Unstructured Viscous Grid Generation by Advancing-Front Method

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BY ADVANCING-FRONT METHOD

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Abstract

A new method of generating unstructured triangular/tetrahedral grids with high-aspect-ratio cells is proposed. The method is based on a new grid-marching strategy referred to as 'advancing-layers' for construction of highly stretched cells in the boundary layer and the conventional advancing-front technique for generation of regular, equilateral cells in the inviscid-flow region. Unlike the existing semi-structured viscous grid generation techniques, the new procedure relies on a totally unstructured advancing-front grid strategy resulting in a substantially enhanced grid flexibility and efficiency. The method is conceptually simple but powerful, capable of producing high quality viscous grids for complex configurations with ease. A number of two-dimensional, triangular grids are presented to demonstrate the methodology. The basic elements of the method, however, have been primarily designed with three-dimensional problems in mind, making it extendible for tetrahedral, viscous grid generation.

Introduction

During the past few years, computational fluid dynamics (CFD) has progressed to the point that routine, accurate inviscid-flow computations are now possible for practical, realistic configurations in short periods of time. The turn-around time required for a typical inviscid-flow calculation (including grid generation) for a complex geometry is now in order of days using the unstructured grid methodology\(^1\)-\(^4\) as compared to weeks or even months with the computational capabilities of only a few years ago. With such rapid progress in development of new CFD methodology (grid generation, flow solution, and graphics), the goal of highly automated CFD (challenged by the industry) seems to be not too far from reality any longer.
A significant part of the recent accomplishment has been due to the emergence of new, efficient grid generation techniques. Unstructured grids, for example, have been extremely successful in simplifying the complexity of discretizing inviscid flow-fields. The growing number of new methods and publications implies the importance and increased popularity of the unstructured grid methodology. Among the various unstructured grid generation methods available, two main classes of grid strategies have been well recognized over the years: Delaunay triangulation\textsuperscript{5–7} and advancing-front\textsuperscript{8,9}. A combination of the two methods has also been reported recently\textsuperscript{10–12}. While the Delaunay-triangulation-based algorithms are usually more efficient, many of them lack the self-sufficiency (point distribution), robustness, and grid quality of the advancing-front grid generators. A detailed comparison of the two strategies is given in Ref. 12.

Generally speaking, many consider the problem of 3D unstructured grid generation resolved for the inviscid-flow computation in view of the recent developments. However, unstructured grids have not been as effective in discretizing complex domains for routine computation of the Navier-Stokes equations. The problem of generating highly stretched cells to resolve the boundary layer, especially in three dimensions, has proven to be non-trivial. Unlike the variety of the Euler grid generation techniques available, there are only a few methods in the literature addressing the problem of viscous, unstructured (semi-unstructured) grid generation. Among these are the techniques reported in Refs. 13-15. The grid generation approach of Ref. 13 is based on a modified Delaunay triangulation in which an initial point distribution is used to define a set of stretching vectors and locally mapped spaces for insertion and reconnection of new grid points. This method, like most Delaunay-based grid generation techniques, relies on an existing point distribution and has been demonstrated for two-dimensional grid generation only. The methods of Refs. 14 and 15 are directionally structured grid generation techniques in which prismatic cells extend from inner to outer triangulated boundaries. This class of grids are only partially unstructured and retain some of the limitations of structured grids such as grid generation complexity at sharp corners, matching prisms which approach from opposite
close boundaries, gridding the wake region, etc. In addition, the total number of cells becomes inefficiently large as the same number of prisms usually extend from the inner to the outer boundaries.

This paper introduces a new approach for generation of highly stretched grids. The method is based entirely on the advancing-front technique and, thus, benefits from the generality, flexibility, and grid quality of the conventional advancing-front-based Euler grid generators. The method is self-sufficient for insertion of grid points in the boundary layer and beyond. Being based on a totally unstructured grid strategy, the new method alleviates the difficulties stemming from the structural limitations of the prismatic techniques. In this paper, the methodology and some results in two dimensions are presented. However, the approach is not specific to two-dimensional problems and should be readily extendible to three dimensions.

**Approach**

The proposed grid generation process is divided into three separate stages: 1) surface grid generation, 2) construction of high-aspect-ratio cells in the viscous region, and 3) generation of regular (ideally equilateral) cells in the inviscid-flow region. An existing robust advancing-front-based Euler grid generation system (VGRID$^{18,19}$) is used to perform steps 1 and 3.

Due to the high-aspect-ratio requirement for cells in the viscous-dominated regions, a regular advancing-front strategy, like that conventionally used for the Euler grid generation, is inadequate for generation of viscous grids without extensive modifications. Any such enhancement would require a mechanism to distribute grid points densely in one direction and sparsely in other(s) to create highly stretched cells in the boundary layer and isotropic point distribution elsewhere. This, in turn, would require additional information such as stretching directions and ratios prescribed in the background grid, distribution of stretching information in the domain, a coordinate transformation which would incorporate multi-dimensional stretching to form cells in an unstretched space, and an inverse transformation
to obtain the final stretched grid. All these additional elements would complicate the entire process of the conventional advancing front considerably.

The process of viscous grid generation is greatly simplified if the generation of high-aspect-ratio cells in the viscous region is performed separately from that of regular unstretched cells outside the boundary layer. To that end, a special advancing-front strategy, referred to here as 'advancing-layers', has been devised for generation of viscous grids (step 2). The goal is to enhance the existing advancing-front technique for specific generation of stretched cells while maintaining the desirable features of the original advancing-front method, e.g., self-sufficiency, robustness, automation, and high degree of grid quality and flexibility. The new method is supplemental to the conventional advancing-front technique. The following sections explain the grid generation stages by the conventional and modified advancing-front methods.

**Advancing-Front**

In a conventional advancing-front method, inviscid grids are typically generated by forming cells starting from the boundaries (initial front) and marching towards the interior of the computational domain. The front (boundary between the gridded and ungridded regions) is advanced into the field in an irregular manner with no predetermined, orderly structure while introducing new grid points in the field. The process is continued until the entire domain is filled with contiguous cells. The basics of the conventional advancing-front grid generation procedure, used in this work, is described in Ref. 16 in detail. A revised version of the method is reported in Ref. 17. A distinctive characteristic of this method is that both the insertion (distribution) of grid points in the field and the process of forming grid connectivities are performed simultaneously. Since the new grid points are consistently positioned at the best possible locations in relation to the current front, the method generally results in good quality grids while enjoying a high degree of flexibility and self-sufficiency.
The generation process starts with discretization of the boundaries of the computational domain to form a surface mesh or initial front. The surface mesh used for viscous grid generation, in this method, is identical to that for generating Euler grids. Consequently, the same surface mesh as well as the Euler volume grid generation routines of Refs. 16 and 17 are employed with no modifications. A surface grid is generated by first defining the configuration of interest in terms of line segments in two dimensions and surface patches in three dimensions. Grid points are then distributed along the lines to form an initial front. In three dimensions, surface patches are further triangulated to construct the surface grid. After a surface mesh is generated, triangular or tetrahedral cells are formed on each ‘face’ (line segments in 2D or surface triangles in 3D) which, in turn, create more faces on the front. The number of faces on the front initially increases as the front advances in the field and then gradually declines, until it eventually goes to zero when the grid is complete.

The desired distribution of grid points on the surface and in the field is controlled by the information stored in a secondary, coarse mesh referred to as the ‘background grid’. The quality of the final grid, to a large extent, depends on the background grid and the spacing distribution which it provides. Since the role of a background grid is only to supply grid information without the requirement of conforming to the configuration, a uniform Cartesian mesh is used as a background grid. Associated with a Cartesian background grid are a number of point and line source elements with prescribed grid characteristics such as spacing parameters. The grid spacings specified at the source elements diffuse throughout the domain, onto the nodes of the background grid, as modeled by solving a Poisson equation. A detailed description of structured background grids is given in Ref. 18. Once the spacing distribution is determined, the background mesh along with the sources control the grid generation process for a smooth clustering of points on the surface and in the field. The use of Cartesian background grids has considerably improved the advancing-front process, resulting in excellent Euler grids as reported in Refs. 4, 17-19. In this work, the same background mesh is used for generation of grids in both viscous and inviscid regions. No modifications, adjustments, or addition of extra information to the
background grid are necessary for generation of viscous grids.

Advancing-Layers

As in the conventional advancing-front technique, cells are formed in the new approach starting from the boundaries and progress into the domain. However, unlike the generation procedure of the conventional method in which cells are added in no systematic sequence, the construction of stretched grid is done by advancing one layer of cells at a time. This strategy is employed to 1) minimize the front congestion, 2) minimize the complexity of search-and-check operations when selecting an optimum cell connectivity, and 3) evenly distribute stretched cells on all 'no-slip' boundaries, causing fronts from opposite, close walls to meet at midway. By bringing this order into the advancing-front process, not only does the operation become less complicated, but also its efficiency improves considerably. As a result, the need for a CPU-intensive face-cross check is totally eliminated for the advancing layers without losing the capability of detecting nearby fronts. The face-cross-check operation of the conventional advancing-front method accounts for about 5% of the total grid generation time in 2D (15% in 3D.)

Due to the concentration of grid points over a small length scale across the boundary layer, the position of each new point being introduced in the field has a significant effect on the quality of the generated viscous grid. Even a modest misplacement of grid points, for example, could adversely affect the side angles of highly stretched triangular (tetrahedral) cells in this region. Likewise, the complexity of the viscous grid generation process, especially at sharp corners, largely depends on the arrangement of grid points in the boundary layer. To ensure a good quality grid and to prevent complications during the advancement of cell layers, the position of grid points are determined at specific locations in the viscous region. Unlike the conventional advancing-front method in which new 'ideal' points are placed along individual 'face' vectors (originated from the centroid and normal to the plane of each 'face' on the front), the new approach positions grid points along a set of surface vectors which pass through the corners of the faces on the cell layers. The result
is a more organized grid structure and a practicable grid-advancement strategy which, in turn, produces better grid quality without complications, especially at corners.

The surface vectors are originally determined at each mesh point by first averaging the normal unit vectors of the faces sharing the point and then smoothing the vectors by a Laplacian smoothing operation through the following iterative scheme.

\[
v_{p}^{t+1} = (1 - \omega)v_{p}^{t} + \omega \frac{1}{N_p} \sum_{n=1}^{N_p} v_{n}^{t}
\]  

where \(v_{p}\) is a surface vector at grid point \(p\), \(v_{n}\) is a vector at the \(n^{th}\) ‘neighboring’ point connected to point \(p\), \(\omega\) is a relaxation parameter (0\(<\omega\)<2), \(t\) is an iteration level, \(t\) represents the latest iteration level at point \(n\), and \(N_p\) is the number of points connected to point \(p\). The number of ‘neighboring’ points, \(N_p\), is a constant equal to 2 in two dimensions and a variable greater than 2 in three dimensions. Equation (1) is solved iteratively on the interior surface grid points for each line segment (or surface patch) with the vectors at the line (patch) boundaries kept fixed. Figure 1 shows surface vectors for a simple configuration before and after smoothing.

The advancing-layers process starts by introducing new grid points in the field along the surface vectors and connecting them to the corresponding faces on the front. The distribution of grid points is determined along the surface vectors by a stretching function rather than a background grid as in the conventional advancing-front method. A geometric function is used, in this work, to determine the grid spacing for each layer of cells,

\[
\delta_{\ell} = \delta_1 (1 + r)^{\ell - 1}
\]  

where \(\delta_{\ell}\) is the spacing for the \(\ell^{th}\) layer, and \(r\) is a prescribed rate of stretching across the boundary layer. The spacing for the first layer of cells, \(\delta_1\), is also prescribed by the user.

As in the conventional method, front faces are successively selected from a list and used to form cells. After a cell is constructed, the old face becomes ‘inactive’ and is removed
from the front, and new faces are created and added to the list. Three types of faces are identified on the front as the layers advance in the field: 1) faces with all nodes on the surface grid or the previously constructed layer (primary faces), 2) faces with one node (in 2D) on the surface grid or the previous layer and one on the current layer, each along a different surface vector (secondary faces), and 3) faces similar to secondary faces, but both nodes along the same vector (cross-sectional faces). Figure 2 shows different types of faces on a two-dimensional front. Only the primary and secondary faces are selected to form cells. Although considered ‘active’, a cross-sectional face is simply skipped as it will eventually be removed along with an adjacent primary or secondary face.

To form a cell on a front face, either an existing point on the front or a new ‘ideal’ point is used. In the conventional advancing-front method, one ‘ideal’ point is provisionally considered for each face to be removed. In the new method, two or one ‘ideal’ points are considered for the primary or secondary faces, respectively. In addition to the ‘ideal’ points, all existing points (those from other fronts which are within a certain distance from the face being removed) are identified and considered as candidate points for forming the next cell. All candidates, including the ‘ideal’ point(s), are then evaluated, and the best possible point is selected.

The criterion by which points are evaluated has a significant impact on the grid quality and the advancement process. In the conventional advancing-front method, criteria such as the cell apex or spherical angles usually perform satisfactorily to form regular cells. However, due to the high-aspect-ratio requirement of cells in the boundary layer, and since there is no coordinate transformation involved in the present method, the conventional criteria are not adequate for generation of highly stretched cells. Instead, a new criterion based on a ‘spring’ analogy is introduced and used in this work. The candidate points, while fixed in space, are assumed to be connected to the corners of the face to be removed by tension springs as shown in Fig. 3. The resulting force exerted on a point is equal to the sum of the tensions of the springs connected to the point which are, in turn, proportional to the spring displacements, i.e.,
where \( F_p \) is the total force exerted on point \( p \), \( s_n \) is the distance by which the \( n^{th} \) spring connected to point \( p \) is displaced (stretched) from its neutral length (i.e., the length when the point is positioned at the centroid of the face resulting in zero tension), and \( N \) is the number of springs connected to point \( p \) (2 in two- and 3 in three-dimensions.) Among the candidate points, the one with the smallest \( F_p \) is selected to form the next cell. Since the new criterion is not based on any angle that the prospective cell would make, a 'close' point forming the best possible stretched cell is always favored regardless of the angles being made. When the selected point is a new 'ideal' point, the criterion ensures that no other existing face is close enough to obstruct the new cell being formed. This eliminates the need for a face-crossing check as mentioned earlier.

The layers continue to advance in the field, while growing in thickness, until certain conditions are locally satisfied on the front. Obviously, when an existing point other than a new 'ideal' point is selected for a face, the advancement of layers stops on that face. This occurs when a face on the front approaches an opposite front. Even though a background grid does not directly determine the distribution of grid points during the generation of cell layers, it controls the extent by which the layers grow in the field. The advancement process locally terminates on a face when the grid spacing, determined by Eq. (2), matches that dictated by the background grid for that location. In two dimensions, the local growth of a layer is limited by any of the following constraints based on the side lengths and areas of the individual cells being formed.

\[
F_p \propto \sum_{n=1}^{N} s_n \tag{3}
\]

\[
\delta_t > \delta_b \tag{4a}
\]

\[
\delta_t > d_f \tag{4b}
\]

\[
\delta_t d_f > \delta_b^2 \tag{4c}
\]
where \( \delta_t \) is the spacing obtained from Eq. (2), \( \delta_b \) is the local spacing determined by the background grid, and \( d_f \) is the length of the face to be removed.

When the 'proximity' and/or 'spacing' criteria are satisfied on all faces on the front, the process automatically switches from the advancing-layers to the conventional advancing-front grid generation to form regular cells in the rest of the domain. The limiters given by Expressions (3) and (4) ensure a flexible convergence of approaching fronts and a smooth transition from a viscous, highly stretched grid to an inviscid, regular grid.

**Results**

Two-dimensional, triangular, viscous grids were generated around two configurations: 1) a simple NACA 0012 and 2) a complex multi-element airfoil to demonstrate the capability of the new approach. Euler grids were previously shown around the same configurations\(^{18} \) to demonstrate the use of structured background grids. Here, the same background grids are used again for generation of viscous grids to emphasize the simplicity and compatibility of input data prepared for generation of Euler grids. The only additional information required are the spacing for the first layer of cells, rate of grid stretching, and identification of the 'no-slip' boundaries. For present examples, a uniform first-layer spacing is specified along the boundaries.

The grid generated for the NACA 0012 airfoil, shown in Fig. 4, contains 6978 cells and 3552 points with 126 of the points on the boundaries. A moderate first layer spacing of 0.001 chord is specified for this example which has resulted in an aspect ratio of order of 20 to 1 for the first layer of cells. A corresponding Euler grid contains 5778 cells and 2952 points using the same background grid. Figure 4a shows the entire grid, representing a smooth distribution of points throughout the field. A close-up view of the grid around the airfoil is shown in Fig. 4b with the nose and tail sections further magnified as shown in Figs. 4c and 4d, respectively. A smooth transition of the grid from one type (generated by the advancing-layers process) to another is evident from these figures which indicates excellent compatibilities and communication among the two grid strategies and the background grid.
To resolve the wake region with viscous grid, 'transparent' line segments may be added to the wake, and stretched cell layers formed on both sides. Such a capability is planned for further work.

An additional grid was generated on the NACA 0012 airfoil with the aspect ratios of cells off the surface increased to an average value of 20000 to 1 to demonstrate the ability of producing extremely stretched cells in the boundary layer region. Figure 5 shows a close-up of the grid on the upper surface of the airfoil.

To further examine the capability of the method, a grid was generated around a multi-element configuration. The geometry consists of four airfoil sections in a high-lift arrangement. The generated grid, shown in Fig. 6, contains 23599 cells, 12034 points, and 475 boundary points with a first-layer spacing of 0.0003 chord. Figure 6a shows a partial grid around the geometry generated using the advancing-layers mode of the grid generator. As indicated, layers have advanced in the field until either the local grid spacings have matched those from the background grid, or the opposite fronts have approached. The complete grid around the components is shown in Fig. 6b. As in the previous example, the layers are smoothly blended with the regular grid, constructed by the advancing-front part of the grid generator, in such a way that the transition from one grid type to another is almost indistinguishable. Figures 6c and 6d show the partial and complete grids, respectively, at the front section. To complicate the grid even further, a small line source (not shown but described in Ref. 18) has been added to simulate a grid adapted to a shock wave on the upper nose region of the main airfoil. Note that the progression of cell layers at the shock location is automatically adjusted to the rapid variation of grid size (Fig. 6c), resulting in high resolution grids of both types coupled smoothly in that location as shown in Fig. 6d. The partial and complete grids, at the sharp trailing corners of the main airfoil, are shown in Figs. 6e and 6f, respectively. The generation of viscous grid, in both the convex and concave corners, is handled appropriately with no distortion or complications introduced. Finally, the grid in the narrow regions between the aft airfoil sections is shown in Figs. 6g and 6h. This is a good example to demonstrate the complexity.
of generating viscous grids between two close walls. A grid strategy based on the structured or semi-structured technique would have the difficulty of matching the opposite grids. In this example, layers from opposite walls are advanced towards each other to the point that gaps of about one cell layer are left open between the fronts at midways as shown in Fig. 6g. The gaps along with the rest of the domain are then filled with regular cells by the advancing-front grid generator as shown in Fig. 6h.

The grids, presented in this paper, were generated using a Silicon Graphics IRIS 4D/210 VGX workstation. Presently, the code's performance is (depending on complexity of the background grid used) about 360 triangles per second on that workstation and 2350 triangles per second on a CRAY YMP. The total turn-around time required for generating a grid around a complex geometry, such as the one presented here, is about an hour including the set-up of line segments defining the configuration, preparing the background-grid information such as the source element properties, a few iterations to produce the desirable grid point distribution on the surface and in the field, and generating the final grid. Being still in a developmental stage, the code is currently under further refinement and improvement for better performance and new capabilities. The inclusion of a graphic interface for setting up the background source elements, currently under way, will improve the applicability of the code considerably.

**Concluding Remarks**

A new method of unstructured viscous grid generation has been introduced. The method benefits from the flexibility and generality of the unstructured grid generation by advancing-front technique and provides smooth, structured-looking grids in the viscous region without producing complications usually associated with the structured grid strategies. Due to a new grid-marching approach, many of the shortcomings of the existing methods are resolved. The new method does not require coordinate transformations between the physical and unstretched spaces for generation of high-aspect-ratio cells. Due to a novel, simple, front-detecting procedure, the need for the expensive face-crossing check

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is eliminated for generation of viscous grids, resulting in an increased efficiency compared to the regular advancing-front method. In general, the proposed grid strategy along with the conventional advancing-front technique has provided a new, robust approach to the problem of unstructured viscous grid generation.

The method has been applied to two-dimensional problems with satisfactory results. Although the complexity of three-dimensional problems is compounded, the implementation of the concept to three dimensions should be straightforward as most of the essential elements are already worked out. The extension of the method to three dimensions as well as the implementation of ‘transparent’ lines in the wake region are planned for future work.

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References


Figure 1- Surface vectors used for the advancing-layers process.
Figure 2- Different face types on a front during the advancing-layers process: 1) primary face, 2) secondary face, and 3) cross-sectional face.

Figure 3- 'Close' points assumed to be connected to a face on the front by tension springs.
Figure 4- Continued.

(c) leading edge section

(d) trailing edge section
Figure 5. Unstructured viscous grid on the upper surface of a NACA 0012 airfoil showing highly stretched cells.
Figure 6- Unstructured viscous grid around a multi-element airfoil.
Figure 6- Continued.

(c) partial grid, front section

(d) complete grid, front section
(e) partial grid, tail section of main airfoil

(f) complete grid, tail section of main airfoil

Figure 6- Continued.
Figure 6- Continued.

(g) partial grid, tail section

(h) complete grid, tail section
A new method of generating unstructured triangular/tetrahedral grids with high-aspect-ratio cells is proposed. The method is based on new grid-marching strategy referred to as 'advancing-layers' for construction of highly stretched cells in the boundary layer and the conventional advancing-front technique for generation of regular, equilateral cells in the inviscid-flow region. Unlike the existing semi-structured viscous grid generation techniques, the new procedure relies on a totally unstructured advancing-front grid strategy resulting in a substantially enhanced grid flexibility and efficiency. The method is conceptually simple but powerful, capable of producing high quality viscous grids for complex configurations with ease. A number of two-dimensional, triangular grids are presented to demonstrate the methodology. The basic elements of the method, however, have been primarily designed with three-dimensional problems in mind, making it extendible for tetrahedral, viscous grid generation.