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FRACTAL IMAGE COMPRESSION: A RESOLUTION INDEPENDENT REPRESENTATION FOR IMAGERY

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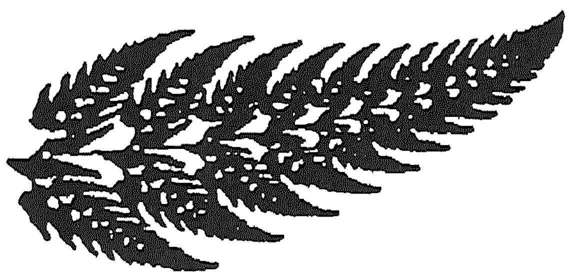
1. Background

A deterministic fractal is an image which has low information content and no inherent scale. Because of their low information content, deterministic fractals can be described with small data sets. They can be displayed at high resolution since they are not bound by an inherent scale. A remarkable consequence follows. Fractal images can be encoded at very high compression ratios. This fern, for example, is encoded in less than 50 bytes and yet can be displayed at resolutions with increasing levels of detail appearing.



The Fractal Transform was discovered in 1988 by Michael F. Barnsley. It is the basis for a new image compression scheme which was initially developed by myself and Michael Barnsley at Iterated Systems. The Fractal Transform effectively solves the problem of finding a fractal which approximates a digital "real world image."

2. The Fractal Hypothesis for Real World Images

Fractal Image Compression, when used in a lossy mode, provides approximations to a large class of images, termed Real World Images.

- Digital real world photographs contain exceptionally high frequency of occurrence of local large scale correlations, under affine transformations.

- Given a digital real world photograph, it can be approximately partitioned into contractive affine transformations of geometrical structures in the image. Affine transformations of these structures, interpreted as probabilistic events, are unlikely to occur in an arbitrary digital image. However, in a specific image, such occurrences are likely events. For digital real world images, the higher the resolution, the better the approximation.

Affine transformations are composed of translation, differential scaling and differential rotation operations. An affine transform, T, is contractive relative to a given metric, d, when

$$d(T(p),T(p')) < c*d(p,p'), \text{ for some constant } c<1 \text{ and all points } p \text{ and } p'.$$

Real world images are distinguished from general digital bit-mapped images by the property that they contain local large scale affine correlations unexpectedly often. Foe example, the probability of a randomly generated 1K x 1K x 24 bit/pixel image containing a face of photographic quality is nearly infinitesimal. Experience shows that whenever a real world image contains one face, it tends to contain additional faces. The additional faces are not exact copies of each other, but to a reasonable approximation, can be mapped into each other by affine transformations.

3. Methodology of the Fractal Transform

The Fractal Transform process consists of distinct encoding methodologies. The encoding step begins by creating two collections of images subsets. The first subset consists of Domain Blocks. The collection of Domain Blocks must satisfy two conditions: (a) Domain Blocks must not overlap; and (b) as a whole, the Domain Blocks must exactly cover the image. The second subset consists of Range Blocks. Range Blocks may or may not overlap and may or may not cover the image. Once the subsets of Domain and Range Blocks have been established, a Range Block and a contractive affine transformation is chosen for each Domain Block. The choice is made so that difference between the affine mapping of the Range Block and the Domain Block is minimal. The information which is retained as the code produced by the Fractal Transform consists of the description of the Domain and Range Block sets and, for each Domain Block, the affine transformation and location of the Range Block. No image data need be saved. For each domain block D, only the Range Block address, R_D, and an affine transformation, T_D, are saved.

Decoding an image from its Fractal Transform code consists of repeatedly transforming images starting from an arbitrary initial image, I_O, to create successive images I_1, I_2, ... I_N. I_j is transformed into I_{j+1} by partitioning I_{j+1} into the Domain Block structure obtained during the encoding step and computing the contents of the Range Block region R_D computed from I_j. These computations continue until the difference between I_j and I_{j+1} differ by less than a specified amount.

No inherent scale is introduced during the Fractal Transform process. Relational information and not resolution specific data is retained. Moreover, code produced is of finite length and so of low information content. We conclude that *the Fractal Transform produces a fractal description of images*.

Summary of the Fractal Transform Methodology

- Search for Affine Transformations
  - Partition Image and Create Collection of Image Fragments
  - Map Image into Itself
  - Contractive Mappings
  - Minimize Distance between Image Fragment and Partition Elements
- Produce Code
  - Save the Transformations
  - Discard All Image and Other External Data
  - Organize Transformations into Fractal Image Format
- Reconstruct Image Algorithmically
  - Iterative Process
  - Repeated Application of Image Processing Transformation

FRACTAL TRANSFORM - TECHNICAL

Step 1 - Partition the image to be encoded creating Domain Blocks.

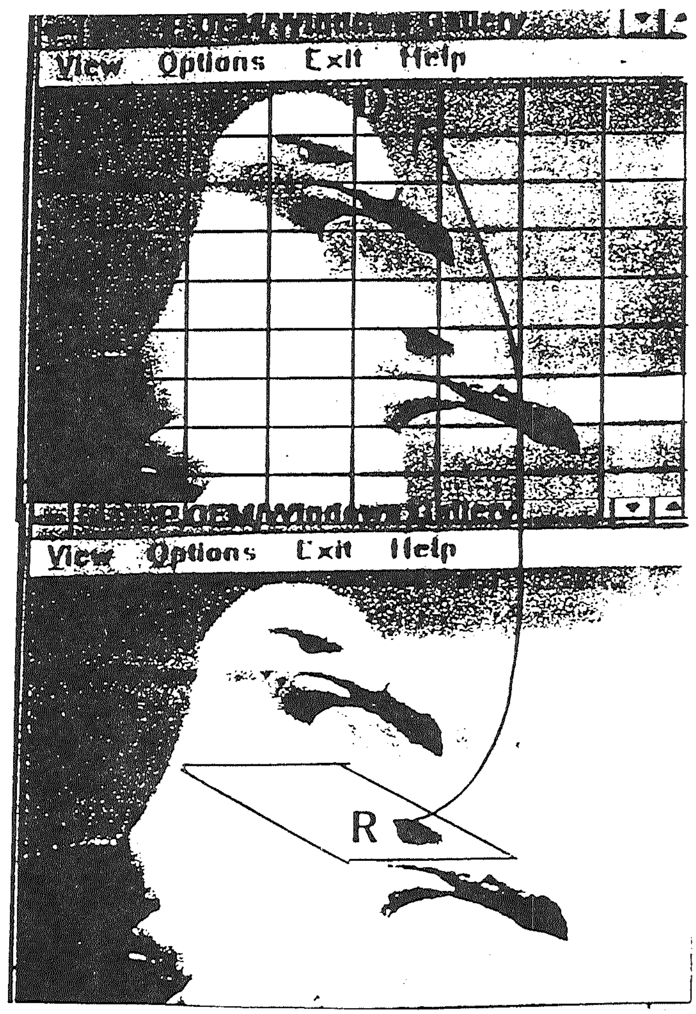
Step 2 - Choose another collection of image subsets, called Range Blocks.

Step 3 - For each Domain Block, D, choose Range Block R_D and contractive affine transformation $T_D : R_D \to D$ so that image distance between D and $T_D(R_D)$ is minimal.

FRACTAL TRANSFORM CODE CONSISTS OF

Domain Block Set,
Range Block sets, and
for each Domain
Block,

the affine
transformation
and location of
the Range Block.

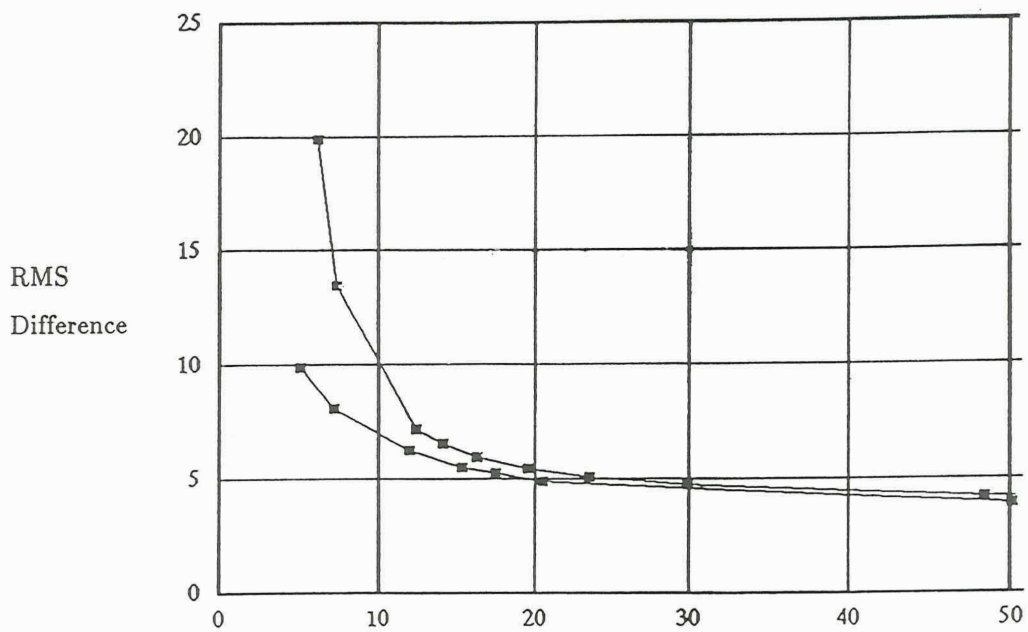


4. Comparison with the Discrete Cosine Transform

Study I

Twenty images input at a resolution of 640 x 400 x 24 bits/pixel were compression by the Fractal Transform and a JPEG implementation of DCT compression. The image set includes data from 9 ISO standard images. Image fidelity is measured by comparing the root-mean-square (RMS) difference between the original digital image and the compressed and then decompressed image. Compressed file sizes range from about 5K to 50K. Typically, for larger file sizes the image fidelity of the techniques are comparable. However, at lower file sizes, the Fractal Transform yields significantly better quality imagery than the DCT technique, as measured by the RMS.

The following data is from the "balloons" ISO image:



"Balloons" ISO Image.

Vertical Axis is RMS Difference between Compressed and Original Data.

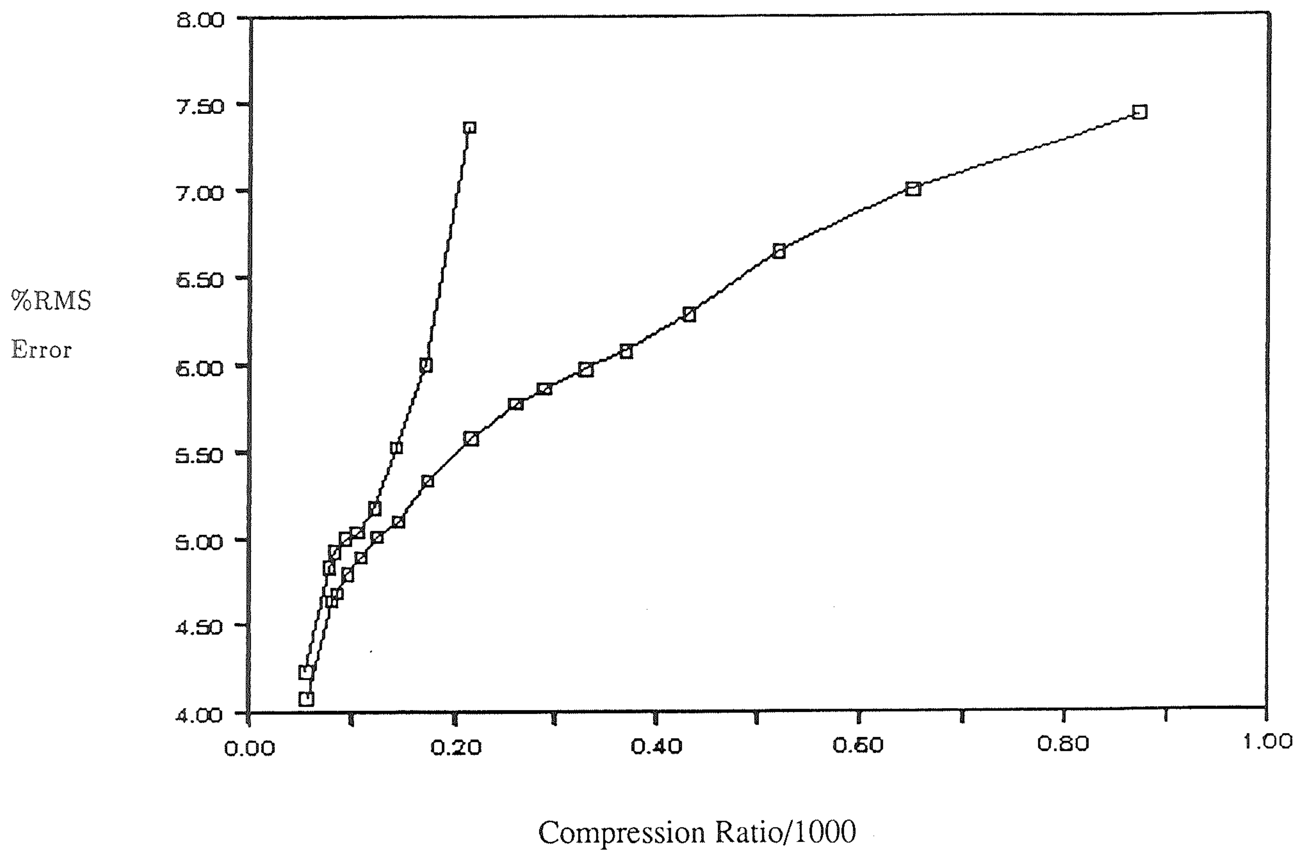
Lower Curve is Fractal Transform Data. Upper Curve is DCT Data.

Vertical Axis is RMS difference. Horizontal Axis is Compressed File Size in Kilobytes.

Study II

Two 5 band AVHRR images were obtained at resolution 1024 x 1024 x 16 bits/pixel for each band. The data pre-processed to retain 8 bits of data. This pre-processed data was compressed by Fractal Compression technology and by a JPEG implementation of DCT by Xing. The data below represents compression of one of the two five band images resulting in file sizes ranging from about 90,000 bytes to 6,000 bytes.

The vertical axis provides values for % root-mean-square error(%RMS). The RMS is computed for the five band image and is then divided by 255, since the RMS can vary between 0 and 255. Using the RMS as a measure of fidelity, higher %RMS values indicate less fidelity. The horizontal axis measures the (compression ratio)/1000, which varies inversely as the file size of the compressed image.



The upper and shorter curve describes the JPEG results. The lower and longer curve describes the results of Fractal Compression on the same digital imagery. The rapid increase in %RMS for the JPEG technique points towards the breakdown of the JPEG methodology. This is borne out by visual inspection in which the JPEG images reveal a blocky nature, the characteristic artifact which results as the limit of the JPEG technique is approached. The Fractal Compression technique permits much smaller file sizes to be reached.

5. Resolution Independence

A straight line in encoded in a computer by a formula, not by a bit map image of a representative line. Output and display questions are, for the most part, independent of the formula for the line. When an output device is attached to a computer, a software program which targets the output specific format (resolution, aspect ratio, pixel/depth, number of colors, etc.) of the output device, is used to generate a suitable display from the resolution independent formula which represents the line. Analogously, the Fractal Image Format provides a description of an image which is independent of the output device.

The Fractal Image Format is a Resolution Independent Representation for Images.

No inherent scale is introduced during the Fractal Transform process. Relational information and not resolution specific data is retained. Therefore Fractal Transform compression produces a resolution independent description of images.

An Evaluation of Resolution Independence

Forty 640 x 400 x 24 bits/pixel satellite images of the earth were compressed to a variety of file sizes between 15K and 5K by both DCT and Fractal Transform technologies. In the case of DCT the original image was compressed and decompressed and a RMS error was computed between the decompressed and original image. To evaluate the nature of the resolution independence of the Fractal Transform technique, each of the 640 x 400 images was first subsampled to create a 320 x 200 image. This smaller image was compressed and then decompressed at the larger 640 x 400 resolution. A RMS error was computed for the 640 x 400 original and fractal zoom image. In each case, the Fractal Zoom RMS was no worse than the DCT RMS. On average, the Fractal Zoom technique reduced RMS errors by 15% from those produced by DCT.