Calculations of Turbulent Separated Flows

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ABSTRACT

A numerical study of incompressible turbulent separated flows is carried out by using two-equation turbulence models of the K-ε type. On the basis of realizability analysis, a new formulation of the eddy-viscosity is proposed which ensures the positiveness of turbulent normal stresses—a realizability condition that most existing two-equation turbulence models are unable to satisfy. The present model is applied to calculate two backward-facing step flows. Calculations with the standard K-ε model and a recently developed RNG-based K-ε model are also made for comparison. The calculations are performed with a finite-volume method. A second-order accurate differencing scheme and sufficiently fine grids are used to ensure the numerical accuracy of solutions. The calculated results are compared with the experimental data for both mean and turbulent quantities. The comparison shows that the present model performs quite well for separated flows.

1. INTRODUCTION

Turbulent separated flows occur in a number of engineering applications. Because of their great practical importance, there is a strong demand for calculation methods to predict such flows. Turbulent flow over a backward-facing step is one of the most extensively used benchmark cases in the study of turbulence models for separated flows. It involves severe adverse pressure gradient, streamline curvature, coexistence of both strong and weak shear layers as well as significant extra strain rates in more than one direction, thereby constituting a severe test for turbulence models. If a turbulence model can correctly simulate this flow, it will be likely to be successful with other complicated flows.

The relevant experimental studies on backward-facing step flows are reported in
Bradshaw and Wong (1972), Driver and Seegmiller (1985), Driver et al. (1987), Durst and Schmitt (1985), Eaton and Johnston (1980), Kim et al. (1978, 1980), Stevenson et al. (1984) and Westphal et al. (1981). Among them, the case of Kim et al. (1978) with a larger expansion was a test case (0421) for the 1980-81 Stanford Conference on Complex Turbulent Flows (Kline et al., 1981), which has extensively been used to validate numerical calculations. However, this case has no turbulent data in the recirculation zone. The case of Driver and Seegmiller (1985) with a smaller expansion provides detailed data, including the wall friction coefficient and the turbulent quantities up to triple correlations.

The recent calculations with turbulence modeling can be found in Avva et al. (1990), Celenligil and Mellor (1985), Obi et al. (1989), So and Lai (1988), Speziale and Ngo (1988), Speziale and Thangam (1992) and Thangam and Hur (1991). The calculations of Celenligil and Mellor, Obi et al., and So and Lai were carried out with second-order closures, and the others with the standard K-\(\varepsilon\) model and its variants. These calculations show that the K-\(\varepsilon\) model largely underpredicts the reattachment point which is a sensitive parameter to assess the overall performance of turbulence models. No definitive conclusion can be drawn with the second-order closures, because Celenligil and Mellor obtained an overprediction, while Obi et al. and So and Lai obtained an underprediction of the reattachment point. The overall improvement achieved with these second-order closures is not strong enough to establish their convincing superiority over the K-\(\varepsilon\) model in calculating separated flows.

In the standard K-\(\varepsilon\) model, all the model coefficients are constant which are determined from a set of experiments for simple turbulent flows. Numerical experience over the last two decades has shown that this set of constants have a broad applicability, but they should not be expected to be universal. Rodi (1972) found that the K-\(\varepsilon\) model's ability to predict weak shear flows can be significantly improved by using \(C_\mu\) as a function of the average ratio of \(P/\varepsilon\) (\(P\) is the production of the turbulent kinetic energy) instead of a constant. Leschziner and Rodi (1981) proposed a function for \(C_\mu\) which takes into account the effect of streamline curvature and obtained improved results in the calculation of annular and twin parallel jets. Recently, Yakhot and co-workers have developed a version of the K-\(\varepsilon\) model using Renormalization Group (RNG) method. This model is of the same form as the standard K-\(\varepsilon\) model, but all the model coefficients assume different values. In the latest version of the RNG based K-\(\varepsilon\) model (Speziale and Thangam, 1992), the coefficient \(C_1\) related to the production of dissipation term is set to a function of \(\eta\), where \(\eta\) is the time scale ratio of the turbulence to the mean flow field. The reattachment point predicted by this model is within 5% of the experimental value for the case of
Kim et al. (1978).

In this study, the realizability principle (Schumann, 1977 and Lumley, 1978) is applied to analyze the K-\( \epsilon \) model. The analysis results in a new formulation of \( C_\mu \) which is a function of time scale ratio of the turbulence to the mean strain rate. The new \( C_\mu \) will ensure the positivity of each component of the turbulent kinetic energy – realizability that most existing eddy-viscosity models do not satisfy. The model validation is made on the basis of applications to the two backward-facing step flows experimentally studied by Driver and Seegmiller (1985) and Kim et al. (1978). Calculations are carried out with a conservative finite-volume method, and a second-order accurate and bounded differencing scheme together with sufficiently fine grids is used to ensure the solution both grid-independent and free from numerical diffusion. The calculated results are compared in detail with experimental data as well as with those obtained using the standard K-\( \epsilon \) model and the RNG K-\( \epsilon \) model.

2. MATHEMATICAL FORMULATION

2.1 Governing Equations

For incompressible steady flows, the non-dimensional governing equations formulated within the framework of the K-\( \epsilon \) model may be written as

\[
U_{j,j} = 0
\]  
(1)

\[
(U_jU_i - \frac{1}{Re}U_{i,j} + \overline{u_iu_j})_j = -p_i
\]  
(2)

\[
[U_jK - (\frac{1}{Re} + \frac{\nu_t}{\sigma_K})K_{,j}]_j = P - \epsilon
\]  
(3)

\[
[U_j\epsilon - (\frac{1}{Re} + \frac{\nu_t}{\sigma_\epsilon})\epsilon_{,j}]_j = C_1 \frac{\epsilon}{K} P - C_2 \frac{\epsilon^2}{K}
\]  
(4)

\[
-\overline{u_iu_j} = -\frac{2}{3}K\delta_{ij} + \nu_t(U_{i,j} + U_{j,i})
\]  
(5)

\[
\nu_t = C_\mu \frac{K^2}{\epsilon}
\]  
(6)
\[ P = -u_i u_j U_{i,j} \]  

where non-dimensionalization is made by using the reference length \( L_{\text{ref}} \) and the reference velocity \( U_{\text{ref}} \). Accordingly, the flow Reynolds number is defined by

\[ Re = \frac{L_{\text{ref}} U_{\text{ref}}}{\nu} \]

In the standard K-\( \epsilon \) model (Launder and Spalding, 1974), the model coefficients \( C_\mu, C_1, C_2, \sigma_K \) and \( \sigma_\epsilon \) assume the following constant values:

\[ C_\mu = 0.09, \quad C_1 = 1.44, \quad C_2 = 1.92, \quad \sigma_K = 1, \quad \sigma_\epsilon = 1.3 \]  

and in the RNG K-\( \epsilon \) model (Speziale and Thangam, 1992), they are:

\[ C_\mu = 0.085, \quad C_1 = 1.42 - \frac{\eta(1 - \eta/4.38)}{1 + 0.015\eta^3}, \quad C_2 = 1.68, \quad \sigma_K = \sigma_\epsilon = 0.7179 \]

where

\[ \eta = SK/\epsilon, \quad S = (2S_{ij}S_{ij})^{1/2} \]

and

\[ S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}) \]

### 2.2 Realizability

Realizability (Schumann, 1977, Lumley, 1978) which requires the non-negativity of turbulent normal stresses is a basic physical and mathematical principle that the solution of any turbulence model equation should obey. It also represents a minimal requirement to prevent a turbulence model from producing unphysical results. In the following, we will apply this principle to derive constraint on the model coefficients.

Consider a deformation rate tensor of the form

\[
\begin{pmatrix}
U_{1,1} & 0 & 0 \\
0 & U_{2,2} & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

The continuity equation (1) gives

\[ U_{2,2} = -U_{1,1} \]
and from Eq. (5), the normal stress $\overline{u_1u_1}$ can be written as

$$\frac{\overline{u_1u_1}}{K} = \frac{2}{3} - C_\mu \eta$$

(15)

Note that in case of Eqs. (13) and (14), $\eta$ can be written as

$$\eta = \frac{2U_{1,1}K}{\varepsilon}$$

(16)

Physically, $\overline{u_1u_1}$ will decrease with an increase in the mean strain rate $U_{1,1}$, but $\overline{u_1u_1}$ cannot be driven to negative values. Therefore, realizability conditions for $\overline{u_1u_1}$ are:

$$\frac{\overline{u_1u_1}}{K} > 0, \quad \text{if } 0 < \eta < \infty$$

(17)

$$\frac{\overline{u_1u_1}}{K} \to 0, \quad \text{if } \eta \to \infty$$

(18)

$$\left(\frac{\overline{u_1u_1}}{K}\right)_\eta \to 0, \quad \text{if } \eta \to \infty$$

(19)

These conditions can be satisfied by specifying $C_\mu$ as:

$$C_\mu = \frac{2/3}{A + \eta}$$

(20)

where $A$ is a positive constant.

Similar analysis on $\overline{u_2u_2}$ also leads to Eq. (20). It should be mentioned that Eq. (20) also holds in the case of three-dimensional pure strain rates

$$\begin{pmatrix} U_{1,1} & 0 & 0 \\ 0 & U_{2,2} & 0 \\ 0 & 0 & U_{3,3} \end{pmatrix}$$

(21)

and that any deformation rate tensor can be written in the form of (21) in the principal axes of deformation rate tensor.

The use of Eq. (20) while keeping the other model coefficients the same as those in the standard K-\(\varepsilon\) model constitutes the present realizable isotropic K-\(\varepsilon\) model. The value of the extra model constant $A$ is taken as

$$A = 5.5$$

(22)

which has been found to work well for both the test cases considered in this study.
3. NUMERICAL SOLUTION

In two dimensions, the transport equations (1) to (4) can be written in the following general form

\[ [U_1 \phi - (\frac{1}{Re} + \frac{\nu}{\sigma_\phi})\phi_1], + [U_2 \phi - (\frac{1}{Re} + \frac{\nu}{\sigma_\phi})\phi_2], = S_\phi \]  

(23)

where \( \phi \) stands for \( U_1, U_2, K \) or \( \epsilon \). For the momentum equations, the source term \( S_\phi \) includes the cross-derivative diffusion terms.

The numerical method used to solve the system of equations (23) is a finite-volume procedure. It uses a non-staggered grid with all the dependent variables being stored at the same geometric center of each control volume. The momentum interpolation procedure of Rhie and Chow (1983) is used to avoid spurious oscillations usually associated with the non-staggered grid, and the pressure-velocity coupling is handled with the SIMPLEC algorithm (Van Doormal and Raithby, 1984). To ensure both accuracy and stability of numerical solution, the convection terms are approximated by a second-order and bounded differencing scheme (Zhu, 1991a), and all the other terms by the conventional central differencing scheme. The strongly implicit procedure of Stone (1968) is used to solve the system of algebraic equations. The iterative solution process is considered converged when the maximum normalised residue of all the dependent variables is less than \( 10^{-4} \). The details of the present numerical procedure are given in Zhu (1991b).

4. RESULTS AND DISCUSSION

The present model together with the standard K-\( \epsilon \) model and the RNG K-\( \epsilon \) model are applied to the two backward-facing step flows experimentally studied by Kim, Kline and Johnston (1978) and Driver and Seegmiller (1985), from here on referred to as KKJ- and DS-cases, respectively. Fig.1 shows the flow configuration and the Cartesian co-ordinate system used. Table 1 gives the flow parameters for both cases; here the experimental reference free-stream velocities and step heights are taken as the reference quantities for non-dimensionalization.

<table>
<thead>
<tr>
<th>Table 1. Flow parameters</th>
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<tbody>
<tr>
<td>case</td>
</tr>
<tr>
<td>KKJ</td>
</tr>
<tr>
<td>DS</td>
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</tbody>
</table>
Boundaries of the flows are inlet, outlet and solid wall. At the inlet, the experimental data are available for the streamwise mean velocity $U$ and the turbulent normal stresses $\overline{w^2}$ and $\overline{v^2}$. $K$ is calculated from these $\overline{w^2}$ and $\overline{v^2}$ with the assumption that

$$\overline{w^2} = \frac{1}{2}(\overline{u^2} + \overline{v^2})$$

and $\epsilon$ by

$$\epsilon = \frac{C_{\mu}^{3/4}K^{3/2}}{L}, \quad L = \min(0.41\Delta y, 0.085\delta)$$

where $\Delta y$ is the distance from the wall and $\delta$ is the boundary-layer thickness given in Table 1. At the outlet, the streamwise derivatives of the flow variables are set to zero. Influences of both inlet and outlet conditions on the solution are examined by changing the locations $L_s$ and $L_e$, and it has been found that in both cases, the distances given in Table 1 are already sufficiently far away from the region of interest. The standard wall function approach (Launder and Spalding, 1974) is used to bridge the viscous sublayer near the wall.

Grid dependence of solutions is examined by using two sets of non-uniform numerical grids which contain $110 \times 52$ (coarse) and $199 \times 91$ (fine) points for the KKJ-case and $106 \times 56$ (coarse) and $201 \times 109$ (fine) points for the DS-case. Fig.2(a) shows the friction coefficient $C_f$ at the bottom wall, calculated with the present model on the two grids in the KKJ-case. It can be seen that the grid refinement from $110 \times 52$ to $199 \times 91$ points does produce a noticeable difference. The same also holds true for the other two models. This indicates that the solutions obtained on the coarse grids have not yet been sufficiently close to the grid-independent stage. Recently, Thangam and Hur (1991) have conducted a highly-resolved calculation in the KKJ-case. They have found that quadrupling a $166 \times 73$ grid leads to only a minimal improvement. Therefore, the results with the fine grids can be considered as grid-independent. In the DS-case, the fine grid computations required 681/766/800 iterations and took approximately 8.3/9.3/9.8 minutes of CPU time for the standard/RNG/present model on the Cray YMP computer. Only find grid results will be presented in the following.

In Fig.2(b) the calculated friction coefficients with the three models are compared with the experimental data in the DS-case. No such experimental data are available in the KKJ-case. It can be seen from Fig.2(b) that all the three models largely underpredict the negative peak of $C_f$, pointing to limited accuracy of the wall function approach in the recirculation region. In the recovery region and downwards, the standard $K$-$\epsilon$ model agrees well with the experimental data, while both the RNG
and the present models basically give the same results which are somewhat underpredicted. For lack of good near-wall turbulence models for separated flows, it is difficult to judge the performance of the models with $C_f$ that is very sensitive to the near-wall turbulence modeling.

Table 2 compares the computed and measured reattachment points. They are determined in the calculation from the point where $C_f$ goes to zero. The reattachment point is a critical parameter which has often been used to assess the overall performance of turbulence models. Table 2 clearly shows that the results of both the present and the RNG models are much better than those of the standard model.

<table>
<thead>
<tr>
<th>Case</th>
<th>Experiment</th>
<th>Standard</th>
<th>RNG</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>KKJ</td>
<td>7.0 ±0.5</td>
<td>6.35</td>
<td>7.47</td>
<td>7.34</td>
</tr>
<tr>
<td>DS</td>
<td>6.1</td>
<td>4.99</td>
<td>6.01</td>
<td>5.77</td>
</tr>
</tbody>
</table>

Figs. 3(a) and 3(b) show the comparison of computed and measured static pressure coefficient $C_p$ along the bottom wall. In both cases, the standard K-ε model is seen to predict premature pressure rises, which is consistent with its underprediction of the reattachment lengths, while both the present and the RNG models capture these pressure rises quite well. The results of both the present and the RNG models are very similar, and only at the lower end of steep gradients can some noticeable difference be seen.

The streamwise mean velocity $U$ profiles are shown in Figs. 4(a) and 4(b) at four different downstream locations. Here again, the present and the RNG models yield essentially the same results. They predict reverse flows better than the standard K-ε model, but result in somewhat slower recovery in regions near the reattachment point. Interestingly enough, such a slower recovery has also been found in the RSM prediction by Obi et al. (1989). Further downstream, say at $z=20$ in Fig.4(b), the results of the three models nearly coincide with each other.

In the KKJ-experiment, a high degree of flow unsteadiness was present, causing the reattachment point to swing constantly within a range of one step height. As a result, no experimental data for turbulent quantities were available in the recirculation region. Conversely, the DS-experiment showed a lower unsteadiness of the flow and a smaller uncertainty of the reattachment location. Detailed turbulent data were provided in the whole region of interest. Therefore, the comparison of turbulent quantities are restricted only to the DS-case. Figs. 5 and 6 show the comparison of predicted and measured turbulent stresses at four $x$-locations, two before and two after the reattachment point. It is seen from Fig. 5 that the standard K-ε model overpredicts the turbulent shear stress all along the flow region, while the
present and the RNG models give a better agreement with the experimental data. The results of the present and the RNG models are virtually the same except in the near-step region \((x=2)\) where the RNG model gives a large underprediction. For the turbulent normal stresses in Fig.6, the RNG profiles differ from the present profiles. The RNG model largely underpredicts the turbulent normal stresses in the recirculation region \((x=2 \text{ and } 5)\). The present model produces the best results of all. These different results of the models may be traced to the different levels of the turbulent eddy-viscosity they predict. Fig.7 shows the turbulent eddy-viscosity profiles of the three models in the DS-case. The present and the RNG models considerably reduce the value of \(\nu_t\), but this reduction is more than enough for the RNG model in the near-step region \((x=2)\), resulting in the large underpredictions of the turbulent stresses there.

5. CONCLUSIONS

A new version of the K-\(\epsilon\) model has been developed in which the model coefficient \(C_\mu\) is related to the time scale ratio of the turbulence to the mean strain rate through the realizability analysis. The new model ensures the positivity of individual turbulent normal stresses, while the standard K-\(\epsilon\) model, like many others, can only ensure the positiveness of the turbulent kinetic energy – sum of the turbulent normal stresses. The present model has been compared with the standard K-\(\epsilon\) model and the recently proposed RNG K-\(\epsilon\) model as well as with the experiments in the calculations of the two backward-facing step flows. The comparison shows that the present model effectively reduces the turbulent eddy-viscosity level, resulting in significant improvement over the standard K-\(\epsilon\) model. The RNG model generally gives very similar predictions to the present model, but overly reduces the turbulent eddy-viscosity level in the recirculation region near the step. It should be noted that the set of model constants in the standard K-\(\epsilon\) model have a broad generality and have stood the test of time. The present model differs from the standard K-\(\epsilon\) model only in one model coefficient, while all the model coefficients in the RNG model are different from the standard values. Therefore, the present model could be expected to be more general than the RNG model.
REFERENCES


Figure 1. Backward-facing step geometry
Figure 2. Friction coefficient $C_f$ along the bottom wall
Figure 3. Static pressure coefficient $C_p$ along the bottom wall
Figure 4. Streamwise mean velocity $U$-profiles (key to symbols as in figure 3)
Figure 5. Turbulent shear stress profiles (key to symbols as in figure 3)

Figure 6. Turbulent normal stress profiles (key to symbols as in figure 3)
Figure 7. Turbulent eddy-viscosity profiles (key to symbols as in figure 3)
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