Experimental Investigation of Crossflow Jet Mixing in a Rectangular Duct

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Prepared for the
29th Joint Propulsion Conference and Exhibit
cosponsored by the AIAA, SAE, ASME, and ASEE
Monterey, California, June 28–30, 1993
EXPERIMENTAL INVESTIGATION OF CROSSFLOW JET MIXING
IN A RECTANGULAR DUCT

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Abstract

An experimental investigation of the mixing of non-reacting opposed rows of jets injected normal to a confined rectangular crossflow has been conducted. Planar Mie-scattering was used to measure the time-average concentration distribution of the jet fluid in planes perpendicular to the duct axis. The mixing effectiveness of round orifice injectors was measured as a function of orifice spacing and orifice diameter. Mixing effectiveness was determined using a spatial unmixedness parameter based on the variance of mean jet concentration distributions. Optimum mixing was obtained when the spacing-to-duct-height ratio was inversely proportional to the square root of the jet-to-mainstream momentum-flux ratio. For opposed rows of round holes with centerlines inline, mixing was similar for blockages up to 75%. Lower levels of unmixedness were obtained as a function of downstream location when axial injection length was minimized. Mixing may be enhanced if orifice centerlines of opposed rows are staggered, but note that blockage must be ≤ 50% for this configuration.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_j/A_m$</td>
<td>jet-to-mainstream area ratio</td>
</tr>
<tr>
<td>$B$</td>
<td>blockage = $y$ projection / $S$</td>
</tr>
<tr>
<td>$C$</td>
<td>$(S/H) \times \sqrt{J}$ (see Eq. 3)</td>
</tr>
<tr>
<td>$C_u$</td>
<td>fully mixed mass fraction $= (w_j/w_m)/(1+w_j/w_m) = \Theta_{EB}$, (Ref. 2)</td>
</tr>
<tr>
<td>$C_d$</td>
<td>orifice discharge coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>orifice diameter</td>
</tr>
<tr>
<td>$H$</td>
<td>duct height at injection plane = 2 in</td>
</tr>
<tr>
<td>$J$</td>
<td>jet-to-mainstream momentum-flux ratio $= (p_j , V_j^2) / (p_m , V_m^2)$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$S$</td>
<td>spacing between adjacent orifice mid-points</td>
</tr>
<tr>
<td>$U_s$</td>
<td>spatial unmixedness parameter (Eq. 2)</td>
</tr>
<tr>
<td>$V_m$</td>
<td>mainstream velocity = 10 ft/s</td>
</tr>
<tr>
<td>$V_j$</td>
<td>jet velocity $= w_j / \rho_j , A_j , C_d$</td>
</tr>
<tr>
<td>$w_j/w_m$</td>
<td>jet-to-mainstream mass flow ratio</td>
</tr>
</tbody>
</table>

Nomenclature (cont.)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>downstream coordinate, $x = 0$ at the upstream edge of the orifice</td>
</tr>
<tr>
<td>$y$</td>
<td>cross-stream coordinate</td>
</tr>
</tbody>
</table>

Introduction

The injection of jets normal to a crossflow is a commonly employed mixing technique. One important application is the dilution zone of a gas turbine combustor which typically uses rows of relatively cool air jets to lower the exit temperature of the combustor. This mixing technique is also under evaluation as a key technology for the development of an advanced low NOx engine based on a Rich-Burn/Quick-Mix/Lean-Burn (RQL) combustor. The RQL combustor depends on an efficient quick mix section that rapidly and uniformly dilutes the rich zone products to minimize emissions.

Extensive cross flow mixing investigations reported by Holdeman have focused on conventional gas turbine dilution zones where up to 30% of the total flow was introduced with the dilution jets. Recently other studies of jets in a rectangular cross flow have been reported by Smith, Bain, Smith, & Holdeman, and Liscinsky et al., while studies of confined jets in cylindrical ducts have been reported by Talpallikar et al., Smith, Talpallikar, & Holdeman, Vranos et al., Hatch et al., Oechsle, Mongia, & Holdeman, and Kroll et al. These studies all conclude that the rate of mixing by a row of jets in cross flow is primarily determined by the jet-to-mainstream momentum-flux ratio ($J$) and the orifice spacing-to-duct height ratio ($S/H$).

In the RQL combustor the jet fluid introduced in the quick mix section accounts for up to 75% of the total flow. Since the available pressure drop is limited, injection of large mass flows through discrete orifices requires jet-to-mainstream area ratios larger than those considered when studying conventional dilution zones. In this investigation the effects of closely spaced orifices ($S/D < 2$) are compared to the conclusions of previous studies where larger orifice spacing was evaluated.
Figure 1 is a schematic representation of the cross flow mixing apparatus. The apparatus consists of 3 parallel contiguous ducts of rectangular cross section, simulating a sector of an annular combustor. Sector width is 12 inches. The inner duct height \( H \) is variable, but was set at 2 inches for the reported experiments. The outer ducts (shrouds), which supply the injectant gas, are 1 inch in height. These are separated from the inner duct by removable, 0.12 inch thick flat plates. The injectant is fed from the shrouds to the inner duct through orifices of various sizes and shapes that are machined into the plates. Mass flow to each of the 3 ducts is controlled independently using venturi flowmeters. The mean mainstream flow velocity was 10 ft/sec with less than 6% variation across the duct and a turbulence level of 1.3%. All tests were conducted with unity density ratio.

Planar digital imaging was used to optically measure concentration distributions in planes perpendicular to the duct axis beginning at the trailing-edge of the orifice. The Mie-scattering technique is applied by marking the jet flow with an oil aerosol (µm sized particles). A light sheet (0.02 inch thick) is created using a 2W argon-ion laser and a rotating mirror. The flow field is illuminated by passing the light sheet through a window in the side wall of the test section. An image intensified thermo-electrically cooled CCD camera, located inside the duct 2.5 ft downstream of the orifice centerline, is focused on the illuminated plane (end-on view). The camera is programmed to make exposures coincident with the sweep of the beam through the flow field. The image is digitized at a spatial resolution of 0.02 x 0.02 inch/pixel in a 576 x 100 pixel format and sent to a computer for storage. The scattered light intensity is proportional to the number of particles in the measurement volume. If only one of two streams is marked, the light intensity of the undiluted marked fluid represents mole fraction unity.

An unmixedness parameter that quantifies temporal fluctuations was defined by Danckwertz\(^1\) as,

\[
U = \frac{c^2}{c(1-c)}
\]

where,

\[
c^2 = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m} (c_{ij} - \bar{c})^2 = \text{concentration variance}
\]

\[
n = \text{number of images in data set}
\]

\[
m = \text{number of pixels in each image}
\]

\[
c_{ij} = \text{instantaneous concentration at a pixel}
\]

\[
\bar{c} = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} = \text{concentration mean}
\]

\[
\bar{c} (1-\bar{c}) = \text{maximum concentration variance}
\]

\[
= 0.188 \text{ for a jet-mainstream flow split of 3:1}
\]

Normalization by \( \bar{c} (1-\bar{c}) \) allows comparison of systems of different \( \bar{c} \) (different \( w_j/w_m \)) and bounds \( U \) between 0 and 1. \( U = 0 \) corresponds to a perfectly mixed system, and \( U = 1 \) a perfectly segregated system.

The objective of this investigation was to rapidly screen a variety of flow and geometric configurations and compare the experimental results to similar numerical studies. Therefore, the suitability of using an unmixedness parameter based on the mean distribution alone was studied. It was found that \( U \) obtained from an ensemble of instantaneous distributions was approximately equal to that obtained from the average distribution\(^9\). Therefore, the unmixedness parameter used in this investigation is \textit{spatial unmixedness}:

\[
U_s = \frac{c_{\text{var}}}{c_{\text{avg}} (1 - c_{\text{avg}})}
\]
where,

\[ C_{\text{var}} = \frac{1}{m} \sum_{i=1}^{m} (C_i - C_{\text{avg}})^2 \]

- spatial concentration variance

\[ \bar{C}_i = \text{time-average concentration at a pixel} \]

\[ C_{\text{avg}} = \text{fully mixed concentration downstream of the trailing edge of the orifice} \]

The measured relative light intensities are converted to measurements of concentration by normalizing so that

\[ \frac{1}{m} \sum_{i=1}^{m} C_i = C_{\text{avg}}, \] (the metered/fully mixed value). Therefore, although \( C_{\text{avg}} = \bar{C} \), the actual value of \( \bar{C} \) is not measured directly and cannot be computed in the same way upstream of the trailing edge of the orifice, i.e. before all of the jet mass is injected. Eq. 1 is still valid, however, if concentration is measured directly or determined by calibration using a supplemental technique (see Ref. 9, p. 4).

### Mixing Configurations

Table 1 identifies 8 orifice plate configurations that were tested. The configuration sketches are drawn approximately 1/4 scale in Table 1. The configurations consisted of round holes with \( D = 0.5, 0.75, \) and \( 0.85 \) and a rectangular slot with a 2:1 aspect ratio. Injection was 2-sided with the midpoints of the orifices on opposite sides either directly inline, or staggered (top wall orifice midpoints bisect the space between adjacent orifices on the bottom wall, i.e. the area ratios of the inline and staggered configurations were equivalent). Discharge coefficients were determined for each orifice plate configuration by measuring the \( \Delta P \) across the plate over a range of mass flows and averaging. The \( C_d \) was used to set momentum-flux ratios of 25 and 50. (For reference,

\[ J = \frac{(w_j/w_m)^2}{(p_j/p_m)(C_d)^2(A_j/A_m)^2} \]

<table>
<thead>
<tr>
<th>Plate</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.50 0.50 2.00 0.20 0.64 0.250 0.50</td>
</tr>
<tr>
<td>1</td>
<td>0.50 0.38 1.50 0.26 0.64 0.250 0.67</td>
</tr>
<tr>
<td>3</td>
<td>0.75 0.75 2.00 0.29 0.65 0.375 0.50</td>
</tr>
<tr>
<td>7</td>
<td>0.75 0.63 1.67 0.35 0.65 0.375 0.60</td>
</tr>
<tr>
<td>2</td>
<td>0.75 0.50 1.33 0.44 0.65 0.375 0.75</td>
</tr>
<tr>
<td>9</td>
<td>0.75 0.50 1.18 0.57 0.65 0.375 0.94</td>
</tr>
<tr>
<td>11</td>
<td>0.85 0.50 1.18 0.57 0.66 0.425 0.85</td>
</tr>
<tr>
<td>12</td>
<td>(0.5 x 1.0) 0.50 (2.00) 0.57 0.75 0.500 0.50</td>
</tr>
</tbody>
</table>

Table 1: Orifice Plate Configurations

* \( x \) projection / \( H \) (\( H = 2 \) inches for all tests)

† \( y \) projection / \( S \) (blockage = the reciprocal of \( S/D \) for the orifice configurations in Table 1)
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Results and Discussion

Orifice Spacing

Mean concentration distributions for two-sided injection from opposing rows of round holes with the top and bottom hole centered opposite each other (inline) are shown in Fig. 2 at $x/H = 0.375$ and 0.500 when $J = 25$. A 10-level color scale is used to represent contours of jet mass fraction from 0 to 1.0 (pure mainstream fluid colored red = 0 and pure jet fluid colored dark blue = 1.0). In each figure the orifice spacing decreases from $S/H = 0.75$ in the top row to $S/H = 0.4$ in the bottom row. Hole diameter is constant at 0.75 inches, consequently $A_j/A_m$ for plate #9 (bottom row) is about 50% larger than plate #3 (top row). Therefore the mass flow ratio also increases from the top row to the bottom row in Fig. 2. The fully mixed concentration, $C_{avg}$, i.e. the color corresponding to the fully mixed condition, is not the same contour for all of the configurations shown in Fig. 2.

From Fig. 2 the effect of orifice spacing on jet penetration is clear: at a given $J$, jet penetration decreases as spacing decreases. At $S/H = 0.75$ the jet trajectories from the top and bottom walls collide at the mid-point of the duct, while at $S/H = 0.4$ the jets remain near the walls of the duct.

In previous studies by Holdeman\textsuperscript{2} jet penetration and centerplane profiles were found to be independent of orifice diameter, when $S/H$ and $\sqrt{J}$ were inversely proportional:

$$S/H = C / \sqrt{J}$$  \hspace{1cm} (3)

An optimum $S/H$ of 0.25 would be predicted using Eq. 3 for opposed inline orifices ($C = 1.25$) and $J = 25$. In Ref. 2 the "optimum" was obtained by visual inspection of the centerplane profiles and therefore depends on $x/H$.

\begin{align*}
\text{Plate #3} \\
S/H = 0.75 \\
C_{avg} = 0.49 \\
\text{Plate #7} \\
S/H = 0.63 \\
C_{avg} = 0.53 \\
\text{Plate #2} \\
S/H = 0.50 \\
C_{avg} = 0.59 \\
\text{Plate #9} \\
S/H = 0.40 \\
C_{avg} = 0.64
\end{align*}

Figure 2: Average Concentration Distributions for Opposed Inline Round Holes ($D=0.75''$) at $J = 25$
In Fig. 3 the effect of S/H on mixing effectiveness is shown in a plot of $U_S$ vs downstream distance for the same configurations in Fig. 2. An optimum spacing is indicated at $S/H = 0.5$, which corresponds to a jet penetration between the case where the jets "over-penetrate" at $S/H = 0.75$ (top row in Fig. 2) and "under-penetrate" at $S/H = 0.4$ (bottom row in Fig. 2). The orifice configuration shown as plate #2 in Table 1 provided the fastest mixing at $J = 25$.

Eq. 3 is evaluated in Fig. 4 at $J = 25$ for the fastest mixing configuration at $D = 0.75$ (#2, shown previously in Fig. 3) and two configurations with $D = 0.50$ and similar values for C. Mixing was most effective for each hole size when $C = 2.5$. The unmixedness curves for both plates #2 and #10 (Table 1) at $J = 25$ support Eq. 3 in that mixing rate is a function of spacing and $J$, but independent of orifice diameter.

The value of 2.5 obtained for C for opposed rows of inline orifices is higher than the value of 1.25 found by Holdeman. It is suspected that two factors influence the variation in C: (1) data analysis in previous studies compared centerplane profiles, while this study measures the nonuniformity of the entire duct cross section, and (2) the orifice configurations of this study are outside of the range of the previous data set, i.e. previous $A_j/A_m < 0.1$ and $S/D > 2$ (widely spaced) while in this study $A_j/A_m > 0.2$ and $S/D < 2$ (closely spaced).

Orifice Diameter and Blockage

Eq. 3 can be used as a design tool, given $J$, to specify the optimum $S$ for a row of inline round holes. However, for durability reasons, it may be necessary to consider the trade-off between blockage (B)(webb between adjacent orifices) and longer injection length (rectangular slots). In the limit, when $B = 1$, a 2D slot is obtained which has been shown to be a poor mixer compared to a row of discrete orifices. This would suggest that in addition to an optimum $S$, B is also an important consideration. Unfortunately, for round holes, B and D cannot be tested independently. A comparison of orifice plates #10, #2, and #11 affords an evaluation progressively greater B, while maintaining a constant S.
In Fig. 6 $U_s$ is plotted as a function of $x/H$ at $J = 25$ for $B = 0.50, 0.75$ and 0.85. Note that for these configurations $B = D$ since $S = 1$. In addition since $C = 2.5$, the configurations are optimized at this $J$. The unmixedness curves for the three configurations are similar, which indicates that $B$ ranging from 0.50 to 0.85 does not affect mixing rate. Even a relatively high blockage still allows the mainstream flow to squeeze between the jets and generate a 3D flowfield. However, the level of $U_s$ in the near-field ($x/H < 0.5$) is influenced by $D$. At $x/H < 0.5$, the lowest levels of $U_s$ are obtained for the smallest diameter holes. Although optimum spacing as specified by Eq. 3 does not appear to be a function of $D$ (same value of $C$ is obtained independent of $D$), the level of $U_s$ at a particular value of $x/H$ is affected by the axial length of the orifice, i.e. the mixing curves shift downstream along with the trailing edge of the orifice. Fig. 6 indicates that the lowest values of $U_s$ are obtained in a minimum $x/H$ when the axial length of the orifice is minimized.

In Fig. 7 $U_s$ is shown as a function of $x/H$ at $J = 25$ for a rectangular slot with a 2:1 aspect ratio (plate #12, Table 1) and a round hole with $D = 0.85$ (plate #11, Table 1). The area ratios of the two configurations are equivalent. Mixing rates of the two configurations are not the same. The levels of unmixedness are significantly higher for the slot configuration and the rate is slower. Fig. 7 further emphasizes that mixing effectiveness diminishes as axial length of injection increases. However, the problem of liner durability could be addressed by the use of rectangular slots if slot length = hole diameter.

**Figure 7: Comparison of Equal Area Equal S Opposed Inline Round Holes and Rectangular Slots at $J = 25$**

**Inline vs. Staggered**

Opposed jet configurations are not limited to inline orientations. In Fig. 8 two examples of inline and staggered configurations are shown at $J = 50$ for round holes. Both configurations are for holes with $D = 0.75$, but the case on the left has an $S = 1.0$ ($S/H = 0.5$) and the one on the right an $S = 1.5$ ($S/H = 0.75$). Note that $A_j/A_m$ (and therefore $w_j/w_m$) for the inline and staggered configurations are equivalent at each $S/D$. When $S/D = 1.33$ (plate #2), the inline and staggered distributions are very similar: the jets overpenetrate and appear to remain relatively unmixed at $x/H = 0.625$. In contrast, at $S/D = 2.00$ (plate #3) the inline and staggered distributions are quite different. The staggered configuration appears better mixed at $x/H = 0.625$. The staggered jets become elongated as they pass by each other. An interaction of the counter-rotating vortex pair from the top and bottom jets is indicated by the loss of the characteristic "horseshoe" shape in the near-field. It appears that the resulting vortex system is less stable than any of the other three configurations. Note that the jet fluid apparent nearest the walls in the lower right figure ($S/D = 2$, staggered) is from jets injected from the opposite wall, whereas the jet fluid nearest the walls in the other figures is from jets injected from the adjacent wall.
In Fig. 9 $U_s$ is plotted as a function of $x/H$ for the four configurations in Fig. 8. As predicted the staggered configuration is the most effective mixer. A systematic inspection of staggered configurations for the plates shown in Table 1 indicated that the staggered orientation does not increase mixing effectiveness until $S/D = 2$, i.e. at $S/D < 2$ $U_s$ curves for staggered and inline configurations were similar. The implication is that there may also be an optimum $S/D$, in addition to an optimum $S/H$, for staggered configurations. If the required $A_j/A_m$ can be achieved with $S/D = 2$, it appears that staggering may produce lower levels of $U_s$ in the near-field if $J$ is sufficient for the jets from opposite walls to penetrate past each other.

![Figure 9: Effect of Orifice Orientation on Spatial Unmixedness at $J = 50$](image)

![Figure 8: Comparison of Staggered and Inline Orientations at $J = 50$](image)
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Conclusions

1. As found in previous studies by Holdeman, mixing effectiveness can be characterized by:

\[ C = (S/H) \cdot \sqrt{J} \]

The optimum value obtained for \( C \) for opposed inline round holes in this study was 2.5.

2. For opposed rows of round holes with centerlines inline, mixing was similar for blockages ranging from 0.5 to 0.75.

3. Lower levels of \( U_s \) were obtained as a function of downstream distance when the diameter, or length, of the orifice was minimized.

4. The vortex pattern formed by jets staggered at \( S/D = 2 \) appears to destabilize more quickly than that from inline jets. Therefore, properly spaced staggered orientations may augment mixing.

Acknowledgments

This work was supported by NASA Contract NAS3-25954, Task Order #12.

References


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