless hinge, the equations of motion of S, when viewed as one system becomes

![Combined System S](Fig.3)
Comparing equations (11) and (12) with equation (13) partitioned as shown, it is immediately evident that the damping matrices in equations (11) and (12) are exactly the diagonal "elements" of the system damping matrix shown in equation (13). There is, thus, a one-to-one mapping between the elements of the damping matrices of components A and B and the elements of the diagonal submatrices of the system damping matrix.

If the connection at P between A and B is modified to include a torsional spring-dashpot system, the equations of motion are modified somewhat and is given as equation (14) below.

Here, both the diagonal and the off-diagonal submatrices of the system damping matrix are affected. Note, however, that if \( b = 0 \) (no damping at the joint), then the one-to-one mapping described
earlier is recovered. In practice, damping is rarely included at such hinge connections of multibody systems. Hence, it is concluded that for hinge-connected systems, changes in component damping matrices have direct effect on the diagonal submatrices of the system damping matrix. These effects are quantifiable following the relationships given in equations (11), (12), and (13).

4.2 Selection of Component Damping Matrices

As stated earlier, a multibody system containing flexible components can be viewed as one structure; and can therefore be represented by equation (2) or equation (4). Given a desired damping matrix for the system as a whole, our goal is to determine the component damping matrices that will produce the desired system damping matrix. The analyses presented in Section 4.1 above indicate a clear path to the solution of the problem if the matrix $C$ of equation (2) is the known or desired system damping matrix. However, this is generally not the case in practice. Normally, it is the diagonal matrix $c$ of equation (4) that is prescribed. Each of its diagonal element is assumed to be equal to $2\zeta_i \omega_i$, where $\omega_i$ is the natural frequency corresponding to the $i$th mode, and $\zeta_i$ is taken to be about 1%. Thus, it is assumed here that

$$c = \begin{bmatrix} d_1 & 0 \\ & d_2 \\ & \ddots \\ 0 & & d_n \end{bmatrix} \quad (15)$$

with all the $d_i$'s known. The matrix $C$ can be found by using equation (8). The elements of $C$ can thus be shown to be

$$C_{ij} = \sum_{k=1}^{n} u_{ik} u_{jk} d_k \quad (16)$$
where the \( u_{ij} \)'s are elements of the system modal matrix. The matrix \( C \) whose elements are given by equation (16), is now partitioned according to the number of degrees of freedom of each of the components (see the partitioning scheme used in equation (13)). The \( i \)th diagonal submatrix of \( C \) contains precisely the elements of the undiagonalized damping matrix of body \( i \). In summary, the selection strategy consists of the following steps:

1. Assign values to system modal damping ratios; this determines the elements \( d_i \) of the system's diagonal damping matrix \( C \);
2. Determine the elements of the undiagonalized system damping matrix \( C \) using equation (16);
3. Partition the \( C \) matrix according to the components' degrees of freedom;
4. The elements of the undiagonalized component damping matrix \( C_i \) for body \( i \) is identical to the \( i \)th diagonal submatrix of \( C \).

Note that the component damping matrices that emerge from this process are the \( C_i \)'s and not \( c_i \)'s. This implies that component information will then have to be supplied to the multibody simulation code in the form of \( M_i, C_i, \) and \( K_i \). All the codes that we know of can accept component data in this form.

5.0 MINORITY EDUCATION COMPONENT

One of the most successful aspects of this project was its education component. It was particularly successful in exposing students and faculty at Tuskegee University to a current NASA research topic. Three faculty members, two graduate students and one undergraduate student participated directly in this project. The two graduate students received their M.S degrees at Tuskegee University with at least partial funding from this project. The undergraduate student turned out to become the computer expert for
the group; he has also graduated with a B.S. degree in Mechanical/Aerospace Engineering (dual Major).

This project also contributed positively to the research infrastructure at Tuskegee University. The grant made it possible to purchase some critical computer hardware and software that were used to start a small Laboratory in Systems and Structural Dynamics.

**Personnel Utilized**

**Senior Personnel**

- Dr. Fidelis Eke (Mechan. Engr.) - Principal Investigator
- Dr. Estelle Eke (Aerosp. Engr.) - Co-Investigator
- Dr. Olusegun Adeyemi (Mech. Engr.) - Senior Investigator

**Graduate Students Supported**

- Mr. Busty Okundaye (Mechan. Engr.)
- Mr. Sheng-Fang Shen (Mechan. Engr.)

**Undergraduate Student Supported**

- Mr. Steven Hill (Mechan/Aero. Engr.)

6.0 PRESENTATIONS AND PUBLICATIONS

A presentation of some early results of this project was made at the Sixty-Seventh Annual Meeting of the Alabama Academy of Sciences in March 1990 at Mobile, Alabama. An abstract of this work is being submitted to the AAS/AIAA Conference Committee for presentation at the August 1991 Astrodynamics Conference in Durango, Colorado. It is planned to submit the same material to the AIAA Journal of Dynamic Systems and Control.
7.0 CONCLUSION

The main goal of this research was achieved. Some insight has been gained into the factors governing the selection of component damping matrices for interconnected multibody systems. Specifically, a workable selection strategy was developed for the case where the interconnection between bodies is through frictionless hinges - this is the normal assumption in most aerospace applications.

This project was also quite successful in exposing students and Faculty in a Historically Black University to a current NASA research effort, and contributed to the development of research infrastructure at the University.

8.0 RECOMMENDATION FOR FUTURE RESEARCH

Like all engineering results and techniques, the component damping characterization method developed as a result of this research effort cannot solve all possible damping characterization problems under any circumstances. The two main limitations are that the desired system damping matrix must be diagonal, and the inter-component connections must be by frictionless hinges. These restrictions do not constitute major shortcomings since many engineering systems (and most aerospace systems) actually satisfy the above conditions. Nevertheless, it is recommended that the results obtained be extended to systems with other than hinge connections. It is our belief that this is not only feasible, but it can be done relatively easily from the current results.

At a more fundamental level, it is recommended that studies be undertaken to quantify the actual impact on multibody simulation results of errors in component damping, with a possible view to developing "robust" simulation packages.
9.0 REFERENCES


