

# Spacecraft Detumbling Through Energy Dissipation

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## Abstract

The attitude motion of a tumbling, rigid, axisymmetric spacecraft is considered. A methodology for detumbling the spacecraft through energy dissipation is presented. The differential equations governing this motion are stiff, and therefore an approximate solution, based on the variation of constants method, is developed and utilized in the analysis of the detumbling strategy. Stability of the detumbling process is also addressed.

## Introduction

As human expectations and scientific frontiers expand, the capabilities of satellites and space platforms must expand to meet these challenges. This results in more expensive satellites and space platforms being designed and launched. These elaborate systems will require on-orbit servicing/repairs and recovery missions to correct system malfunctions. In the past, on-orbit servicing and recovery missions have been uncommon operations since the cost of a replacement satellite was far less than the cost of these missions. However, today's high cost of manufacturing and launching of space systems make servicing and recovery missions an economical alternative to spacecraft replacement.<sup>1-4</sup> For example, the INTELSAT 6 communication satellite with an initial cost of \$265M will be repaired on orbit at a total cost of \$150M.<sup>5</sup>

In addition to monetary costs, there is the "cost" of human lives when manned space flights are involved. For these missions, recovery is not an alternative, it is the only choice. Finally, the growing concern over space debris mandates that at the end of a spacecraft's useful life, it must be retrieved and properly disposed of.

Malfunctioning of a spacecraft could result in a wildly gyrating, uncontrolled system. In the case of a manned spacecraft, it may not be feasible to wait for a period of several days while the spacecraft settles into a state of pure spin<sup>6</sup> before a rescue mission is attempted. It is also reasonable to assume that the manned spacecraft may be a module from a larger system, and as such, does not possess the degree of flexibility necessary to dissipate energy at a sufficiently high rate in order to quickly detumble itself. Consequently, it can be expected that during some recovery missions, the uncontrolled spacecraft will have non-zero precession and nutation rates which must be reduced to zero as quickly as possible.

The dynamic interactions involved in detumbling one spacecraft (uncontrolled vehicle) by another spacecraft (rescue vehicle), of perhaps comparable mass, are non-trivial. The task of grasping the uncontrolled (tumbling) spacecraft poses quite a challenge to the recovery

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vehicle since the grappling point on the disabled spacecraft may traverse a cone in space. In addition to the grasping task, the disabled spacecraft must be stabilized in a manner which maintains the motion of both vehicles within some safe bounds. This requires detailed knowledge of the dynamic characteristics of the spacecraft and places greater demands on the rescue vehicle's attitude control system, as well as fuel reserves. More importantly, this situation involves great safety hazards, particularly if either the disabled spacecraft or the rescue vehicle is manned. Yet the philosophy behind current approaches to the spacecraft retrieval problem is to "grapple and wrestle" the spacecraft,<sup>2-4</sup> since the time required for the spacecraft to settle into a state of pure spin may well exceed the time available during a retrieval mission.

An alternative to the current "grapple and wrestle" retrieval approach is a retrieval strategy which first reduces the motion of the spacecraft to that of pure spin and then despins the spacecraft. In this paper, a process by which this may be achieved, within the time frame of a retrieval mission, is presented. In the following section, a strategy for detumbling the spacecraft is presented. The equations governing the detumbling motion are developed and presented. An approximate solution to the governing equations is presented and used to investigate the proposed detumbling strategy. The paper concludes with suggestions for future work.

### **Detumbling Strategy**

It is well known that the general rotational motion of a torque-free rigid body involves spin, nutation and precession.<sup>7-9</sup> Also, when dissipative effects are present (e.g., a flexible body), the rotational motion of the body eventually reduces to a state of pure spin about the axis of maximum moment of inertia. This state of pure spin rotational motion is a result of energy dissipation, and therefore represents the steady state rotational motion of all real spacecraft (i.e., non-rigid bodies). It is worth noting that for an axisymmetric body (one where the two smaller principal moments of inertia are equal) the nutation rate as measured with respect to the (constant) angular momentum vector is zero.

In practice, the time required for this state of rotational motion to occur is typically on the order of several days.<sup>6</sup> It is proposed that in order to decrease the required time, the energy dissipation rate of the spacecraft should be increased. This would be accomplished by attaching a dissipative device to the spacecraft; that is, retroactively fitting the spacecraft with external precession and nutation dampers.

The dissipative device consists of a flexible rod with an end mass as shown in Fig. 1. Damping effects in the rod would be tailored to dissipate energy at a rate which decreases the nutation angle within the time frame of the mission. The length and stiffness of the rod, and the size of the end mass are design criteria which are governed by stability requirements. As depicted in Fig. 1, usage of this device requires only a slight modification of the Tumbling Satellite Retrieval (TSR) Kit developed by Grumman.<sup>4</sup> It is proposed that the arm of the device be constructed from "smart" materials such that, during instances when the device is attached to the rescue vehicle, it will be sufficiently stiff to allow rapid maneuvers. However,

once grappling has been accomplished, the device will be detached from the rescue vehicle, eliminating its source of power, thereby rendering the device passive.

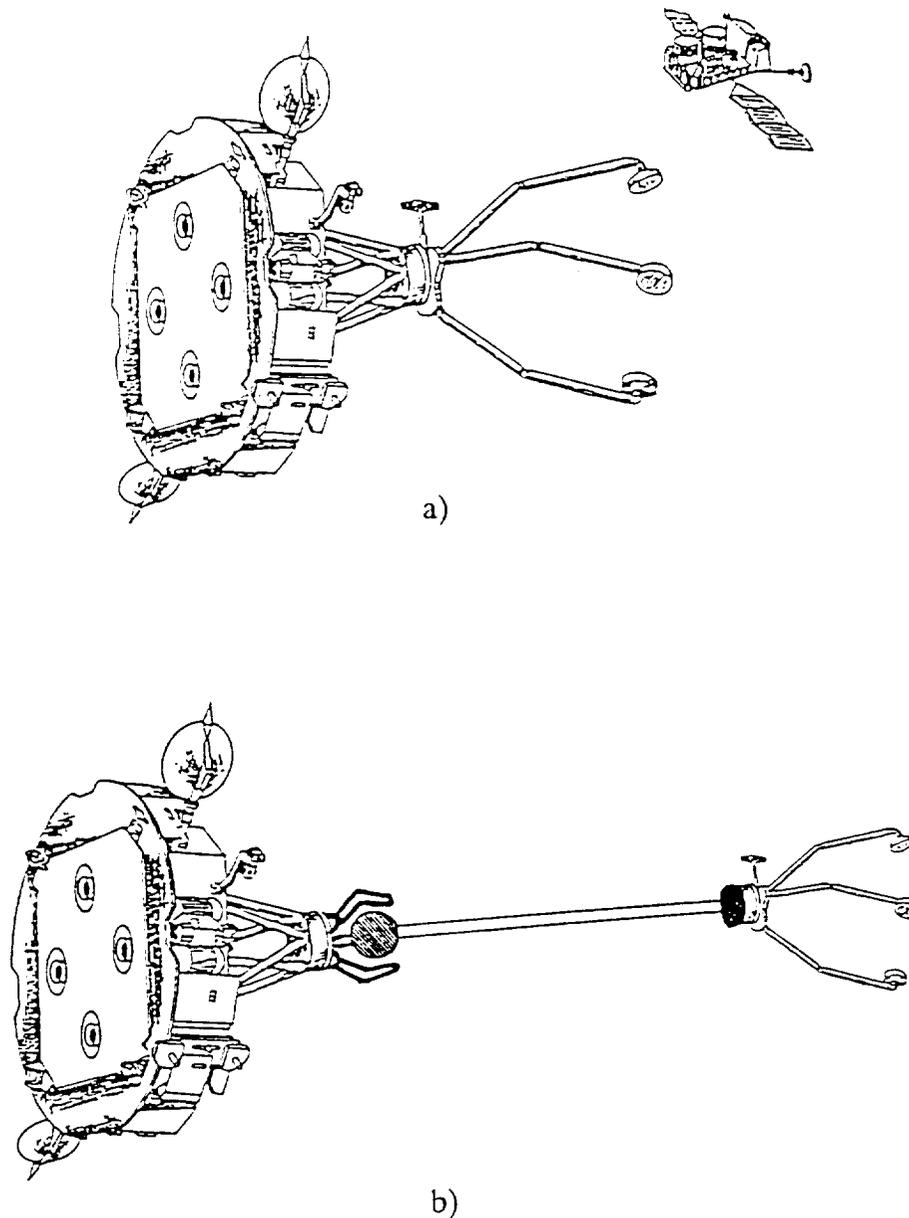


Fig. 1. Retrieval vehicle: a) current concept (Grumman),  
b) proposed concept

In this paper, the issues associated with attaching the device to a tumbling spacecraft (i.e., locating/tracking/spin-rate matching and gripping), or the actual design of the dissipative device are not addressed. In what follows, it is assumed that the device has been successfully attached to the spacecraft. The following sections present an analysis of the dynamic performance of the device.

## Equations of Motion

The spacecraft model adopted for the present study is shown schematically in Fig. 2. It consists of an axisymmetric rigid spacecraft, S, an end-mass, E, and a flexible link, L. The end-mass is attached to the spacecraft via the flexible link in such a manner that when the link is in its undeformed state, both the link and the end-mass are along the axis of symmetry of the spacecraft.

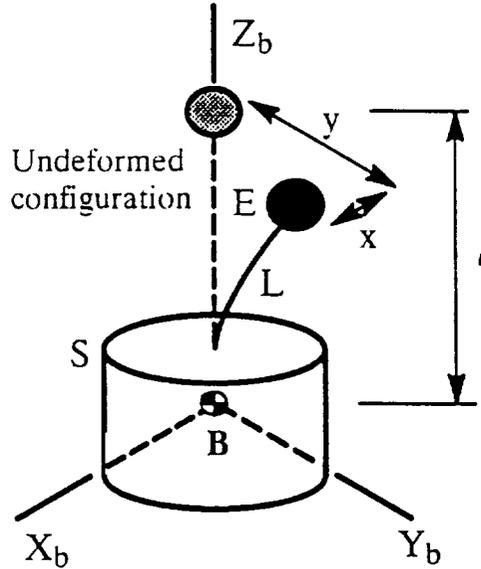


Fig. 2. Spacecraft Model

The dextral orthogonal coordinate system  $BX_b Y_b Z_b$  is fixed in the spacecraft. The axes are centroidal principal axes for the spacecraft. The  $Z_b$  axis lies along the axis of maximum inertia which is also the axis of symmetry. The displacements of the end-mass in the  $Z_b$ -direction are assumed small and therefore are neglected. That is, the end-mass is assumed to move parallel to the  $X_b Y_b$ -plane; in this plane, the displacements of the end-mass relative to the spacecraft are denoted by  $x$  and  $y$  as shown in Fig. 1.

The centroidal moments of inertia of the spacecraft are  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$ , where  $I_{xx} = I_{yy} < I_{zz}$ . The contribution of the end-mass to the overall system mass is neglected since its mass,  $m$ , is significantly smaller than the mass,  $M$ , of the spacecraft. The flexible link connecting the end-mass to the spacecraft is assumed "massless." Under these assumptions, the center of mass location,  $B$ , is unchanged by the the addition of the dissipation device.

Denoting the stiffness and the damping of the flexible rod by  $K$  and  $C$ , respectively, then the equations governing the motion of the system can be expressed as

$$\ddot{x} + c\dot{x} + (k - \omega_z^2 - \omega_y^2)x - 2\omega_z\dot{y} + (\omega_x\omega_y - \dot{\omega}_z)y = -(\omega_x\omega_z + \dot{\omega}_y)l \quad (1)$$

$$\ddot{y} + c\dot{y} + (k - \omega_z^2 - \omega_x^2)y + 2\omega_z\dot{x} + (\omega_x\omega_y + \dot{\omega}_z)x = -(\omega_y\omega_z - \dot{\omega}_x)l \quad (2)$$

$$l\dot{\omega}_x = -\omega_y\omega_z(\mu - 1)l - l(ky + c\dot{y}) - F_{zy} \quad (3)$$

$$l\dot{\omega}_y = \omega_x\omega_z(\mu - 1)l + l(kx + c\dot{x}) + F_{zx} \quad (4)$$

$$\mu l\dot{\omega}_z = c(\dot{y}x - \dot{x}y), \quad (5)$$

where  $l$  is the distance of the end-mass from B,  $c$ ,  $k$ , and  $l$ , are mass “normalized” quantities,  $\mu$  is a nondimensional inertia ratio ( $\mu > 1$ ), and  $F_z$  is the  $z$ -direction inertia force associated with the end-mass. Note that the mass normalized stiffness,  $k$ , is actually the square of the fundamental frequency for the dissipative device.

$$c = \frac{C}{m}; \quad k = \frac{K}{m} = \omega_n^2 \quad (6.a)$$

$$l = \frac{I_{xx}}{m} = \frac{I_{yy}}{m}; \quad \mu = \frac{I_{zz}}{I_{xx}} \quad (6.b)$$

$$F_z = \dot{\omega}_x y - \dot{\omega}_y x + 2(\omega_x \dot{y} - \omega_y \dot{x}) + \omega_z(\omega_x x + \omega_y y) - l(\omega_x^2 + \omega_y^2) \quad (6.c)$$

Equations (1) through (5) represent a set of stiff differential equations since there are two disparate time scales. An approximate analytical solution for these equations is developed in the next section.

### Approximate Solution

Assuming a small attached end-mass  $E$  ( $m \ll M$ ) and relatively small dissipation rates, the rotational motion of the system (spacecraft and device) can be approximated for a few cycles of oscillation by Euler's equations for an axisymmetric, torque free, rigid body. These equations are

$$\dot{\omega}_x = -\omega_y\omega_z(\mu - 1), \quad (7.a)$$

$$\dot{\omega}_y = \omega_x\omega_z(\mu - 1), \quad (7.b)$$

$$\dot{\omega}_z = 0, \quad (7.c)$$

where  $\mu$  is as defined in Eq. (6.b). Euler's equation (Eq. (7)) has a solution

$$\omega_x = A \cos \Omega(t + t_0), \quad (8.a)$$

$$\omega_y = A \sin \Omega(t + t_0), \quad (8.b)$$

$$\omega_z = \text{constant} \quad (8.c)$$

where  $A$  represents the tangential angular velocity (i.e., the resultant of  $\omega_x$  and  $\omega_y$ ; see Fig. 4), and  $\Omega = (\mu - 1)\omega_z$ . Equations (1) and (2) can now be rewritten as

$$\ddot{x} + c\dot{x} + (k - \omega_z^2 - A^2 S_\Omega^2)x - 2\omega_z \dot{y} + A^2 S_\Omega C_\Omega y = -\mu\omega_z l A C_\Omega \quad (9)$$

$$\ddot{y} + c\dot{y} + (k - \omega_z^2 - A^2 C_\Omega^2)y + 2\omega_z \dot{x} + A^2 S_\Omega C_\Omega x = -\mu\omega_z l A S_\Omega \quad (10)$$

where

$$S_{\Omega} = \sin \Omega(t + t_0); \quad C_{\Omega} = \cos \Omega(t + t_0). \quad (11)$$

Equations (9) and (10) have time varying coefficients; thus, a study of stability via the Routh–Hurwitz criterion is inapplicable. To circumvent this problem, a coordinate system,  $B\xi\eta\zeta$ , which rotates relative to the spacecraft–fixed  $Z_b$ –axis with angular rate  $\Omega$  is defined (see Fig. 3). The counterparts of Eqs. (9) and (10) in this coordinate system are constant

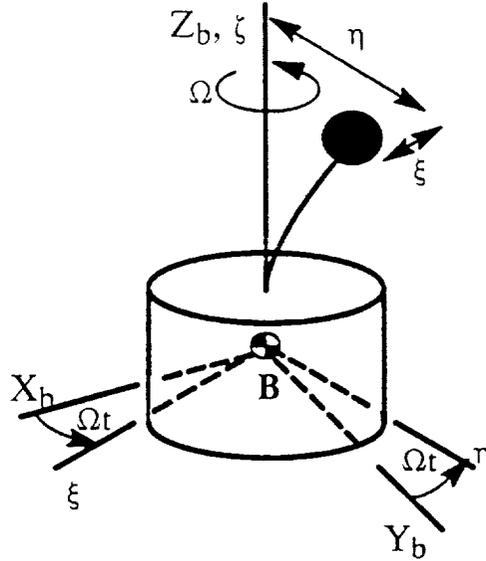


Fig. 3.  $B\xi\eta\zeta$  coordinate system

coefficient differential equations for  $\xi$  and  $\eta$ . Routh–Hurwitz criterion can now be applied to show that the complementary solutions of  $\xi$  and  $\eta$  decay provided that the normalized stiffness satisfies

$$k - \mu^2 \omega_z^2 - \frac{A^2}{2} > 0, \quad (12)$$

and the normalized damping is positive (i.e.,  $c > 0$ ). Note that if  $H$  is the magnitude of the angular momentum of the tumbling spacecraft, then

$$H^2 = m^2 I^2 (\mu^2 \omega_z^2 + A^2), \quad (13)$$

which implies  $\mu^2 \omega_z^2 + \frac{A^2}{2} < \frac{H^2}{m^2 I^2}$  is bounded at all times for any given set of initial conditions. Therefore, proper selection of  $k$  will always satisfy Eq. (12).

It can also be shown that in the  $B\xi\eta\zeta$  coordinate system, the particular solutions for the counterparts of Eqs. (9) and (10) are constants  $\xi_1$  and  $\eta_1$ . Since the complementary solutions decay to zero and the particular solutions are constant, then steady state solutions for Eqs. (9) and (10) can be expressed as

$$x = \zeta_1 C_{\Omega} - \eta_1 S_{\Omega} \quad (14.a)$$

$$y = \zeta_1 S_{\Omega} + \eta_1 C_{\Omega} \quad (14.b)$$

The energy dissipation rate  $D$  then becomes

$$D = mc(\dot{x}^2 + \dot{y}^2) = mc\Omega^2(\zeta_1^2 + \eta_1^2) \quad (15)$$

where the entire energy dissipation is considered as energy lost by the tumbling spacecraft. That is,

$$\frac{dT}{dt} = -D \quad (16)$$

where  $T$  is the spacecraft's (rotational) kinetic energy.

$$T = \frac{mI}{2}(\mu\omega_z^2 + A^2) \quad (17)$$

Now, the quantity  $A$  is a measure of how far the spacecraft is from a state of pure spin. When  $A$  is zero, the nutation angle is zero, therefore, the spacecraft is in a state of pure spin. The angle that the axis of symmetry makes with the direction of the constant angular momentum vector is given by (see Fig. 4)

$$\tan \theta = \frac{A}{\mu\omega_z} \quad (18)$$

where  $\theta$  is the nutation half angle.

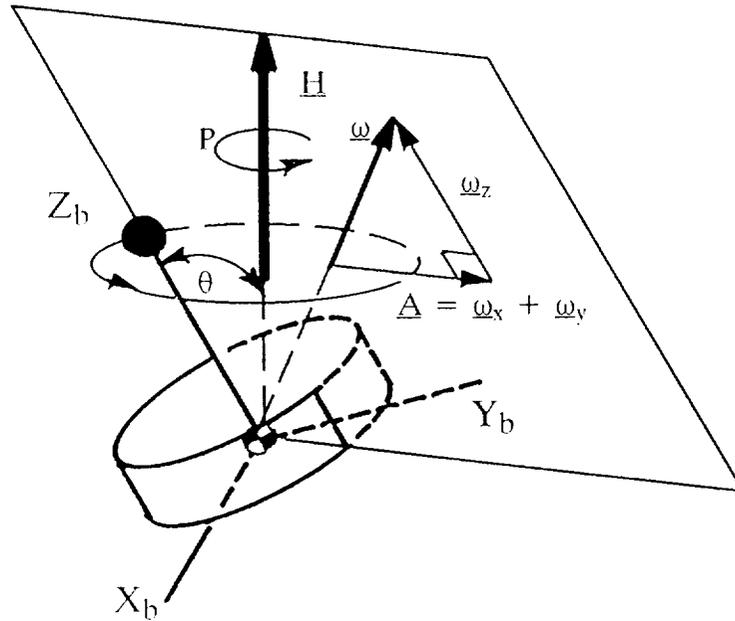


Fig. 4. Precessing spacecraft

Via Eqs. (13) through (17), a constant coefficient, ordinary differential equation for the  $A^2$  can be developed. Omitting the algebra, this equation is

$$\frac{d}{dt}(A^2) = -\frac{2c\varrho l^2 A^2 (P^2 - A^2)^2 (Q^2 - c^2 \varrho^2 A^2)}{I(Q^2 + (k - P^2 - c^2 \varrho^2) A^2)^2} \quad (19)$$

where  $\rho$ ,  $P^2$ , and  $Q^2$  are defined respectively as

$$\rho = \frac{\mu - 1}{\mu},$$

$$P^2 = \frac{H^2}{m^2 I^2} = \mu^2 \omega_z^2 + A^2,$$

$$Q^2 = (k - P^2)^2 + c^2 \varrho^2 P^2.$$

Note that the  $P$  is the precession rate of the spacecraft (see Fig. 4).

Equation (19) is of the form

$$\frac{dx}{dt} = -\frac{\alpha x(\beta - x)^2(\gamma - x)}{(\delta + x)^2},$$

which may be rewritten as

$$dt = -\frac{1}{\alpha} \frac{(\delta + x)^2}{x(\beta - x)^2(\gamma - x)} dx$$

Using a partial fraction expansion, an analytical solution can be obtained, resulting in a solution of Eq. (19) of the form  $t = f(A^2)$ . For studies of settling time versus  $A^2$ , this form of the anti-derivative of Eq. (19) is quite convenient. However, when  $A^2$  as a function of time is required, it is more convenient to numerically integrate Eq. (19).

## Results

In order to validate the approximate solution, Eqs. (1) through (5) and Eq. (19) were numerically integrated using the MATLAB<sup>10</sup> function "ODE45." Initial conditions for the approximate solution were  $A = 6$ ,  $\omega_z = 3$  whereas initial conditions for the complete solution were  $\omega_x = 6$ ,  $\omega_y = 0$ ,  $\omega_z = 3$ ,  $\dot{x} = \dot{y} = 0$ , and  $x = y = 0$ . (Note, any combination of  $\omega_x$  and  $\omega_y$  resulting in  $A = 6$  is applicable since the transients decay rapidly.) In both cases,  $\mu = 1.5$  resulting in  $P^2 = 56.25$ . The results of these numerical integrations for two different scenarios are shown in Figures 5 and 6. While the accuracy of the approximate solution is quite acceptable, its computational requirement is typically three to four orders of magnitude less than that required for the "complete" solution. Figures 5 and 6 show that the relative error for the approximate solution decreases as the detumbling time becomes longer (i.e., the energy dissipation rate decreases). This is expected since the approximation becomes more accurate as the dissipation rate decreases.

Figures 7 through 11 demonstrate the dependence of energy dissipation rate, hence settling time, on system the parameters  $c$ ,  $k$ ,  $l$ ,  $\mu$ , and  $H$ , respectively. For an effective

comparison, an initial value of  $A = 6$  is used in each investigation. In Figs. 7 through 10, the parameter  $P^2$  remains unchanged at  $P^2 = 56.25$  (i.e.,  $\mu = 1.5$ ,  $\omega_z = 3$ ), whereas, in Fig. 11,  $P^2$  changes as  $\omega_z$  is varied from 2 to 5. That is, in Fig. 11 the angular momentum is varied while keeping the inertia properties constant. In contrast, in Fig. 10 the angular momentum is held constant while the inertia properties are varied ( $0.5 < \mu < 2$  and  $\omega_z$  adjusted such that  $P^2$  is maintained at 56.25). In Figs. 7 through 10, the label by each curve represents the value of the parameter which was varied; in Fig. 11, the label by each curve represents the value of  $\omega_z$ .

Damping obviously has a strong effect on the dissipation rate and generally an increase in damping leads to a decrease in settling time (Fig. 7). However, a relatively high damping value ( $c = 500$ ) causes the settling time to increase indicating that for a given configuration, there is an optimal value of  $c$ . Increasing  $c$  beyond this optimal value will result in increasing settling times. As expected, decreasing the length of the rod (Fig. 8) or increasing its stiffness (Fig. 9) increases the settling time since both of the processes decrease the energy dissipation rate. Values of  $1 < \mu < 2$  are required for stability about the  $Z_b$ -axis (Fig. 10). For cases in which  $\mu < 1$ , the system is unstable about the  $Z_b$ -axis, therefore  $A$  increases instead of decreasing. Note that  $\mu < 1$  does not violate the assumptions used in formulating the problem, but represents an inappropriate configuration. Figure 11 demonstrates that the settling time increases as the initial spin rate of the spacecraft decreases.

Figure 12 shows contours of constant settling time for various combinations of normalized damping and stiffness. Settling time was defined as the time required for  $A^2$  to decrease to 1% of its original value. The contours of Fig. 12 were developed using values of  $I = 400$ ,  $\mu = 1.5$ ,  $l = 1$ , and  $\omega_z = 3$ ; the parameter  $P^2$  was 56.25 (i.e.,  $A = 6$  initially). Each contour is labelled with the settling time in hours. These contours again demonstrate that for a given stiffness, there is an optimal value of  $c$ , beyond which the settling time increases. For large values of  $k$ , the optimal points on each contour lie approximately on a straight line. It can be shown that for cases where ( $A^2 \ll k$ ), the "optimal" value of  $c$  is proportional to  $(k - P^2)$ ; the constant of proportionality depends on the choice of the initial and final values of  $A^2$  used in the definition of settling time. Superimposed on the contours of Fig. 12 is the theoretically derived straight line. Excellent agreement is observed for cases involving large stiffness values. It should be noted that the "optimal" values of  $c$  are unrealistically large; therefore we may assume as a general rule of thumb that the damping should be made as large as possible.

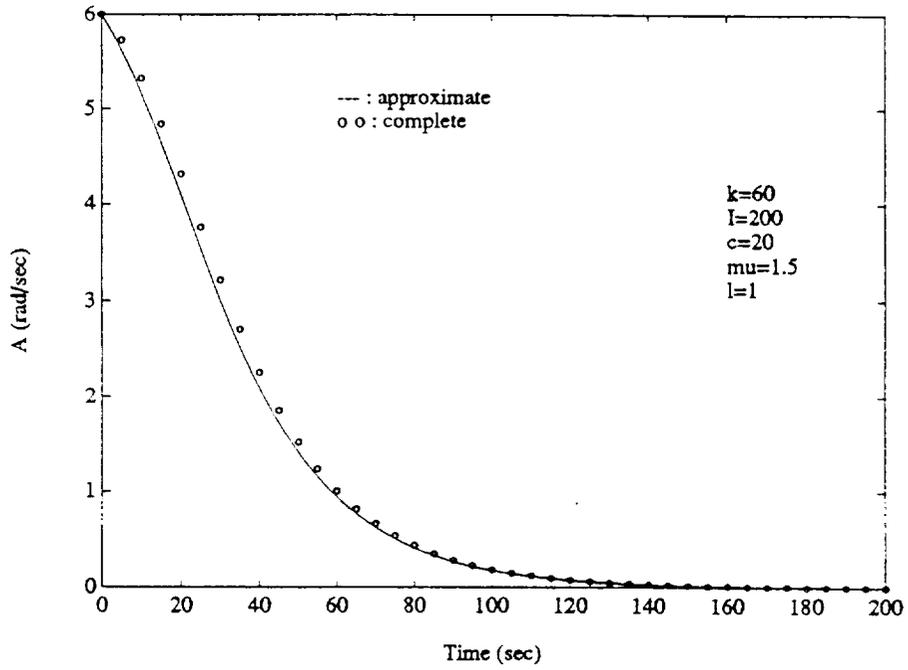


Fig. 5. Spacecraft's state of spin; Complete vs. Approximate (Case 1)

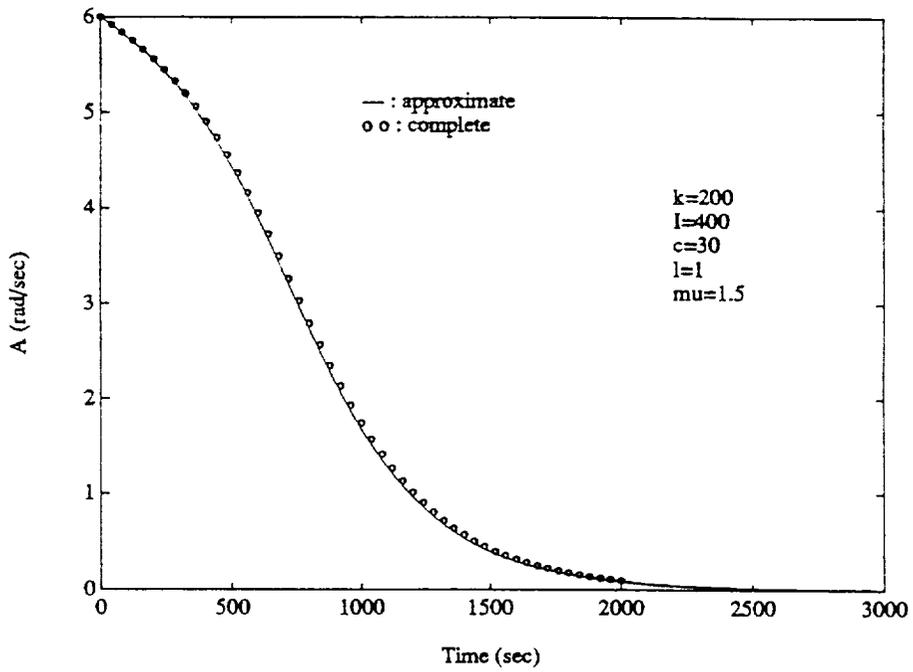


Fig. 6. Spacecraft's state of spin; Complete vs. Approximate (Case 2)

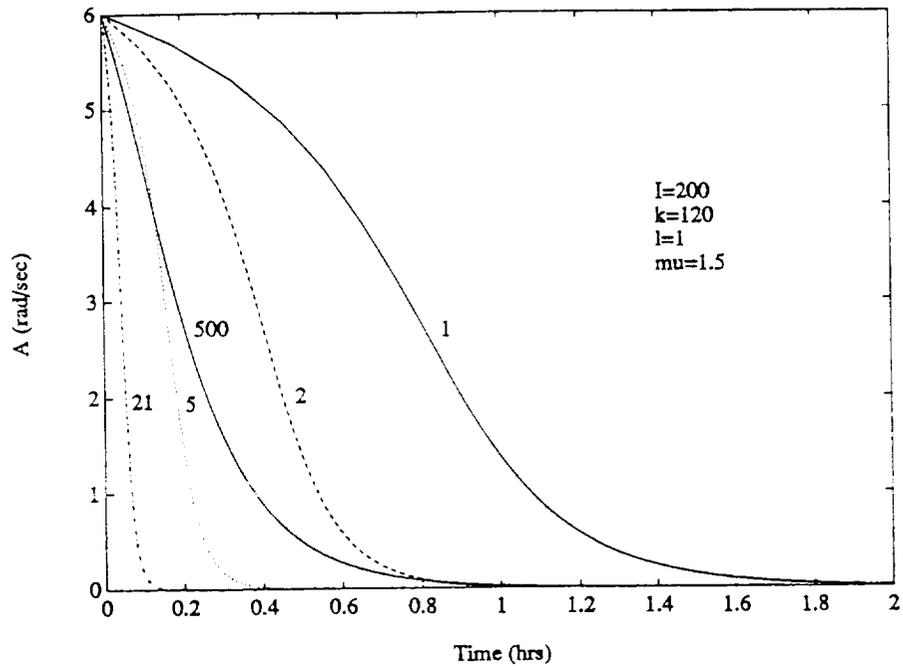


Fig. 7. Effects of damping on dissipation rate

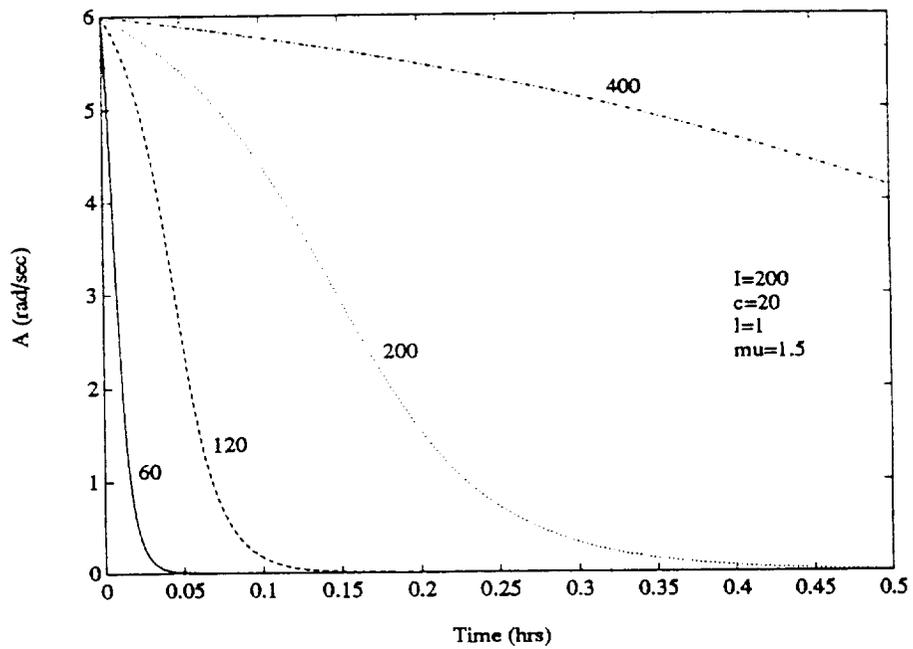


Fig. 8. Effects of stiffness on dissipation rate

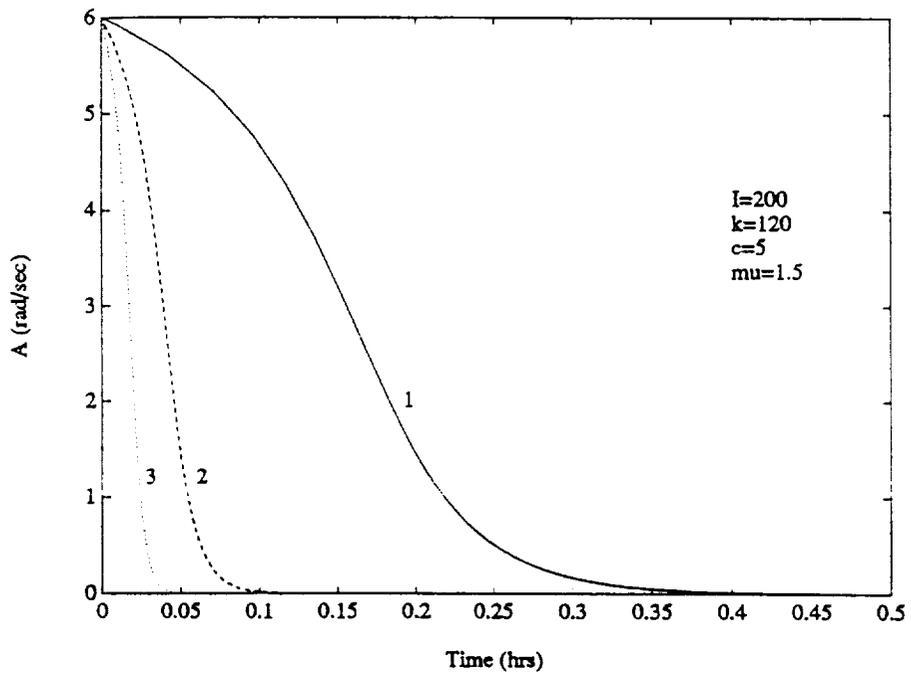


Fig. 9. Effects of length on dissipation rate

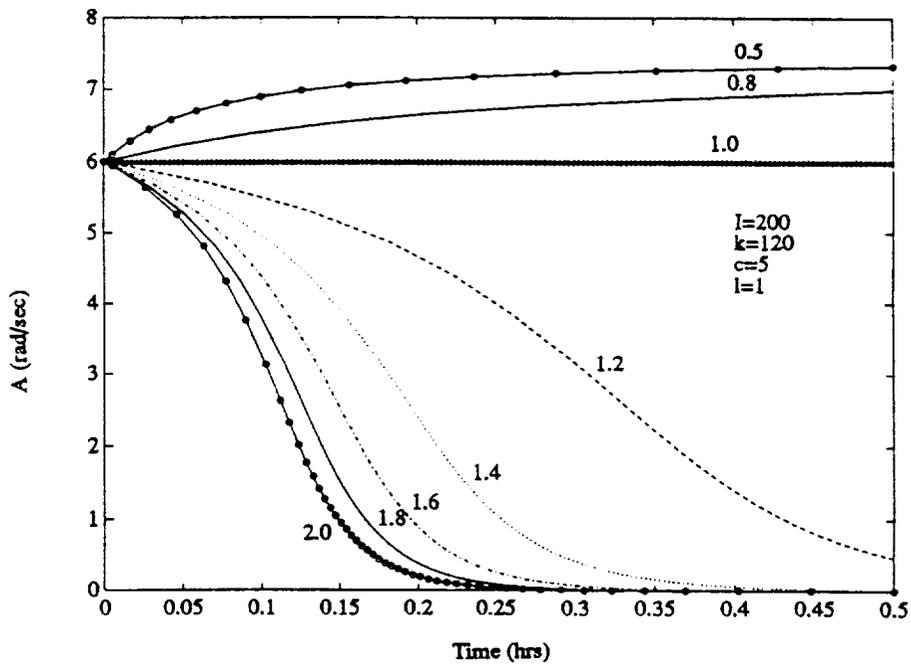


Fig. 10. Effects of spacecraft inertia properties on dissipation rate

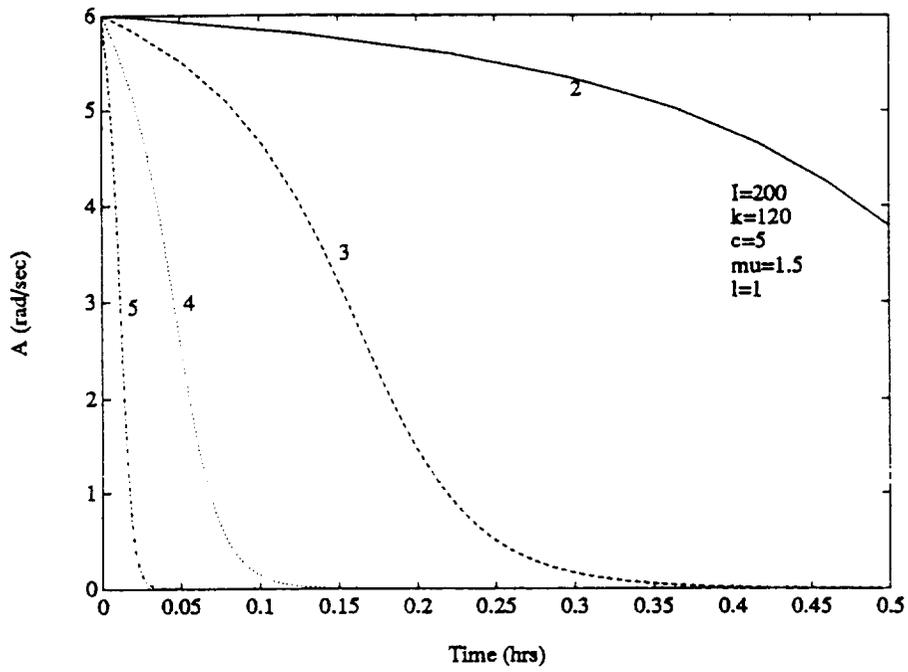


Fig. 11. Effects of angular momentum on dissipation rate

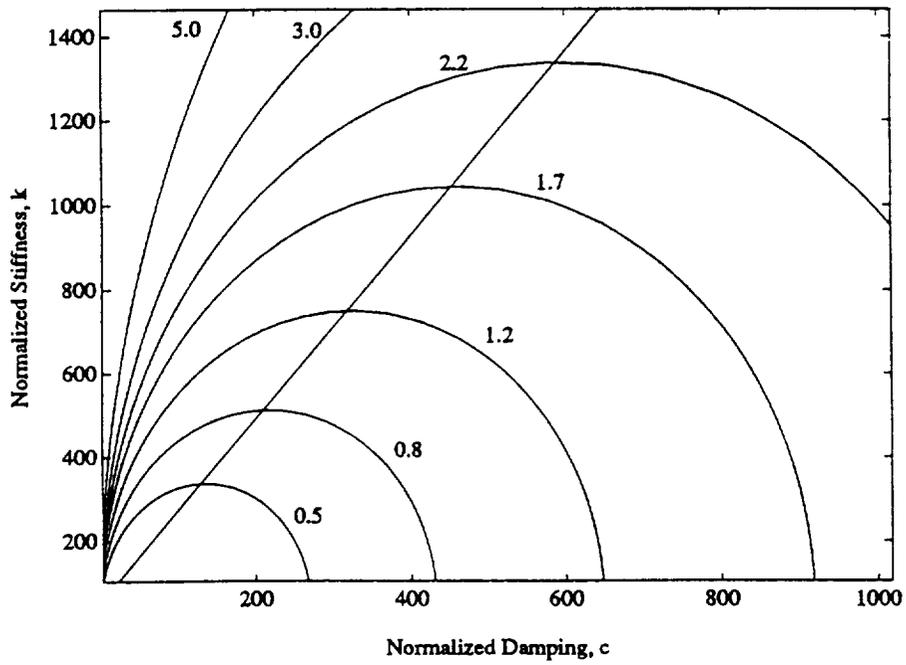


Fig. 12. Constant settling time contours

## Summary

The approximate solution developed closely parallels the energy-sink approach.<sup>11</sup> The device presented in this paper is an extension of the one-degree-of-freedom (dof) ball-in-tube precession dampers studied by previous authors;<sup>11,12</sup> this device represents a two dof damper.

The problem addressed in this paper is an important part of the bigger problem of devising safe and efficient spacecraft detumbling and retrieval strategies. Although the results presented in this paper are based on somewhat higher than normal initial rotational rates and normalized damping characteristics, the usefulness of the proposed device is well demonstrated. Currently, the “optimal” normalized damping coefficients are not realizable; however, with developments in the area of material sciences, these “optimal” damping coefficients may eventually be achievable. Future work of direct practical utility will include (1) a detailed study of desired settling time as a function of system parameters, (2) stability analyses associated with misalignment of the device, and (3) despin strategies.

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