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Fatigue Criterion to System Design, Life and Reliability — A Primer

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Fatigue Criterion to System Design, Life and Reliability - A Primer

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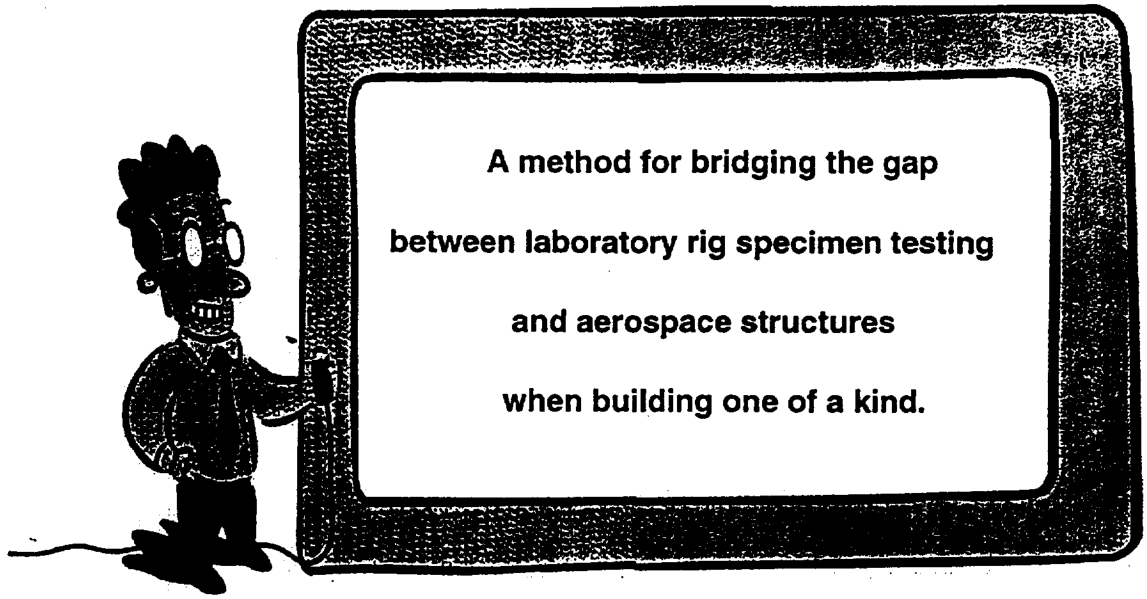
ABSTRACT

A method for estimating a component's design survivability by incorporating finite element analysis and probabilistic material properties was developed. The method evaluates design parameters through direct comparisons of component survivability expressed in terms of Weibull parameters. The analysis was applied to a rotating disk with mounting bolt holes. The highest probability of failure occurred at, or near, the maximum shear stress region of the bolt holes. Distribution of material failure as a function of Weibull slope affects the probability of survival. Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume Weibull parameters, as well as the stress-life exponent, in order to determine the qualitative effect of disk speed on the probability of survival.

INTRODUCTION

Finite-life component design requires a probabilistic approach that couples operating life with an expected rate of survival. Weibull (1939) demonstrated a statistical analysis that was particularly effective in describing experimental fatigue data. Lundberg and Palmgren (1947) applied Weibull analysis to contact stress, high-cycle-fatigue bearing problems. Grisaffe (1965) demonstrated the applicability of a Weibull analysis to other types of durability problems. Ioannides and Harris (1985) and Zaretsky (1987) proposed a generalized Weibull-based methodology for structural fatigue life prediction based on a discrete-stressed volume approach. Zaretsky et al. (1989) coupled this methodology to a stress field determined by finite element methods to predict the life and reliability of a generic rotating disk. They also demonstrated the applicability of the methodology to the design process through parametric studies that showed a component's life sensitivities to such design variables as disk diameter and thickness, and bolt hole size, number, and location.



Nemeth et al. (1990) developed a computer program for quantifying reliability that is based on inherent flaws found in ceramics. The program calculates the fast fracture reliability of macroscopically isotropic ceramic components. This method is also based on the component's entire stress state, not just the maximum stress point. August and Zaretsky (1991) extended the method of Zaretsky (1987) to allow for calculating the local probability of failure within any component's stressed volume as well as within the entire component. They demonstrated the technique on a generic disk by examining the sensitivity of stressed-volume survivability to uncertainties in the material properties. This method can be used to bridge the gap between laboratory rig specimen testing and aerospace structures when building one of a kind.

ANALYTICAL METHODOLOGY

Experimental fatigue data plotted on Weibull probability paper can be represented as a straight line. For a constant stress level the number of stress cycles to failure (i.e., life) is plotted on the abscissa. The percentile of specimens that survive at a given life is plotted on the ordinate. The transformation of stress cycles and survivability into Weibull coordinates is given by the equation.

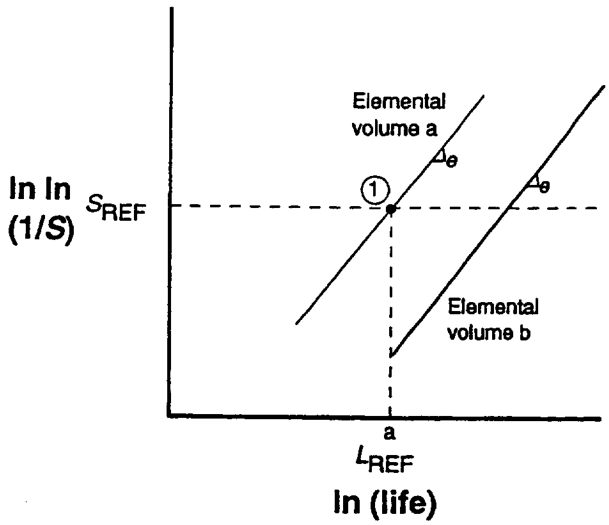
$$\ln \ln \frac{1}{S} = e \ln (L - L_u) - e \ln (L_o - L_u) \tag{1}$$

Element Survivability Methodology

Step 1 - Reference Element

Three parameter Weibull equation: $\ln \ln \frac{1}{S} = e \ln (L - L_u) - e \ln (L_o - L_u)$

Two parameter Weibull equation: $\ln \ln \frac{1}{S} = e \ln L - e \ln L_o$

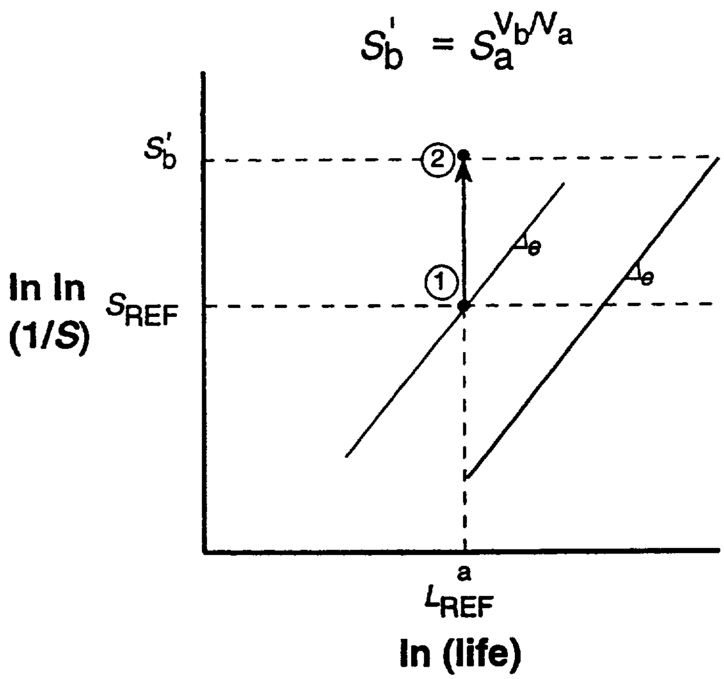


Where S is statistical fraction of specimens that survive a given number of stress cycles, e is the Weibull slope, L is stress cycles to failure, L_u is stress cycles where all specimens survive, and L_o is stress cycles where 36.8 percent of specimens survive. This is referred to as a three-parameter Weibull equation. A two-parameter Weibull equation is obtained by setting $L_u = 0$. The Weibull slope provides an indication of the scatter of the statistical properties, with e = 1,2, or 3.57 being representative of exponential, Rayleigh, and Guassian distributions, respectively.

A methodology for predicting elemental survivability at a fixed life is illustrated in the figure. The critical element's life and survivability is set at unity and 90 percent, respectively (point 1). This is the analysis point to which all the other elements' lives and survivability will be referenced. Other designs can also be referenced to this point by using the same critical volume and stress. The expected survivability at any number of stress cycles can be determined by using Eq. (1). This is shown in the figure by drawing a line through the reference point with a slope equal to e, the material's Weibull slope.

Element Survivability Methodology

Step 2 - Volume Effect



From coupon testing the distribution on the Weibull plot for L_{REF} is obtained based upon a stress τ_a and stressed Volume V_a. The size effects on survivability for different volumes equally stressed can be expressed as

$$S_b' = S_a^{V_b/V_a} \qquad (2)$$

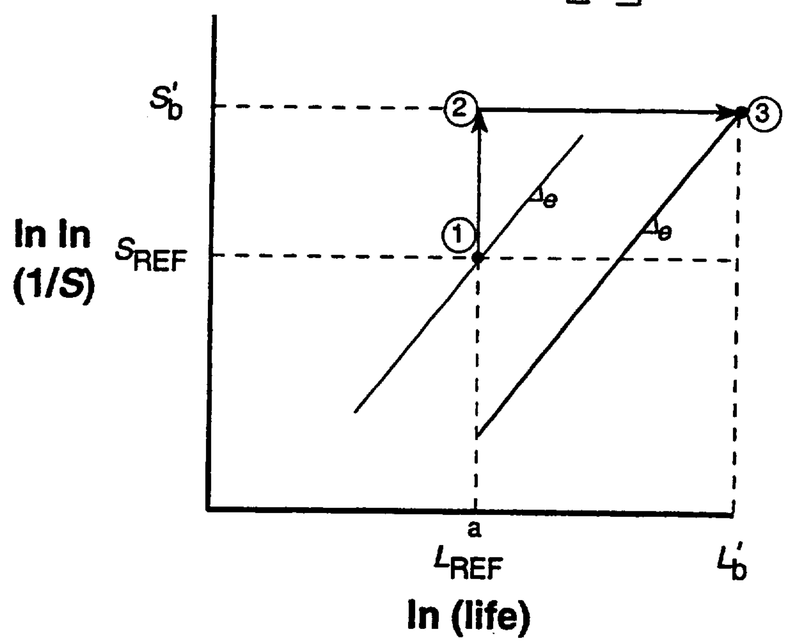
The elements survivability is adjusted based strictly on volume effects by using Eq. (2). The probability of survival for volume b at equal stress is moved from S_{REF} (point 1) to S_b' (point 2) in the figure. This allows for valid comparison of relative component survivability at equal life.

This adjustment was done independently of the stress levels. Because stress is not involved, elemental life was not affected, and the adjustment resulted in a purely vertical displacement from the reference point. In the example shown, the second element's volume was greater than the critical volume and contained more potential crack initiation sites. Therefore, the adjusted survivability was lower.

Element Survivability Methodology

Step 3 - Stress Effect

$$L_b' = L_{REF}\left[\frac{\tau_a}{\tau_b}\right]^c$$



Assuming that life is exponentially related to stress to a negative power, the number of stress cycles to failure at a given stress level can be related to stress cycles to failure at a new stress level by the equation.

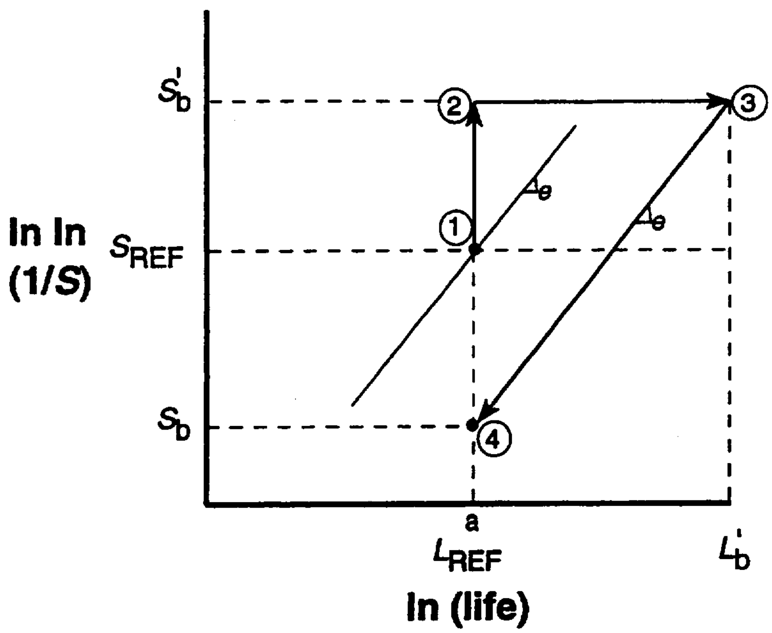
$$L_b' = L_{REF}\left[\frac{\tau_a}{\tau_b}\right]^c \qquad (3)$$

Where L_a is the known life at stress level τ_a, and c is the stress-life exponent. These values can be obtained from coupon testing. Based upon the above, the life is moved from L_{REF} (point 2 to L_b (point 3) in the figure. This allows for comparison of relative component life at equal survivability.

Element Survivability Methodology

Step 4 - Elemental Volume Survivability at L_{REF}

$$S_b = S_b'^{\,(L_{REF}/L_b')^e}$$



The element's life was adjusted strictly on the basis of elemental stress relative to the critical stress by using Eq. (3) (point 3). This adjustment was done independently of volume considerations. Consequently, elemental survivability was not affected, and the life adjustment results in purely horizontal displacement at constant survivability. In the example shown, the second element's stress is lower than the critical stress. Therefore, its life is greater and shifts to the right.

Zaretsky's method can be extended further to examine relative component survivability at equal stressed volume but different life. This is given by

$$S_b = S_b'^{\,(L_{REF}/L_b')^e} \tag{4}$$

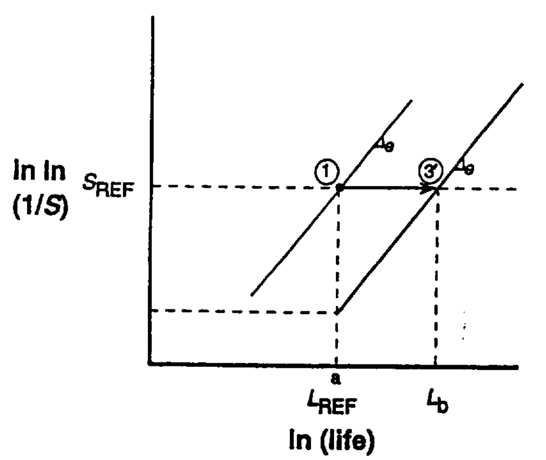
By using Eq. (4), the compared element's survivability is adjusted once more to the critical element's assigned life of unity. (point 4). This time the displacement is along the Weibull line and gives the survivability of an element relative to the critical element, with both elements having the same life.

Equation (4) can be used to identify critical components within a system and to optimize noncritical ones. Because a minimum life is required of all components in the system, individual components survivability at that life can be assessed relative to a predetermined critical component. In this manner, component survivability at that life can be assessed relative to a predetermined critical component. In this manner, components whose survivability is judged to be too low can be redesigned for greater survivability. Also, components whose survivability is much greater than that of the critical component can be redesigned for smaller size or lower weight while still maintaining adequate protection against failure. Repeating this procedure for all the elements in the finite element model gives the survivability for any section of the design at any time.

Element Survivability Methodology

Step 3' - Stress and Volume Adjustment

$$L_b = L_{REF}\left[\frac{\tau_a}{\tau_b}\right]^c\left[\frac{V_a}{V_b}\right]^{1/e}$$



Steps 2 through 3 can be combined and the method shortened by combining stress and volume adjustments according to the following equation (Zaretsky, 1987).

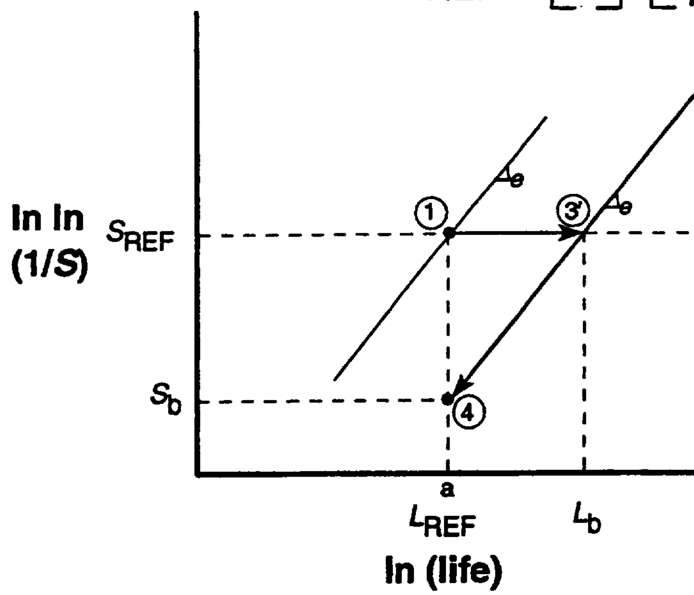
$$L_b = L_{REF}\left[\frac{\tau_a}{\tau_b}\right]^c\left[\frac{V_a}{V_b}\right]^{1/e} \tag{5}$$

Point 1 is moved horizontally to point 3' from L_{REF} to L_b.

Element Survivability Methodology

Step 4' - Elemental Volume Survivability at L_{REF}

$$S_b = S_{REF}^{(L_{REF}/L_b)^e} \quad \text{where:} \quad \frac{L_b}{L_{REF}} = \left[\frac{\tau_a}{\tau_b}\right]^c \left[\frac{V_a}{V_b}\right]^{1/e}$$

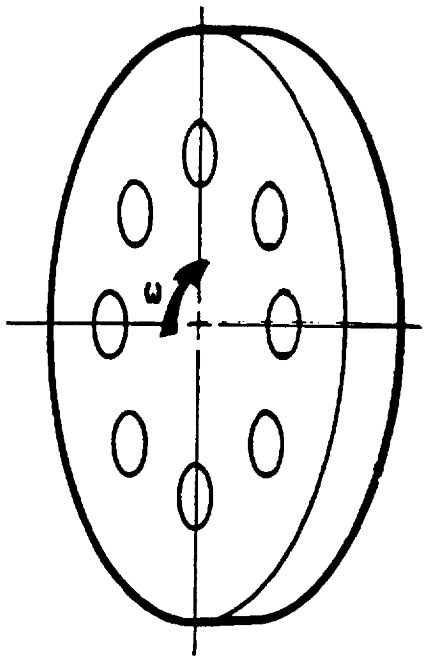


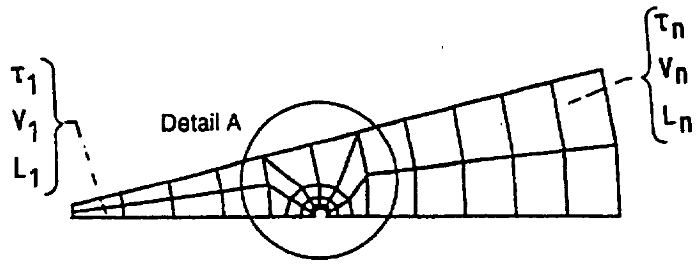
Step 4 is identical to Step 4 where the time displacement is along the Weibull line from point 3 to point 4 and gives the survivability of an element relative to the critical element, with both elements having the same life.

RESULTS AND DISCUSSION

Parametric analytical studies were conducted to calculate speed effects on the survivability of a generic disk and to examine the probabilistic effects of material properties on disk survivability. The generic disk represents the first step in the investigation of aerospace propulsion turbine disks. It is 61 cm (24 in.) in diameter and has twelve 1-cm (0.4-in.) diameter bolt holes.

Finite Element Model





For a designated probability of survival, the life of an elemental volume is:

$$L_n = L_{REF}\left(\frac{\tau_{REF}}{\tau_n}\right)^c\left(\frac{V_{REF}}{V_n}\right)^{1/e}$$

The life of the entire disk is:

$$\left(\frac{1}{L_s}\right)^e = \sum_{i=1}^{n}\left(\frac{1}{L_i}\right)^e$$

The figure shows the finite element model of the disk used in these studies. From the axisymmetric conditions of the disk a 15 degree section of the disk is modeled with 62 grid points and 42 eight-noded solid elements. Boundary conditions, defined in terms of the model's cylindrical coordinates, constrain displacement in the circumferential direction and allow displacement in the radial and axial directions. The disk stresses are result of the centrifugal loads due to disk rotation. No thermal loads were included.

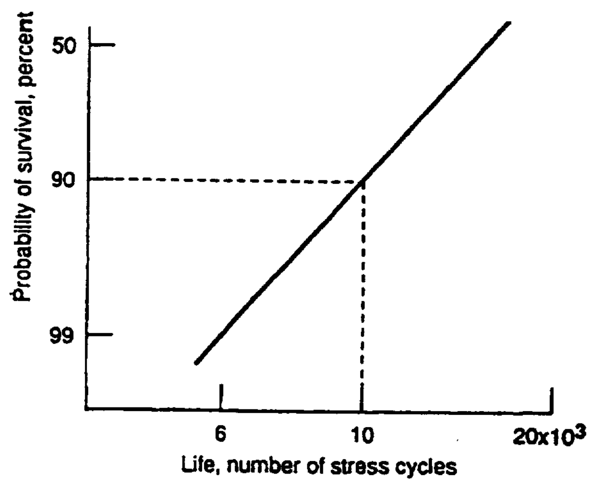
MSC/NASTRAN was used for the finite element code. The linear elastic option, SOL 24, was used to calculate the stresses. This analysis is limited to stress levels below the yield point. It can be used for high cycle fatigue where stresses are low and the occurrence of failure is less deterministic and more probabilistic. However, because the statistical failure theory is based on stress levels and distribution, the life and reliability methodology can also be adapted for nonlinear analysis. The system life can then be calculated from the individual component lives as follows:

$$\left(\frac{1}{L_s}\right)^e = \sum_{i=1}^{n}\left(\frac{1}{L_i}\right)^e \qquad (6)$$

where L_s is the total system life and L_i is the i^{th} component life (Zaretsky, 1987).

Assumed Isotropic Material Properties

Elastic modulus, kN/m^2 (psi)	1.10x10^{11} (16x10^6)
Poisson's ratio	0.33
Density, kg/m^3 (lb/in^3)	4.61x10^{-5} (0.16)
Gage volume, m^3 (in^3)	5.74x10^{-6} (0.35)
L_{10} life, number of cycles	10,000
Shear stress at L_{10}, kN/m^2 (ksi)	275,800 (40)

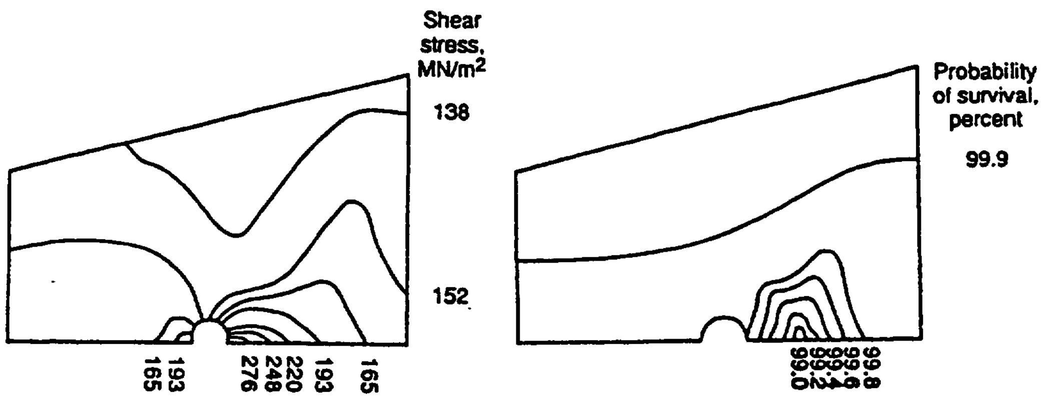


The figure lists the assumed isotropic material properties used in the analyses. The probabilistic aspect of the material strength is given in the L_{10} life of the material. This L_{10} life represents the life in the number of stress cycles in which 90 percent of the tested coupon survive at a constant maximum shear stress. By using these values, a finite element stress analyses was performed on the generic disk for a speed of 12 800 rpm.

Disk FEM Analysis of Detail A

(L_{REF}, 10,000 cycles; Speed, 12,800 rpm)

$$S_s = \prod_{i=1}^{n} S_i$$



Shear Stress Contours   Probability of Survival

Several different criteria can be considered for normalizing the critical structural element. These criteria are:

(1) Maximum shear stress failure theory
(2) Maximum normal stress failure theory
(3) Maximum element strain energy density
(4) Maximum element volume

The maximum shear stress should generally be used in identifying the critical element. Justification is based on the applicability of the maximum shear stress failure theory for ductile materials. Independent of the selected criteria, for all calculation purposes the element with maximum critical stress is given a relative life of unity at a probability of survival of 90 percent, or a 10 percent probability of failure (Zaretsky et al. 1989). The figure shows the results of the stress analysis. The maximum shear stress was approximately 275 800 KN/m^2 (40 ksi) at the bolt hole.

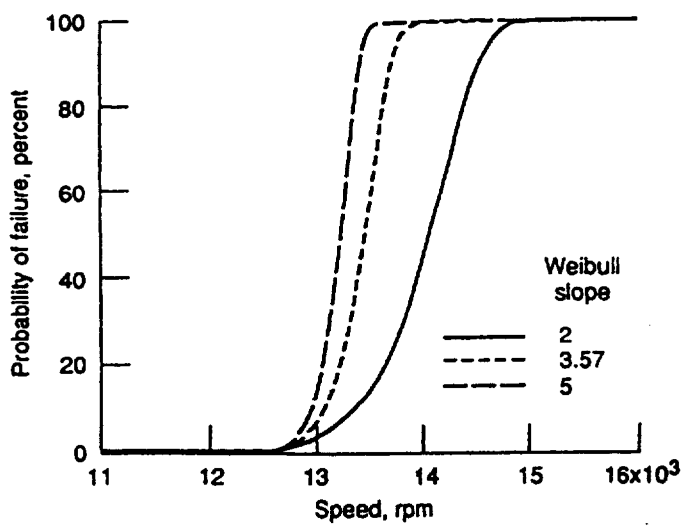
The failure probability analysis for each of the model's elements was then calculated by the following equation:

$$S_s = \prod_{i=1}^{n} S_i \tag{7}$$

The results shown indicate that the region with the lowest reliability (i.e., the highest probability of failure) occurred at, or near, the maximum shear stress region at the bolt hole. However, outside this region the disk had relatively high regions of survivability.

Effect of Speed and Weibull Slope on Survivability

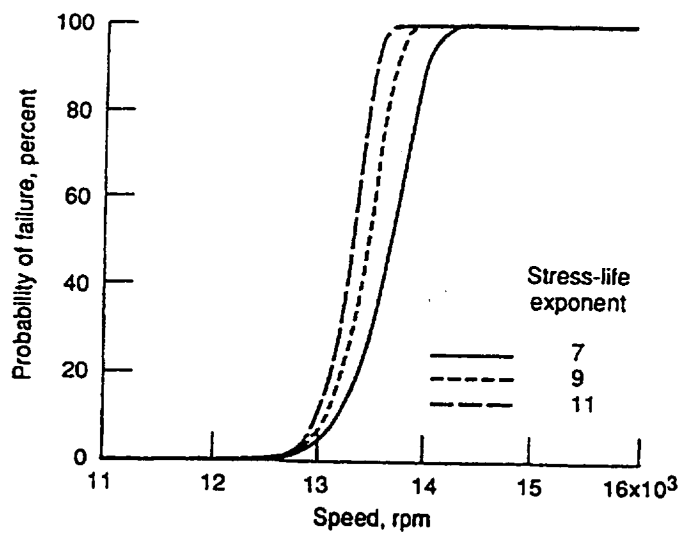
(L_{REF}, 10,000 cycles)



This figure shows the relation between life and survivability for the disk at an assumed Weibull slopes. For a Weibull slope 3.57, 99 percent of the samples survived 6000 cycles, 90 percent survived 10 000 cycles, and 10 percent survived 20 000 cycles. A higher value of Weibull slope would demonstrate less scatter, with failure occurring over a smaller number of stress cycles. A lower value would show failure over a broader range of lives and therefore might be indicative of more uncertainty in the disk lives.

Effect of Speed and Stress-Life Exponent on Survivability

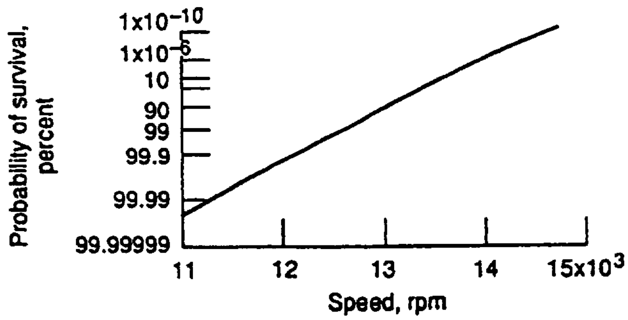
(L_{REF}, 10,000 cycles)



The figure shows the effect of speed on the cumulative probability of failure with varying stress-life exponents. There is not the degree of divergence of the three curves above 12 800 rpm as there was with Weibull slope variations. The same general trends hold, higher failure rates with higher stress-life exponent. At high survivability values, the value of the stress-life exponent had little effect at any given speed. Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume Weibull parameters as well as the stress-life exponent in order to determine the disk speed at which the probability of survival will be highest.

Effect of Speed on Survivability

(L_{REF}, 10,000 cycles; Stress-life exponent, 9; Weibull slope, 3.57)



The figure shows the effect of a disk's rotational speed on its probability of achieving a life of 10 000 stress cycles. The speed effect was plotted for an assumed Weibull slope 3.57 and a stress-life exponent of 9. The disk first experienced a maximum shear stress of 275 800 kN/m_2 (40 ksi) at a speed of 12 800 rpm. This disk had a probability of survival of about 96 percent at this speed, which was higher than the coupon survival rate of 90 percent at the same stress and number of cycles.

CONCLUSION

The methodology described can easily be incorporated into the finite element method to determine the life and survivability of a structural component. The entire component is analogous to a system. The finite element model of the component discretizes its geometry into elemental volumes. These elements are considered to be the base members of the "system." The time to crack initiation for each element is calculated from the elemental stress levels and volume as described previously to determine the component's incipient failure time. Elemental survivability is also calculated to find areas that have either too low survivability or are overdesigned.

A relative comparison approach is used in this methodology. A critical element in the model is selected on the basis of a maximum stress state. The selected stress failure criterion can be based on the material used in the component. If no fatigue data are available, the stressed element can be arbitrarily assigned a life and a survivability that is used to normalize the life and survivability of the other elements. This approach allows easy, qualitative comparisons between designs. Also, only one set of coupon fatigue tests at operating temperature is necessary to fix the Weibull parameters and to establish quantitative component lives if the stress-life exponent of the material is already known. However, if the stress-life exponent is not known, at least two additional sets of coupon fatigue tests will be necessary.

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