Configuration Optimization of Space Structures

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Third Annual Symposium
November 21 & 22, 1991

A NASA Space Engineering Research Center at the University of Colorado
Objective

✓ Develop a Computer Aid for the Conceptual/Initial Design of Aerospace Structures, Allowing Configuration and Shape to be a priori Design Variables.
Approach

✓ **Kikuchi’s Homogenization Method:**
A “Design Domain Block,” filled initially with homogenized finite elements, is gradually “sculpted” into an optimal structure under control of an optimization driver.

✓ A sequence of such structures may be obtained. This can help the conceptual designer.
Example: A Classical Shape Design Problem
Design Domain May Contain Predetermined Holes:
Homogenization Method Steps

✓ Set up a Design Domain.

✓ Fill it with Homogenized Finite Elements.

✓ Define Loads and Support Conditions.

✓ Minimize an Objective Function (e.g. Compliance) under Maximum-Volume Constraint.

✓ Changing Maximum Volume Yields a Sequence of Designs.

✓ If Satisfied with a Design, Body-Fit-Remesh it, and Proceed with Standard Finite Element Analysis.
Example: 3D Mechanical Component Design
Element-Level Design Variables: MicroHole Dimensions

In two dimensions: $a$, $b$, $\theta$ in each element (3)

In three dimensions: $a$, $b$, $c$, $\theta_1$, $\theta_2$, $\theta_3$ in each element (6)

100 $\times$ 100 2D mesh: 30,000 Design Variables
30 $\times$ 30 $\times$ 30 3D mesh: 162,000 Design Variables

Taking Advantage of Design-Variable Locality Essential
Forming a Homogenized Finite Element

\[ K^e = \int_A hB^T C_H B dA \]

\[ C_H = C_H(a, b, \theta) \quad \text{homogenized material response matrix} \]

\[ C = C_H(0, 0, 0) \quad \text{full element; no microhole} \]

\[ C = C_H(1, 1, \theta) = 0 \quad \text{void; microhole fills element} \]
\[
\triangleright \text{ 2-D Optimization Problem}
\]

\begin{itemize}
\item \textbf{Objective Function (Compliance \equiv Inverse Stiffness)}
\[\Pi(a, b, \theta) = p^T v\]
\item \textbf{Stiffness Relation (Discrete FE Equation)}
\[v(a, b, \theta) = K^{-1}(a, b, \theta) p, \quad K = \sum_{e} L^{eT}K^e(a^e, b^e, \theta^e)L^e\]
\item \textbf{Volume Inequality Constraint}
\[V(a, b) \leq V_T = \kappa V_{domain}, \quad 0 < \kappa \leq 1\]
\item \textbf{Microhole Constraints}
\[0 \leq a^e \leq 1, \quad 0 \leq b^e \leq 1, \quad -45^\circ \leq \theta^e \leq 45^\circ, \quad e = 1, \ldots N_e\]
\end{itemize}
Treatment of Volume Inequality Constraint

✓ Augmented Lagrangian Formulation

\[ L = \Pi - \lambda_V C_\nu + \sigma_V C^2_\nu \]

where

\[ \lambda_V = \text{Lagrangian multiplier estimate} \]
\[ \sigma_V = \text{penalty weight} \]
\[ C_\nu = \begin{cases} V_T - V, & \text{if } V_T < V; \\ 0, & \text{otherwise}. \end{cases} \]
Algorithm for the Volume Inequality Constraint

i) Set $\lambda^{(1)}_V = \lambda^0_V$, $\sigma^{(1)}_V = \sigma^0_V$, $k = 1$

ii) Minimize $\Pi(a, b, \theta, \lambda^{(k)}_V, \sigma^{(k)}_V)$ keeping $\lambda_V$ and $\sigma_V$ fixed, with $(a, b, \theta)$ subjected to limit constraints.

iii) Compute $C = C^{(k)} = V_T - V(a, b, \theta)$.
    If $C < 0$ and $|C| > \frac{1}{4}|C^{(k-1)}|$ set $\sigma_V = 10\sigma_V$ and go to ii)

iv) else set

\[
k = k + 1
\]

\[
\lambda^{(k)}_V = \lambda^{(k-1)}_V - \sigma_V C
\]

If $C < 0$ go to ii) else done
Object Function Derivatives: Taking Advantage of Design Locality

Objective Function Gradients

\[
\frac{\partial p^T v}{\partial a^e} = -v^T \frac{\partial K}{\partial a^e} v
\]

\[
\frac{\partial p^T v}{\partial b^e} = -v^T \frac{\partial K}{\partial b^e} v
\]

Stifness (Discrete Equilibrium) Constraints

\[
\frac{\partial v}{\partial a^e} = -K^{-1} \frac{\partial K}{\partial a^e} v
\]

\[
\frac{\partial v}{\partial b^e} = -K^{-1} \frac{\partial K}{\partial b^e} v
\]
Stiffness Variations

For Element Stiffness

\[ K^e = \int_{V^e} B^T C(a^e, b^e, \theta^e) B \, dV^e \]

\[ \frac{\partial K^e}{\partial a^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial a^e} B \, dV^e \]

\[ \frac{\partial K^e}{\partial b^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial b^e} B \, dV^e \]

For Global Stiffness

\[ \frac{\partial K}{\partial a^e} = L^e \frac{\partial K^e}{\partial a^e} L^e \quad \frac{\partial K}{\partial a^e} = L^e \frac{\partial K^e}{\partial a^e} L^e \]
Variations of the Potential

\[ \Pi = v^T K v \]

\[ \delta \Pi = -v^T \delta K v \equiv - \int_A \epsilon^T \delta C \epsilon \, dA = 0 \]

\[ \delta^2 \Pi \simeq 2 \int_A \epsilon \delta C C^{-1} \delta C \epsilon \, dA - \int_A \epsilon^T \delta^2 C \epsilon \, dA \]
Schematics of the Optimization Program.
Progress

✓ Simple $C_H$ developed and implemented.

✓ Homogenized F.E. Model of Design implemented.

✓ Optimization method:

  ✓ Simulating Annealing: did not work.

  ✓ Augmented Lagrangian with Conjugate Gradient:

    works for simple problems (next slide)

  ✓ Augmented Lagrangian with Newton/Projected Gradient:

    implemented; under testing.
Validation Problem (First Successful Solution)

Design Domain

uniform load
q = 8

Removed Material

\[ V_{\text{ref}} = 50 \]
\[ E = 10,000 \]
\[ v = 0 \]
\[ R = 1 \]

2x2 mesh over D.D.

Solution for 50% volume reduction

Target volume \( V = \frac{1}{2} V_{\text{ref}} = 25 \)

Computed solution agrees with analytical solution from Lagrangian function

Minimization Method: AL + CG + CPT

189 object function evaluations
Computational Issues

- Coping with Large Number of Design Variables (10^2–10^6):
  Adaptive Hierarchical Optimization, Domain Decomposition, "Hole Dropping"

- Handling Design-Following Loads.

- Handling Different Materials over Design Domain.

- Parallel Computations.
RESEARCH ISSUES

✓ DIFFERENT OPTIMALITY CRITERIA:

CONCURRENT OBJECT FUNCTIONS OVER DOMAIN
(e.g. MULTIPLE LOAD CASES)

DIFFERENT OBJECT FUNCTIONS OVER SUBDOMAINS
(e.g. MAXIMUM ENERGY ABSORPTION ON ONE,
MINIMUM COMPLIANCE ON ANOTHER)

✓ TENSION/compression Design — Cables, Brittle Materials.

✓ ANISOTROPIC Design — Composites.

✓ VIBRATION/Stability CONSTRAINTS.
Telerobotic Rovers for Extraterrestrial Construction

Jim Avery

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