Vibration and Noise Analysis of a Gear Transmission System

F.K. Choy and W. Qian
The University of Akron
Akron, Ohio

and

J.J. Zakrajsek and F.B. Oswald
Lewis Research Center
Cleveland, Ohio

Prepared for the
29th Joint Propulsion Conference and Exhibit
cosponsored by the AIAA, SAE, ASME, and ASEE
Monterey, California, June 28–30, 1993
VIBRATION AND NOISE ANALYSIS OF A GEAR TRANSMISSION SYSTEM

F.K. Choy and W. Qian
Department of Mechanical Engineering
The University of Akron
Akron, Ohio 44325-3903

and

J.J. Zakrajsek and F.B. Oswald
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

Abstract

This paper presents a comprehensive procedure to predict both the vibration and noise generated by a gear transmission system under normal operating conditions. The gearbox vibrations were obtained from both numerical simulation and experimental studies using a gear noise test rig. In addition, the noise generated by the gearbox vibrations was recorded during the experimental testing. A numerical method was used to develop linear relationships between the gearbox vibration and the generated noise. The hypercoherence function is introduced to correlate the nonlinear relationship between the fundamental noise frequency and its harmonics. A numerical procedure was developed using both the linear and nonlinear relationships generated from the experimental data to predict noise resulting from the gearbox vibrations. The application of this methodology is demonstrated by comparing the numerical and experimental results from the gear noise test rig.

Nomenclature

- \{F(t)\} external and mass-imbalance excitations
- \{F_G(t)\} gear force
- \{F_c(t)\} force acting on casing structure
- \{F_s(t)\} shaft bow force
- \[G_A\] rotor angular acceleration
- \[G_v\] gyroscopic
- \[\mathcal{G}_A\] \[\Phi\]^T \[G_A\] [\Phi]
- \[\mathcal{G}_v\] \[\Phi\]^T \[G_v\] [\Phi]
- \[K_b\] bearing stiffness
- \[K_c\] casing structure stiffness
- \[K_s\] shaft bow stiffness
- \[K_A\] \[\Phi\]^T \[K_A\] [\Phi]
- \[K_b\] \[\Phi\]^T \[K_b\] [\Phi]
- \[K_c\] \[\Phi\]^T \[K_c\] [\Phi]
- \[K_{bx}, K_{by}\] bearing stiffness in x and y directions
- \[M\] mass matrix of rotor
- \[M_c\] mass matrix of casing structure
- \[W\] generalized displacement vector of rotor
- \[W_c\] generalized displacement vector of casing
- \[x\] rotor x-displacement
- \[x_c\] casing x-displacement
- \[y\] rotor y-displacement
- \[y_c\] casing y-displacement
- \[z\] rotor z-displacement
- \[z_c\] casing z-displacement
- \[\mu\] friction coefficient between the gear teeth surface
In response to the importance of the application and the limited amount of work performed in the area of gearbox noise and vibration research, a program was started with the main objective of developing the methodology required to predict gear transmission system noise and vibration. This paper describes the global dynamic model that was developed to simulate the noise and vibration of a multistage gear system. Results predicted by the numerical model are compared to experimental results obtained from the gear noise test rig at NASA Lewis Research Center.

Analytical Procedure for Gear System Vibration

To achieve the numerical solution, the generalized equations of motion developed in a previous paper (Choy, 1993) presents the numerical procedure using the modal synthesis method. Using the undamped modes for both the rotor-bearing-gear system and the gearbox structure, the generalized equations of motion for the system are transformed into the corresponding modal coordinates. This procedure significantly reduces the degrees of freedom of the system, and provides an estimate of the modal excitations at the various modes.

The modal transformation procedure can be developed by assuming that the generalized displacements can be represented by a linear combination of the modal characteristics, (Choy, 1991, 1993) as

\[
\{W\} = \begin{bmatrix}
\{\Phi_x\} \{A\} \\
\{\Phi_x\} \{B\} \\
\{\Phi_y\} \{B\} \\
\{\Phi_z\} \{D\} \\
\{\Phi_t\} \{D_t\}
\end{bmatrix}
\]

where A, B, D, and D_t are the time-dependent modal excitation functions, and \([\Phi_x], [\Phi_y], [\Phi_z], [\Phi_t]\), and \([\Phi_t]\) are the matrices containing the orthonormal modes of the corresponding rotor system. Similarly, for the gearbox housing structure,
The modal equations for the rotor system (Choy, 1991, 1993) can be written in modal coordinates as:

\[
\{W_c\} = \begin{bmatrix}
\{\Phi_c\} & \{A_c\} \\
\{\Phi_{cx}\} & \{A_c\} \\
\{\Phi_{cy}\} & \{B_c\} \\
\{\Phi_{cx}\} & \{B_c\} \\
\{\Phi_{cy}\} & \{D_c\} \\
\{\Phi_{cx}\} & \{D_c\}
\end{bmatrix} \begin{bmatrix}
\{\epsilon\}
\end{bmatrix}
\]

using

\[
\{Z\} = \begin{bmatrix}
\{A\} \\
\{B\} \\
\{D\} \\
\{D_t\}
\end{bmatrix} = \begin{bmatrix}
\{A_c\} \\
\{B_c\} \\
\{D_c\}
\end{bmatrix} \begin{bmatrix}
\{\epsilon\}
\end{bmatrix}
\]

The modal equations for the rotor system (Choy, 1991, 1993) can be written in modal coordinates as:

\[
+ [\Phi]^T \{F(t) + F_G(t) + F_b(t)\}
\]

and for the casing

\[
[I] \{\ddot{Z}_c\} = [G_c] \{\dot{Z}_c\} - [\omega^2] \{Z_c\} - [K_b] \{Z_c\} - [C_b] \{\dot{Z}_c\} \\
+ [\Phi_c]^T [K_b] [\Phi_c] \{Z\} + [\Phi_c]^T [C_b] [\Phi_c] \{\dot{Z}\} + [\Phi_c]^T \{F_c(t)\}
\]

Using the appropriate initial conditions for the modal velocity \{\dot{Z}\} and displacement \{Z\}, the modal accelerations \{\ddot{Z}\} can be solved. The solution procedure can be summarized as follows:

1. Input initial conditions to calculate modal accelerations.

2. Use variable time-stepping integration scheme to evaluate modal velocity and displacement at next time step.

3. Calculate velocity and displacement in generalized coordinates.

4. Calculate nonlinear gear mesh forces due to relative motion between the two gears.

5. Calculate bearing forces due to relative motion between the rotor and the casing.

6. Calculate modal bearing and gear forces.

7. Repeat from step number 2.

**Experimental Studies**

Test results from the gear noise test rig (Fig. 1(a)) at NASA Lewis were used in this study to refine and validate the global dynamic model developed. Three phases of experimental studies were performed: (1) the determination of static modal characteristics using the impulse-response technique; (2) the transient vibration of both rotor and the gearbox structure during operation; and (3) measurement of the noise generation at the gearbox surfaces. An outline of the experimental studies is presented below:

**Evaluation of Experimental Modal Characteristics**

The gear transmission system used in this test is shown in Fig. 1(b). The gear system consists of a pair of identical parallel axis spur gears supported by rolling element bearings. The top and four sides of the gearbox were divided into a grid of 116 node points. The transfer functions of the system were found from the impulse-response technique by using an impact hammer. Three accelerometers, one on the surface in the x-direction, another in the y-direction, and the last in the z-direction, were used to obtain the responses from the impacts. Natural frequencies and their corresponding mode shapes excited during the experiment were evaluated using a two-channel dynamic signal analyzer with PC-based modal software. The modal characteristics found during the experiment were compared with those generated by the numerical model for verification purposes.

**Experimental Investigations of System Dynamics**

The test rig is powered by a 150-kW (200-hp) variable speed electric motor. An eddy-current dynamometer...
is used to absorb power at the opposite end. During this test, accelerometers were placed on the top surface of the gearbox to monitor the vibrations of the gearbox structure. In addition, two sets of noncontacting eddy-current proximitors monitored motion of the gear shafting. The vibration signals of the gearbox surface and the shafts were recorded over a range of operating speeds from 3000 to 5000 rpm. A two-channel dynamic signal analyzer computed the vibration frequency spectra which are displayed in a waterfall diagram. The lateral vibration frequency spectra of the gears were also computed. The results of this phase of the experimental work were compared with those from the model for verification of the numerical procedure and improvement of the modeling of the gear transmission system.

Investigations of Gearbox Noise

Two acoustic microphones were used in this experiment. Both microphones were placed over the top cover of the gearbox with one in the vertical direction and the other at a 45° angle from the vertical. While rotor and gearbox vibration are recorded as described in the last section, noise was obtained by the microphone system at various running speeds. A correlation between the gearbox vibrations and the noise level was performed in the frequency domain using linear transfer and cross-correlation functions. The nonlinear hypercoherence function (Jong, 1986) was also used to generate a relationship between the noise and the vibration data in order to develop a noise prediction algorithm for the global dynamic model.

Correlation and Benchmarking of System Dynamics By Numerical Procedures with Experimental Results

The modal hammer excited many vibration modes of the gearbox structure. Eight of these modes in the frequency range 0 to 3000 Hz seem to dominate the transfer function spectra. These eight modes, shown in Fig. 2, were chosen to represent the vibration of the system. In order to produce an accurate analytical simulation of the test gearbox, a similar set of modes was generated using a NASTRAN finite element model of the gearbox structure. Out of 25 modes in the analytical model in the 0 to 3000 Hz frequency range, the eight dominant modes (similar to those from the experimental studies) were used to represent the gearbox dynamic characteristics. These analytically simulated modes are shown in Fig. 3. Table I lists the natural frequencies of the simulated modes. The calculated natural frequencies are within 5 percent of the measured modes. The three-dimensional analytical mode shapes (Fig. 3) are very similar to the experimental mode shapes (Fig. 2). The correlation of the results between the analytical model and the experimental measurements confirms the accuracy of the dynamic representation of the test gearbox using only a limited number of modes. In addition, this correlation of the experimental modal characteristics with the numerical model helps to reduce the number of modes being used in the numerical simulation to achieve an accurate result.

A slow roll of the rotor-gear assembly revealed that a substantial residual bow was present in the rotor as shown by its large orbital motion given in Fig. 4. Figure 4(a) represents the orbit of the driver rotor at the gear location and Fig. 4(b) represents the orbit of the driven rotor. The circular orbit in the driver rotor at slow roll represents the residual bow deformation of the rotor. The elliptical orbit in the driven rotor is due to a combination of the residual bow effects and the vertical gear force from the torque of the driving gear. In order to analytically simulate the influence of this effect, a residual bow of 0.05 mm (2 mil) was incorporated into the numerical model. Figure 5 represents the frequency spectra of the driven gear vibration generated from both the experimental and the numerical procedures. The excellent agreement in both results further verifies the validity of the numerical study.

In order to predict the noise of the transmission system during operation, the vibration of the gearbox must be accurately modeled. Figures 6 and 7 present the time domain vibration signal and corresponding frequency spectra of the signal for the gearbox top surface from both experimental and numerical studies. Figure 6(b) shows the analytically predicted vibration signal (acceleration) at the top of the casing structure at an operating speed of 3000 rpm while the corresponding experimental results are given in Fig. 6(a). Note that the amplitudes and general shapes of the two time signals are very similar. The comparison of their frequency contents are given by the frequency spectra in Fig. 7. Figure 7(a) shows the frequency components of the measured vibration at the top of the gearbox. Note that the major vibration components occur at the mesh frequency of 1250 Hz and at 3-times (3750 Hz) and 4-times (5000 Hz) the mesh frequency. The very large amplitude in the frequency components at 3750 and 5000 Hz are due to the excitation of the gearbox natural frequencies near the 3-times and 4-times multiples of the mesh frequency. The fundamental and the 2-times mesh frequency component are substantially smaller due to the lack of any gearbox natural frequencies near 1250 and 2500 Hz. In addition, frequency components of substantial magnitude are also noticed at 700 and 2900 Hz, which are also close to the natural frequencies of the gearbox. Figure 7(b) depicts the results from the numerical simulations of the system. The numerical results correspond closely to those from the experimental study with the exception of the 4-times mesh frequency.
component. The magnitude and the general distribution of the frequency content of the spectrum is very similar to the experimental results.

Correlation of Noise and Vibration

In order to predict the noise generated in a gear transmission system, a two-phase procedure was developed to correlate the noise and vibration data obtained during experimental studies: (1) linear correlation through transfer functions, and (2) nonlinear correlation through hypercoherence functions. The relationships derived in this study were used to predict gearbox noise from the numerical simulation of the gearbox vibrations. A description of the procedures used is presented in the following paragraphs:

Correlation of Experimental Noise and Vibration Data

In this experiment, a new set of two identical 25-tooth gears were installed in the gearbox shown in Fig. 1(b). The vibration of the gearbox surfaces was monitored by accelerometers, and the noise data was obtained by microphones, as described in the previous section. The frequency and power spectra of both the vibration and noise signals at 3000 rpm running speed are shown in Fig. 8. Note that in the vibration spectrum, a small frequency component exists at the mesh frequency of 1250 Hz with two very large components at 3-times the mesh frequency (3750 Hz), and 4-times mesh frequency (5000 Hz). In the noise spectrum, while frequency components of considerable magnitudes are observed at the mesh frequency of 1250 Hz, and at 2 and 3-times the mesh frequency (2500 and 3750 Hz), a large amplitude frequency component is also noticed at 400 Hz. This 400-Hz frequency component does not coincide with any of the major vibration frequencies at the top surface of the gearbox. A closer examination of the system dynamic characteristics reveals that the 400-Hz frequency is an excitation of one of the natural frequencies of the rotor system, which has a very small amplitude in the gearbox vibration spectrum. A logical explanation is that noise is being produced at the rotor-gear system at 400 Hz and is not being transferred to the vibration of the gearbox surface. This phenomenon is further verified by the very large amplitude of the vibration-to-noise transfer function at 400 Hz as given in Fig. 9(a). A linear coherence function between the noise and vibration signals is also given in Fig. 9(b).

From the experimental data, it can be observed that in the frequency domain, most of the noise components, other than the 400-Hz contributions, exist at multiples of the mesh frequency. The nonlinear hypercoherence function was used for the second phase of this study. The hypercoherence function (Jong, 1986) establishes a nonlinear relationship between the basic frequency component and its multiple harmonics. Figure 10 shows the hypercoherence functions of the noise signals for the 3000-rpm case. The magnitudes of the harmonic components form the nonlinear relationship of each component to the fundamental gear mesh frequency. These relationships provide a picture of the relative contribution of the higher harmonics with respect to the fundamental frequency. These hypercoherence functions were used on the numerical simulation data to examine the accuracy of the noise prediction.

In order to predict the noise level of the transmission system during operation, the vibrations of the gearbox structure were simulated using the modal synthesis method. The transfer function for microphone number 1 (vertical to the plate), as given in Fig. 9(a), was used to predict the noise spectrum of the system (Fig. 11(b)). Figure 11 shows the comparison of the results of the experimental noise (Fig. 11(a)) with the predicted noise (Fig. 11(b)) for the vertical microphone. Although all the major frequency components excited in both cases are very similar, one can see that the predicted noise spectrum (Fig. 11(b)) has a somewhat higher amplitude at mesh frequency (1x) and a very small 4-times mesh frequency (4x) component as in the vibration spectrum.

Another way of examining the validity of the predicted noise signal is through the nonlinear relationships derived by the hypercoherence function. Figure 12 shows the comparison of the hypercoherence functions from both experimental and numerical solutions for microphone number 1. Note that both the experimental and numerical results show a very small 2-times component, with a substantial amplitude at 3, 4, and 5 times fundamental frequency. However, a 50-percent coherence is found in the 3, 4, and 5 times components from numerical predictions (Fig. 12(b)) while only about a 25-percent coherence is seen from the experimental results (Fig. 12(a)). This difference is possibly due to the limitations of the numerical model in simulating various details in the gearbox transmission system. An explanation of the much higher (nearly double) coherence amplitudes in the numerical study is that only the major frequency-related vibration factors are included in the model, which results in a predicted vibration signal consisting mostly of frequency components that are closely related to the fundamental frequency.

Conclusions and Summary

A methodology was developed using the modal synthesis technique to simulate the global dynamics of a gear transmission system. Good correlation has been obtained using results from the global dynamic model
and measured dynamic characteristics from the NASA gear noise test rig. A numerical relationship was also developed from the experimental data to correlate noise and vibrations in the frequency domain. This relationship was applied to the vibration simulations to predict the noise generated by the gearbox. Results of the predicted noise were compared to the experimentally measured data linearly using the frequency spectra and nonlinearly using the hypercoherence function. Some conclusions of this study can be summarized as follows:

1. The dynamics of a gear transmission system coupled with the vibrations of the gearbox structure during operation can be simulated within 5 percent of the modal frequencies using the modal synthesis technique.

2. Predicted vibrations at the housing surface using the global dynamic model are in good agreement with measured values.

3. Both the frequencies and magnitudes of the predicted noise components are similar to those obtained from experimental study.

4. The hypercoherence function was found to be a valuable tool in comparing the nonlinear relationships in the predicted noise to the corresponding nonlinear relationships in the measured noise.

References


<table>
<thead>
<tr>
<th>Experimental, Hz</th>
<th>Analytical, Hz</th>
<th>Difference, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>658</td>
<td>658</td>
<td>0</td>
</tr>
<tr>
<td>1049</td>
<td>1006</td>
<td>-4.1</td>
</tr>
<tr>
<td>1710</td>
<td>1762</td>
<td>3.0</td>
</tr>
<tr>
<td>2000</td>
<td>2051</td>
<td>2.6</td>
</tr>
<tr>
<td>2276</td>
<td>2336</td>
<td>2.6</td>
</tr>
<tr>
<td>2536</td>
<td>2536</td>
<td>0</td>
</tr>
<tr>
<td>2722</td>
<td>2752</td>
<td>1.1</td>
</tr>
<tr>
<td>2962</td>
<td>3012</td>
<td>1.7</td>
</tr>
</tbody>
</table>
(a) Schematic of NASA gear noise rig.

(b) Test gearbox.

Figure 1.—Gear noise test rig at NASA LeRC.
Figure 2.—Experimentally determined mode shapes of the gear box (0 to 3000 Hz).
Figure 3.—Analytically determined mode shapes of the gear box (0 to 3000 Hz).
Figure 4.—Orbital motion of rotor during slow roll.

(a) Driving rotor orbit.  (b) Driven rotor orbit.
Figure 5.—Frequency spectra of the driving rotor vibration.
Figure 6.—Time vibration signal at top of gear box for operating speed of 3000 rpm.

Figure 7.—Vibration frequency spectra at top of gear box for operating speed of 3000 rpm.
Figure 8.—Frequency and power spectrum at the operating speed of 3000 rpm.

Figure 9.—Functions between noise (microphone number 1) and vibration at the operating speed of 3000 rpm.

Figure 10.—Hypercoherence function for the noise signal from microphone number 2 at 3000 rpm.
Figure 11.—Frequency spectra results from microphone number 1 at the operating speed of 3000 rpm.

Figure 12.—Hypercoherence function of the noise signal from microphone number 1 at 3000 rpm.
This paper presents a comprehensive procedure to predict both the vibration and noise generated by a gear transmission system under normal operating conditions. The gearbox vibrations were obtained from both numerical simulation and experimental studies using a gear noise test rig. In addition, the noise generated by the gearbox vibrations was recorded during the experimental testing. A numerical method was used to develop linear relationships between the gearbox vibration and the generated noise. The hypercoherence function is introduced to correlate the nonlinear relationship between the fundamental noise frequency and its harmonics. A numerical procedure was developed using both the linear and nonlinear relationships generated from the experimental data to predict noise resulting from the gearbox vibrations. The application of this methodology is demonstrated by comparing the numerical and experimental results from the gear noise test rig.