An Analytical Optimization of Electric Propulsion Orbit Transfer Vehicles

Steven R. Oleson  
*Sverdrup Technology, Inc.*  
*Lewis Research Center Group*  
*Brook Park, Ohio*

May 1993

Prepared for  
*Lewis Research Center*  
*Under Contract NAS3–25266*
AN ANALYTICAL OPTIMIZATION METHOD FOR ELECTRIC PROPULSION ORBIT TRANSFER VEHICLES

Steven R. Oleson
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio 44142

Abstract

Due to electric propulsion's inherent propellant mass savings over chemical propulsion, electric propulsion orbit transfer vehicles (EPOTVs) are a highly efficient mode of orbit transfer. When selecting an electric propulsion device (ion, MPD, or arcjet) and propellant for a particular mission, it is preferable to use quick, analytical system optimization methods instead of time intensive numerical integration methods. It is also of interest to determine each thruster's optimal operating characteristics for a specific mission. Analytical expressions are derived which determine the optimal specific impulse (Isp) for each type of electric thruster to maximize payload fraction for a desired thrusting time. These expressions take into account the variation of thruster efficiency with specific impulse. Verification of the method is made with representative electric propulsion values on a LEO-to-GEO mission. Application of the method to specific missions is discussed.

INTRODUCTION

In the coming decades, electric propulsion promises to become a frequently used and highly efficient mode of space propulsion. This is due to the inherently higher specific impulse (Isp) of electric propulsion (800 to 10,000 s verses 100 to 450 s for chemical propulsion) which produces significant propellant mass savings. These savings can then be used to increase deliverable payload mass or allow the use of smaller, less expensive expendable launch vehicles. However, these savings usually come at the expense of longer mission times. Electric propulsion is already being used to provide station keeping propulsion for Earth satellites. In addition, current NASA studies have shown the potential for electric propulsion's use as primary propulsion on interplanetary probes, lunar and Mars cargo vehicles, and Mars piloted vehicles. As part of the evolutionary process towards the interplanetary application, the initial use of electric propulsion (EP) for primary propulsion is likely to be as a highly efficient Earth orbit transfer vehicle (OTV).

The purpose of this paper is to provide this quick analytical method for predicting EP performance, optimal Isp's, and the corresponding EP vehicle parameters. This method can be used to quickly determine the performance of electric propulsion orbit transfer vehicles (EPOTV) for any circular orbit to circular orbit transfer mission. By using the predicted optimal Isp method, parametric variation of Isp is not necessary thus eliminating one variable from a parametric trade.

The analysis of EP trajectories is different from chemical impulsive trajectories since the electric thrusters operate continuously at very small thrust levels. As a consequence of this, an EP vehicle performs a spiral trajectory from orbit to orbit near a planetary body. To accurately analyze this spiraling, numerical integration methods are usually used, especially when optimal vehicle steering is desired. These methods usually require specific vehicle assumptions which do not allow for quick parametric assessments of various thrusters, power systems, power levels, and specific impulses. Alternately, by using certain simplifying mission assumptions, analytical methods can be used to perform a quick parametric scoping of potential thruster/power combinations, without requiring more detailed inputs or specialized user training. In most cases, the efficiency of EP thrusters is highly dependent upon the Isp. This variation of efficiency with Isp must be considered before choosing thrusters and power levels since it greatly affects payload performance. Gilland (1991) did account for this variation for lunar and Mars missions. A similar derivation
will be made here for near-planetary EP missions.

Specifically, this analytical performance technique determines the optimal Isp and maximum payload for EP vehicles considering specific thrusters and desired thrusting time. Assumptions are presented describing the general characteristics of power source and the thruster systems needed to perform a parametric trade. The low thrust mission analysis is simplified to a required low thrust velocity increment, ΔV, to allow a simple analytical relation of the mission requirements to the EP vehicle system parameters. EP vehicle parameters are defined and the dependency of optimal Isp, optimal power level and trip time on these parameters is then shown. Optimal Isp equations are derived for each type of thruster using curve-fit efficiency/Isp relationships and mathematical simplifications. The accuracy of these analytically predicted optimal Isp’s is compared to the numerically calculated optimal Isp’s using sample value for the mission and vehicle parameters. A comparison of the resulting payload fractions is also made.

ASSUMPTIONS

The analysis in this paper assumes a single parameter, the specific mass, to characterize the power and thruster system scaling. The independent variable is assumed to be the vehicle thrusting time. The analysis considers the available power to be constant, which is representative of nuclear powered EPOTVs. For solar powered craft, two other affects make the actual trip time longer than the thrusting time; (1) the Van Allen belt radiation degrades the power, and (2) shading limits power production time to sunlit parts of the orbit. Other methods must be used to assess these affects and are not addressed here.

The efficiency of some electric thrusters is highly dependent upon the effective exhaust velocity, c or specific impulse Isp. Depending upon the type of thruster (ion, MPD, or arcjet), the efficiency - Isp relation differs. All of the following relations are based upon projected near-term technology. The coefficients of these relations are derived from curve fits of empirical data. As long as the mathematical form of the efficiency/Isp relationship is valid, however, the method derived in this paper is applicable with the appropriate efficiency coefficients.

For ion thrusters utilizing inert gases (argon, krypton, xenon) the relationship is second-order as shown by Equation (1). For MPD thrusters using hydrogen fuel, the relation is the same as (1) for Isp’s greater than 7800 s (Gilland, 1991). For Isp’s less than 7800 s the efficiency more closely follows relation (2).

\[ \eta_{\text{ion}} = \frac{b c^2}{c^2 + d^2} \quad [1] \]
\[ \eta_{\text{MPD}} = \frac{b c}{c + d} \quad [2] \]

The near-term coefficients b and d for each propellant as well as their feasible Isp ranges are presented in Table 1, the corresponding efficiencies predicted by Equation (1) are shown in Figure 1. Recent H\textsubscript{2} arcjet data (NASA, 1991) indicate that arcjet efficiency does not have a strong correlation with Isp. Consequently, it is assumed that H\textsubscript{2} arcjets have a constant efficiency. The near-term Isp range and efficiency for H\textsubscript{2} arcjets are also shown in Table 1.

The nominal range over which an electric thruster’s Isp can be varied is limited. Table 1 shows the accepted ranges for each type of thruster. For some orbit transfer missions the optimal Isp may be below this Isp range depending upon efficiency, specific mass and desired thrust time. Recent research has shown that “derated” xenon and krypton ion thrusters can be run at lower Isp’s with a different efficiency relationship than for high Isp operation (Equation 1) (Patterson, 1992). The implications of using derated thrusters are not considered in this paper.
MISSION ANALYSIS

An analytically calculable parameter is used to relate the mission performance requirements to vehicle parameters. This mission parameter is the velocity increment required or \( \Delta V \). This \( \Delta V \) is related to the vehicle performance through the rocket equation: final mass / initial mass = \( e^{-\Delta v/c} \), where \( c \) is the effective exhaust velocity of the EP thruster.

Since an EP vehicle thrusts throughout most of its mission, an impulsive \( \Delta V \), (an assumption that all thrusting burns change the vehicle’s velocity instantaneously), can not be accurately used. However, past work done by Edelbaum (1961) gives an analytical expression which approximates the \( \Delta V \) required by a low thrust vehicle performing an orbital transfer and a plane change near the Earth. This expression is,

\[
\Delta V = \left[ \frac{V_o^2 + V^2 - 2V_o V \cos (\pi/2 \theta)}{2} \right]^{1/2}
\]

where \( V_o \) and \( V \) are the circular velocities of the original and desired orbits, respectively, and \( \theta \) is the desired plane change. Some of the assumptions accompanying this equation are: (1) the orbit must always remain quasi-circular during the transfer, (2) the thrust angle is assumed constant during each revolution, and (3) the thrust magnitude is assumed constant throughout the transfer. In addition, the thrust-to-weight ratio of the EP system must be \( 10^{-2} \) or less.

EP VEHICLE ANALYSIS

An EP vehicle is modeled by four main elements each of which contributes to the total mass (\( M_o \)) of the EP vehicle: the propellant, the propellant tankage, the propulsion system, and the payload. The following derivation is similar to that done by Stuhlinger (1964). The propellant mass (\( M_p \)) is the mass of propellant required for the mission. The propellant tankage mass is directly related to the mass of the fuel by a tankage factor: \( K_t = \text{mass of tankage / mass of propellant.} \) The propulsion system mass (\( M_w \)) is defined as the power subsystem mass (main power generation and power management and distribution-PMAD) and thruster subsystem mass (including thrusters, gimbals, power processing units -PPUs, and propellant feeds but excluding fuel and tankage). Finally, the payload mass (\( M_L \)) includes useable payload mass, fixed support system masses not scalable to the thruster power (for example structures, guidance and control, and communications), and any contingency mass. These combine to make the total initial EP vehicle mass:

\[
M_o = M_L + M_w + M_p + K_t M_p.
\]

Such parameterization of an EP vehicle allows for each component to be compared to the total initial mass of the EP vehicle. A system mass fraction (or just system fraction) can be calculated as system mass / total initial mass = \( \mu \). The propellant fraction, \( \mu_p \), is determined easily using the rocket equation with the mission’s Edelbaum \( \Delta V \) (Equation 3) and the exhaust velocity of the electric thrusters (c):

\[
\mu_p = 1 - e^{-\Delta v/c}.
\]

The tankage mass fraction \( \mu_{K_t} \) is:

\[
\mu_{K_t} = K_t \mu_p.
\]
The propulsion system fraction \( \mu_w \) is derived by considering the thrust power \( (P_t) \): \( P_t = \frac{1}{2} \left( \frac{dm}{dt} \right) c^2 \), where the mass flow rate is obtained from the propellant required and the given thrusting time, \( \tau \): \( \frac{dm}{dt} = \frac{M_p}{\tau} \). The thrust power can be related to the electric power \( (P_e) \) input to the thrusters by an efficiency factor, \( \eta \): \( P_t = \eta P_e \), where the efficiency accounts only for the actual efficiency of the thruster. Line losses, PMAD and PPU efficiencies, and housekeeping power requirements are included in the specific mass of the propulsion system. As discussed previously, the efficiency of the thruster is known to vary with exhaust velocity or Isp. The specific efficiency/Isp relationships for each thruster are described previously.

Finally, this electric power \( (P_e) \) provided to the thrusters is related to the mass of the power subsystem by: \( M_w = P_e \kappa \), where the term \( \kappa \) represents the specific mass of the propulsion system as: \( \kappa = \text{propulsion system mass [kg]} / \text{electric power sent to thrusters [kWe]} \). These equations are combined to determine the propulsion system fraction:

\[
\mu_w = \frac{(\alpha c^2)}{(2 \eta \tau)} \left(1 - e^{-\Delta vz/c}\right).
\]  

Finally, noting that the values of the system fractions must add to one: \( \mu_L + \mu_w + \mu_p + \mu_{Kt} = 1 \), the payload fraction, \( \mu_L \), is defined by the other fractions as:

\[
\mu_L = e^{-\Delta vz/c} - \left[ K_t + \frac{(\alpha c^2)}{(2 \eta \tau)} \right] \left(1 - e^{-\Delta vz/c}\right).
\]  

It is evident that the EP system payload fraction is dependent upon the mission parameters \( \Delta V \) and specified thrust time \( \tau \), as well as the system parameters \( c, \alpha, \eta \), and \( K_t \). Consequently, the performance of an EP vehicle can be easily predicted for various mission and system assumptions.

**ANALYTICALLY OPTIMAL \( \mu_L \)**

Now that an analytical expression for the payload fraction is available, the various relations for efficiency (Equations (1) and (2)) are used to find a thruster specific payload fraction:

\[
\mu_L = e^{-\Delta vz/c} - \left[ K_t + \frac{(\alpha c^2)}{(2 \eta \tau)} \right] \left(1 - e^{-\Delta vz/c}\right) \quad [\eta = \text{constant}]
\]  

Hydrogen Arcjets

\[
\mu_L = e^{-\Delta vz/c} - \left[ K_t + \alpha \frac{(c^2 + d^2)}{(2 b \tau)} \right] \left(1 - e^{-\Delta vz/c}\right) \quad [\eta = b \frac{c^2}{(c^2 + d^2)}]
\]  

Ion and MPD Isp>7800s

\[
\mu_L = e^{-\Delta vz/c} - \left[ K_t + \alpha \frac{(c^2 + c d)}{(2 b \tau)} \right] \left(1 - e^{-\Delta vz/c}\right) \quad [\eta = b \frac{c}{(c + d)}]
\]  

MPD Isp<7800s

The payload fraction for specific choices of \( \Delta V \), thrust time, specific impulse, tankage, propulsion system specific mass, and thruster efficiency factors \( b \) and \( d \) are calculated using these equations. With the choice of a specific mission and a specific system only the thrust time and the constant exhaust velocity (or specific impulse) of the thrusters can be varied to alter payload performance.

Payload optimizations are presented by various authors including Stuhlinger (1964). The optimal Isp for maximum payload fraction is shown to be the same as the Isp for the shortest trip time and a fixed payload fraction. Stuhlinger developed an approximation of optimal exhaust velocity for
maximum payload fraction as: $c_{opt} = (\tau / \alpha)^{1/2}$. However, this parameter does not address the dependency of EP thruster efficiency on specific impulse. Gilland (1991) takes these variable efficiency equations into account for Mars and Lunar mission scoping. A similar derivation is presented here for near-planetary applications. The figures of merit that this method addresses are payload fraction and the flight time since both are directly linked to the economics of any mission.

Equations (9)-(11) are easily optimized in terms of exhaust velocity by taking the partial derivative of the payload fraction with respect to the exhaust velocity and setting it equal to zero: $\partial \mu_t / \partial c = 0$. Equation (10) is addressed first since the solution for optimal exhaust velocity or Isp should be the same as the optimal for Equation (9) if the efficiency is constant and the efficiency factor $d$ is set to zero. The relation giving the optimal exhaust velocity for maximum payload fraction is,

$$c_{opt} = -\Delta v / 2 + [2 \eta \tau (1 + K_t) / \alpha - d \Delta v^2 / 12]^{1/2},$$

(13)

which can only be solved numerically in its present form. Consequently, the substitution, $x = \Delta v / c$, and the truncated Taylor series: $e^x = 1 + x + x^2 / 2! + x^3 / 3!$, are used to simplify Equation (12) so that the optimal exhaust velocity, $c_{opt}$, is the real root of a quadratic:

$$c_{opt} = -\Delta v / 2 + [2 b \tau (1 + K_t) / \alpha + d^2 - \Delta v^2 / 12]^{1/2},$$

(12)

Equation (13) gives the optimal exhaust velocity for thrusters having the efficiency relationship shown in Equation (1) (current ion thrusters and MPD thrusters with Isp > 7800 s). As a consequence of the truncated Taylor series, Equation (13) is not exact. (However, for EP thruster Isp ranges of 800 s to 10,000 s and mission ΔVs below 10 km/s the method’s error is negligible.) The approximation loses accuracy at low exhaust velocities and large mission ΔV’s and thus should be checked before using this method.

By letting thruster efficiency factors $d = 0$ and $b = \eta$ in (13), the optimal exhaust velocity for a constant efficiency is expressed as:

$$c_{opt} = -\Delta v / 2 + [2 \eta \tau (1 + K_t) / \alpha - \Delta v^2 / 12]^{1/2},$$

(14)

which may be used for arcjets. Finally, for the efficiency relationship Equation (2), the optimal exhaust velocity is:

$$c_{opt} = -\Delta v / 2 + [2 b \tau (1 + K_t) / \alpha - d \Delta v / 2 + \Delta v^2 / 4]^{1/2},$$

(15)

which may be used for MPD thrusters with Isp < 7800 s. Due to algebraic complexity, only a second order expansion of $e^x$ was used, thus making this approximation slightly less accurate.

**VERIFICATION OF THE PREDICTED OPTIMAL ISP**

The accuracy of the predicted optimal exhaust velocity relation for ion propulsion Equation (13) as compared to the numerically calculated and the Stuhlinger predicted optimal specific impulses is now presented. The accuracy of the other two optimal equations (Equations 14 and 15) is similar. The sample mission is a transfer from a low Earth parking orbit to geostationary orbit. It is assumed that the initial parking orbit is a circular Earth orbit at an altitude of 500 km with an inclination of 28.7°. The desired geostationary orbit altitude is 35683 km with a 0° inclination. For this orbit transfer, the Edelbaum equation produces a required low thrust ΔV of 5.86 km/s. The thruster used to verify the method’s optimal Isp is xenon ion. Xenon ion efficiencies and Isp
ranges are listed in Table 1. A tankage and reserves fraction of 10% is assumed for xenon (Welle, 1990). The sample power system specific mass is assumed to be 40 kg/kWe based upon the goal of the 100 kWe SP-100 reactor program (Truscello, et al, 1992) plus an additional 4 kg/kWe for the PMAD (Gilland, George 1992). A sample thruster specific mass of 6 kg/kWe (including thruster, PPU, gimbals, and propellant feeds) is added to the power system specific masses (Gilland, George 1992). Thus the sample propulsion system specific mass is 50 kg/kWe.

For the sample mission and system, the optimal Isp's predicted by Equation (13) are shown in Figure 2 as a function of thrusting time. The optimum Isp's predicted by Stuhlinger and the numerically calculated optimum Isp's are also shown. The equations derived in this paper show almost exact agreement with the actual optimum specific impulse. Stuhlinger's Isp varies significantly from the actual optimum Isp.

The corresponding maximum payload fractions found using the predicted, Stuhlinger, and actual optimum Isp's are shown in Figure 3. It is clear, at least for the sample case chosen, that the maximum payload fraction predicted by the analytical method is very close to the actual optimum payload fraction. While the maximum payload fraction predicted by Stuhlinger shows only a small difference, the resulting system parameters, such as required power and amount of propellant, are misleading.

APPLICATION OF THE METHOD

The payload fractions for the sample mission using the three thruster types are shown together in Figure 4. Figure 5 shows corresponding optimal Isp's. By using these figures, a preliminary scoping of the mission performance of each thruster type is available. For EP systems a trade exists between mission duration and payload delivered. While no 'optimal' payload and trip time is evident, designing a vehicle with attributes from the 'knee' of the curve, (where the return of more payload for longer trip time diminishes) is suggested. In further analyzing the mission a cost analysis should be done balancing the operations cost against reduced launch costs.

It is evident that for short thrusting times, some of the optimal Isp’s are below the thruster Isp range provided with Table (1). This indicates that either the lowest Isp boundary value must be used or research must be done to permit operation in this Isp range, if the performance advantage warrants it.

CONCLUSIONS

A quick analytical method for predicting mission performance for EPOTV vehicles is shown. By using the optimal exhaust velocity (Isp) equations derived in this paper, one parameter of defining the mission (Isp) need not be parametrically varied. The optimal Isp equations are shown to predict the actual optimal Isp for maximum payload fraction for a desired thrust time quite accurately.

ACKNOWLEDGMENTS

The author wishes to thank Jim Gilland, Jeff George, John Riehl, Tim Wickenheiser, Jim Sovey, and Mike Patterson for their insight during the development of this paper. Research for this paper was done at NASA Lewis Research Center's Advanced Space Analysis Office (Contract NAS 3-25266) by Sverdrup Technology, Inc.

References


---Nomenclature---

**English**

c: Exhaust Velocity

g_0: Reference Gravitational Acceleration

Isp: Specific Impulse, \( c / g_0 \)

b: Thruster variable efficiency factor

d: Thruster variable efficiency factor

V: Circular Orbit Velocity

K_t: Tankage Fraction

M: Mass

P_t: Thrust Power, \( 1/2 \) mass flow rate * \( c^2 \)

\( \frac{dm}{dt} \): Propellant Mass Flow Rate

P_e: Electric Power Produced by Power Subsystem

**Greek**

\( \eta \): Propulsion system efficiency, \( P_t / P_e \)

\( \alpha \): Propulsion (Thruster and Power) System Specific Mass, \( M_w / P_e \)

\( \theta \): Inclination change (°)

\( \tau \): Thrusting Time

\( \mu \): Mass Fractions, subsystem Mass / \( M_0 \)

\( \Delta V \): Velocity increment

**Subscript**

\( o \): initial

L: Payload, Support, and Contingency

w: Propulsion Subsystem

p: Propellant

opt: Optimal
TABLE 1. Various Thruster Efficiencies and Isp Ranges.

<table>
<thead>
<tr>
<th>Thruster Type</th>
<th>Efficiency Factor b</th>
<th>Efficiency Factor d (m/s)</th>
<th>Nominal Isp Operation Range</th>
<th>&quot;Derated&quot; Isp Operation Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xenon Ion</td>
<td>0.81</td>
<td>13500</td>
<td>3500 - 5000 s</td>
<td>800 - 3300 s</td>
</tr>
<tr>
<td>Krypton Ion</td>
<td>0.81</td>
<td>17000</td>
<td>5000 - 8000 s</td>
<td>1500 - 5100 s</td>
</tr>
<tr>
<td>Argon Ion</td>
<td>0.81</td>
<td>24500</td>
<td>7000 - 10000 s</td>
<td>5000 - 7000 s</td>
</tr>
<tr>
<td>H2 MPD: Isp &lt; 7800 s</td>
<td>0.86</td>
<td>25900</td>
<td>2000 - 7800 s</td>
<td></td>
</tr>
<tr>
<td>H2 MPD: Isp &gt; 7800 s</td>
<td>0.92</td>
<td>50300</td>
<td>7800 - 10000 s</td>
<td></td>
</tr>
<tr>
<td>Hydrogen Arcjet</td>
<td>Constant efficiency</td>
<td>.33</td>
<td>1000 - 1500 s</td>
<td></td>
</tr>
</tbody>
</table>
An Analytical Optimization of Electric Propulsion Orbit Transfer Vehicles

Steven R. Oleson

Sverdrup Technology, Inc.
Lewis Research Center Group
2001 Aerospace Parkway
Brook Park, Ohio 44142

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135–3191

Due to electric propulsion’s inherent propellant mass savings over chemical propulsion, electric propulsion orbit transfer vehicles (EPOTVs) are a highly efficient mode of orbit transfer. When selecting an electric propulsion device (ion, MPD, or arcjet) and propellant for a particular mission, it is preferable to use quick, analytical system optimization methods instead of time intensive numerical integration methods. It is also of interest to determine each thruster’s optimal operating characteristics for a specific mission. Analytical expressions are derived which determine the optimal specific impulse (Isp) for each type of electric thruster to maximize payload fraction for a desired thrusting time. These expressions take into account the variation of thruster efficiency with specific impulse. Verification of the method is made with representative electric propulsion values on a LEO-to-GEO mission. Application of the method to specific missions is discussed.

Electric propulsion; Orbit transfer; Ion engines; Arcjets; MPD thrusters; Analytic mission design; Optimal Isp; Optimal power