Configuration Optimization of Space Structures

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Third Annual Symposium
November 21 & 22, 1991

A NASA Space Engineering Research Center at the University of Colorado
Objective

✓ Develop a Computer Aid for the Conceptual/Initial Design of Aerospace Structures, Allowing Configuration and Shape to be 
a priori Design Variables.
Approach

✓ Kikuchi’s Homogenization Method:
A “Design Domain Block,” Filled Initially with
Homogenized Finite Elements, is Gradually
“Sculpted” into an Optimal Structure under
Control of an Optimization Driver.

✓ A Sequence of such Structures may be Obtained.
This can Help the Conceptual Designer.
Design Domain May Contain Predetermined Holes:
Homogenization Method Steps

✓ **Set up a Design Domain.**

✓ **Fill it with Homogenized Finite Elements.**

✓ **Define Loads and Support Conditions.**

✓ **Minimize an Objective Function (e.g. Compliance) under Maximum-Volume Constraint.**

✓ **Changing Maximum Volume Yields a Sequence of Designs.**

✓ **If Satisfied with a Design, Body-Fit-Remesh it, and Proceed with Standard Finite Element Analysis.**
Example: 3D Mechanical Component Design
Element-Level Design Variables: MicroHole Dimensions

In two dimensions: \(a\), \(b\), \(\theta\) in each element (3)
In three dimensions: \(a\), \(b\), \(c\), \(\theta_1\), \(\theta_2\), \(\theta_3\) in each element (6)

100 \times 100 \ 2D \ mesh: 30,000 Design Variables
30 \times 30 \times 30 \ 3D \ mesh: 162,000 Design Variables

Taking Advantage of Design-Variable Locality Essential
Forming a Homogenized Finite Element

\[ K^e = \int_A hB^T C_H B \, dA \]

\[ C_H = C_H(a, b, \theta) \quad \text{homogenized material response matrix} \]

\[ C = C_H(0, 0, 0) \quad \text{full element; no microhole} \]

\[ C = C_H(1, 1, \theta) = 0 \quad \text{void; microhole fills element} \]
2-D Optimization Problem

✓ **Objective Function (Compliance ≡ Inverse Stiffness)**

\[ \Pi(a, b, \theta) = p^T v \]

✓ **Stiffness Relation (Discrete FE Equation)**

\[ v(a, b, \theta) = K^{-1}(a, b, \theta) p, \quad K = \sum_e L^e \mathcal{K}^e(a^e, b^e, \theta^e) L^e \]

✓ **Volume Inequality Constraint**

\[ V(a, b) \leq V_T = \kappa V_{domain}, \quad 0 < \kappa \leq 1 \]

✓ **Microhole Constraints**

\[ 0 \leq a^e \leq 1, \quad 0 \leq b^e \leq 1, \quad -45^\circ \leq \theta^e \leq 45^\circ, \quad e = 1, \ldots N_e \]
Treatment of Volume Inequality Constraint

✓ Augmented Lagrangian Formulation

\[ L = \Pi - \lambda_V C_- + \sigma_V C_-^2 \]

where

\[ \lambda_V = \text{Lagrangian multiplier estimate} \]
\[ \sigma_V = \text{penalty weight} \]

\[ C_- = \begin{cases} 
V_T - V, & \text{if } V_T < V; \\
0, & \text{otherwise.}
\end{cases} \]
Algorithm for the Volume Inequality Constraint

i) Set $\lambda_V^{(1)} = \lambda_V^0$, $\sigma_V^{(1)} = \sigma_V^0$, $k = 1$

ii) Minimize $\Pi(a, b, \theta, \lambda_V^{(k)}, \sigma_V^{(k)})$ keeping $\lambda_V$ and $\sigma_V$ fixed, with $(a,b,\theta)$ subjected to limit constraints.

iii) Compute $C = C^{(k)} = V_T - V(a, b, \theta)$.
    If $C < 0$ and $|C| > \frac{1}{4}|C^{(k-1)}|$ set $\sigma_V = 10\sigma_V$ and go to ii)

iv) else set

$$k = k + 1$$

$$\lambda_V^{(k)} = \lambda_V^{(k-1)} - \sigma_V C$$

If $C < 0$ go to ii) else done
Object Function Derivatives: Taking Advantage of Design Locality

- Objective Function Gradients
  \[
  \frac{\partial p^Tv}{\partial \alpha_e} = -v^T \frac{\partial K}{\partial \alpha_e} v
  \]
  \[
  \frac{\partial p^Tv}{\partial \beta_e} = -v^T \frac{\partial K}{\partial \beta_e} v
  \]

- Stifness (Discrete Equilibrium) Constraints
  \[
  \frac{\partial v}{\partial \alpha_e} = -K^{-1} \frac{\partial K}{\partial \alpha_e} v
  \]
  \[
  \frac{\partial v}{\partial \beta_e} = -K^{-1} \frac{\partial K}{\partial \beta_e} v
  \]
Stiffness Variations

\[ K^e = \int_{V^e} B^T C(a^e, b^e, \theta^e) B \, dV^e \]

\[ \frac{\partial K^e}{\partial a^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial a^e} B \, dV^e \]

\[ \frac{\partial K^e}{\partial b^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial b^e} B \, dV^e \]

\[ \frac{\partial K}{\partial a^e} = L^e \frac{\partial K^e}{\partial a^e} L^e \quad \frac{\partial K}{\partial a^e} = L^e \frac{\partial K^e}{\partial a^e} L^e \]
Variations of the Potential

\[ \Pi = v^T K v \]

First Variation

\[ \delta \Pi = -v^T \delta K v \equiv - \int_A \epsilon^T \delta \epsilon dA = 0 \]

Second Variation

\[ \delta^2 \Pi \approx 2 \int_A \epsilon \delta C^{-1} \delta \epsilon dA - \int_A \epsilon^T \delta^2 \epsilon dA \]
Schematics of the Optimization Program.
Progress

✓ Simple $C_h$ developed and implemented.

✓ Homogenized F.E. Model of Design implemented.

✓ Optimization method:
  ✓ Simulating Annealing: did not work.

✓ Augmented Lagrangian with Conjugate Gradient:
  works for simple problems (next slide)

✓ Augmented Lagrangian with Newton/Projected Gradient:
  implemented; under testing.
Validation Problem (First Successful Solution)

\[ V_{\text{ref}} = 50 \]
\[ E = 10,000 \]
\[ v = 0 \]
\[ R = 1 \]

Solution for 50\% volume reduction
Target volume \( V = \frac{1}{2} V_{\text{ref}} = 25 \)
Computed solution agrees with analytical solution from Lagrangian function
Minimization Method: AL + CG + CPT
189 object function evaluations

Design Domain

uniform load \( q = 8 \)

Removed Material
Computational Issues

- Coping with Large Number of Design Variables ($10^2$–$10^6$): Adaptive Hierarchical Optimization, Domain Decomposition, “Hole Dropping”

- Handling Design-Following Loads.

- Handling Different Materials over Design Domain.

- Parallel Computations.
> Research Issues

✓ Different Optimality Criteria:

Concurrent Object Functions Over Domain
(e.g. Multiple Load Cases)

Different Object Functions over Subdomains
(e.g. Maximum Energy Absorption on one,
Minimum Compliance on Another)

✓ Tension/compression Design — Cables, Brittle Materials.

✓ Anisotropic Design — Composites.

✓ Vibration/Stability Constraints.
Telerobotic Rovers for Extraterrestrial Construction

Jim Avery

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