CONTROLLED STRUCTURES TECHNOLOGY
M.I.T. SPACE ENGINEERING RESEARCH CENTER
STEERING COMMITTEE WORKSHOP AGENDA

January 22 and 23, 1992
(Given Room)
Arrive at Room 37-355 (253-8809), directions will be given

Wednesday, January 22, 1992

9:30 am Coffee
10:00-10:30 Welcome and Overview Crawley

INTERFEROMETER TESTBED
10:30-10:45 Problem Description Shea/Nisbet
10:45-11:20 Damping and Identification How/Blackwood
11:20-11:40 Closed-Loop Control Jacques
11:40-12:00 Low Authority Control Hall/MacMartin
12:00-1:00 Lunch
1:00-2:00 Lab Tour Meet at Rm. 37-372

MIDDECK 0-GRAVITY DYNAMICS EXPERIMENT
2:15-2:45 Motivation/Background Crawley
MODE Program Highlights:
Schedule
Hardware
On-Orbit Operations

2:45-3:00 STA Crawley/Masters
3:00-3:15 FTA van Schoor
3:15-3:30 Break

MIDDECK ACTIVE CONTROL EXPERIMENT
3:30-3:50 Motivation/Objectives Miller
MACE Flight Program
Development Model
Related Research:
3:50-4:10 Gravity Effects Crawley/Rey
4:10-4:30 Identification for Robust Control Karlov
4:30-5:00 FUTURE PLAN-OVERVIEW Crawley

Thursday, January 23, 1992

8:00 am Coffee

ADDITIONAL RESEARCH IN PROGRESS
8:30-8:50 Multivariable Identification for Control Athans/Lublin/Douglas
8:50-9:10 Strain Actuated Aeroelastic Control Crawley/Lazarus
9:10-9:30 Sensor/Actuator Technology Development Hagood/Anderson
9:30-9:50 Input Command Shaping Seering/Chang
9:50-10:10 Other Ongoing Research Crawley/Miller
10:10-10:20 Break
10:20-11:00 FUTURE PLAN-DETAIL Crawley
11:00-11:45 Workgroups
11:45-12:15 Steering Committee Executive Session
12:15-1:00 Lunch
1:00-2:00 Steering Committee Recommendations
2:00 Conclusion

NASA-CR-192992) CONTROLLED STRUCTURES TECHNOLOGY STEERING COMMITTEE WORKSHOP (MIT) 249
M.I.T.
SPACE ENGINEERING RESEARCH CENTER

FOCUSED ON CONTROLLED STRUCTURES TECHNOLOGY

1992 OVERVIEW

Prof. Edward F. Crawley       Center Director
THE OBJECTIVE

To develop and demonstrate a unified technology of controlled structures, to codify and disseminate this technology, and to train a generation of skilled engineers.
**APPROACH**

- Form a cohesive focused interdepartmental faculty and student group.
- Identify goals by examining shortcomings of current technology and requirements of future scientific and exploration missions.
- Develop technology base research program to advance toward these goals.
- Develop hardware testbeds which mimic the requirements of relevant NASA missions.
- Promote reality by requiring students to demonstrate concepts on hardware testbeds.
- Insure coordination by working with NASA CSI field centers.
- Enable technical transfer by actively involving industrial participants.
CENTER STEERING COMMITTEE

- Chairman - Vacant
- MIT Administration
- Science Advisory Representative
- Corporate Representative
  (LMSC, ITEK, Draper, MDSSC, Harris, TRW)
- Center Management
TECHNICAL REPRESENTATIVE COMMITTEE

- Robert Hayduk, NASA Code RS- Chairman
- Jet Propulsion Laboratory
- Langley Research Center
- Marshall Space Flight Center
THE SCIENCE ADVISORY COMMITTEE

- Prof. Claude Canizares - Chairman
  X-Ray Astronomy

- Prof. Bernard Burke
  Interferometry

- Prof. Davis Staelin
  Radio Astronomy

- Dr. Mike Shao
  SAO/JPL

- Dr. Steve Synnott
  Jet Propulsion Laboratory
A CST DESIGN METHODOLOGY

1. Design structure.

2. Design disturbances and control objective.

3. Design actuator and sensor.

4. Design local/low authority control.

5. Design high authority control.
EVALUATION PLAN

- Internal review meeting - faculty and students present work to date, proposals for next year. Internal consensus on direction.

- Science Advisory Committee - review by leading scientists of relevance and direction of program.

- Steering/Technical Representative Committee - one day workshop presents work-to-date and consensus plans. Steering Committee (non-NASA) and TRC (NASA) caucus individually and give input to program. Director and TRC Chair concur on budget.

- Mid-summer internal review meeting - faculty and students check on progress and direction.

- Annual Symposium - mid-summer, at or near NASA center or MIT.
MAJOR INTERACTIONS WITH GOVERNMENT AND INDUSTRY

- Coordination of Interferometer Testbeds (MIT, JPL)
- Design and development of MODE flight experiment (MIT, LaRC, MDSSC, Payload Systems, Inc.)
- Development and demonstration of command prefiltering for vibration reduction on Shuttle RMS (MIT, JSC, Draper)
- Guest investigator on ASTREX facility (LaRC, AFAL)
- Research funding to develop robust active vibration suppression of the AMASS program (TRW)
GOVERNMENT, INDUSTRY PERSONNEL AT MIT

- Short Visits from JPL, LaRC, MSFC
- Repeated Short Visits on MACE from LaRC
- Visiting Scholar from Harris (2 weeks)
- Visiting Scholar from ISI (1 week)
- Visiting Scholar from MDSSC (multiple 1x2 visits)
- Visiting Scholar from U of Colorado (4 months)
- Visiting Scholar from Penn State (6 months)
- Visiting Scholar from MAI (6 months)
MIT PERSONNEL IN GOVERNMENT AND INDUSTRY

- Active at major conferences (CSI, SDM, ACC, CDC, etc.)
- Summer symposium at MIT
- Undergraduate students in residence at JPL
- MIT faculty participation in JPL interferometer design reviews
- COOP students at LaRC, JPL, ...
- Multiple students in residence at LMSC
- In the next year, hope to have longer visit at LaRC
INTERFEROMETER TESTBED:
OVERVIEW AND HARDWARE SUMMARY

Andrew Nisbet

January 22, 1992
Reference Mission & Science Requirements

- Needed scientific reference mission to define stringent CST goals: Optical Interferometer Spacecraft
- Consultation with astronomers produced the set of mission objectives shown below. The science requirements were then derived from the mission objectives.

<table>
<thead>
<tr>
<th>Mission Objectives</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>description</td>
<td>Value</td>
</tr>
<tr>
<td>resolution</td>
<td>3 m-arcsec (0.014 μrad)</td>
</tr>
<tr>
<td>wavelength</td>
<td>0.5 micron</td>
</tr>
<tr>
<td>stellar magnitude</td>
<td>10</td>
</tr>
<tr>
<td>imaging</td>
<td>100% u-v plane coverage 95% image quality</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Science Requirements</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>description</td>
<td>Value</td>
</tr>
<tr>
<td>baseline</td>
<td>35 meters</td>
</tr>
<tr>
<td>high freq. DPL error</td>
<td>25 nm &gt; 0.005 Hz</td>
</tr>
<tr>
<td>low freq. DPL error</td>
<td>10 nm &lt; 0.005 Hz</td>
</tr>
</tbody>
</table>

Schematic of a two dimensional interferometer
**Science Requirements Derivation**

- Baseline derived from resolution objective

\[ \rho = \frac{\lambda}{D} = 0.003 \text{ arcsec} \]

- High frequency DPL limit derived from maximum intensity function

\[ I_{\text{max}} = I_T + V(u) \cos \left( 2\pi \frac{\partial \ell}{\lambda} \right) \]
\[ \approx I_T + V(u) \left( 1 - \frac{1}{2} \left( 2\pi \frac{\partial \ell}{\lambda} \right)^2 \right) \]

- Low frequency DPL limited derived from phase error

\[ \phi_e = 2\pi \frac{\partial \ell}{\lambda} \]
**Reference Spacecraft Design Summary**

<table>
<thead>
<tr>
<th>Description</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>35 meters</td>
</tr>
<tr>
<td>Architecture</td>
<td>Deployable tetrahedral truss structure</td>
</tr>
<tr>
<td>Spacecraft Mass</td>
<td>3285 kg (truss)</td>
</tr>
<tr>
<td></td>
<td>12325 kg (total) incl. 25% margin</td>
</tr>
<tr>
<td>Payload Architecture</td>
<td>8 siderostats in non-redundant 2-D array</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>TDRSS S-band compatible</td>
</tr>
<tr>
<td>Power Source</td>
<td>Body mounted solar arrays</td>
</tr>
<tr>
<td>Attitude Control</td>
<td>8 1200 Nms RWAs &amp; mag. torque momentum dumping</td>
</tr>
<tr>
<td>Science Mode</td>
<td>217 sec integration time, continuous rotation about LOS</td>
</tr>
</tbody>
</table>
Testbed Requirements & Performance Metric

- Structure is 1/10th scale truss-work tetrahedron with three siderostat locations. Performance requirement of 50 nm RMS differential pathlength error (DPL) above 10 Hz is an order of magnitude improvement over expected disturbance environment.

- Decision made to address only a subset of the DPL problem. Errors due only to internal flexible motion will be considered, that due to external flexible and rigid body motion are not measured.
The performance metric as defined by internal flexible motion is measured by laser legs AF, BF, and CF. The performance goals for the testbed are:

$$\max \begin{pmatrix} (AF - BF)_{RMS} \\ (BF - CF)_{RMS} \\ (CF - AF)_{RMS} \end{pmatrix} \leq 50 \text{ nm over 10 - 200 Hz Bandwidth}$$
*Disturbance Source & Signal*

- Reference mission design identified typical spacecraft disturbances and determined DPL response of ~500 nm RMS on structure with nominal level of damping (~1%).
  - Narrow band spikes (reaction wheel imbalances)
  - Broadband (fluid flow noise)
  - Transient (solar array or antenna drives)

- Disturbance Signals Applied To Piezo-Shaker:
  - A signal to represent low broadband noise with slowly varying spikes produced insufficient excitation in structure.

![Force PSD Graph](image)

**Response:** only 80 - 100 nm RMS predominantly in 20 - 50 Hz band
- Disturbance Signal (con’t):
  - Signal representing RWA spike envelope which increases with the square of the frequency up to wheels speed limit at 70 Hz. Some tail on the spectrum due to harmonics.

- This signal produces the following response with the desired level of disturbance (770 nm RMS). 96% of the total is below 200 Hz.
Testbed Architecture

Real-Time Computer
33 MHz 68030 µP
Supercard Vector Processor

Software
VaxWorks Real-Time Operating System
MatCon - LQG Digital Control

VME
Backplane

Data Acquisition System
16 A/Ds, 10 D/As
6 HP Laser Axis Boards
16 Digitally Controlled Amplifiers with 4-pole Bessel Anti-aliasing Filters

Sensor & Actuator Signal Conditioners

Tetrahedral Truss

Sensors
Accelerometers
Load Cells
Strain Gauges
Laser & Optics

Actuators
Piezo-electric Struts
Disturbance Source
Active Mirror Mounts

Tektronix Spectrum Analyzer

Space Engineering Research Center
Passive Damping in the MIT SERC Controlled Structures Testbed

Jonathan How, Gary Blackwood, and Eric Anderson

SERC Steering Committee Meeting
22 January, 1992
Finite Element Model and Identification Procedure

Jonathan P. How, Gary Blackwood, Eric Anderson, and Etienne Balmes

SERC Steering Committee Meeting
22 January, 1992
Interferometer Finite Element Model

- ADINA model, with 1500 degrees of freedom.
- Important attributes:
  - 1 beam element per strut
  - consistent mass matrix used
  - node flexibility incorporated through measured strut component test data
  - wires modelled as distributed masses
  - damping not modelled directly, included as modal damping in post processing
  - closely spaced modes due to near symmetries in structure
  - requires approximately 2 mins of Cray CPU time for the first 40 flexible modes.
Testbed Mode Shapes

Mode 1 (25.8 Hz)  Mode 2 (27.2 Hz)
Testbed Mode Shapes

Mode 6 (36.1 Hz)  Mode 7 (36.3 Hz)
Finite Element Model Update

- Large discrepancy between finite element model and identified frequencies indicate that update required.

![Old ID and FE frequency comparison graph](image)

- Agreement of modal frequency distribution:
  - poor at high frequencies
  - better for lower frequency modes dominated by first leg bending modes.

- Better model needed for sensor, actuator, damper placement, and initial control designs.
Identification Procedure

- **Hardware:**
  - 29 Kistler, 9 Sunstrand accelerometers
  - Bruel and Kjaer electromagnetic shaker
  - Tektronix scanner used to simultaneously measure all 38 channels.

- **Selection of shaker locations:**
  - 2142 possible locations reduced to 24 based on rankings using mean and maximum modal controllability.
  - goal: maximize controllability of least-controllable mode

\[
\max_i \left( \min_r |A_i^r| \right) \quad \begin{cases} A = \text{modal residue at input} \\ i = \text{input dof (24)} \\ r = \text{mode number (20)} \end{cases}
\]
Shaker Locations

- Analysis resulted in one shaker location in each truss leg.

<table>
<thead>
<tr>
<th>Residue</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>6</td>
<td></td>
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</tbody>
</table>

**Shaker Tip:**

- Connection to armature of electromagnetic shaker

**Shaker locations:**

- [Diagram of shaker locations]
Data Analysis

- Transfer functions fit with modified least squares approach (R. Smith, UCSB).
- Leads to state space representation of measured data:

\[
G_{fit}(s) := \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]

- state space representation of each mode:

\[
A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

\[
C_i = \begin{bmatrix} c_{1i} & c_{2i} \end{bmatrix} (38 \times 2)
\]

- full model:

\[
A = \text{BlockDiag} (A_i) \quad B = \text{Col} (B_i) \quad C = \text{Row} (C_i) \quad D
\]
Data Analysis

- The A and B matrices held fixed for each shaker location.
- Full C matrix adds extra flexibility to approach.
- Modal frequency and damping computed with invfreqs function in MATLAB on several transfer functions.
- Note: good fits require good estimates of the frequency and damping of every mode in the frequency range of interest.
- One of several curve fitting approaches employed at SERC.
Modal Frequency and Damping Comparison

- Six A matrices agree well in frequency, less so in damping.
Computational Procedure

- C and D matrix rows independently selected for each sensor.
- Example: Pick D

\[ E_r = \left( \sum_{i=1}^{m} ||Y(j\omega_i) - G(j\omega_i)U(j\omega_i)||^2 \right)^{\frac{1}{2}} \]

\[ Z(j\omega_i) = Y(j\omega_i) - C_b(j\omega_i - A_b)^{-1}B_bU(j\omega_i) \]

\[ E_r(j\omega_i) = Z(j\omega_i) - D(j\omega_i)U(j\omega_i) \]

Then \( D_a = \begin{bmatrix} \text{Re}(Z) & \text{Im}(Z) \end{bmatrix} / \begin{bmatrix} \text{Re}(U) & \text{Im}(U) \end{bmatrix} \)

where \( \bar{Z} = [Z(1) \ Z(2) \ \ldots \ Z(m)] \)

- Similar for the C matrix. Several iterations required.
- Software exists to perform an overall A matrix update.
Fit Comparison

- Final fit comparison:

![Graph showing magnitude and phase vs. frequency](image)

- Procedure effectively fits hundreds of transfer functions. Results good enough for control designs.
Residue Analysis

- Need to compute displacement residues from approximate accelerometer transfer function.

\[
G_{fit}(s) = \sum_{i=1}^{m} \frac{c_{1i} + c_{2i}s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + d \approx \ddot{y} \frac{\ddot{y}}{f}
\]

\[
\overline{G}_{fit}(s) = \sum_{i=1}^{m} \frac{b_{1i} + b_{2i}s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + \frac{h(s)}{s^2} \approx \frac{y}{f}
\]

where \( b_{1i} = -\frac{(1 - 4\zeta^2)}{\omega_i^2} \ c_{1i} - \frac{2\zeta}{\omega_i} \ c_{2i} \)

\( b_{2i} = \frac{2\zeta}{\omega_i^3} \ c_{1i} - \frac{1}{\omega_i^2} \ c_{2i} \)

Residue: \( \phi_i(x_{act})\phi_i(x_{sens})^H = (b_{1i} + b_{2i}s) \bigg|_{s=j\omega_i} \)
Typical Residues

- Residues rotated by phase at sensor collocated with shaker.
Current Status

- Frequency comparison after structural and model updates:

![Current ID and FE frequency comparison graph]

- Modifications:
  - eigenvector studies illustrated importance of plate flexibility, inclusion in the FEM lead to improved frequency agreement (44 % error in first 9)
  - fourth vertex stiffened to improve optical alignment, and better agreement indicates prior presence of local modes.
Identification/FEM Residual Comparison

- Correlate identified and FEM residues for first 14 modes.
- Modal Assurance Criterion:

\[
\text{mac}(x_1, x_2) = \frac{\| \sum_{i=1}^{m} \phi(x_1)_i \phi(x_2)_i^H \|^2}{(\sum_{j=1}^{m} \phi(x_1)_j \phi(x_1)_j^H)(\sum_{j=1}^{m} \phi(x_2)_j \phi(x_2)_j^H)}
\]

- Issue: FEM modes real, measured residues complex.
Future Work

- Continue coarse FEM changes to correct plate flexibility and mass distribution assumptions.
- Apply gradient type updates on the stiffness and mass matrices to match residues of higher frequency modes.
- Improve FEM suspension model with ID data.
- Develop state space model that can be used for sensor, actuator, damper placement, and initial control designs.
Pathlength Control Using Isolation Mounts

Input: piezo voltage
Output: pathlength C-E (microns)
Control Design for the SERC Experimental Testbeds

Steering Committee Presentation

Robert Jacques
Gary Blackwood
Douglas MacMartin
Jonathan How
Eric Anderson

January 22, 1992
Approaches

- Global control loops from lasers to active struts
- Local control loop wrapped around active struts
- Isolation at the performance outputs
- Phase 0 testbed
SISO Control Design and Results

- Hardware:
  - Actuator: active strut
  - Sensor: Laser Leg

- Assumptions
  - Disturbance source and actuator collocated
  - Performance metric and sensor collocated

- Control design method
  1. Measure transfer function from actuator to sensor
  2. Use nonlinear curve fitting technique to obtain state space model of system
  3. Reduce model order
  4. Design LQG controller
  5. Remove dynamics from controller which do not contribute to stability and affect the performance only slightly
**Sensor and Actuator Locations**

- **Sensor**: Laser from fourth vertex to plate A
- **Actuator**: Active strut placed to maximize modal residues in transfer function from strut to laser
Model Identification

- Averaged empirical transfer function estimate from actuator to sensor obtained using Tektronix box.

- Nonlinear algorithm used to fit poles and zeros to transfer function. Comparison of measured and modelled transfer functions.
Control Design

- Model truncated at 80 Hz
- LQG controller had some slight notching which did not improve stability and affected the cost slightly. These dynamics were removed from the controller.
Experimental Results

- Controller digitized and implemented on real time computer at 2000 Hz.
Preliminary LAC Experimental Results

- Single collocated rate feedback loop closed around active strut to demonstrate active damping.

- Open and closed loop transfer functions from disturbance source to siderostat acceleration.
Active Vibration Isolation

Problem Statement

Problem A: Given the prescribed displacement disturbance of a flexible base structure, design an active interface to reduce transmissibility of displacement to a rigid payload. 214z

Problem B: Given the prescribed clamped force and output impedance of a vibrating machine, design an active interface to reduce transmissibility of force to a flexible base structure.

Thesis will study:

- relative performance of passive and active isolation designs
- choice of feedback sensors and passive design to decouple flexible base dynamics
- active isolation as an element of CST process:
  - disturbance attenuation
  - simultaneous isolation mounts
  - isolation and pathlength control together
- implementation of MDOF isolators on testbed

MIT Space Engineering Research Center
Model: Base Flexibility Coupling into Isolation Feedback Loop

- Plant transfer function: \( y(s)/f(s) \)
- Non-dimensional parameter governing flexible coupling of \( i^{th} \) mode:

\[
\begin{align*}
\omega_r &\ll \omega_o: \quad \beta_L = \frac{m(\phi_i)^2}{2\zeta_i} \\
\omega_r &\gg \omega_o: \quad \beta_H = \frac{m(\phi_i)^2}{2\zeta_i} \left(\frac{\omega_r}{\omega_o}\right)^2 \\
g_\beta &= (1 + \beta)^{1/2} \quad \varphi_\beta = 2\tan(\beta/2)
\end{align*}
\]

\[
H_k(s) = \sum_{i=1}^{\infty} \frac{(\phi_i)^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}
\]

\[
\omega_o = \frac{k}{m}
\]
A Phase 0 Testbed

- 33 cm cantilevered beam, initially developed as a deformable mirror testbed.

- Designed to test the sensor, actuator, modelling, and control issues associated with nanometer resolution requirements.

- Sensors/actuators: tip laser displacement, PZT, and PVDF.

- High Authority/Low Authority controller:
  - passive damping (resistive piezoelectric shunting)
  - LAC: strain rate feedback (PVDF to PZT), applied to same actuators as HAC, analog
  - HAC: constrained order LQG (4), two loop feedback (laser measurement to PZT), digital.
Closed Loop Results

- Transfer function from disturbance (mid beam) to tip displacement:

![Graph showing frequency response with magnitude on the y-axis and frequency on the x-axis, comparing open loop and closed loop responses.]

--- Open loop
.... Closed loop
A Stochastic Approach to Robust Broadband Structural Control

Douglas G. MacMartin
Steven R. Hall

January 22, 1992
Goal

- Active broadband control of uncertain modally dense structures.
- Use collocated feedback.
  - Positive real controller guarantees stability.
  - Low authority or local control ("active damping.")
- Use local acoustic or statistical model of structure.
- Maximize power dissipation.
  - Equivalent to impedance matching.
  - Cannot match impedance exactly at all frequencies due to causality constraint.
- Experimental demonstration on complex structures.
Local Models

Travelling wave model

Dereverberated mobility model

Disturbance

\( w_i \): Incoming Disturbances

\( w_o \): Outgoing Disturbances

Actuator/Sensor Pair
Computation of Dereverberated Mobility

- From averaging transfer function:
Power Flow

- Use local model only.
- Power dissipation related to input mobility $G$:

\[
\begin{bmatrix}
  z \\
  y
\end{bmatrix} = \begin{bmatrix}
  G_1 I & G_1 G_0^* \\
  G_0 & G
\end{bmatrix} \begin{bmatrix}
  w \\
  u
\end{bmatrix}
\]

- Define $H(s) = \frac{z(s)}{w(s)}$.
  - Analogous to reflection coefficient
  - Reflected power is $(H^*H)w^*w$
  - Dissipated power is $(I - H^*H)w^*w$
Impedance Matching

- Maximum power dissipation is obtained if the compensator is the conjugate of the structural impedance.
  - This is noncausal ⇒ need best causal approximation.
- $\mathcal{H}_2$ optimization: No guarantee of stability on actual structure.
- $\mathcal{H}_\infty$ optimization: guarantees stability, but doesn’t minimize desired performance metric.
- Goal: minimize actual rms cost and guarantee stability using only local information.
Stochastic Systems

- Investigate systems of the form
  \[ \dot{x} = A(\sigma)x + w \]
  - \( \sigma \) is a random variable with known probability distribution.
  - Uncertainty is only in the imaginary part of the eigenvalues.

- The average covariance \( \langle xx^T \rangle_{\sigma,w} \) satisfies
  - Incoherence: the amplitudes of distinct modes are uncorrelated.
  - Equipartition: the average kinetic and potential energy of each mode is the same, and conservation of energy (for an undamped system).

- These are the key assumptions made in Statistical Energy Analysis.
Control Problem

- Minimize "global" mean-square performance metric $\langle y^Ty \rangle$
  using only local knowledge, local control.
- Use characteristics of stochastic systems.
- Use conservation of energy!
Control of Stochastic Systems

- Incoherence ⇒
  \[ \langle y^T y \rangle = \sum_{n=0}^{\infty} C_n E_n \simeq \int_{-\infty}^{\infty} C(\omega) E(\omega) \]

- Incoherence & Equipartition ⇒ incoming structural power is proportional to structural energy.
  - Total power dissipated is therefore
    \[ \Pi_{diss} = (I - H(j\omega)^* H(j\omega)) E(\omega) \]
    \[ \text{Relative Dissipation} \]

- Conservation of energy ⇒
  \[ \Pi_{diss} = \Pi_{in} \]

- The average (over uncertainty) cost is therefore:
  \[ J = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(\omega) \left[ (I - H^* H)^{-1} H^* H \right] \Pi_{in}(\omega) \, d\omega \]
Properties

- Guarantees $\mathcal{H}_\infty$ norm bound, $\|H\|_\infty < 1$.
  - Guarantees \begin{align*}
  \text{positive real} \\
  \text{dissipates power} \\
  \text{stable closed loop}
  \end{align*}

- Overbounds weighted $\mathcal{H}_2$ cost.
  - Minimizes desired performance metric.
Using Cost Functional

- Cost is related to other $\mathcal{H}_2/\mathcal{H}_\infty$ approaches, and to differential games.
- Can evaluate cost for a state space system with solution to a Riccati equation and a Lyapunov equation.
- Can optimize cost using Lagrange multiplier approaches
  - Yields coupled Riccati and Lyapunov equations.
  - Cannot be solved explicitly.
  - Can be solved using numerical optimization.
Example: Bernoulli-Euler Beam

- Free-free beam, force actuator and velocity sensor at left end, disturbance force at right end.
- Minimize difference between end-point displacements.

\[ y = w \]

- Dereverberated mobility: (describes local dynamics)

\[ G(s) = \frac{\sqrt{2}}{(\rho A)^{3/4}(EI)^{1/4}} \cdot \frac{1}{\sqrt{s}} \]

- Non-causal impedance match: (maximum dissipation)

\[ K(s) = \frac{(\rho A)^{3/4}(EI)^{1/4}}{\sqrt{2}} \cdot \sqrt{-s} \]
Compensator Design

- Choose $\Pi_{in}(\omega)$ based on disturbance spectrum $V(\omega)$ and dereverberated input mobility at disturbance location $G_d(\omega)$.

  $$\Pi_{in} = (G_d + G_d^*)V$$

- Choose $C(\omega)$ to approximate modal cost:

- Choose desired compensator order and use numerical optimization.
Performance

- Compare cost versus length of beam for various compensators.
  - LQG compensator unstable for small changes in length.

![Graph showing cost versus length for different compensators.](attachment:image.png)
“Power” Dual Variables

- Force into structure and relative velocity across active strut are dual.
- Piezo stack stiffness is high ⇒ commands displacement.
  - Can also command relative velocity.
- Want compensator $K(s)$ such that

$$\dot{x} = K(s)f$$

$$\Rightarrow x = K(s)\left(\frac{f}{s}\right)$$

- Use integral of force feedback.
Dereverberation of Complex Structure

- Wave approach: truss behaves like a beam at low frequencies.
- Compute "best" fit of log magnitude using only real poles and zeroes.
- Fit transfer function using complex poles and zeroes, and add damping to resulting model.
Dereverberated Transfer Function

- Open loop transfer function from displacement to integrated force.
- Three (real) pole fit of log magnitude.
Conclusions

- Use local model of structure for broadband control.
  - Dereverberated mobility or travelling wave model.
  - Ideal compensator is a non-causal impedance match.
- Parameter-robust control for structures must take advantage of conservation of energy.
- Average covariance exhibits equipartition and incoherence.
- A cost functional can be obtained that uses these properties, and guarantees both stability and performance robustness.
MODE FLIGHT HARDWARE ELEMENTS

Experiment Support Module

EXPERIMENT CONTROL → EXCITATION

DATA RECORDER → DATA MEASUREMENT

Test Articles

Fluid Test Article (FTA)

Structural Test Article (STA)
MODE SCIENCE OBJECTIVES

- Investigate two gravity-dependent nonlinear dynamic phenomena affecting future spacecraft:

  Fluid Test Article (FTA)
  Fluid slosh in a tank, uncoupled and coupled to spacecraft motions.

  Structural Test Article (STA)
  Nonlinear dynamics of truss structures.

- Both investigations complement OAST R&D base and enhance future space station/exploration missions.
Science objectives for both test articles include:

- Establishing a data base of dynamic response data in the ground and orbital gravity regimes.
- Understanding truss joint and fluid interactions with spacecraft motion.
- Using test results to verify nonlinear computer models developed at MIT.
- Using the knowledge obtained to more efficiently design spacecraft and their control systems.
**MODE TEAM**

Flight experiment funded by the NASA HQ In-Space Technology Experiments Program (In-Step)

<table>
<thead>
<tr>
<th>Principal Investigator and science development team</th>
<th>M.I.T. Space Engineering Research Center</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsor</td>
<td>NASA Office of Aeronautics, Science, and Technology</td>
</tr>
<tr>
<td>Program monitor</td>
<td>NASA Langley Research Center</td>
</tr>
<tr>
<td>Hardware fabrication and integration team</td>
<td>Payload Systems Inc.</td>
</tr>
<tr>
<td>Co-Investigator</td>
<td>McDonnell Douglas Space Systems Co.</td>
</tr>
</tbody>
</table>
MODE IN THE UNIVERSITY ENVIRONMENT

- MODE provided focus and motivation for engineering education:
  
  20 undergraduate students
  7 graduate students
  2 postdoctoral students

- Student participation involved basic research and actual flight hardware fabrication and testing
FLIGHT OPERATIONS

- MODE was launched on-board Discovery, STS-48, on Sept. 12, 1991. Hardware was deployed in the Middeck by the crew during operations.
- Flight operations began on Friday, September 13.
- MODE operations occurred on Flight Days 2, 4 and 5.
- Operations were supported at JSC by a 10 person MIT/PSI/MDSSC/LaRC team.
- All planned test operations plus an additional 3 hours of testing were successfully completed yielding over 600 Mbytes of on-orbit data.
- Hardware was recovered within 24 hours after landing.
- Hardware is currently being used at MIT for ground science tests.
FLIGHT OPERATIONS (CONT.)

- Fluid data was obtained on:
  silicone oil, uncoupled, flat bottom tank
  silicone oil, uncoupled, spherical bottom tank
  water, uncoupled, flat bottom tank
  water, coupled, flat bottom tank
  water, uncoupled, spherical bottom tank
  water, coupled, spherical bottom tank.

- Structural data was obtained on:
  straight, 3 amplitudes, 3 pretension settings
  straight with α-joint, 3 amplitudes, 2 friction
  settings
  L-shape, 2 amplitudes, 2 friction settings
  L-shape with flex. appendage
SUMMARY

- MODE exploits the unique 0-g laboratory environment provided by the Space Shuttle.
- Provides NASA with a reusable dynamic test facility for testing dynamic systems in space.
- Complements OAST R&D base and enhances future Space Station/exploration missions.
- Team represents a unique consortium of university, industry, and government.
- Cost-effective flight experiment developed on schedule by a small core group of scientists, engineers, and students.
- Provides mission relevant focus for engineering education.
MODE: STRUCTURAL TEST ARTICLE (STA)

Prof. E. Crawley
Mr. Brett Masters

MIT
MIT
MODE: Structural Test Article Motivation

• DETAILED MODEL AND UNDERSTANDING OF ON-ORBIT STRUCTURAL DYNAMICS IS IMPORTANT SINCE:
  Resonant and transient response influence on-board vibration / acoustic environment.
  Incorrect modeling of dynamics can cause inadvertent CSI with attitude dynamics.
  Detailed modelling is vital for robustness / performance of precision controlled structures.

• NEED TO CORRECTLY MODEL AND UNDERSTAND NON-LINEAR EFFECTS ON A COMPONENT AND SUB-COMPONENT LEVEL.

• UNDERSTANDING ON-ORBIT DYNAMICS WILL REDUCE UNCERTAINTIES BY:
  comparison of earth test results with 0-gravity test results.
  verifying and validating analytical models.
  adding to the scant data base of quality data available on the dynamics of large flexible space structures in 0-gravity.

Space Engineering Research Center
Why Test STA In Space?

- GROUND BASED SIMULATIONS HAVE BEEN EXPLOITED

  Options

  (a) Suspended in air
  (b) Suspended in vacuum
  (c) Lofted in vacuum (Free-fall)

BUT:

  (a) Air damping undesirable, suspension systems corrupt modal measurements, and the gravity field causes pre-loads and pre-deflections in the structure.
  (b) Suspension and gravity!
  (c) Short time periods of free-fall reduce accuracy of modal identification due to
      (i) Uncertain initial conditions (inhomogeneous terms)
      (ii) Difficulty in exciting the structure
      (iii) Poor signal-to-noise ratio
Hardware

Four Test Configurations of the STA

KEY
1: Erectable Bay
2: Deployable Bays (4)
3: Alpha Joint
4: Rigid Appendage
5: Flex. Appendage
Sensors and Actuator

Sixteen sensor channels arranged and conditioned as full-bridge resistive gages

- four strain gage pairs located on one face of adjustable preload bay
- eleven accelerometers (piezoresistive) at predetermined locations
- one load cell located in the proof-mass actuator housing

Sensor and actuator locations for Straight and L configurations.
Experimental Support Module

ESM Driven Flow Diagram
Data

Ground (1 Hz Suspension) vs. Space Torsion Mode (baseline, lowest preload)

LEGEND
- space, resonance force input 4.88 lbf
+ space, resonance force input 2.70 lbf
* space, resonance force input 0.45 lbf
-- ground, resonance force input 3.56 lbf
--- ground, resonance force input 1.99 lbf
---- ground, resonance force input 0.39 lbf
Data (cont.)

Ground (1 Hz Suspension) vs. Space Torsion Mode (alpha tight)

LEGEND
- space, resonance force input 5.00 lbf
+ space, resonance force input 2.78 lbf
* space, resonance force input 0.46 lbf
-- ground, resonance force input 4.37 lbf
- ground, resonance force input 2.39 lbf
- ground, resonance force input 0.31 lbf
Preliminary Results

- Modes generally appear softer in 0-gravity.

- Resonances exhibit similar shifts, on the ground and in 0-gravity, relative to input forcing level.

- Modes are generally more damped in 0-gravity.

- Data exhibit some anomalies, to be explained by non-linear analysis?

- Some modes out of 0-gravity test windows!
Supporting Analysis Program

Component Testing

Bay testing
Single joint testing

Analytical Model

Use force-state results to generate nonlinear model
Use results to verify nonlinear on component level or to build "component" nonlinear model

Ground Modal Testing

Determine linear modal characteristics
Determine Nonlinear Modal characteristics
Understand suspension effects

Use to update FEM
Use to verify analytical model
Identify limitation of earth testing

Space Modal Testing

Determine linear modal characteristics
Determine Nonlinear Modal characteristics

Space Engineering Research Center
Modelling Approach

- Modal Test
- Finite Element Model
  - Linear Structural Dynamic Model
  - Analytical Model
    - Force-State Map of Nonlinear Sub-components
      - Describing Function of Force-State Map
    - Nonlinear Structural Model (Equivalent beam representation or Multi-degree-of-freedom Model)
      - Forced Response Characteristics using Harmonic Balance Method
      - MODE Flight and Ground Experimental Results
- Compare to Verify Model
MIDDECK 0-GRAVITY DYNAMICS EXPERIMENT

FLUID SLOSH EXPERIMENTS

Dr. Marthinus C. van Schoor
University of Pretoria
South Africa

January 1992
Summary Preliminary Results
- More benign nonlinear behavior than observed on earth
- Modal damping ratios and frequencies different from predicted

Typical Results
Silicone Oil (3.1 cm Cylindrical Tank with a Flat Bottom):

Earth:
Silicone Oil (3.1 cm Cylindrical Tank with a Flat Bottom):
Water (3.1 cm Cylindrical Tank with a Flat Bottom):

Earth:
Earth: Water in a Flat Bottom Tank

Planar Slosh Force (N)

Frequency (Hz)

Planar Slosh Force Phase Angle (Degrees)

Frequency (Hz)
Water (3.1 cm Cylindrical Tank with a Flat Bottom):

Space:
Water in a 3.1 cm Diameter Flat Bottom Tank

Planar Slopsh Force (N)

Frequency in Hz

Phase of Planar Slopsh Force (Degrees)

Frequency in Hz

Space Engineering Research Center
PROGRAM OBJECTIVES

- Science Objective
  To develop a verified set of methods that will allow designers of CSI/CST spacecraft, which cannot be dynamically tested on the ground in a sufficiently realistic 0-g simulation, to have confidence in the eventual orbital performance of such spacecraft.

- Implications
  Understand direct and indirect gravity effects and the relation between control authority and manifestation.
  Develop procedures for predicting on-orbit performance.
  Quantify prediction accuracy achievable through analysis and ground tests.
  Develop techniques for on-orbit identification.
  Quantify performance improvement through control redesign based upon on-orbit identification.
PROGRAM FEATURES

Milestones

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Design Review (PDR)</td>
<td>April, 1992</td>
</tr>
<tr>
<td>Critical Design Review (CDR)</td>
<td>December, 1992</td>
</tr>
<tr>
<td>Launch</td>
<td>July, 1994</td>
</tr>
</tbody>
</table>

Participants

<table>
<thead>
<tr>
<th>Participant</th>
<th>Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.I.T. SERC</td>
<td>PI and science development</td>
</tr>
<tr>
<td>Payload Systems Inc.</td>
<td>Fabrication and Integration</td>
</tr>
<tr>
<td>Lockheed, Sunnyvale</td>
<td>Co-Investigator</td>
</tr>
<tr>
<td>McDonnell Douglas</td>
<td>Co-Investigator</td>
</tr>
<tr>
<td>Integrated Systems Inc.</td>
<td>AC-100 and design tools</td>
</tr>
<tr>
<td>CSA</td>
<td>Suspension</td>
</tr>
</tbody>
</table>

Hardware Phases

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Development Model</td>
<td>science development</td>
</tr>
<tr>
<td>Engineering Model</td>
<td>flight prototyping</td>
</tr>
<tr>
<td>Flight Model</td>
<td>two flight hardware units</td>
</tr>
</tbody>
</table>
FLIGHT EXPERIMENT FEATURES

- Middeck experiment stored in four lockers
- Test article deployed on middeck with umbilical connection to Experiment Support Module (ESM)
- ESM contains:
  - experiment and realtime control computers
  - actuator power amplifiers and sensor signal conditioning
  - data storage
  - human and host computer interfaces
- Duration of three 8-hour days
- On-orbit timeline
  - Identification
  - Downlink ID data
  - Implement same algorithms as on ground
  - Implement algorithms adjusted for lack of gravity
  - Uplink ID based control algorithms
  - Implement new measurement based algorithms
- ESM provides reusable realtime control facility
CURRENT ACTIVITIES

- Preliminary flight hardware design (PSI)
- Engineering Model gimbal acquisition (PSI, LMSC)
- Selecting on-orbit test configurations based upon desire to have gravity effects cause significant performance deviation from ground near midrange of control authority (Sepe)
- Incorporating gravity effects in Finite Element model (Rey)
- Development Model (DM) modern closed-loop control (Saarmaa, Miller)
- Command input shaping tests on DM (Chang)
- Formulation of 1-g and 0-g identification procedures (Karlov, Douglas)
- Study of multibody gravity effects (Quadrelli)
- Fabrication of active strut (PSI)
MIDDECK ACTIVE CONTROL EXPERIMENT (MACE)
Development Model Lab Testing
(Flight unit will have smaller torque wheels and gimbal motors)
Finite Element Modelling and Measurement Verification

Data Channel

0-g model, 1-g data

1-g model, 1-g data

Space Engineering Research Center
DEVELOPMENT MODEL TESTING

Component Control Work

Gimbal Control
Torque Wheel Control
Baseline Control

<table>
<thead>
<tr>
<th>Torque Input Axis</th>
<th>Z</th>
<th>Z</th>
<th>Z</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage RMS (volts)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Input Torque RMS (Nm)</td>
<td>0.0069</td>
<td>0.0206</td>
<td>0.0069</td>
<td>0.017371</td>
</tr>
<tr>
<td>Payload Output Axis</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td>X</td>
</tr>
<tr>
<td>Payload Angle RMS (degrees)</td>
<td>0.0389</td>
<td>0.0698</td>
<td>0.0387</td>
<td>0.1430</td>
</tr>
<tr>
<td>Payload Angle 3RMS (degrees)</td>
<td>0.1168</td>
<td>0.2004</td>
<td>0.1070</td>
<td>0.4629</td>
</tr>
<tr>
<td>Payload Clamped Angle</td>
<td>+45 deg</td>
<td>+45 deg</td>
<td>-45 deg</td>
<td>0 deg</td>
</tr>
<tr>
<td></td>
<td>about Z</td>
<td>about Z</td>
<td>about Z</td>
<td>(down)</td>
</tr>
</tbody>
</table>
DEVELOPMENT MODEL TESTING

Single Input, Single Output Control (localized topology)

Description:
Penalize payload inertial angle
Feed payload inertial angle to gimbal outer stage motor
Torque disturbance additive with gimbal control signal
Formulate Linear Quadratic Gaussian Control (LQG)

Measurement Model

Performance

Transfer Functions of Measured and Curve Fit Data

Open-loop and Closed-loop Payload Angle Response

Space Engineering Research Center
Description:

Feed relative gimbal and inertial bus angles to gimbal outer stage motor

Torque disturbance additive with gimbal control signal

Since poles common to all transfer functions, must be averaged

measurement model
DEVELOPMENT MODEL TESTING

Single Input, Two Output Control (cont'd)

Description:

Formulate Linear Quadratic Gaussian control (LQG)
penalize payload inertial angle  penalize payload and bus inertial angles
FUTURE DEVELOPMENT MODEL WORK

- Closed-loop control based on finite element model (Grocott, Glaese, Miller)
- Development of multi-input, multi-output measurement models (Karlov, Douglas, Athans)
- Classical control design using successive loop closure (Campbell, Crawley)
- Robust control using multi model and other uncertainty approaches (MacMartin, Hall)
- Constrained order and topology control (Mercadal, Vander Velde)
- Slew command shaping for minimizing excitation of flexibility (Chang, Seering)
THE MIDDECK ACTIVE CONTROL EXPERIMENT:

GRAVITY AND SUSPENSION EFFECTS

Dr. E. F. Crawley  MIT
Dr. H. Alexander  MIT
Mr. Daniel Rey  MIT

SERC Steering Committee Workshop
January 22 and 23, 1992
OUTLINE

- Motivation and Objectives
- Approach
- Overview of Gravity and Suspension Effects
- Modelling of Gravity and Suspension Effects
  - Effects on Structure
  - Effects on Sensors and Actuators
- Application to MACE
  - EM Configuration Study and DM Modelling
  - Objectives
  - Approach
  - Results
- Conclusions
- Future Work
GRAVITY AND SUSPENSION INFLUENCES

Motivation:

The need to perform closed-loop tests for performance verification of controlled structures and the reality of largely being limited to ground-based tests motivate the study of the impact of gravity and suspension effects on the closed-loop system behavior.

Objectives:

To identify and understand the effects of gravity and of a suspension system on the dynamics of a controlled structure.

To develop and codify modelling techniques for the prediction of potential changes in the plant dynamics due to the presence or absence of gravity.

To develop rule-of-thumb predictions for the magnitude of the various gravity effects based on beam equivalent approximations of the suspended structure.
APPROACH

Motivate research with a simple example of gravity field perturbation to closed-loop performance.

Identify and categorize gravity and suspension system effects.

Develop an understanding of how to incorporate gravity effects into the system variational principle and its finite order approximation.

Verify modelling techniques by applying them to sample problems.

Perform parametric variation studies and experimental validation.

Develop rules-of-thumb for when gravity and suspension system effects become important for suspended structures.
OVERVIEW: Gravity and Suspension Effects on Structure

The following are direct effects on the structure and its model (i.e. the A matrix of the controlled structure model.)

<table>
<thead>
<tr>
<th>GRAVITY FIELD EFFECTS</th>
<th>SUSPENSION SYSTEM EFFECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) FINITE DEFLECTIONS</td>
<td>3) STATIC B. C. PERTURBATIONS</td>
</tr>
<tr>
<td>- distributed gravity loads will cause initial deformations</td>
<td>- static translational stiffnesses in the horizontal and vertical</td>
</tr>
<tr>
<td>of the suspended structure.</td>
<td>directions are prescribed by the suspension system at each</td>
</tr>
<tr>
<td></td>
<td>attachment point.</td>
</tr>
<tr>
<td>2) GRAVITY STIFFENING/DESTIFFENING</td>
<td>4) DYNAMIC B.C. PERTURBATIONS</td>
</tr>
<tr>
<td>- distributed gravity loads stress the deformed structure</td>
<td>- modal coupling with the suspension dynamic modes results</td>
</tr>
<tr>
<td>which leads to eigenfrequency shifts and eigenmode</td>
<td>in dynamic impedances at the attachment points.</td>
</tr>
<tr>
<td>coupling.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5) DYNAMIC LOADING DUE TO GRAVITY FIELD AND SUSPENSION</td>
<td></td>
</tr>
<tr>
<td>CONSTRAINTS</td>
<td></td>
</tr>
<tr>
<td>- dynamic torques which result from center of mass axis</td>
<td></td>
</tr>
<tr>
<td>offsets with respect to the suspension support plane(s).</td>
<td></td>
</tr>
</tbody>
</table>
OVERVIEW: Gravity Effects on Sensors and Actuators

Sensors and actuators are also sensitive to gravity field effects. Perturbations to their performance result in changes to the control, output and feed-forward matrices (i.e. B, C and D matrices) of the model. Two types of effects can be identified:

1) Direct Effects:

The harmonic rotation of a translating mass device in a gravity field results in an additive perturbation to its ideal performance. To date, the only devices identified as being susceptible to this effect are the accelerometer and the proof-mass actuator.

Gravity loading can also result in non-ideal sensor and actuator performance; e.g. gravity induced friction in device.

2) Indirect Effects:

Perturbations to the structure (i.e. the A matrix) can result in perturbations to the sensor and actuator performance depending on their location and the extent of the perturbation to the structure.
MODELLING OF GRAVITY AND SUSPENSION EFFECTS ON STRUCTURE

Gravity

The effect of a constant stress on a structure in equilibrium is to alter its stiffness.

In a FEM this change to the stiffness matrix can be called the differential stiffness matrix or the geometric stiffness matrix. The geometric stiffness matrix is a function of the applied loading and to comprehensively incorporate the effect of gravity loading on the homogeneous system one should include its effects in the initial deformation calculations.

Suspension System

The effects of the suspension system on the structural dynamics are captured by including the suspension system in the system model before incorporating the effects of gravity.
Geometric stiffness theory and initial static deformation calculation

The geometric stiffness matrix is obtained from the non-linear strain terms in the potential energy expression for an elastic structure in equilibrium:

\[
U_p = \int_V \left\{ \frac{1}{2} \left( \varepsilon_i^T E \varepsilon_i + 2 \varepsilon_i^T E \varepsilon_q + \varepsilon_q^T E \varepsilon_q \right) - \left( \varepsilon_i^T E \varepsilon_i^{00} - \varepsilon_i^T E \varepsilon_i^{01} - \varepsilon_q^T E \varepsilon_q^{10} - \varepsilon_q^T E \varepsilon_q^{11} \right) \right\} dV \\
+ \int_V \left\{ \varepsilon_i^T \sigma_0 + \varepsilon_q^T \sigma_0 \right\} dV - \int_V u^T F dV - \int_S u^T \phi dS
\]

For finite deflections the initial static deformation calculation is a non-linear problem as both the deflection and the stiffness are functions of the loads. Iterations are therefore required to solve for the final equilibrium state:

\[
q_{i+1} = \left[ K(q_i) + Kg(q_i, Q(q_i)) \right]^{-1} Q(q_i)
\]
MODELLING GRAVITY'S EFFECT ON ACCELEROMETERS AND PMAs

Harmonic rotation of an accelerometer or proof-mass actuator about an axis other the vertical axis or the device's sensitivity/actuation axis results in an additive perturbation to the device's output/input.

The harmonic rotation can be as a result of harmonic bending or torsional vibration of the supporting structure.

The output/input is either attenuated or amplified depending on the relative phasing of the coupled translation and rotation of the supporting structure, (with a possible phase reversal in the attenuation case).

In the case of torsion the output/input perturbation is about zero such that gravity makes torsional modes observable and gravity causes the PMA force actuator to induce torques.

The effects of gravity on the B and C matrices have been identified and non-dimensional sensitivity measures show that these effects are especially important at low frequencies but are only significant with near-horizontal oriented devices and at those points where the rotations are large with respect to the displacements.
MODELLING GRAVITY'S EFFECT ON ACCELEROMETERS AND PMAs

The output of an accelerometer mounted to a beam in uncoupled bending and torsion vibration is given by:

\[
a_i = \sum_{r=1}^{N_t} \left\{ (g \sin (\Phi_r^t(x_i)) \, \kappa_i^t) \, \eta_r^t(t) \right\} + \sum_{r=1}^{N_b} \left\{ (g \sin (\Phi_r^b(x_i)) \, \kappa_i^b - \omega_j^2 \Phi_r^b(x_i)) \, \eta_r^b(t) \right\}
\]

In modal modelling terms this yields the following C matrix type terms:

\[
X = \begin{bmatrix} \eta^t & \dot{\eta}^t & \eta^b & \dot{\eta}^b \end{bmatrix}^T
\]

\[
C_{ij}^T = g \kappa_i^t \Phi_j^t(x_i)
\]

\[
C_{ij}^B = g \Phi_j^b(x_i) \kappa_i^b - \omega_j^2 \Phi_j^b(x_i)
\]

\[
C = \begin{bmatrix} C^T & 0 & C^B & 0 \end{bmatrix}
\]

The non-dimensional gravity effect ratio is thus:

\[
\Gamma = \frac{g \frac{\Phi_j^b}{\omega_j^2 \Phi_j^b} \left( \frac{x_i}{l} \right) \kappa_i^b}{\omega_j^2 \Phi_j^b \left( \frac{x_i}{l} \right)}
\]

Similar expressions hold for the effective input of a PMA mounted to a beam in bending and torsion vibration.
APPLICATION TO MACE

MACE EM Configuration Study

Objectives:

Select a MACE Engineering Model baseline configuration based on open-loop and closed-loop susceptibility to gravity and suspension effects.

MACE DM Modelling

Objectives:

Develop a high fidelity model of the MACE DM for use in control system design.
Experimentally verify the gravity and suspension modelling techniques.
MACE EM CONFIGURATION STUDY

Approach:

Develop set of 0g and 1g models of various fundamental MACE EM geometries.
Study open-loop impact of gravity and suspension system using eigensystem and transfer function references.
Study closed-loop impact of gravity and suspension system by applying LQG to system with PD stabilized bus attitude and gimbal pointing loops.

Configurations studied:

Baseline - circular section struts, 1.7 Hz fundamental, payloads at 45° from vertical, planar system (i.e. suspension plane).

- \( f = 1 \text{ Hz} \) - stiffness change to obtain 1 Hz fundamental.
- \( \frac{E_{lz}}{E_{ly}} = 3 \) - rectangular section struts, stiffened about z.
- \( \frac{E_{lz}}{E_{ly}} = 1/3 \) - rectangular section struts, destiffened about y.
- Out-of-plane - performance payload is swung 45° out-of-plane.
- Flex. App. - flap-type flexible appendages added to node 1.
- L-shaped - downward 90° bend is put in bus at node 2.
MACE EM CONFIGURATION STUDY

Sample Open-Loop Results:

- Case 5 - ONE g: MACE test article with Flexible Appare Pair
  
  Mode 17, Frequency 5,910 Hz.
MACE EM CONFIGURATION STUDY

Sample Closed-Loop Results:
MACE DM CONFIGURATION STUDY

Approach:

Add a basic model of suspension system to 0g model and incorporate static pre-deformation and geometric stiffening effects.

Model pneumatic-electric suspension devices as soft tuned springs between ceiling and suspension carriages, constrain suspension carriages to vertical translation and attach structure to carriages via stiff pinned-pinned rods.

Compare low-frequency modal I.D. data with predicted transfer functions and tune if required.

MACE 1g DM Model Schematic:
MACE DM CONFIGURATION STUDY

Status:

First iteration complete. Low frequency predictions have improved but some tuning is required including the addition of a higher order suspension model.

Sample Transfer Function Results:
CONCLUSIONS

The MACE test article is a valuable and effective test-bed for the research of gravity and suspension system effects.

The result of this research will be:

• a set of verified gravity effect and suspension system modelling techniques for application to detailed models of suspended controlled space structures, and

• a set of general gravity effect rules-of-thumb based on non-dimensional parameter descriptions of a suspended space structure.
FUTURE WORK

Paper - “Direct Effects of Gravity on the Control and Output Matrices of Controlled Structure Models.”

Investigate impact of not reforming the mass matrix in the case of large deflections.

Tune the 1g DM model and continue experimental validation using the MACE test article.

Derivation of gravity/suspension effect rules of thumb.

S.M. Thesis - “Gravity and Suspension Effects on Controlled Structure Models”
THE MIDDECK ACTIVE CONTROL EXPERIMENT (MACE): IDENTIFICATION FOR ROBUST CONTROL

Dr. Valery I. Karlov

Moscow Aviation Institute (MAI)
Identification For Robust Control

Stages of Design

- Finite Element Method
- Inputs
- ID
- Model
  - Structure and nominal parameters
  - Uncertainty (bounds)
  - "Expert" (arbitrary) bounds
- Optimization
- Robust stability
- Robust performance

no need any more!
<table>
<thead>
<tr>
<th>Level</th>
<th>Methods</th>
<th>Equations</th>
<th>Parameters</th>
<th>Realistic Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Empirical Transfer Function Estimate</td>
<td>SISO</td>
<td>$A(q)y(t) = B(q)u(t)$</td>
<td>(\tilde{y} = \Phi(\alpha, \beta))</td>
</tr>
<tr>
<td>2</td>
<td>Least Square</td>
<td>MIMO (deterministic)</td>
<td>$\dot{x} = A(x) + B(u) + \frac{\epsilon(t)}{t}$</td>
<td>(\alpha, \text{frequencies})</td>
</tr>
<tr>
<td>3</td>
<td>Extended Kalman-type filters (state and parameter estimation)</td>
<td>State-space</td>
<td>$x = A(x) + B(u) + \frac{\epsilon(t)}{t}$</td>
<td>(\alpha, \text{frequencies})</td>
</tr>
</tbody>
</table>
Basic Elements of The Approach

1. Non-linear problem of Riccati equation control for augmented covariance matrix:

\[
\begin{bmatrix}
P_x & P_{x\alpha} \\
P_{\alpha x} & P_{\alpha}
\end{bmatrix}
\]

2. Equivalent linear problem
(Received on the basis of non-traditional usage of RE analitical properties)

3. Converge numerical algorithm of optimization

4. Extended Kalman filter
(Solution on the basis of decomposition with respect to frequencies)

5. Robust control problem
- Cost averaging techniques (use the "Post-ID" bounds directly)
- Petersen-Hollot's bounds (need modification)

   a). \[SA_0 + A_0^T S + (K + \beta \gamma NWN^T) - S(BR^1 B^T - \beta \gamma^1 LVL^T)S = 0\]
   \[\beta < 1, \quad VW = P_{\alpha}\]

   b). Duality principle for design of dynamical feedback
What the Approach Provides

- **Realistic statistical model of uncertainty**
  (accuracy characteristics are received in the state-space model with "separated" noises in sensors and actuators)

- **Active ID**: Optimization of open- and close-loop inputs directly with respect to robust control performance

- **Taking into account constraints on excitation**
  (desirable ID accuracy can be achieved with much less excitation, extremely important for experiments in the space)

- **Possibility to identify time-varying parameters**
  (in case of moving rigid payloads)
Advantages of "Post-ID" Model of Uncertainty

- \( \alpha \) is Gaussian vector with covariance matrix \( P_\alpha \)

\[ \Delta \alpha \]

- Quadratic bounds
- Interval bounds
- Realistic bounds

"Worst combination" (causes conservatism of robust control, for large \( N \) dramatically)

- Reveals "cost" of different errors
- Reveals covariances between parameters
- Prevents non-realistic "worst combination" of parameters
  (degrades conservatism of robust control)
Advantages of Optimization

- Further degrading the conservatism

- Better coping with "difficulties" in the model, e.g. close modes (excitation in optimal directions amplifies the difference between modes)

- The best compromise between excitation and robust control performance

\[
J = J_R + pJ_E \quad \text{where } p \text{ is a "price" of ID}
\]

All $J$ are quadratic forms
Practical Realization

- Simulation of identification and robust control processes for MACE
  (important for confirming convergence of parameter estimates to "true" ones)

- Ground experiment

- Experiment in space

\[
\text{Optimal inputs } u = u^*(t) + L(t)y
\]

\[
\text{Experiment} \rightarrow \text{Data (y)}
\]

\[
\text{Data processing (EKF)} \rightarrow \text{Optimal parameter estimates; Model of MACE for robust pointing control}
\]
SUMMARY OF ADDITIONAL RESEARCH IN MULTIVARIABLE IDENTIFICATION AND CONTROL

Michael Athans
Frank Agguiero
Edward Bielecki
Josef Bokor
Joel Douglas
Kirk Gilpin
Leonard Lublin

January 23, 1992
ROBUST MULTIVARIABLE CONTROL WITH PARAMETRIC UNCERTAINTY

- Novel Robust LQR control strategy derived by J. Douglas (SM thesis, two ACC papers) under assumption of full state feedback.

- Uncertain energy interpretation continuing by J. Douglas and E. Bielecki.

- Extension to Robust $H_2/H_\infty$ version, with output feedback and dynamic compensation, topic of J. Douglas Ph.D. thesis with applications to MACE.
MULTIVARIABLE IDENTIFICATION

- Systematic procedure for MIMO identification, together with required software, developed (K. Gilpin SM thesis; Gilpin, Athans, Bokor 1992 ACC paper). Blends:
  - Model reduction using Hankel matrix tools.
  - Initial SISO models using ordinary least squares and sinusoidal inputs.
  - Selective model refinement using nonlinear least squares.
  - Integration into MIMO model via residue matrix methods.
  - MIMO zero refinement using maximum likelihood identification based on white noise signals. Constrain MIMO poles, refine MIMO zeros.
  - Generation of singular values vs frequency.

- Methodology tested on interferometer testbed using accelerometer measurements. Extended to laser measurements (R. Jacques, F. Agguiero, and others). Will be used by Douglas for MACE.
MIMO DESIGN UNDER PARAMETRIC AND OTHER DYNAMIC UNCERTAINTY

- Goal is how to deal with complex dynamic plant errors in MIMO feedback control designs (e.g. interferometer) and to study impact of scaling and frequency weights.

- Developed $H_2$ (frequency-weighted LQG) MIMO controller for a 2D truss using truncated beam model. Topic of L. Lublin SM thesis (being written right now). Simulation study; no hardware tests as yet.

- Parametric and high-frequency dynamics model errors limit superior disturbance-rejection performance.

- Phase information from Singular Value Decomposition provides partial information for stability-robustness.
  - We could not obtain significant reduction in conservatism associated with unstructured stability-robustness tests using this MIMO phase information.
Multivariable Identification for Control

Joel Douglas

January 23, 1992
MOTIVATION

- We want to study the design of robust multivariable controllers.
- Need a state-space model of system for design.
- How do we derive a minimal multivariable model?
SISO Identification

• Well known techniques for Identifying SISO models.
  – Output Error/Nonlinear Least Squares
    \[ Y(z) = G(z, \theta_f)U(z) + e_{oe}(z) \]
  – Equation Error/Linear Least Squares
    \[ G(z) = G(z, \theta_f) + e(z) \]
  – Results in a set of poles/zeros for each SISO loop.

• Example on MACE
  – Inputs: gimbal torque, torque wheel
  – Outputs: payload angle, encoder angle, bus angle
  – Nonlinear method with new cost function (Jacques):
    \[ J = \| \log G - \log G_m \|_2 \]
MACE EXAMPLE:
TORQUE WHEEL TO ENCODER

---

Some text and graphs are present here, showing data in different formats. The graphs appear to represent frequency responses or similar analyses, possibly related to the performance characteristics of a torque wheel encoder system.
MACE EXAMPLE:
TORQUE WHEEL TO ENCODER

$|\delta|, \phi

$\delta$, $\phi$

$E_{\text{Mag}}$

$E_{\text{4}}$

$F(\text{Hz})$

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MIMO STATE SPACE MODEL

- State Space desired for ease of implementing control algorithms
- Goal is to determine minimal representation w/o truncation
- SM thesis by Gilpin
FUTURE WORK

- Identification
  - Combine SISO transfer functions using Gilpin’s procedure.
  - Learn about identifying bounds on errors in frequencies, damping, mode shapes.

- Multivariable Control
  - Close a multivariable loop on MACE.
  - Extend robust full state feedback procedure of SM thesis to output feedback.

- Combine ID and robust control for overall procedure.
STRAIN ACTUATED AEROELASTIC CONTROL

Kenneth B Lazarus

January 23, 1992

Project Sponsors:

General Dynamics Corporation
NASA Langley Research Center
National Science Foundation
PROJECT GOAL

- Develop and Demonstrate Strain Actuated Lifting Surface Technology for Aeroelastic Control

  — Induced Strain Actuation, rather than conventional articulated methods, allows for:
    
    Control of the Lifting Surface Shape for Altering the Aerodynamic Forces
    
    Direct Control of the Strain in the Structure and Dynamic Mode Shapes

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SPECIFIC OBJECTIVES

- Develop a Capability for Analyzing Plate-Like Aeroelastic Lifting Surfaces
- Develop MIMO Control Laws for the Strain Actuated Adaptive Wing
- Demonstrate that Strain Actuation is an Effective means of Achieving Aeroelastic Control
STRUCTURAL AND AERODYNAMIC MODELLING

- Plate Geometric and Material Properties
- Wind Tunnel Operating Conditions
- Actuators / Sensors
- Model Lifting Surface with Piezoelectrics as Integrated Composite Plate
- Choose Assumed Mode Shapes
  - In Vacuo Ritz Analysis Including Strain Energy
    - Natural Vibration Modes
    - Natural Frequencies
    - Piezoelectric Forcing
    - Mass and Stiffness
    - 20 Modes
    - 40 States
  - Kernal Function Unsteady Aerodynamic Code
    - Unsteady Aerodynamic Forces
  - Non-Linear Least Squares Optimization
    - Rational Approximation of Aerodynamic Forces
    - 3 Lag Sets
    - 18 States
  - Actuator / Sensor Dynamics
    - 9 Sensor States
    - 24 Actuator States
- Full Order State Space Plant Model
  - 91 States

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CONTROL LAW DESIGN METHODOLOGY

• Reduce "Full" Order Model to "Design" Model
  — Obtain Minimum Realization
  — Find Hankel Singular Values
  — Retain Modes with Largest Hankel SV's and DC Components of Others

• Design Linear Quadratic Gaussian Compensator
  — Cost Minimization
  — Loop Shaping

• Reduce "Design" Model to "Controller" Model
  — Same Procedure as Above
  — Optimal Projection
**SYSTEM BLOCK DIAGRAM**

- Plant Model from Raleigh-Ritz and Unsteady Aerodynamic Analysis
- Sensor, Amplifier and Filter Dynamics Included in “Full” System
- Magnetic Shaker (Bench) or Gust Generator (WT) Disturbance Source
- MIMO Compensators Designed using Reduced Order LQG or Optimal Projection Theory
- Compensators Implemented by a Real Time Digital Control Computer

---

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ADAPTIVE WING TEST ARTICLE

- Cantilever Plate Configuration: Actuators Cover 71% of Plate

![Diagram of cantilever plate configuration with dimensions and power bus cross section.]
**BENCH - TOP EXPERIMENTS**

- Correlate Analytic Model and Check Hardware Functionality

- Verify Control Law Design Procedure and Gain Necessary Controller Design Experience

- Demonstrate High-Authority Large-Bandwidth Disturbance Rejection Capabilities
BENCH-TOP DISTURBANCE REJECTION: OPEN AND CLOSED LOOP RESPONSE

- Aluminum Bench Mark Specimen
- Reduced Order LQG Design: $\rho = 1e^{-2}$  Sensor Noise = 3.0%

---

**Analytic Model**

![Graph showing frequency response of analytic model.]

**Bench-Top Experiments**

![Graph showing frequency response of bench-top experiments.]

---

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BENCH-TOP DISTURBANCE REJECTION: OPEN AND CLOSED LOOP RESPONSE

- Graphite/Epoxy Bend/Twist Coupled Specimen
- Reduced Order LQG Design: \( \rho = 1e^{-2} \)  Sensor Noise = 3.0%

---

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**Bench-Top Disturbance Rejection: State Cost Versus Control Cost**

- Aluminum Bench Mark Specimen
- Reduced Order LQG & OPT Designs: Sensor Noise = 3.0%

**State Cost Reduced by 96% (14 db RMS)**
BENCH-TOP DISTURBANCE REJECTION: STATE COST VERSUS CONTROL COST

- Graphite/Epoxy Bend/Twist Coupled Specimen
- Reduced Order LQG Design: Sensor Noise = 3.0%

- State Cost Reduced by 96% (14 db RMS)

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WIND TUNNEL EXPERIMENTS

- Aeroelastic Control Issues

  Performance Objectives
  - Flutter Suppression
  - Vibration Suppression
  - Gust Alleviation
  - Maneuverability

  Control Law Objectives
  - Stability
  - Plant Regulation
  - Disturbance Rejection
  - Low Frequency Command Following

- Result: A Well Regulated Plant with High Loop Gain in the Low Frequency Regime is Desired
WIND TUNNEL SET-UP

- 1 Foot Low Turbulence Tunnel
  - Test Section: 8" x 12"
  - Maximum Speed: 100 MPH

- Gust Generator 1 Semi-Chord Ahead of Leading Edge

- Laser Displacement Sensors Built Into Side of Test Section
WIND TUNNEL GUST ALLEVIATION:
OPEN AND CLOSED LOOP RESPONSE AT 60 MPH

- Aluminum Bench Mark Specimen
- Reduced Order LQG Design: $\rho = 1e^{-1}$  
  Sensor Noise = 1.0%
WIND TUNNEL GUST ALLEVIATION:
OPEN AND CLOSED LOOP RESPONSE AT 60 MPH

- Graphite/Epoxy Bend/Twist Coupled Specimen
- Reduced Order LQG Design: \( \rho = 1 \times 10^0 \)  
  Sensor Noise = 0.5%

---

**Analytic Model**

**Bench-Top Experiments**
WIND TUNNEL GUST ALLEVIATION:
STATE COST VERSUS CONTROL COST AT 60 MPH

- Aluminum Bench Mark Specimen
- Reduced Order LQG Design: Sensor Noise = 0.5%

- State Cost Reduced by 84% (8 db RMS)

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WIND TUNNEL GUST ALLEVIATION: STATE COST VERSUS CONTROL COST AT 60 MPH

- Graphite/Epoxy Bend/Twist Coupled Specimen
- Reduced Order LQG Design: Sensor Noise = 1.0%

- State Cost Reduced by 84% (8 db RMS)
WIND TUNNEL COMMAND FOLLOWING:
OPEN AND CLOSED LOOP ERROR AT 60 MPH

Graphite/Epoxy Bend/Twist Coupled Specimen: Low Bandwidth

Analytic Model

Bench-Top Experiments
WIND TUNNEL COMMAND FOLLOWING:
OPEN AND CLOSED LOOP ERROR AT 60 MPH

Graphite/Epoxy Bend/Twist Coupled Specimen: High Bandwidth

Analytic Model

Bench-Top Experiments

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WIND TUNNEL FLUTTER SUPPRESSION: OPEN LOOP FLUTTER SPEED

- Aluminum Plate Original Flutter Speed About 125 MPH

- Flutter Speed Lowered to 88 MPH by:
  - Adding 1.6x Original Weight
  - 0.8 Semi-Chords Behind the TE
WIND TUNNEL FLUTTER SUPPRESSION: CLOSED LOOP STATE COST CURVES

- Finite State Cost (stable system) for Any Control Weight
- High Frequency Modes Are Destabilized as Gain Becomes Large

---

Space Engineering Research Center
Sensor and Actuator Technology Development

Eric Anderson and Nesbitt W. Hagood

January 23, 1992
Outline

- Sensor and Actuator Placement for Robustness
- Self-Sensing Actuators
- Nonlinear Actuation Models
**Sensor/Actuator Selection**

- Sensor and actuator selection/placement sets an *a priori* limit on closed loop performance.
- Correct placement can improve nominal performance for any specific control design technique.
- Placement problem has been investigated previously
  - open loop vs. closed loop algorithms
  - optimal vs. heuristic algorithms
- Degrees of freedom for sensor and actuator suite design:
  - Number
  - Type
  - Location
  - subject to design constraints
- Placement is typically done using initial inaccurate model.
Sensor/Actuator Selection for Robustness

- **Concept**: Select sensors and actuators to minimize impact of model inaccuracies on achievable performance and stability.

- **Motivation**:
  - Placement and resulting closed-loop performance/stability are a strong function of model.
  - Only have uncertain model on which to base placement decisions.
  - Implies large uncertainties in *achievable* closed-loop performance or robustness.

- **Method**: Incorporate model uncertainty information into open or closed loop placement algorithms.
Achievable Performance Robustness

- Control design must use actuator and sensors it is given.
  - Example: Loss of controllability when actuator is unwittingly placed at a node.

- Can enable control task by introducing performance robustness through s/a set.
Representing the Uncertain System

- All system matrices affected by model uncertainty
- Focus on finite element errors, not ID errors
- Determine eigenvector uncertainty to expected errors:
  - Stiffness of components
  - Boundary conditions
  - Mass distribution
- Two approaches:
  - Range of possible plants/systems over all uncertain parameters
  - Sensitivity of nominal plant/system to uncertain parameters
Figures of Merit

- Open loop analysis of sensor/actuator options used to reduce number of choices to manageable number

- Use controllability and observability gramians
  \[ W_c(t_0,t_1) = \int_{t_0}^{t_1} \phi(t_1, \tau) B(\tau) \phi^T(t_1, \tau) d\tau \]

- Calculate with Lyapunov equation for each value of uncertainty
  \[ 0 = A_i W_{ci} + W_{ci} A_i^T + B_i \sum_{ww} B_i^T; \]

- Closed loop cost
  \[ J_{cl} = \text{tr} \left[ \langle Q C_i^T C_i \rangle \right] \]
  where
  \[ 0 = A_i Q_i + Q_i A_i^T + B_i B_i^T \]
Design Algorithms

- Open loop
  - Compute expected value of gramians over entire uncertainty set
  - Reduce number of s/a options by straight ranking

- Closed loop
  - Use existing techniques for optimization
  - Cost is expected value over uncertainty set

- Trade off degree of open loop reduction vs. size of set for closed loop optimization
**Current Efforts**

- **Analytical sample problem: cantilevered beam**
  - 6 sensors, 6 actuators
  - LQG control (SISO and TITO)

- **Interferometer testbed**
  - Analysis based on finite element model
  - Uncertainty description provided from system ID data
  - Main focus is active strut placement problem
  - Experimental demonstration of improved closed loop performance based on sensor/actuator location and type
Piezoelectric Actuation and Sensing

- Structure with piezoelectric actuators/sensors.

- Governing Equations of Motion:

  \[
  \left( M_s + M_p \right) \ddot{r} + \left( K_s + K_p \right) r - \Theta v = B_f f \quad \text{Actuator Eq.}
  \]

  \[
  \Theta^T r + C_p v = B_q q \quad \text{Sensor Eq.}
  \]
**Simultaneous Sensing and Actuation**

- **Concept:** Use the same piece of piezoelectric simultaneously as both a structural sensor and an actuator.

- **Motivation:**
  - Eliminates need for separate sensor. Reduced signal conditioning.
  - Perfectly collocated dual sensor useful for structural control.

- **Modelling:** If the applied current and piezoelectric electrode voltage is known, one can reconstruct the mechanical strain or strain rate.

  \[
  \Theta^t r = q - C_p v \\
  \Theta^t \dot{r} = i - C_p \dot{v}
  \]

- The \(\Theta^t r\) term is proportional to averaged strain state as for the charge based sensor.

  \[
  \Theta^t r = \int_{V_p} \left[ e_{31} S_1 + e_{31} S_2 + e_{33} S_3 \right] dv
  \]

- More insight can be gained on the physical significance of the terms by using a piezoelectric circuit analogy.
**Physical Interpretation**

- The piezoelectric transformer analogy is useful for determining the physical significance of the terms. The piezoelectric element is represented as a transformer converting mechanical energy to electrical and vice versa.

- The sensor equation can be interpreted physically as measuring the difference in between applied current and the capacitance current.

\[
\Theta^T \dot{r} = i - C_p \dot{v} \\
\underline{i_{\text{mech}}} \quad \underline{i_{\text{tot}}} \quad \underline{i_{\text{elec}}}
\]
Simple Circuit Implementation

- Strain Rate Circuit

![Diagram of Strain Rate Circuit]

B) Strain-Rate Sensing Configuration

- Also possible to implement simple strain sensing circuit by measuring applied charge rather than current.
**System for Experimental Demonstration**

- Cantilevered beam with PZT wafers on the surface.
Open Loop Results

- Model compares well with measurement
- Zero location matching hindered by PZT hysteresis
Closed Loop Results

- Tip displacement/force input with "sensuator" loop closed

- Positive position feedback (PPF)

- LQG

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Applications

- Retrofit of sensing capability on existing actuator systems
  - Information on local deformation
  - Information for collocated control (addition of damping)
- Linearization of actuator response
- Health monitoring and system identification
- Active structural control (high gain collocated loops)
Nonlinear Actuation Models

Concept: Develop models of actuated structures capable of handling material nonlinearities.

Motivation:
- Piezoelectric material properties are nonlinear at high strains.
- Higher actuation performance available from inherently nonlinear materials.
- Electrostrictive materials.
- New high-strain, shape-memory ceramics.

Approach:
- Microscopic material models for capturing relevant physics.
- Macroscopic phenomenological models for nonlinear structural response using energy methods.
Conclusions

- Ongoing work in three areas:
  - Robust Actuator and Sensor Placement
  - Self-Sensing Actuation
  - Nonlinear Actuation Modeling

- Robust actuator and sensor placement addresses a clear need but faces the difficulty of good error model development.

- Self-sensing actuation has been demonstrated and modeled, works well in active control systems for simple structures, and is being applied to built up structures.

- New research on nonlinear actuation models holds promise for high fidelity modeling of high strain actuation materials.
IMPLEMENTATION OF INPUT COMMAND SHAPING TO REDUCE VIBRATION IN FLEXIBLE SPACE STRUCTURES

Kenneth W. Chang
Professor Warren P. Seering
B. Whitney Rappole

SERC Steering Committee Meeting
January 23, 1992
GOALS OF RESEARCH

- Explore theory of input command shaping to find an efficient algorithm for flexible space structures.

- Characterize MACE test article.

- Implement input shaper on the MACE structure; interpret results.
OUTLINE

• Background on Input Shaping.

• Simulation Results.

• Experimental Results.

• Future Work.
WHAT IS INPUT COMMAND SHAPING?

An **OPEN-LOOP** method of reducing residual vibration by manipulating the input to a dynamic system.
IMPULSE COMMAND SHAPING ASSUMES
SECOND ORDER LINEAR RESPONSE.

\[ y_i(t) = A_i e^{-\zeta \omega (t - t_i)} \sin((t - t_i) \omega \sqrt{1 - \zeta^2}) \]

- \( y_i \): Response to Impulse \( i \)
- \( A_i \): Magnitude of Impulse \( i \)
- \( t_i \): Time of Impulse \( i \)
- \( \omega \): System Natural Frequency
- \( \zeta \): System Damping Ratio
BY SUPERPOSITION, WE CAN CALCULATE THE RESPONSE TO MULTIPLE IMPULSES.

\[
y_i(t) = \sum_{i=1}^{N} A_i e^{-\zeta \omega (t - t_i)} \sin((t - t_i) \omega \sqrt{1 - \zeta^2})
\]

- $i$: Impulse Counter
- $N$: Number of Impulses

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AN EXPRESSIONS FOR THE AMPLITUDE OF RESIDUAL VIBRATION.

\[ \text{Amp} = \left[ \left( \sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \sin(t_i \omega \sqrt{1 - \zeta^2}) \right)^2 + \left( \sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \cos(t_i \omega \sqrt{1 - \zeta^2}) \right)^2 \right]^{1/2} \]

Expression for envelope amplitude at \( t_N \), the time of the final impulse.
TO ELIMINATE RESIDUAL VIBRATION, THESE CONSTRAINTS MUST BE MET.

\[
\sum_{i=1}^{N} A_i e^{-\zeta \omega t} \sin \left( t_i \omega \sqrt{1 - \zeta^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t} \sin \left( t_i \omega \sqrt{1 - \zeta^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i e^{-\zeta \omega t} \cos \left( t_i \omega \sqrt{1 - \zeta^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t} \cos \left( t_i \omega \sqrt{1 - \zeta^2} \right) = 0
\]
WE CAN EXTEND THIS METHOD TO MULTIPLE MODES.

\[
\sum_{i=1}^{N} A_i \, e^{-\zeta_j \, \omega_j \, t_i} \sin \left( t_i \, \omega_j \, \sqrt{1 - \zeta_j^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i \, e^{-\zeta_j \, \omega_j \, t_i} \cos \left( t_i \, \omega_j \, \sqrt{1 - \zeta_j^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i \, t_i \, e^{-\zeta_j \, \omega_j \, t_i} \sin \left( t_i \, \omega_j \, \sqrt{1 - \zeta_j^2} \right) = 0
\]

\[
\sum_{i=1}^{N} A_i \, t_i \, e^{-\zeta_j \, \omega_j \, t_i} \cos \left( t_i \, \omega_j \, \sqrt{1 - \zeta_j^2} \right) = 0
\]

These four equations are repeated for each mode "j"
AN EXAMPLE SOLUTION OF IMPULSES FOR A 4 MODE PROBLEM.
THE SHAPER IS IMPLEMENTED BY CONVOLVING THE INPUT WITH THE IMPULSES.
RESPONSE TO INPUTS FOR A SIMPLE 1 MODE SYSTEM.
DISCOS SIMULATION RESULTS.
DISCOS MODEL OF MACE.

Body 1 = Bus + Gimbals + Torque Wheels
Body 2 = Payload 1 (Rigid)
Body 3 = Payload 2 (Rigid)
**IDENTIFIED CLOSED LOOP FREQUENCIES**
**(NON-LINEAR DISCOS MODEL)**

<table>
<thead>
<tr>
<th>60° Outboard (Hz) (Beginning of 120° Slew)</th>
<th>60° Inboard (Hz) (End of 120° Slew)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.18</td>
<td>1.88</td>
</tr>
<tr>
<td>14.25</td>
<td>13.40</td>
</tr>
<tr>
<td>15.25</td>
<td>14.20</td>
</tr>
<tr>
<td>15.90</td>
<td>15.90</td>
</tr>
</tbody>
</table>
TIME TRACE OF 120° PAYLOAD SLEW

---

Response to Unshaped Input

Response to Shaped Input

---

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ENDPOINT RESPONSE TO UNSHAPED SLEW

Graph showing endpoint displacement (mm) over time (sec) with a line indicating response to unshaped input.
ENDPOINT RESPONSE TO SHAPED SLEW

![Graph showing endpoint response to shaped input over time.]

- Time (sec)
- Endpoint Displacement (mm)

- Response to Shaped Input
SHAPED RESPONSE DETAIL

![Graph showing the response to shaped input over time. The x-axis represents time in seconds (3 to 6), and the y-axis represents endpoint displacement in millimeters (0.00 to 0.05). The graph shows a oscillatory pattern with a peak at around 3.5 seconds and a trough at around 4.5 seconds. The legend indicates the response to shaped input.]
RESPONSE DETAIL FOR SHAPER WITH ADDITIONAL MODES.

- Response to Shaped Input

Endpoint Displacement (mm)

Time (sec)
EXPERIMENTAL RESULTS
MACE PLANT FREQUENCY RESPONSE

FREQUENCY (HZ)

MAGNITUDE

1.0000e-04

0.001

0.01

0.1

1

10

100
IMPULSE SHAPERS CALCULATED FOR FIRST FOUR MODES.

1.82 hz
6.82 hz
8.79 hz
9.29 hz

FIRST MODE ONLY

FOUR MODES
EXPERIMENTAL SETUP

Tektronix Fourier Analyzer

IN

OUT

White Noise

AC-100 real-time controller.

Interface

Input Shaper

Sensor Amp

Accelerometer Output

Motor Amp

Current Command
MACE PLANT WITH SHAPER FOR 1ST MODE

Magnitude vs Frequency (Hz)

Space Engineering Research Center
INSENSITIVITY CURVE FOR ONE MODE SHAPER
FREQUENCY RESPONSE WITH SHAPER FOR FIRST FOUR MODES

MAGNITUDE

100
10
1
0.1
0.01
0.001
1.0000e-04

FREQUENCY (HZ)

0 2 4 6 8 10 12 14 16 18 20

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CONCLUSIONS

- Input shaping can reduce vibration in multiple-mode systems (MACE).
- Real system non-linearities decrease effectiveness of impulse shaping.
FUTURE WORK

- Identify relevant non-linearities in MACE test article. (friction, kinematic changes in frequency, gravity effects ...)

- Implement an input shaper which most efficiently reduces residual vibration in the presence of non-linearities.

- Generalize observations for all systems.
Embedded Electronics for Intelligent Structures

Recent work in the center has established potential advantages of distributing and embedding large numbers of sensors, actuators, and processors for precision control of flexible structures.

A technique to embed electronic devices in addition to sensors and actuators in structural members has been demonstrated.

Current research is focusing on the distribution of power and signals to embedded electronic components.

- Thermal, structural, and electrical design models of a prototypical intelligent structure are being developed to explore functional requirements for embedded components.
- These requirements will drive the custom design of an efficient, single-chip amplifier suitable for driving piezoelectric actuators.

The work is expected to culminate in a graphite/epoxy beam test article in which both an actuator and an amplifier are embedded, possibly with additional control circuitry.
METHODS OF IDENTIFICATION AND MODELING OF THE INTERFEROMETER TESTBED

• OBJECTIVE:

To become familiar with the data acquisition and system identification techniques of the interferometer testbed, and to integrate suspension modes of the interferometer into the finite element code.

• APPROACH:

Use interferometer data acquisition equipment to record four transfer functions. The transfer functions correspond to two active strut inputs to the plant and two laser displacement measurements, as the output.

Linearize displacement dynamics of suspended interferometer in order to come up with a first cut model of suspension modes to incorporate in finite element code.

• STATUS:

Transfer function data taken and analyzed

Verification of predicted suspension modes in progress
PRESSURE ACTUATOR WITH VISCOSOUS FLUID DAMPING

- OBJECTIVE: The design, manufacture, and testing of a pressure-controlled actuator with viscous fluid damping
- PRINCIPLE: Controlled pressurization will alter the dimensions of the structure while eliminating high frequency excitations
- APPROACH:
  -- Application of theory to a first generation design including actuator size, shape and critical subcomponents
  -- Design and manufacture of an actuator based on an isotropic (Aluminum) pressure cavity
  -- Testing and analysis of first generation design
  -- Second generation design and manufacture based on an anisotropic (composite) cavity to maximize axial stroke and hoop stress tolerance
  -- Incorporation of actuator into a controlled truss structure
- STATUS: Initial component design completed
Passive Control/Damping: Spangler, Hall

- Objective: Design and implementation of optimal structural control using integral piezoelectrics shunted with strictly passive electrical networks.

- Approach
  - Modeling for Control Design:

  ![Diagram](image)

  * Controller input/output are voltage/current at piezo electrodes.
  * Piezo is both sensor and actuator, truly collocated.
  * $G_{22}$ is an electrical impedance which exhibits the combined effects of the piezo capacitance and the structure's dynamics.
Passive Control: Spangler, Hall (cont.)

- Control Design:
  * Passivity constraint is formulated in terms of constraints on the state-space realization of the controller network.
  * Cost (some functional of $z$) is optimized numerically over the constrained controller.

- Status:
  - Frequency domain modeling technique developed - extension of [Hagood '91] - and examples worked.
  - $\mathcal{H}_2$ controllers developed for benchmark examples, with networks restricted to RC.

- Future:
  - Develop a more fundamental, elegant solution technique for the $\mathcal{H}_2$/RC design problem (riccati equations).
  - Experiment on the Interferometer Testbed.
END-POINT CONTROL OF FLEXIBLE MANIPULATOR ARMS

Objective:
- To achieve high-bandwidth, robust, end-point position control of a two-link, planar manipulator with distributed flexibility in the links.

Approach:
- Model distributed flexibility using assumed modes.
- Investigate maximum performance achievable.
- Obtain analytical stability and performance results for desired controllers.
- Design controllers for robustness to model uncertainties, both structured and unstructured.
- Exploit the use of simplified models when possible.
- Conduct simulations.
Status:

- Simulation of planar, two-link flexible manipulator has been created.
- Investigation of nonlinear system dynamics, degree of nonlinearity of desired trajectories, and choice of mode shapes for control design are under way.
- Theory of output regulation for nonlinear systems used as a framework to elucidate tradeoffs between Feedforward and Feedback control.
- Performance limitations inherent in nonlinear nonminimum phase (NMP) systems made transparent in this way and are reducible locally to limits of performance on linear NMP systems.
- Concepts from Lyapunov-based Sliding Control and Adaptive control for rigid robots and robots with flexible joints are combined with differential geometric concepts to extend results to flexible link robots.
- Hamiltonian approach to Lyapunov function selection promises stabilizing control results without neglecting higher order dynamics through modal truncation.
- Extensions to tracking control and to controllers with robustness to parameter uncertainty seem possible.