Configuration Optimization of Space Structures

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Objective

✓ Develop a Computer Aid for the Conceptual/Initial Design of Aerospace Structures, allowing Configuration and Shape to be a priori Design Variables.
Approach

✓ Kikuchi's Homogenization Method:
A "Design Domain Block," Filled Initially with Homogenized Finite Elements, is Gradually "Sculpted" into an Optimal Structure under Control of an Optimization Driver.

✓ A Sequence of such Structures may be Obtained. This can Help the Conceptual Designer.
Design Domain May Contain Predetermined Holes:
Homogenization Method Steps

✓ Set up a Design Domain.
✓ Fill it with Homogenized Finite Elements.
✓ Define Loads and Support Conditions.
✓ Minimize an Objective Function (e.g. Compliance) under Maximum-Volume Constraint.
✓ Changing Maximum Volume Yields a Sequence of Designs.
✓ If Satisfied with a Design, Body-Fit-Remesh it, and Proceed with Standard Finite Element Analysis.
Example: 3D Mechanical Component Design
> Element-Level Design Variables: MicroHole Dimensions

In two dimensions: \(a, b, \theta\) in each element (3)

In three dimensions: \(a, b, c, \theta_1, \theta_2, \theta_3\) in each element (6)

100 \(\times\) 100 2D mesh: 30,000 Design Variables

30 \(\times\) 30 \(\times\) 30 3D mesh: 162,000 Design Variables

> Taking Advantage of Design-Variable Locality Essential
Forming a Homogenized Finite Element

\[ K^e = \int_A hB^T C_H B \, dA \]

\[ C_H = C_H(a, b, \theta) \quad \text{homogenized material response matrix} \]

\[ C = C_H(0, 0, 0) \quad \text{full element; no microhole} \]

\[ C = C_H(1, 1, \theta) = 0 \quad \text{void; microhole fills element} \]
2-D Optimization Problem

✓ **Objective Function (Compliance ≡ Inverse Stiffness)**

\[ \Pi(a, b, \theta) = p^Tv \]

✓ **Stiffness Relation (Discrete FE Equation)**

\[ v(a, b, \theta) = K^{-1}(a, b, \theta)p, \quad K = \sum_{e} L^eK^e(a^e, b^e, \theta^e)L^e \]

✓ **Volume Inequality Constraint**

\[ V(a, b) \leq V_T = \kappa V_{domain}, \quad 0 < \kappa \leq 1 \]

✓ **Microhole Constraints**

\[ 0 \leq a^e \leq 1, \quad 0 \leq b^e \leq 1, \quad -45^\circ \leq \theta^e \leq 45^\circ, \quad e = 1, \ldots N_e \]
Treatment of Volume Inequality Constraint

√ Augmented Lagrangian Formulation

\[ L = \Pi - \lambda V C_\neg + \sigma V C^2_\neg \]

where

\[ \lambda V = \text{Lagrangian multiplier estimate} \]

\[ \sigma V = \text{penalty weight} \]

\[ C_\neg = \begin{cases} V_T - V, & \text{if } V_T < V; \\ 0, & \text{otherwise}. \end{cases} \]
Algorithm for the Volume Inequality Constraint

i) Set $\lambda^{(1)}_V = \lambda^0_V$, $\sigma^{(1)}_V = \sigma^0_V$, $k = 1$

ii) Minimize $\Pi(a, b, \theta, \lambda^{(k)}_V, \sigma^{(k)}_V)$ keeping $\lambda_V$ and $\sigma_V$ fixed, with $(a, b, \theta)$ subjected to limit constraints.

iii) Compute $C = C^{(k)} = V_T - V(a, b, \theta)$.
    If $C < 0$ and $|C| > \frac{1}{4}|C^{(k-1)}|$ set $\sigma_V = 10\sigma_V$ and go to ii)

iv) else set
    
    $k = k + 1$
    $\lambda^{(k)}_V = \lambda^{(k-1)}_V - \sigma_V C$

    If $C < 0$ go to ii) else done
Object Function Derivatives: Taking Advantage of Design Locality

\[ \frac{\partial p^T v}{\partial a^e} = -v^T \frac{\partial K}{\partial a^e} v \]

\[ \frac{\partial p^T v}{\partial b^e} = -v^T \frac{\partial K}{\partial b^e} v \]

Stifness (Discrete Equilibrium) Constraints

\[ \frac{\partial v}{\partial a^e} = -K^{-1} \frac{\partial K}{\partial a^e} v \]

\[ \frac{\partial v}{\partial b^e} = -K^{-1} \frac{\partial K}{\partial b^e} v \]
\[ K^e = \int_{V^e} B^T C(a^e, b^e, \theta^e) B \, dV^e \]
\[ \frac{\partial K^e}{\partial a^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial a^e} B \, dV^e \]
\[ \frac{\partial K^e}{\partial b^e} = \int_{V^e} B^T \frac{\partial C(a^e, b^e, \theta^e)}{\partial b^e} B \, dV^e \]

\[ \frac{\partial K}{\partial a^e} = L^e^T \frac{\partial K^e}{\partial a^e} L^e \quad \frac{\partial K}{\partial a^e} = L^e^T \frac{\partial K^e}{\partial a^e} L^e \]
Variations of the Potential

\[ \Pi = v^T K v \]

\[ \delta \Pi = -v^T \delta K v \equiv - \int_A \epsilon^T \delta C \epsilon dA = 0 \]

\[ \delta^2 \Pi \simeq 2 \int_A \epsilon \delta C C^{-1} \delta C \epsilon dA - \int_A \epsilon^T \delta^2 C \epsilon dA \]
Progress

✓ Simple $C_H$ developed and implemented.

✓ Homogenized F.E. Model of Design implemented.

✓ Optimization method:
  
  ✓ Simulating Annealing: did not work.

✓ Augmented Lagrangian with Conjugate Gradient:
  
  works for simple problems (next slide)

✓ Augmented Lagrangian with Newton/Projected Gradient:
  
  implemented; under testing.
Validation Problem (First Successful Solution)

Design Domain

uniform load
q = 8

\[ V_{ref} = 50 \]
\[ E = 10,000 \]
\[ v = 0 \]
\[ R = 1 \]

2x2 mesh over D.D.

Solution for 50% volume reduction
Target volume \( V = \frac{1}{2} V_{ref} = 25 \)
Computed solution agrees with analytical solution from Lagrangian function
Minimization Method: AL + CG + CPT
189 object function evaluations
Computational Issues

✓ Coping with Large Number of Design Variables ($10^2$–$10^6$):
  Adaptive Hierarchical Optimization, Domain Decomposition, "Hole Dropping"

✓ Handling Design-Following Loads.

✓ Handling Different Materials over Design Domain.

✓ Parallel Computations.
Research Issues

✓ Different Optimality Criteria:

Concurrent Object Functions over Domain
(e.g. Multiple Load Cases)

Different Object Functions over Subdomains
(e.g. Maximum Energy Absorption on one,
Minimum Compliance on Another)

✓ Tension/compression Design — Cables, Brittle Materials.

✓ Anisotropic Design — Composites.

✓ Vibration/Stability Constraints.
Telerobotic Rovers for Extraterrestrial Construction

Jim Avery

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