Optical Model Calculations of 14.6A GeV Silicon Fragmentation Cross Sections

Lawrence W. Townsend, Ferdous Khan, and Ram K. Tripathi

MAY 1993
Optical Model Calculations of 14.6A GeV Silicon Fragmentation Cross Sections

Lawrence W. Townsend  
Langley Research Center  
Hampton, Virginia

Ferdous Khan  
Old Dominion University  
Norfolk, Virginia

Ram K. Tripathi  
ViGYAN, Inc.  
Hampton, Virginia

NASA Technical Memorandum 4461
Symbols

$A$ nuclear mass number

$B(e)$ average slope parameter of nucleon-nucleon scattering amplitude, fm$^2$

$b$ projectile impact parameter vector, fm

$\binom{D}{d}$ binomial coefficient

$E$ energy, GeV

e two-nucleon kinetic energy in their center-of-mass frame, GeV

FSI frictional spectator interaction

$N$ total number of projectile nucleus neutrons

$n$ number of abraded neutrons

$p$ number of abraded protons

$T(b)$ probability for not removing single nucleon by abrasion

$y$ two-nucleon relative position vector, fm

$Z$ total number of projectile-nucleus protons

$z$ number of abraded protons

$z_0$ position vector of projectile along beam direction, fm

$\xi_T$ collection of constituent relative coordinates for target, fm

$\rho$ nuclear single-particle density, fm$^{-3}$

$\sigma$ cross section, fm$^2$ or mb

$\sigma(e)$ average nucleon-nucleon total cross section, fm$^2$ or mb

Subscripts:

abr abraded

$NN$ nucleon-nucleon

nuc nuclear

$P$ projectile

PF prefragment

$T$ target
Abstract

An optical potential abrasion-ablation collision model is used to calculate hadronic dissociation cross sections for a 14.6 GeV $^{28}$Si beam fragmenting in aluminum, tin, and lead targets. The frictional-spectator-interaction (FSI) contributions are computed with two different formalisms for the energy-dependent mean free path. These estimates are compared with experimental data and with estimates obtained from semiempirical fragmentation models commonly used in galactic cosmic ray transport studies.

Introduction

As the era of career astronauts who will crew Space Station Freedom, establish lunar bases, and explore the solar system approaches, concern is mounting over the possible harmful effects to these crews from the high-energy heavy ion component of solar and galactic cosmic rays (refs. 1 and 2). To adequately assess these risks, knowledge of cosmic ray interactions in bulk matter is required. A major source of uncertainty in these radiation risk assessments is the fragmentation cross-section data base (ref. 3) used as input into the studies. The present experimental data base is sparse, and many of the data are of questionable accuracy (ref. 4). Hence, accurate theories of nuclear fragmentation are hampered in their development by the paucity of relevant, high-quality experimental data.

The Langley Research Center is currently developing nuclear fragmentation models from two perspectives. At the most fundamental level, an abrasion-ablation fragmentation theory is being developed. The theory uses a detailed quantum-mechanical optical potential formalism to describe the abrasion (knockout) stage of the collision (refs. 5 and 6). No arbitrarily adjusted parameters are included in this model. The ablation stage of the collision theory uses a Monte Carlo cascade-evaporation model (ref. 7) to describe the prefragment deexcitation through particle emission channels. The details of the optical potential abrasion-ablation model are described in the next section.

Although this quantum-mechanical formalism is based upon quantum scattering theory, it is still under development and is far too complex and computationally inefficient to use within a transport code. Instead, a less sophisticated, semiempirical abrasion-ablation fragmentation model (ref. 8) has been developed for use in the Langley galactic cosmic ray transport codes. This semiempirical formalism treats the abrasion stage classically by using geometric arguments to estimate the prefragment cross sections. The ablation stage is described by a simple nuclear evaporation process. The model has one adjustable parameter—a second-order correction to the prefragment excitation energy.

In this work, the quantum-mechanical optical potential fragmentation model is described. Estimates are presented of fragmentation cross sections for $^{28}$Si beams at a total energy of 14.64 GeV (kinetic energy of 13.7 GeV) colliding with aluminum, tin, and lead targets. These predictions are compared with experimental measurements of semi-inclusive cross sections for 1n, 1p, 2n, and 2p productions reported by the E814 collaboration at Brookhaven National Laboratory (ref. 9). The effects on the results of modifying the frictional-spectator-interaction (FSI) contribution to include an energy-dependent mean free path obtained from a nonlocal optical model are investigated. For comparison, predictions from the semiempirical fragmentation models of the Langley Research Center and the Naval Research Laboratory are also reported. (See refs. 8 and 10.) The agreement between theory and experiment is poor. The main source
of disagreement appears to be the lack of detector acceptance corrections being applied to the data.

Optical Potential Fragmentation Model

In an abrasion-ablation collision model, the projectile nuclei, moving at relativistic speeds, collide with stationary target nuclei. At small impact parameters, portions of their nuclear volumes overlap and are sheared away in the collision. This is the abrasion process. The remaining piece of projectile matter, sometimes called a prefragment, continues its trajectory with essentially its precollision velocity. Because of the nuclear dynamics of the abrasion process, the projectile prefragment is in an excited state after the collision. This excess energy is removed in the ablation process by the emission of gamma radiation and/or the emission of one or more nuclear particles (e.g., nucleons or composites). The remaining nucleus is the nuclear fragment species which is experimentally detected and whose cross section is measured.

In the optical potential formalism (ref. 6), the cross section for producing, by abrasion, a prefragment of charge \( Z_{PF} \) and mass \( A_{PF} \) is given by

\[
\sigma_{abr}(Z_{PF}, A_{PF}) = \binom{N}{n} \binom{Z}{z} \int d^2b [1 - T(b)]^{n+z} [T(b)]^{A_{PF}} \tag{1}
\]

where

\[
T(b) = \exp[-A_T \sigma(e) I(b)] \tag{2}
\]

and

\[
I(b) = [2 \pi B(e)]^{-3/2} \int dz_0 \int d^3 \xi_T \rho_T (\xi_T) \int d^3 y \rho_P (b + z_0 + y + \xi_T) \exp\left[-\frac{y^2}{2B(e)}\right] \tag{3}
\]

In equations (1) through (3),

- \( b \) = impact parameter vector
- \( e \) = two-nucleon kinetic energy in their center-of-mass frame
- \( z_0 \) = target center-of-mass position in projectile rest frame
- \( \xi_i \) = internal coordinates of colliding nuclei, \( i = P \) or \( T \)
- \( A_i \) = mass numbers of colliding nuclei, \( i = P \) or \( T \)
- \( y \) = projectile-nucleon–target-nucleon relative separation vector

The nuclear number densities \( \rho_i (i = P \text{ or } T) \) are obtained from the appropriate charge densities by an unfolding procedure (ref. 5). The constituent-averaged nucleon-nucleon cross sections \( \sigma(e) \) are given in references 1, 5, and 6. Values for the diffractive nucleon-nucleon scattering slope parameter \( B(e) \) are obtained from the parameterization in reference 11.

In equation (1), a hypergeometric charge dispersion model is chosen to describe the distribution of abraded nucleons. The model assumes that \( z \) out of \( Z \) projectile protons and \( n \) out of \( N \) projectile neutrons are abraded where

\[
N + Z = A_P \tag{4}
\]

\[
A_{PF} = A_P - n - z \tag{5}
\]

and \( \binom{A}{B} \) denotes the usual binomial coefficient expression from probability theory.
Prefragment excitation energies are estimated from

\[ E_{\text{exc}} = E_s + E_{\text{FSI}} \]  

(6)

where the surface energy term \( E_s \) is calculated by using the clean-cut abrasion model of reference 12. The frictional-spectator-interaction energy \( E_{\text{FSI}} \) contribution is calculated with either the model of Rasmussen and collaborators (ref. 13) or a modified form which includes an energy-dependent nuclear mean free path obtained from the nonlocal optical model results of Negele (ref. 14).

In the Rasmussen model, the rate of energy transfer is

\[ \frac{dE}{dx} = -\frac{\alpha}{\lambda} E = -\frac{E}{4\lambda} \]  

(7)

where

\[ \lambda = \frac{1}{\rho \sigma_{NN}} \left( \sigma_{NN} \approx \frac{300}{E} \right) \]  

(8)

yields

\[ \frac{dE}{dx} = -12.75 \text{ MeV/fm} \]  

(9)

If a spherical nucleus of uniform density is assumed, the average energy deposited per interaction is

\[ \langle E_{\text{FSI}} \rangle \approx 10.2A^{1/3} \text{ MeV} \]  

(10)

which yields 31 MeV/FSI for a silicon projectile.

In the modified \( E_{\text{FSI}} \) model, the rate of energy transfer is also given by equation (7); however, the mean free path (eq. (8)) is replaced by

\[ \lambda = 16.6E^{-0.26} \]  

(11)

which parameterizes the nonlocal optical model results of Negele (ref. 14). Therefore, equation (9) is replaced by

\[ \frac{dE}{dx} = -0.015E^{1.26} \]  

(12)

For a spherical nucleus with a uniform density, the average energy deposited per FSI becomes

\[ \langle E_{\text{FSI}} \rangle \approx 0.0126A^{1/3}E^{1.26} \]  

(13)

Assuming a relative kinetic energy of \( \approx 85 \text{ MeV} \) for each FSI nucleon gives

\[ \langle E_{\text{FSI}} \rangle \approx 3.4A^{1/3} \]  

(14)

which yields 10 MeV/FSI for the silicon projectile.

Therefore, the abrasion cross section for a prefragment species \((Z_{PF}, A_{PF})\) which has undergone \( q \) frictional spectator interactions is (ref. 15)

\[ \sigma_{\text{abr}} (Z_{PF}, A_{PF}, q) = \binom{n + z}{q} (1 - P_{\text{esc}})^q (P_{\text{esc}})^{n+z-q} \sigma_{\text{abr}} (Z_{PF}, A_{PF}) \]  

(15)
where \( P_{\text{esc}} \) is the probability that an abraded nucleon escapes without undergoing any frictional spectator interactions. In the present report, the choice of \( P_{\text{esc}} = 0.5 \) follows from the original work of Rasmussen (ref. 13).

Depending upon the magnitude of its excitation energy, the prefragment will decay by emitting nucleons, composites, and gamma rays. The probability \( \alpha_{ij}(q) \) that a prefragment species \( j \), which has undergone \( q \) frictional spectator interactions, deexcites to produce a particular final fragment of type \( i \) is obtained with the EVA-3 Monte Carlo cascade-evaporation computer code (ref. 7). Therefore, the final hadronic cross section for production of the type \( i \) isotope is obtained from

\[
\sigma_{\text{nuc}}(Z_i, A_i) = \sum_{j} \sum_{q=0}^{n+z} \alpha_{ij}(q) \sigma_{\text{abr}}(Z_j, A_j, q)
\]

where the summation over \( j \) accounts for contributions from different prefragment isotopes \( j \), and the summation over \( q \) accounts for the effects of different FSI excitation energies.

**Exclusive Channel Results**

Exclusive (specific) channel cross-section estimates obtained with the optical potential abrasion-ablation fragmentation model described in the previous sections are given separately in tables 1 through 3 for each target. For each fragmentation channel, cross-section predictions are displayed for the present work (with two different \( E_{\text{FSI}} \) formalisms given by eqs. (10) and (14)). Also displayed are previously published estimates (ref. 15) obtained with this optical model but which treated \( E_{\text{FSI}} \) as a free parameter. Finally, for comparison purposes, results obtained from two semiempirical fragmentation models (refs. 8 and 10), commonly used for cosmic ray shielding studies, are also shown.

It is apparent from these calculations that significant variations in the cross sections are obtained from different models and even from the same model with different underlying assumptions. For the present work (optical-model results), the cross-section variations (e.g., \( ^{27}\text{Al} + p \) or \( ^{26}\text{Al} + p + n \)) probably arise because particle-emission thresholds in the ablation stage of the reaction result in a sensitivity to variations in \( E_{\text{FSI}} \), which are readily apparent when exclusive channels are considered. Therefore, these variations suggest that exclusive channel laboratory measurements must be made so that the underlying physical assumptions in the theories can be tested and verified or rejected.

**Inclusive Cross-Section Results**

Although exclusive channel laboratory measurements for hadronic dissociation of \( ^{28}\text{Si} \) beams at kinetic energies of 13.7 GeV have not been made, semi-inclusive measurements for \( 1n \), \( 1p \), \( 2n \), and \( 2p \) removal have been made and reported (ref. 9). To compare our predictions with these semi-inclusive measurements, we sum the exclusive channels listed in the tables for each target for each of the relevant nucleon-emission reactions. For example, the \( 1n \) semi-inclusive calculation for an aluminum target is the sum of the \( ^{27}\text{Si} + n \), \( ^{26}\text{Al} + p + n \), \( ^{25}\text{Mg} + 2p + n \), and \( ^{24}\text{Na} + 3p + n \) exclusive channels from table 1. Similarly, the \( 1p \) semi-inclusive calculation for the lead target is the sum of the exclusive channel cross sections for the \( ^{27}\text{Al} + p \), \( ^{26}\text{Al} + p + n \), \( ^{25}\text{Al} + p + 2n \), and \( ^{24}\text{Al} + p + 3n \) reactions from table 3. The results for all semi-inclusive cross-section calculations from tables 1 through 3 and the experimental measurements reported in reference 9 are listed in table 4. The experimental data are approximate values obtained directly from figure 4 of reference 9. In general, except for isolated cases, none of the calculated values agree with the reported measurements within reasonable accuracy. The typical cross-section differences are on the order of 50 percent or greater. These results are not surprising. As
discussed in reference 15, the experimental data have not been corrected for detector acceptance. Therefore, the observed trend that the estimated cross sections tend to generally overestimate the measured ones is expected because the experimental data underestimate the actual cross sections by some unknown amount. In addition, the measured cross sections include a “trigger bias” (ref. 9) which requires a heavy fragment to be detected simultaneously with the light products (1p, 1n, 2p, 2n, etc.). Therefore, it is possible that the measured data underestimate the actual cross sections even further. Resolution of these theory-experiment differences awaits the application of relevant corrections to the experimental measurements.

From table 4, we observe that the inclusive cross sections also exhibit a moderate sensitivity to $E_{FSI}$ but are difficult to use for accurate analyses of the physics in the fragmentation model because they each include contributions from several exclusive channels.

Concluding Remarks

An optical potential abrasion-ablation-frictional-spectator-interaction fragmentation theory has been described and applied to the problem of the breakup of relativistic silicon beams in heavy targets. The predictions of exclusive channel cross sections were found to be sensitive to the assumed FSI energies. Predictions of inclusive cross sections were found to be only moderately sensitive to the assumed FSI energies. Comparisons of theory to experiment were hampered by the lack of exclusive channel cross-section measurements and by the lack of detector-acceptance corrections for the inclusive 1n, 1p, 2n, and 2p cross-section data. Clearly, exclusive channel measurements and proper detector acceptance corrections are required before more meaningful, quantitative comparisons between theory and experiment can be made.

NASA Langley Research Center
Hampton, VA 23681-0001
March 22, 1993

References


Table 1. Calculations of Exclusive Channel Hadronic Dissociation Cross Sections for Silicon Beams at 13.7 A GeV Kinetic Energy Fragmenting in Aluminum Targets

<table>
<thead>
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<th>Fragment channel</th>
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Table 2. Calculations of Exclusive Channel Hadronic Dissociation Cross Sections for Silicon Beams at 13.7A GeV Kinetic Energy Fragmenting in Tin Targets

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Table 3. Calculations of Exclusive Channel Hadronic Dissociation Cross Sections for Silicon Beams at 13.7A GeV Kinetic Energy Fragmenting in Lead Targets

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Table 4. Inclusive Channel Cross Sections for 13.7A GeV Silicon Beams
Fragmenting in Aluminum, Tin, and Lead Targets

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