A Proposed Study of Multiple Scattering Through Clouds up to 1 THz

G.C. Gerace
E.K. Smith

Campus Box 425
Department of Electrical and Computer Engineering (ECEN)
University of Colorado
Boulder CO 80309-0425

Abstract—A rigorous computation of the electromagnetic field scattered from an atmospheric liquid water cloud is proposed. The recent development of a fast recursive algorithm (Chew algorithm) for computing the fields scattered from numerous scatterers now makes a rigorous computation feasible. We present a method for adapting this algorithm to a general case where there are an extremely large number of scatterers. We also propose extending a new binary PAM channel coding technique (El-Khamy coding) to multiple levels with non-square pulse shapes. The Chew algorithm can be used to compute the transfer function of a cloud channel. Then the transfer function can be used to design an optimum El-Khamy code. In principle, these concepts, can be applied directly to the realistic case of a time-varying cloud (adaptive channel coding and adaptive equalization). A brief review is included of some preliminary work on cloud dispersive effects on digital communication signals and on cloud liquid water spectra and correlations.

I. Introduction

The high variability of clouds makes it difficult to predict their contributions to a specific channel transfer function even if time varying functions are allowed. Some form of an adaptive design is the most probable approach for future systems that must account for effects due to clouds. Such adaptive systems may be designed to either tone down (e.g. communications) or enhance (e.g. cloud microphysical remote sensing) the consequences of a cloudy medium. One can imagine an adaptive system using current technology that works in the following way:

1) A microwave radiometer detects the non-precipitating liquid water along a earth-space propagation path [Westwater, 1978].

2) The liquid water measurement is used to infer cloud droplet size and spatial distributions and cloud extent.

3) The droplet distributions are used to compute the absorption and scattering (including multiple scattering) of an electromagnetic wave propagating through the suspended droplets.
4) The appropriate channel code or channel filter can be computed for the specific application.

Depending on the desired accuracy, these computations could be performed in either "real-time" for less refined estimates, or processed off-line for more precise results. In any event, these are the ideas we have in mind as we continue our research on millimeter wave propagation through clouds. What follows is a brief summary of some preliminary work regarding cloud dispersive effects on digital communication signals; a quick look at a way of spectrally analyzing radiometric measurements of liquid water content; and then the main part of this paper which presents an overview of our current work on rigorous electromagnetic computations of multiple scattering from cloud droplets and matched channel coding schemes.

II. Cloud Dispersive Effects on Digital Communication Signals

Using the Liebe formulation [Liebe, 1989] of the single scatter Rayleigh approximation for computing the attenuation and phase shift of a coherent signal propagation through a liquid water cloud, we analyzed pulse distortion and pulse group delay of digital signals propagating through clouds of various sizes and densities [Gerace and Smith, 1992]. Since Mie scattering becomes a factor above 300 GHz, the results in the Mie region must be refined using the approach described later in this paper. Using numerical simulations we obtained the following results:

1) The Bedrosian-Rice effect [Bedrosian and Rice] is clearly evident in FM and PM signals. This effect is essentially a convolution (linear filtering) of the signal phase modulation with the cloud impulse response.

2) Mie scattering distorted results above 300 GHz and for extremely short pulses (i.e. large bandwidths).

3) Measurements are desired for verification and comparison with numerical results.

4) Pulse spreading is not a significant factor for pulse widths greater than 100 picoseconds.

5) Excess pulse delay is essentially independent of modulation scheme but slightly dependent on carrier frequency. The delay ranged from 1 to 5 picoseconds over a frequency range of 10 to 1000 GHz.

We anticipate that multiple scattering computations will increase the cloud dispersive effects. The hypothesis is that multiple scattering decreases the coherence of the signal and decreases the linearity of the cloud transfer function. Both of these effects will increase signal distortion which changes the effective pulse width and absolute pulse position in time.
III. Cloud Liquid Water Spectra and Correlations

In any cloud radiative transfer calculation, we require some knowledge or assumptions regarding the liquid water content of the cloud. The more details we know about how the water is distributed within the cloud, the better we can characterize the transfer of electromagnetic radiation through the cloud. The Wave Propagation Laboratory of the National Oceanic and Atmospheric Administration (NOAA) has been making microwave radiometric measurements of liquid water content for a number of years. Given this immense amount of data, the challenge is to find insightful methods of analysis.

We are using specific goals and a working hypothesis to guide our approach. Our main goal is to determine if spectral analysis of cloud liquid reveals any underlying physical phenomena. The original hypothesis was that stable clouds would exhibit relatively long correlation times compared to unstable clouds of the same duration. Using data collected over Denver CO in July 1988, we designed algorithms to isolate cloud events, interpolate liquid water values to eliminate sampling jitter, and estimate the mean, variance, power spectral density, autocorrelation function, and correlation time (the time it takes for the correlation to decay to $e^{-1}$ of its maximum value) for each cloud event. The results were subjectively compared with satellite images taken twice a day during the same month.

The cloud detection subroutine segregated 45 cloud events in the July 1988 dataset. Figure 1 through Figure 4 summarize the results. The plots were drawn with continuous lines to make "peaks" and "valleys" easier to see. Figure 1 shows the mean and variance for all 45 cloud events. Note that higher variances are associated with higher means. In other words, the more cloud there is, the higher the variability. Figure 2 shows the time duration and correlation time for all 45 cloud events. Here we see that clouds of longer duration have longer absolute
Figure 1

Figure 2
Figure 3

Figure 4
compared to unstable clouds of the same duration. Unfortunately, stratus type clouds are not common in Colorado summer skies. Essentially no stratus type clouds were observed in satellite data during July 1988. However, light scattered cloud conditions are associated with relatively stable conditions and heavy thunderstorm clouds are usually indicative of unstable conditions. Thus although we can not conclude anything about stratus cloud conditions, we can conclude that the lightly scattered cloud conditions exhibited relatively long normalized correlation times compared to heavy thunderstorm type clouds. So the results clearly lend support to our hypothesis. This encourages us to continue our analysis with other datasets and in particular for winter months when stratus type clouds are more prevalent.

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Now we present some background information and an outline of our proposal for a rigorous computation of how clouds effect electromagnetic wave propagation for frequencies up to 1 THz and some channel filtering and coding techniques that could exploit the results of such a precise computation.

IV. Radiative Transfer Through Clouds

The study of electromagnetic energy propagation through clouds has been studied by researchers in numerous disciplines. It is probably fair to say that there is no single treatise that addresses this topic across a large range of frequencies for a large variety of clouds. The most general solution for propagation of electromagnetic energy of arbitrary bandwidth and polarization through a generalized cloud containing varying quantities of water vapor, liquid, and ice is not currently possible. In attempting such a solution, we immediately run into
physicists, March 1992]. In fact, cloud classification methods are rarely based on anything more than subjectively observed structure and altitude.

However, much is understood about individual processes that occur within clouds. For electromagnetic propagation problems, a statistical description of cloud ice and droplet sizes (spectra) and spacings must suffice. The specifics of how the distributions arise are not of special concern in this study; but how distributions vary for different cloud types and how well they agree with measurements is of interest in order to qualify the application of our results. We are also interested in the time evolution of the distributions to propose extensions of a static model to the full duration of a cloud event.

Once the cloud is characterized by droplet size and spacing distributions (possibly time dependent), the next most natural division of the general problem is to subdivide the electromagnetic spectrum. No specific divisions are generally accepted but the following breakpoints seem natural. The Rayleigh approximation to Mie scattering from spherical droplets is valid for most clouds for frequencies up to 300 GHz [Liebe, 1989]. Also, this is the approximate breakpoint for non-resonant and resonant absorption by water molecules [Zufferey, 1972]. Another breakpoint occurs when the phase shift through the droplet is no longer negligible (Born Approximation). For typical cloud drop sizes, this occurs roughly around 1 THz. This is also approximately the beginning of the infrared region. At this point, Rayleigh-Gans [Van de Hulst, 1981] or Rayleigh-Debye [Kerker, 1969] scattering occurs. Above 100 THz and into the visible region of the electromagnetic spectrum, the size of the cloud droplets are considerably larger than the wavelength causing numerous rays to be simultaneously refracted and reflected by different portions of a single droplet, including possible multiple internal reflections within the droplet. This can significantly distort phase relationships between the incident and scattered fields.

Before using these concepts to guide and bound the formulation of this research proposal, here is a quick review of some selected topics related to electromagnetic propagation through clouds. [Slobin, 1982] was the first major attempt to use meteorological data in conjunction with cloud attenuation equations to formulate practical models for engineering design calculations. [Gerace, et al., 1990] compared numerous microwave band cloud attenuation models and found that most computations agreed for frequencies below 40 GHz for light to medium cloud conditions but diverged for heavier cloud conditions. Most of these models require knowledge of the liquid water content of a cloud which is seldom known but can be estimated with the Slobin models.

Precise values of the refractive index of water is important in cloud scattering and absorption calculations. [Zufferey, 1972] was an impressive study of water refractive index for frequencies up to 600 GHz and [Liebe et al., 1991] used a double relaxation model to extend calculations up to 1 THz.

It is generally believed that multiple scattering plays a role in propagation through clouds and rain but the extent of that role is somewhat controversial. [Ishimaru, 1978] alludes to a number of multiple scattering formalisms. Ishimaru's treatise is the only work we are aware of that thoroughly delineates transport theory (so called "radiative transfer" theory) [Chandraseker,
1960] from analytic theory [Twersky, 1962]. The transport theory does not account for field coherence and deals strictly with intensities (power). Two standard approximations are due to Born and Rytov. The former expands the field in a series and the latter expands the exponent of the field. The first two terms of the Rytov solution are identical to the Born solution and thus the Rytov method is considered superior. On the other hand, analytic theory seeks rigorous solutions to the wave equation and accounts for the coherence of field quantities. The Twersky approach is particularly rigorous and includes all multiple scattering paths except those that go through the same scatterer more than once. Controversy regarding the conditions and applicability of the above theories can be found in [Rogers et.al., 1983], [Brown, 1980], [Brown, 1981], [Ishimaru, 1982], [Crane, 1971], [Tsolakis and Stutzman, 1982], [Oguchi and Ito, 1990], [Ya-Qiu Jin, 1989] to name just a few.

Most of the latest work in this area involves some kind of numerical methods for computing transmission through or reflection from clouds. [Evans and Stephens, 1991] decompose the cloud into layers, solve the radiative transfer equation for a layer, and use transmission and reflection matrices to compute interactions between the layers. A formidable study of the effects of time dependent cloud microphysical structure on radiative transfer [Mugnai and Smith, 1988] showed that absorption and emission processes dominate the initial stages of cloud development but scattering plays a significant role in the latter stages. Another study implicates radiation as a factor in diffusional droplet growth [Sievers and Zdunkowski, 1990], [Dave, 1970] and more recently [Wiscombe, 1980] have developed efficient algorithms for performing complete Mie scattering computations. Looking for practical shortcuts, [Lui et.al., 1991] express Mie scattering and absorption coefficients as polynomials in temperature and frequency for rapid retrieval. Parameterizations based on empirical data were derived by [Derr et.al., 1990]. Yet another study based on empirical data [Ajvazyan, 1991] characterized anomalous radar reflected emissions. Other methods recently put forth include a diffraction-scattering subtraction method [Kamiuto, 1989], a Fourier decomposition of the radiative transfer equation [Garcia and Siewert, 1989], and some admirable attempts to bound the radiative transfer solutions were performed by [Ajvazyan, 1991] and [O’Brien]. We close this review by noting that Monte Carlo methods are often employed to verify the models listed above.

**Research Goals**

Numerous technologies could be enhanced by a better understanding of electromagnetic propagation through clouds. Satellite systems of almost all types depend on information transfer through a cloudy atmosphere. Some ordinance guidance systems are extremely hampered by cloudy conditions often to the point where pilots must return from dangerous missions without deploying their weapons on designated targets. Radars and remote sensing systems can be hampered or enhanced by cloudy conditions depending on the application. For one application, detailed knowledge of propagation through clouds could lead to better sensing of cloud microphysical processes; or for another application, it may be desirable to remove the cloud effects. With these varied applications in mind, we seek to characterize the cloud channel in sufficient detail to make robust signal processing techniques conceivable for managing cloud effects.

Our specific goals are to characterize the fields in the forward, backward, and one
perpendicular direction resulting from a monochromatic elliptically polarized field incident on a cloud of square shape (1 km\(^3\)). The analysis will be performed at discrete frequencies in the oxygen and water vapor window regions between 10 GHz and 1 THz. Using the Khrgian-Mazin droplet spectra distribution and a probabilistically derived droplet spatial distribution, we will vary the total liquid water content over a range of 0.1 to 10 g/m\(^3\). Finally, we hope to demonstrate an application of channel "matched" coding schemes and deconvolution filters to the cloud channel. The details pertinent to achieving these goals are discussed in the later sections and in the references.

Next, we will highlight some fundamental results in radiative transfer theory, cloud physics, channel matched coding, and deconvolution filters. We have adopted a survey format with the details provided in the references. This was done so lengthy derivations and esoteric details would not detract from an understanding of how these theoretical results will be used to achieve the goals stated above. This background information will be followed by "research map" or flow diagram of how we expect the research to proceed.

Scattering of Electromagnetic Fields

Since the photon energies are sufficiently low for the frequencies we are interested in, namely 10 GHz to 1 THz, a wave formulation is an acceptable approach to our cloud scattering problem. Typically one starts with the vector wave equation and uses Hertz or Debye potentials to convert to a scalar wave equation of the form

\[ (\Delta^2 + k^2)\phi(r) = Q(r) \]  \hspace{1cm} (1)

One can use a variety of methods to solve this equation and then apply boundary conditions either directly to the potential solution or convert back to the desired field solutions and apply appropriate boundary conditions for the fields.

For our work, we are only concerned with the scattering from spherical water droplets imbedded in an absorbing atmospheric medium. The problem of scattering from a sphere was solved by numerous people around the turn of the century. In fact, the name attached to the solution depends on the approach taken [Kerker, 1969]. In addition to the original papers, numerous authors, [Kerker, 1969], [Van de Hulst, 1957], [Stratton, 1941], [Newton, 1966], [Ishimaru, 1991] and many others, offer detailed derivations of this classic problem. Since the details are available elsewhere, suffice it to say that the solutions for the scattered field are of the form

\[ \phi_{\text{scat}}(r) = \sum_{n=1}^{\infty} f_n \xi_n(r) \]  \hspace{1cm} (2)
where \( \xi_n(r) \) are products of Hankel functions of index \( n \) and Legendre functions of the first kind. The unperturbed incident field has a similar form because it must also satisfy the wave equation (Helmholtz eq.).

\[
\phi_{\text{inc}}(r) = \sum_{n=1}^{\infty} a_n \psi_n(r)
\]

(3)

where \( \psi_n(r) \) are products of Ricotti-Bessel functions of index \( n \) and Legendre functions of the first kind.

It's worth noting that these equations have the general form of a vector "dot" product and could be written in vector form with the understanding that the vectors are infinitely long or truncated for practical computations [Chew, 1990]. Also recall that Hankel functions are singular at the origin but Bessel functions are regular everywhere. In fact the regular part of a Hankel function is a Bessel function. These ideas are important to an understanding of how Chew's results can be applied to the problem of scattering from multiple spheres which in our case are water droplets.

Using Chew's results, we can relate the amplitude functions in eqs. (2) and (3) by a matrix equation.

\[
f = Ta
\]

(4)

where \( T \) is the transition matrix.

Following [Chew, 1991], this idea can be extended to the case of \( j \) scatterers and using a fast recursive algorithm we can compute \( T_{ij} \) which relates the total scattered field due to the \( i \)-th scatterer to the original incident field when \( j \) scatterers are present. The total field is then the sum of these individual fields.

Despite the elegance and speed of Chew ingenious algorithm, a number of simplifications must be applied to our cloud scattering problem to ensure the computations can be performed in a reasonable amount of computer time.

(1) Only a finite number of drop sizes can be allowed. Thus the droplet spectra distribution must be discretized into a reasonable number of allowed values.

(2) The cloud must be subdivided into sections containing on the order of a billion droplets. The aggregate \( T \) matrix can be computed for this representative section. Then we can allow each section to act as a single scatterer. If the cloud is divided up into \( N \) sections, we can solve the "new" problem of scattering from these \( N \) "scatterers". The implicit assumption here is that each section contains identical droplet distributions. If such an assumption is not desirable, we can compute a \( T \) matrix for each section, and then iterate over the \( N \) sections.
All of this sounds good, but, we have avoided one very important aspect of the problem - polarization. Thus we must extend Chew's algorithm to keep track of vector components and add the fields vectorially.

Cloud Physics

To perform the scattering computations suggested previously, cloud droplet size and
Channel Coding and Equalization

An understanding of propagation through clouds can be exploited in numerous applications. For example, one might be interested in solving the "inverse" problem: given a received signal that has interacted with a cloud, determine certain characteristics of the cloud or an understanding of some underlying physical processes. In most other applications, one is generally interested in compensating for any changes the cloud may have affected to the original signal. Even if the cloud is used as an intentional scattering volume, one is generally interested in recovering an undistorted version of the original signal.

To demonstrate that the results of our cloud scattering calculations can be used to aid in the design of useful signal processing techniques, we will try two approaches for improving signal reception over a cloudy channel. One approach is a coding scheme and the other is an equalization or deconvolution filter. We recognize at the outset that in practical systems both of these methods must be adaptive to be optimally effective because clouds in a particular channel will develop, move, and dissipate over time. However, here we are only concerned with proving the utility of our basic results and hence will save the adaptive problem for another thesis and focus solely on the time-invariant solutions. These solutions will be reasonable valid over some finite time for which the physical state of the cloud varies no more than is allowed in order to remain within the stated performance requirements of the signal processing algorithm or method. We will seek a measure of how much cloud variation is possible before the signal degrades below a stated criterion.

Both of the proposed techniques, coding and filtering, will require knowledge of the cloud transfer function. In the coding method, we will seek a code that is matched to the channel in the sense described in [El-Khamy, 1991]. Specifically, it will be a binary code that is positive (negative) over time intervals when the cloud impulse response is positive (negative) for the full time decision interval. The code is superimposed on the original digital signal and the cloud channel itself acts as the correlation portion of the receiver in a manner similar to a matched filter. Thus the receiver design can be relatively simple. The mathematical details of this approach are in [El-Khamy, 1991]. We hope to extend El-Khamy's work to multi-level pulse amplitude modulation (PAM) codes for improved channel matching and to seek optimum (possibly non-square) pulse shapes.

This coding technique should work in applications requiring "real time" processing. Of course, this ultimately depends on the efficiency of any associated adaptive methods. In contrast, an equalization filter may require more off-line processing. The filter problem can be stated as follows [Roberts and Mullis, 1987]:

\[ V(h) = \|f-g*h\|^2 \]

\[ = \sum_{k=0}^{\infty} |f(k) - (g*h)(k)|^2 \]

\[ = \frac{1}{2\pi} \int |F(e^{j\theta}) - G(e^{j\theta})H(e^{j\theta})|^2 \, d\theta \]  

(7)

108
where $F$ is the desired transfer function, $G$ is the cloud transfer function, and $H$ is the filter we are trying to design. The lower case letters represent the corresponding impulse responses. $V$ is the mean square error we are trying to minimize. The procedure for determining $h$ (or $H$) is to recast this equation into the form of the so-called normal equations and then solve the resulting system of Toeplitz equations using some variation of the Levinson algorithm.

For the equalization problem, $F = 1$. In this case, if $G$ is minimum phase (all zeros inside the unit circle), then $H = 1/G$ is a simple yet stable solution (all poles inside the unit circle). However, in most cases, $G$ is not minimum phase and we must resort to solving the normal equations.

**Research Map**

The problem we are proposing to solve can be summarized as follows:

*Given:*

1. Cloud parameters (droplet spectra, droplet spacing, liquid water content, cloud dimensions, and cloud temperature

   A) Separ_.te coherent and non-coherent signal components
   B) Deterraine the polarization

2. Dispersive effects (pulse propagation)

3. Filters and codes to compensate for undesired signal distortions resulting from propagation through the cloud

We propose bounding the problem in the following way:

**Cloud Definition:**

1) Non-precipilating
2) only liquid droplets (no ice particles)
3) droplet radii < 100 microns

4) cloud elevated far enough above the surface so reflections from the surface back through the cloud can be ignored.

5) cloud liquid water content < 10 g/m$^3$

6) cloud temperature = 10°C

7) assume the droplets are stationary (ignore possible spectral broadening effects due to doppler shifts)

**Frequency Range: 10 GHz - 1 THz**

The anticipated steps proceeding towards a solution are:

**Flow Chart:**

1) Compute Mie scattering results for a single droplet over the full range of allowed droplet sizes

2) Generate droplet sizes and spacings from distributions (Monte Carlo)

3) Use Chew algorithm to parameterize scattering from a small subdivision of the cloud. If necessary parameterize all subdivisions of the cloud if the liquid content of the cloud is allowed to vary over the volume of the cloud.

4) Use Chew algorithm to solve the new scattering problem consisting of N scatterers where N is the total number of subdivisions of the cloud.

5) Repeat steps 2-4 numerous times (Monte-Carlo) and compute mean and variance of the results

6) Change frequencies and repeat

7) Change liquid water content and repeat

8) For coherent signal, interpolate between frequencies and estimate a transfer function around a given carrier frequency

9) Compare results to a current model (Liebe) for a "degenerate case" (i.e. a frequency well below the Rayleigh cutoff)

10) Specify a code and a filter to compensate for the cloud channel and characterize their performance
Summary

Recognizing the seemingly unpredictable nature of clouds, we have proposed a cloud parameterization based on liquid water content and droplet size and spatial distributions. To limit the problem from a radiative transfer or electromagnetic perspective, we have proposed keeping the frequency below 1 THz which allows us to use a continuous wave approach rather than considering individual wavelets (Rayleigh-Gans) or even photons. The principle basis for our scattering analysis is the recently developed recursive Chew algorithm which exploits the concept of a transition matrix that relates scattered amplitude functions to incident amplitude functions. We seek to demonstrate the utility of these results by proposing code and filter designs based on an estimated cloud transfer function. A research map was presented to show the specific steps that will hopefully lead us to some useful results.

Bibliography


