3rd Annual Symposium

July 1, 1991

MIT Space Engineering Research Center

Controlled Structures Technology

(NASA-CR-192993) THE 3RD ANNUAL
CONTROLLED STRUCTURES TECHNOLOGY
SYMPOSIUM (MIT) 251 P

G3/39 0160322
Controlled Structures Technology
M.I.T. Space Engineering Research Center

3rd Annual Symposium
July 1, 1991 - 8:00 am to 5:00 pm

Location: Building 9, Rm. 150
Massachusetts Institute of Technology
Cambridge, MA 02139

AGENDA

8:00  Registration
     E. Crawley

8:30  Welcome and Introduction
     G. Blackwood

9:00  Optical Interferometer Testbed
     D. MacMartin

9:30  Active Impedance Matching of Complex Structural Systems

10:00 Break

10:15 Narrow and Broad Band Isolation for Uncertain Dynamic Systems
      A. von Flotow

10:45 Application of CST to Adaptive Optics
      E. Anderson

11:15 Middeck 0-G Dynamics Experiment (MODE)
      M. van Schoor

11:45 Lunch

1:00  LABORATORY TOUR (MEET IN 37-372)
      W. Seering

2:00  Shaping System Commands to Reduce Residual Vibration
      D. Miller

2:30  Middeck Active Control Experiment (MACE)

3:00  Break

3:15  Robust Control for Uncertain Structures
      M. Athans

3:45  Cost Averaging Techniques for Robust Structural Control
      N. Hagood

4:15  Intelligent Structures Technology

4:45  Conclusion
      E. Crawley
OPTICAL INTERFEROMETER TESTBED

Gary H. Blackwood

MIT Space Engineering Research Center
3rd Annual Symposium

July 1 1991
OPTICAL INTERFEROMETER TESTBED:
OUTLINE

1. Motivation for a laboratory testbed of a space based interferometer

2. Description of testbed in context of Controlled Structures Technology
   - performance metric
   - control hardware

3. Overview of testbed research in context of Controlled Structures Technology
   - structural design
   - disturbances and performance
   - sensor/actuator design
   - local/low authority control
   - global/high authority control
OPTICAL INTERFEROMETER TESTBED PROGRAM

Objective: to provide a versatile environment for well controlled experiments on complete controlled structure systems

Testbed is designed to capture the essential configuration, physics, and performance metric of actual spacecraft

Testbed was designed and constructed by students, staff, and faculty as a facility class experiment

Students will conduct their thesis experiment on the testbed by changing out structural components, control hardware and software

Process provides a realistic evaluation/demonstration of new approaches in controlled structure design
A Testbed Based on a Space-Based Optical Interferometer

- CST

- SERC Scientific Mission Orientation

- Candidate Missions

- Optical Interferometry

- Optical Interferometer Testbed Design Project

- robotics
- reflectors
- masking
- platforms
- materials proc.
A Space-Based Interferometer

- used for astrometry:
  - measure baseline and delay lines using metrology system

- used for imaging
  - measure intensity (mag) and phase (via delay line distance) of central fringe of interference pattern
  - vary baseline and rotate siderostats about LOS to target star by rigid body motion
  - reconstruct image from 2-D spatial IFT of the measured intensity
OPTICAL METROLOGY

Unique feature of testbed is multi-axis laser metrology

At 3 mock siderostat locations are precision 3 axis active mirror mounts holding common endpoint retroreflectors (cat's eyes)

Fourth vertex holds laser and other optics

Use commercially available 670 μWatt laser from Hewlett-Packard

VME based fringe counting provides seamless link to real time controller

Optical components provide 5 laser pathlength measurements:
  • defines baseline for metrology
  • define “total starlight differential pathlength error” metric, simulating both internal and external error sources
  • a subset of these measurements are available for feedback
OPTICAL INTERFEROMETER TESTBED
(OVERVIEW)

- Testbed based on scientific mission for focus of graduate student theses.
- Testbed modelled on a 35 meter baseline earth-orbiting optical interferometer.
- Precision alignment requirement between three onboard optical elements is 50 nanometers RMS above 1/10th of a hertz.
SENSORS AND ACTUATORS

Sensors for Identification and Control

- 32 Kistler accelerometers for modal identifications testing
- 9 Sunstrand micro-g accelerometers mounted at performance-critical locations
- 5 channels of laser measurement
- strain gages and load cells collocated with active struts

Actuators for Identification and Control

- 3 active struts capable of 60 microns of stroke and 250 Newtons of force at high bandwidth (Physik Intrumente)
- 3 three-axis precision mirror actuators (custom) at each mock siderostat location
- Passively shunted piezoelectric struts
Real Time Control Hardware

- VME based digital control hardware
  - 68030 processor
  - CSPI vector processor
- Capability:
  - 16 inputs
  - 10 outputs
  - 32 states at 1000 Hz; scales by \((\text{ns + ni}) \times (\text{ns + no})\)
- Direct link to six HP laser measurement boards
- Control design in MATLAB on Sun SparcStation

- **Analog:** circuits for displacement and velocity feedback to active struts
A CST DESIGN METHODOLOGY

1. Design structure.
2. Design disturbances and control objective.
3. Design actuator and sensor.
4. Design local/low authority control.
5. Design high authority control.
INTERFEROMETER: EXAMPLE OF CST DESIGN

The interferometer will be used not only as a testbed for the elements of Controlled Structures Technology but also for the evolution of the design process.

Step 0 - Mission Requirement Specification
Disturbance selected from spacecraft experience.
Performance metric established which captures challenge of real spacecraft mission.

Step 1 - Design Structure
Structure chosen for “rigid” alignment of primary performance measures - optical elements of the interferometer
Structure serves as “host” to robust control
Passive damping augmentation for performance and robustness

Step 2 - Design disturbances and control objective
Mount onboard disturbances at locations of lower disturbability
Passive and active isolation at source and output
INTERFEROMETER: EXAMPLE OF CST DESIGN

Step 3 - Design Actuator and Sensor
      pole-zero analysis
      actuator and sensor combinations for active isolation
      induced strain actuators
      actuator and sensor placement for global control
      quasi-static shape control

Step 4 - Local/Low Authority Control
      impedance matching
      wave control
      active isolation at disturbance input and at quiet payloads

Step 5 - Global/High Authority Control
      modelling and MIMO identification for control
      global control using distributed active struts
      heirarchic control formulation to simultaneously optimize
      local and global controllers
IDENTIFICATION FOR MIMO CONTROL

SISO Block 1
- Experimental measurement
- LLS Estimation
- NLLS Estimation
- Partial Fraction Expansion

SISO Block n

MIMO Synthesis
- Matrix Partial Fraction Expansion
- Dyadic Decomposition
- MIMO State Space

Strut 3 to Accel. 8

Model Validation
- experiment
- model

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# FEM/ID Correlation for the Naked Truss

<table>
<thead>
<tr>
<th>Measured</th>
<th>FEM 1</th>
<th>FEM 2</th>
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</thead>
<tbody>
<tr>
<td>31.35 Hz</td>
<td>suspension</td>
<td></td>
</tr>
<tr>
<td>31.75 Hz</td>
<td>suspension</td>
<td></td>
</tr>
<tr>
<td>(35.1) Hz</td>
<td>3.5 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>35.1 Hz</td>
<td>3.5 %</td>
<td>0.4 %</td>
</tr>
<tr>
<td>38.9 Hz</td>
<td>4.7 %</td>
<td>1.8 %</td>
</tr>
<tr>
<td>(38.9) Hz</td>
<td>4.7 %</td>
<td>1.8 %</td>
</tr>
<tr>
<td>39.4 Hz</td>
<td>3.5 %</td>
<td>0.6 %</td>
</tr>
<tr>
<td>43.3 Hz</td>
<td>4.0 %</td>
<td>0.9 %</td>
</tr>
<tr>
<td>43.7 Hz</td>
<td>0.2 %</td>
<td>0.1 %</td>
</tr>
<tr>
<td>(43.7) Hz</td>
<td>3.2 %</td>
<td>0.1 %</td>
</tr>
<tr>
<td>52.1 Hz</td>
<td>3.3 %</td>
<td>0.2 %</td>
</tr>
<tr>
<td>54.7 Hz</td>
<td>3.3 %</td>
<td>0.6 %</td>
</tr>
<tr>
<td>55.2 Hz</td>
<td>2.4 %</td>
<td>-0.3 %</td>
</tr>
<tr>
<td>55.6 Hz</td>
<td>2.4 %</td>
<td>-0.4 %</td>
</tr>
<tr>
<td>62.7 Hz</td>
<td>suspension</td>
<td></td>
</tr>
<tr>
<td>63.4 Hz</td>
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<td>94.1 Hz</td>
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<td>1.8 %</td>
</tr>
<tr>
<td>102.0 Hz</td>
<td>2.6 %</td>
<td>1.6 %</td>
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</tbody>
</table>

## Modeshapes

Overall agreement with FEM modeshapes

## Current Issues

- Assessment of accuracy of measured residues for structure with high modal density
- Correlation for incomplete ID and degenerated modes
- Correction, based on ID results, of global parameters, to match the frequencies
- Correction, based on ID results, of local parameters, to match the modeshapes

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DISTURBANCE MODELLING

Performance degradation due primarily to disturbances from reaction wheels and other on-board machinery. Disturbance level at output expected to be 500 nm (rms) on full scale.

Performance metric is not a function of baseline. Scale lab disturbance to same level of 500 nm (rms)

Disturbance source is piezo actuator mounted to vertex; two disturbances modelled are:

- broadband disturbance
- slowly time varying narrowband disturbance

Pathlength error due to broadband disturbance modeled using Statistical Energy Analysis, assuming a typical disturbance input location.

Indicates that 99% of the performance degradation occurs between 20 and 60 Hz.
**DISTURBANCE SOURCE IMPLEMENTATION**

**Goal:** Use 3-axis piezoelectric disturbance source to implement the broadband disturbance spectrum resulting in 500 nm (rms) pathlength error

**Test:** Measured optical pathlength from the fourth vertex to siderostat A with the disturbance source on and off. Computed the RMS change in length over 1/3 Octave bands.

**Results:** Disturbance source does degrade the pathlength as expected while concentrating energy in vicinity of first few structural modes.
STRUCTURE DESIGN: PASSIVE DAMPING

- damping provides some performance within bandwidth
- adds robustness to plant in rolloff region
- improves performance of high authority controllers within bandwidth, which are sensitive to frequency modelling errors

Option: Constrained Layer Viscoelastic struts

- advantages: inexpensive, easy to make
- disadvantages: temperature sensitive, loss factor not high (.05) many struts are required

Typical damped strut frequency response

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STRUCTURE DESIGN: PASSIVE DAMPING

Option: Honeywell D-Strut Passive Damper

- can be tailored for specific structural impedances
- struts have high loss factor (1.5); fewer struts will be necessary
- analytical study: D-Struts placed in locations of highest weighted strain energy; spectrum of disturbance at pathlength output is improved.

Figure 14: Locations of Struts with Highest Weighted Strain Energies
Shunted Piezoelectric Damping

![Diagram of a shunted piezoelectric device with labels for the PZT stack, preload bolt, and circuit diagrams for resistive shunting and resonant shunting.]
LOW AUTHORITY CONTROL

- adds some performance in control bandwidth
- adds robustness in rolloff region of high authority control
- similar benefits to passive damping

Options:
- velocity feedback using active struts
- impedance matching/power flow approaches using external power source
- same formulation, but using passive elements only (shunted piezoelectrics)
Active Isolation of Lightweight Mirrors on Flexible Structures

- Plant transfer function is
  
  \[
  \frac{\text{mirror displacement}}{\text{piezo voltage}}
  \]

- Active mirrors located at locations A, B, C

- Non-dimensional parameter for flex coupling of the ith mode:
  \[
  \beta = \left( \frac{1}{\alpha_i} \right) \left( \frac{m\phi_i^2}{2} \right)
  \]
Open Loop Transfer Function of Mirror

- mirror actuated in piston (z) direction only

mirror mounted on rigid base

Piston & Y on Table  

<table>
<thead>
<tr>
<th>Hz</th>
<th>10^1</th>
<th>10^2</th>
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</table>

mirror mounted at truss vertex  
(plate C)

Piston & Y on Plate C  

<table>
<thead>
<tr>
<th>Hz</th>
<th>10^1</th>
<th>10^2</th>
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mirror mounted on truss beam  
(plate A)

Piston on Plate A  

<table>
<thead>
<tr>
<th>Hz</th>
<th>10^1</th>
<th>10^2</th>
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GLOBAL/HIGH AUTHORITY CONTROL

Current areas of research:

Active strut placement
Sensor placement for control
Static shape control problem
Hierarchic control architectures that combine two or more levels of control design
Probabilistic formulation of control design
SUMMARY

Optical Interferometer testbed captures essential configuration, physics, and performance metric of a class of spacecraft

Student theses directed at different aspects of controlled structures design

Near-term plans are demonstration of local and global control approaches on testbed and evaluation with full optical performance metric
Active Impedance Matching of Complex Structural Systems

Douglas G. MacMartin
David W. Miller
Steven R. Hall

July 1, 1991
Goal

- Active broadband control of uncertain modally dense structures.
- Use collocated feedback.
  - Positive real controller guarantees stability.
  - Low authority or local control ("active damping.")
- Use local acoustic or statistical model of structure.
- Maximize power dissipation.
  - Equivalent to impedance matching.
  - Cannot match impedance exactly at all frequencies due to causality constraint.
- Experimental demonstration on complex structures.
Travelling Wave Model

- Describe junction dynamics using waves.
- Relate physical variables to wave modes:
  \[
  \begin{bmatrix}
  q \\
  f
  \end{bmatrix} = \begin{bmatrix}
  Y_{qi} & Y_{qo} \\
  Y_{fi} & Y_{fo}
  \end{bmatrix}
  \begin{bmatrix}
  w_i \\
  w_o
  \end{bmatrix}
  \]
- Scattering and generation of waves:
  \[
  w_o(s) = S(s)w_i(s) + \Psi(s)u(s)
  \]
- Relate physical variables to control:
  \[
  q = (Y_{qo}\Psi)u + (Y_{qi} + Y_{qo}S)w_i
  \]
Dereverberated Mobility Model

- Total Response = Direct Field + Reverberant Field

- Response of form:
  \[ y(s) = G(s)u(s) + d(s) \]

- Gives accurate model of local dynamics.

- Approach applicable to arbitrary structure.
Computation of Dereverberated Mobility

- From wave model:
  \[ G = s \mathcal{T} \mathcal{L}_s \]
  \[ \mathcal{T} = \mathcal{L}_s \]

- From averaging transfer function:
Control Problem: Optimal Impedance Matching

- Design compensator based on local model and apply to real structure.
- Minimize power flow into structure.
  \[ \Pi(\omega) = \text{tr} \{ \Phi_{uy}(\omega) + \Phi_{yu}(\omega) \} \]
- Maximum dissipation is obtained if the compensator is the conjugate of the structural impedance:
  \[ K(s) = (G(-s)^T)^{-1} \]
- This is noncausal!
- Problem is to match impedance as well as possible, subject to causality.
$H_2$-Optimal Solution

- Minimize rms power flow using Wiener-Hopf or LQG to guarantee causality.

$$J = \int_{-\infty}^{\infty} \mathbf{IP}(\omega)d\omega$$

$$= \int_{-\infty}^{\infty} \text{tr} \{ \Phi_{uy}(\omega) + \Phi_{yu}(\omega) \} d\omega$$

- Requires knowledge of disturbance spectrum $\Phi_{dd}$.

- Local model is not conservative: departing energy does not return.

- No guarantee of stability on actual structure.

  - May add power at certain frequencies to achieve greater dissipation at others.
\( \mathcal{H}_\infty \)-Optimal Solution

- Guarantee stability by guaranteeing power dissipation at all frequencies:

\[
\mathbf{P}(\omega) = \text{tr} \left\{ \Phi_{uy}(\omega) + \Phi_{yu}(\omega) \right\} < 0 \quad \forall \omega
\]

- Instead, guarantee that the reflected power into the structure is less than the incoming power.
  - This is a standard \( \mathcal{H}_\infty \) control problem.

- Guaranteeing \( \|T_{zw}\|_\infty < 1 \) guarantees power dissipation at all frequencies.

- This also guarantees a positive real compensator.
Statistical Energy Analysis (SEA) Solution

- Model structure with average driving point mobility (a generalization of the dereverberated mobility.)
- Include knowledge that structure conserves energy.
- Guaranteed finite energy $\Rightarrow$ Guaranteed stability.
- Minimize desired rms cost, expressed in terms of power flow.
Experimental Transfer Functions

a) Rate  
b) $\mathcal{H}_\infty$  
c) $\mathcal{H}_2$  
d) Weighted $\mathcal{H}_\infty$
Interferometer Actuator and Sensor Locations

Disturbance Source

Active Strut 3

Accelerometer
Active Strut Configuration
"Power" Dual Variables

• Force into structure and relative velocity across active strut are dual.

• Piezo stack stiffness is high $\Rightarrow$ commands displacement.
  – Can also command relative velocity.

• Want compensator $K(s)$ such that

$$ \dot{x} = K(s)f $$

$$ \Rightarrow x = K(s)\left(\frac{f}{s}\right) $$

– Use integral of force feedback.
Dereverberation of Complex Structure

- Wave approach: truss behaves like a beam at low frequencies.
- Compute "best" fit of log magnitude using only real poles and zeroes.
- Fit transfer function using complex poles and zeroes, and add damping to resulting model.
Dereverberated Transfer Function

- Open loop transfer function from displacement to integrated force.
- Three (real) pole fit of log magnitude.
Compensators

![Graph of magnitude and phase vs. frequency for different compensators: Wtd Hinf, Hinf, Rate, Noncausal.](image)

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Preliminary Experimental Results

- Open and closed loop transfer functions from disturbance source to siderostat acceleration.
- Single constant gain loop closed around active strut # 3.
Conclusions

- Local model can be used for control design for uncertain modally dense systems.
  - Travelling wave or dereverberated mobility model.
- Ideal compensator for power dissipation is usually noncausal.
  - This is an impedance matching problem.
- $\mathcal{H}_2$ or $\mathcal{H}_\infty$ optimal matches give good performance.
  - $\mathcal{H}_\infty$ approach guarantees stability.
  - Weighting functions introduce flexibility.
- Concepts can be applied to arbitrarily complex structural systems.
  - Some damping added with simple compensator.
  - Expect more damping possible with better impedance match.
ACTIVE VIBRATION ISOLATION

- NOISY SOURCES, SENSITIVE DESTINATIONS

- OPPORTUNITIES FOR ACTIVE CONTROL

- CURRENT WORK / FORECAST
Applications

Noisy Machine
Flexible structure

Noisy Helicopter Transmission
Flexible Fuselage

Sensitive Instrument

Noisy Space Station

Common Features:

- The mount represents a "bottle neck" in the disturbance path.
- Structural Response is poorly known.
DYNAMIC MACHINERY ISOLATION

FD

noisy machine

FT

mount

flexible structure

FD = disturbance force

FT = transmitted force

STIFFNESS OF MOUNT

ideal

passive compromise

frequency

disturbance spectrum
TRADITIONAL PASSIVE MOUNT DESIGN

DESIGN MODEL:

Rigid Machine
Soft Mount
Rigid Vehicle

\[ f_D \] (disturbance)
\[ f_T \] (transmitted)

\( \frac{f_T}{f_D} \)

well isolated disturbance frequencies

EXAMPLE: Naval Machinery Raft
EXAMPLES OF PASSIVE ISOLATION

Gravity Wave Interferometer

Space Station PPS

Fig 4 Configuration of Gimbal Pointer Model with Passive Base Isolator

Fig 6 Isolator Performance vs Secondary Stiffness K
RESPONSE FREQUENCIES:

Machine:

Mount:

Vehicle

MODELING APPROACHES:

Machine:

Vehicle

DISTURBANCE SPECTRUM

0 Frequency

broadband with "spikes"
HOW MANY INDEPENDENT MOUNT AXES?

If $\omega_{\text{Dist}} < \omega_{\text{Flex}}$, then a flexible body is rigid.
OVERVIEW OF ACTIVE APPROACHES FOR MACHINERY ISOLATION

- Broadband / Semi-passive
- Narrowband
ACTUATOR OPTIONS

GEOMETRY

ACTUATOR IN THE LOAD PATH:

ACTUATOR PARALLEL TO THE LOAD PATH

ACTUATOR REQUIREMENTS
1) Variable viscous damping:

Graf & Shoureshi, 1987
(also much work on vehicle suspensions)
BROADBAND/ACTIVE

Newport Corp EVIS Vibration Table (1988)

Figure 1. EVIS of vibrations from the surface as a function of frequency.

Sperry Corp. FEAMIS Magnetic Isolation System

Fig. 5 Isolator Response Comparison
BROADBAND SINGLE AXIS FEEDBACK
(WATTERS ET. AL. 1989)

Fig. 7. Open-Loop Frequency Response for Uncompensated System

BROADBAND FEEDBACK DOES NOT SELECTIVELY ATTENUATE NARROWBAND DISTURBANCES
**IMPEDANCE MATCHED SENSOR/ACTUATORS**

(For robust broadband active isolation)

**FORCE (ZERO IMPEDANCE)**

- Flex. blob
- Force actuator
- Passive spring
- Force sensor

**DISPLACEMENT (INFINITE IMPEDANCE)**

- Flex. blob
- Acceleration
- Stroke actuator
A PASSIVE NARROW BAND ISOLATION MOUNT

\[ f_D = \sin(\omega_1 t) \]

(A tip-weighted beam in bending)

USE: VIBRATION ISOLATION MOUNTING OF A HELICOPTER FLOOR
Basic principle: Provide compensator gain only at (well-known) disturbance frequency. Avoid destabilizing unmodelled dynamics.

MIMO implementation?
CLASSIFICATION OF APPLICATIONS AND ACCOMPLISHMENTS

Generic

Disturbance mechanism

Adaptation

Disturbance Feedforward

Actuators

Physical System

Important Responses

Feedback

Sensors
NARROWBAND/ACTIVE CONTROL SYNTHESIS TECHNIQUES

Adaptive Signal Processing:

Tuned Oscillator in Feedback Loop:
Allen & Calcaterra (1972)

Figure 21: Vertical Transmissibility of Active Seat Isolator in Isolate Mode Measured During Laboratory Tests
NARROWBAND/ACTIVE

Fig. 3.

White & Cooper (1984)

Fig. 7. Sensor 1 autopower spectra.
NARROWBAND/ACTIVE

**Fig. 5** The active engine mount prevents engine vibration reaching the seating but ignores all other forces. It thus holds the engine rigidly aligned. Any vibration from other parts of the structure which may be transmitted via the seating to the mount (i.e., the reverse direction) is treated identically—it is ignored and not cancelled. This is an essential feature of any active mount since cancellation would not be occurring at "source" and would result in actual enhancement in other parts of the structure.

**Fig. 6** Typical periodic spectrum from the active vibration cancellation rig — (a) before cancellation, (b) after cancellation.

EGHTESADI & CHAPLIN (1987)
TRACKING A TIME-VARYING PERIODIC DISTURBANCE

Narrowband Vibration Isolation

\[
\frac{S + a}{S^2 + 2\xi \omega_n S + \omega_n^2}
\]

\[\omega_n = \omega_{\text{dist}} \quad \text{(time-varying)}\]

\[\zeta = \zeta_{\text{plant}} \quad \text{(best)}\]

(Track the disturbance frequenies through resonances)
Narrowband Vibration Isolation

Apparatus

Active Isolation Performance:
(Time Varying Sinusoidal Disturbance)

Light Machine
\[ \text{swept in one second} \]

Massive Machine
\[ \text{swept in one second} \]
UNKNOWN PLANT PHASE

Compromise between performance and stability:

Possible root - loci (local):

(Collocated force sensor, piezo actuator)

Compromise:

Make compensator damping ratio equal to plant damping ratio

(Limits theoretical active disturbance attenuation to $1/\zeta$)
A Study of Error Sensors and Important Axes

Figure 1. Details of the mount design and laboratory experimental set-up.

Figure 2. Section of the experimental system used to measure the reductions in PSD for the diesel installation, also shown is a plan view of the receiver and measurement locations.

Figure 3. Reductions in cost and energy functions for the system with vertical primary excitation.

Figure 4. Reductions in cost and energy functions for the system with horizontal primary excitation.

Figure 5. Reductions in cost and energy functions for the system with vertical primary excitation and the error sensors located on the intermediate plates.

Figure 6. Reductions in cost and energy functions for the system with horizontal primary excitation and the error sensors located on the intermediate plates.

98  JENKINS ET. AL., ASME WAP, DALLAS, TX, 1990
SUMMARY

- Error sensors should be carefully selected
- Colocated force transducers make good error sensors
- Flanking paths may be important
- Uncontrolled axes are flanking paths
- Active isolation of periodic disturbances is ready for widespread implementation:

  Each application:
  - how many axes?
  - flanking paths?
  - actuators?
  - sensors?
  - plant dynamics?
MOUNTING OF INSTRUMENTS

The influence of disturbances on the cost can be written in terms of modal parameters

$$J = \frac{1}{\delta \omega_0} \left( \Phi^T \beta_u \omega \omega^T \beta_u^T \Phi \right) \gamma_q$$

- \( \Phi \): mass normalized eigenvector at input location
- \( \gamma_q \): state penalty at output location

Sensible mounting locations are those for which \( \phi \) is small (nodes) for the appropriate coupling degrees of freedom

The MIT SERC interferometer testbed:

- closed tetrahedral topology
- vertices provide good mounting locations (20 dB improvement)
- midspan of beams exhibit flexible coupling
REACTUATION OF GIMBALED PAYLOADS

- used to articulate a payload with minimal reaction torques on base structure
- payload is pointed by reacting against a rigid reaction wheel using gimbal motor
- in ideal situation, no net torque is applied to structure
- in real situation, gimbal friction, cabling stiffness limit torque reduction

ref: Laskin et. al., AAS, 1987
(with mass center offset)
BROADBAND ACTIVE ISOLATION

Problem: isolate lightweight mirror from a broadband base disturbance. The base is also flexible.

- wish to design controllers that can ignore the flexibility of the base
- take advantage of a mass ratio that is small compared to the damping ratio to bound gain and phase excursion in the control loop
Lightweight Mirror Positioning on Interferometer Testbed

![Diagram of active mirror and interferometer testbed](image)

![Graph of piston on plate C](image)

![Graph of piston on plate A](image)
FORECAST: (ACTIVE VIBRATION ISOLATION)

- Active control of periodic noise (narrowband) is easy and important.

- Active techniques will have major impact upon narrowband machinery isolation when
  - Design requirements prohibit soft mounts
  - Performance demands justify cost and maintenance

- Broadband isolation will remain passive or quasi-passive, particularly in presence of important plant dynamics.

- Clever sensor/actuator selection enables broadband active isolation; structural dynamics do not contribute to plant
The Application of Controlled Structures Technology to Adaptive Optics

Eric H. Anderson
Jonathan P. How

3rd Annual SERC Symposium
July 1, 1991
Historical Note

- Archimedes (287-212 BC)
- Roman warships burned during siege of Syracuse (open to historical debate)
Outline

- Objectives
- Review of current approaches to optics
- Constraints on weight and performance imposed by control-structure interaction (CSI)
- Potential benefits of CST methodology
- Testbeds for demonstration of approach
- Experimental results from a simple testbed
- Open issues and conclusions
Objectives

- Understand current approaches in optics
- Investigate potential limitations imposed by flexibility in existing and planned optical systems
- Determine and demonstrate benefits of designing large optical systems within the Control Structures Technology (CST) framework.
  - Deformable surfaces
  - Flexible support structures
A Typical Large Optical System (LDR)

Major components:
- Primary
- Secondary/other optics
- Support structure
Overview of Current Optical Programs

• Current design methodologies
  • mirror and support structure designed to avoid interaction of flexible modes with disturbances
  • controller bandwidth limited to avoid any interaction with flexible modes

• Note on terminology
  • Adaptive control
  • Active and adaptive structures
  • Active and adaptive optics
Current Approaches

- Passive devices
  - designed for mechanical and thermal stability
  - typically heavy, thick, very stiff (high natural frequency)
  - not directly designed to perform active or adaptive optics
  - possible gravity sag in secondary support structure
  - recent advance: large lightweight mirror (borosilicate and thin meniscus)

- Example: Hale Observatory/Mt. Palomar (1949)
Current Approaches, cont.

- Active devices
  - quasistatic shape correction
  - demonstrated benefits over passive devices:
    - improved performance with wind disturbances.
    - requires smaller blanks and less stringent polishing
  - backplane stiffness (weight) major issue for monolithic mirrors (depends on actuator type)
  - control has proved effective for segmented mirrors (Keck), but CSI limits bandwidth and performance
  - employ thin meniscus mirrors (40:1)

- Examples: HST, Keck, New Technology Telescope (NTT)
Current Approaches, cont.

- Adaptive devices
  - atmospheric compensation requires very high frequency control.
  - current design methodology results in structures with natural frequencies of kHz.
  - leads to size and weight restrictions
  - pixel processing limit exists as well
  - recent innovation: laser guide star beacon (Starfire)

- Examples: SDI, Very Large Telescope (VLT)
Typical Adaptive Optics System

(Merkle, 1986)
Actuation Approaches for Deforming a Mirror

(Hardy, 1980)
Control Approach Comparison

- Current design:
  - stiff structure
  - static displacement feedback

- CST approach:
  - performance demands lead to higher bandwidths
  - weight limitations lower natural frequency
  - design structural loop to augment WFS, using passive and active feedback
CSI Control Bandwidth Limitations

- Simple models [Hardy 1977], [Robertson 1970] can predict CSI instability from displacement feedback.

- In this case, can increase control bandwidth by including rate feedback and/or passive damping [Pearson and Hansen].

- CST design methodology provides unified approach to control design for significantly more complex structures

  $$\Rightarrow$$ potential performance improvements.
Applications of CST Concepts to Optics

- Examples from literature:
  - Pearson and Hansen [1977] - viscous damping oil extends control bandwidth beyond first mode
  - Greene and Pope [1980] discuss importance of flexible modes on optical performance
  - Perkin Elmer JOSE proposal - truss control bandwidth includes natural frequency of primary mirror
  - segmented mirror designs with active elements in support structure [Chen et al., 1989]
  - ASCIE - closed loop verification of CST on a segmented precision optical device
  - Combined CST/optics modelling (Redding)

- Emphasis of previous CST work has been on support structure
- Application to flexible mirror surface control not fully explored.
Deformable Mirror Testbed Concept

- Main features:
  - self-straining lightweight mirror surface, no backplane
  - elliptical shape for nearby source and sensor (or use distrib. point retroreflectors)
  - 1D testbed captures some relevant science and dynamics.
  - tip/tilt and isolation from supporting truss structure (not shown)
**Testbed Functional Block Diagram**

- **Main components:**
  - mirror (actuators/sensors)
  - isolation and controller
  - optical measurement and performance evaluation

- For design flexibility, disturbances enter acoustically, mechanically, and optically.


**Distributed Control Approaches**

- Distribute control to complement dynamic behavior of structure.

- Efficiency and performance of multi-level architecture required to handle large number of sensors and actuators.

- Low authority controllers designed for stability robustness and local performance.
Available Testbeds

- Phase 0 deformable mirror testbed
  - Initial step towards more comprehensive testbed
  - Self-straining (PZT actuators)
  - Optical and structural (PVDF) measurements
- Interferometer multi-point alignment testbed
Deformable Mirror Testbed
Results from the DMT

- Static and quasistatic considerations
- Noncollocated transfer functions and non-minimum phase zeros (Fleming, Spector and Flashner)
- Overall approach to dynamic control
  - Passive damping (resistive shunting)
  - Low authority control (collocated strain rate feedback)
  - High authority control (reduced order LQG, noncollocated optical measurement, low bandwidth)
- RMS performance

<table>
<thead>
<tr>
<th>Test</th>
<th>Description</th>
<th>Tip RMS (microns)</th>
<th>Perf. Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Open Loop</td>
<td>23.0</td>
<td>1.0</td>
</tr>
<tr>
<td>B</td>
<td>Shunt</td>
<td>21.9</td>
<td>.95</td>
</tr>
<tr>
<td>C</td>
<td>CRF(1,2)</td>
<td>19.9</td>
<td>.86</td>
</tr>
<tr>
<td>D</td>
<td>CRF(1,2), HAC(2)</td>
<td>16.8</td>
<td>.73</td>
</tr>
<tr>
<td>E</td>
<td>CRF(1), HAC(1,2)</td>
<td>11.8</td>
<td>.51</td>
</tr>
</tbody>
</table>

Table 1: Test summary. Numbers in correspond to sensor/actuator pairs for collocated rate feedback (CRF), or the actuator used for high authority control (HAC).
Quasistatic Error Correction

- Sources of error: initially wrong shape; thermal distortion; actuator material nonlinearities
- Solutions: alternative actuators; charge control; linearizing feedback

![Graph showing proportional feedback / actuator 3](image)
Low Authority Control (LAC)

- Measuring strain rate with PVDF
- Rolloff issues
- Implemented at locations 1 and 2
- Target 3rd and 4th modes (400 Hz, 780 Hz)
- Disturbance at actuator 3
High Authority Control (HAC) Design Methodology

- Designed for first 2 modes (freq.-weighted rms performance)
- 10th order design model based on measured data
- Reduced order LQG
  - Technique developed by Mercadal
  - Possible robustness improvement
  - 2nd and 4th order compensators
- Disturbance at actuator 3
Results of HAC

- Implemented on "stealth" real-time control computer at 4 kHz

HAC: actuator 2
CRF: locations 1,2

HAC: actuators 1,2
CRF: location 1
Open Issues

- Must demonstrate concepts on a realistic optical system
  - 2-D mirror
  - wavefront sensor
  - disturbances
    - aircraft borne, atmospheric, wind, thermal, gravity gradient
- Structural sensor options
  - number, location, size, shape
  - fiber optics, strain gauges, piezo-polymers (PVDF), piezo-accelerometers
  - resolution for small surface deflections
  - benefits of "ganging"
- Actuator options
  - number, location, size, shape
  - amplifier power consumption and potential of charge storage devices.
  - membrane mirrors/electrostatic actuation
Open Issues

- Control
  - efficient architecture for numerous sensors
  - implementation: digital/digital vs. digital/analog
  - integration of WFS information into control loop
Conclusions

- Deformable mirrors built to date have been constrained by flexibility.

- Optical systems/deformable mirrors can benefit immediately from application of CST.

- Both weight reduction and performance improvement are possible.

- Application to surface control complements multi-point alignment goal
**Middeck 0-gravity Dynamics Experiment (MODE)**

Marthinus C van Schoor

MIT Space Engineering Research Center
3rd Annual Symposium

July 1991
OUTLINE

- MODE and its Rationale
- Program Objectives
- Science Overview
  - Truss structures
  - Contained fluids
- Experimental Design
  - Structures
  - Contained Fluids
- Progress to date
  - Component Tester
  - ESM, STA and FTA
  - Schedule
MODE and its Rationale

- An experiment that investigates the nonlinear characteristics of two important components of spacecraft
  - Nonlinear dynamics of truss structures
  - Nonlinear dynamics of contained fluids
• Why investigating the dynamics of truss structures?
  - Nonlinear dynamics of jointed space structures can alter the vibrational/acoustical characteristics of a space structure
  - This behavior is important for:
    • On-board micro-gravity experiments
    • Passive damping characteristics of "open-loop" structures
    • Closed-loop stability and performance of controlled structures
  - Little experimental data is available on how gravity effects the dynamic characteristics of jointed space structures and models are not verified
MODE and its Rationale (Continued)

- Why investigating the dynamics of contained fluids?
  - Large fluid/spacecraft mass fractions are desirable
  - Dynamics of contained fluids in space are inherently different from their behavior in 1-g
  - The traditional "linearized or small amplitude" approach cannot be used since
    The motion resulting from large amplitude vibrations significantly departs from the linear behavior
    Bifurcation instabilities and non-deterministic motion also exist
    Nonlinear fluid motion interacts with the spacecraft degrees-of-freedom to yield nonlinear spacecraft modal behavior
  - The existing linear/quasi-nonlinear models are inadequate and new nonlinear models are not validated for zero-gravity conditions. This leads to conservative attitude control designs for spacecraft's with on-board fluids to avoid instabilities
Program Objectives

- For space structures?
  - To establish a database of the dynamic response behavior of structures with typical space structure-components
  - To develop a nonlinear model for the spacecraft's zero-gravity nonlinear structural resonant and transient response characteristics
  - To use the results/model to understand and model how the nonlinear characteristics will alter the spacecraft's vibration/acoustics characteristics
  - Identify the limitations of earth modal testing given the influence of gravity effects on the modal characteristics
  - Use the knowledge and models to design optimal structures and robust and optimal structural controllers.
Program Objectives (Continued)

- For the contained fluids?
  - To obtain the "missing" data point. The measurement of the nonlinear dynamic characteristics of contained fluids in zero-gravity
  - To understand how the nonlinear fluid dynamics interact with the motion of the spacecraft
  - To use the experimental results to verify the nonlinear model developed at MIT
  - To establish a design tool with which designers can with confidence design optimal and robust attitude controllers - even for spacecraft with high fluid/spacecraft mass fractions
Science Overview (Truss Structures)
Gravity effects that alter the modal characteristics of truss structures

- Gravity loading which scales with:

\[
\frac{Gravity \ Load}{Pre-Load}
\]

Nonlinear Joint

Earth

Varying "Stiffness" with Amplitude
Operating Point

Space

Increasing "Stiffness" with Amplitude
Operating Point

Gravity alters the operating point and, therefore, the apparent stiffness and damping of joints and tensioning wires.

- Similar for tensioning wires
• Gravity field also alters the modal characteristics (frequency and mode shapes) of the structure. This effect scales with:

\[
\frac{g}{L_{Suspension}} \frac{\omega^2}{\omega^2} = \frac{\omega_{Pendulum}^2}{\omega_{1st}^2}
\]

where \(\omega_{1st}^2\) is the 1st modal frequency of interest. For example; significant changes in the modal characteristics are observed for a 6 foot long structure if the natural frequency is less than 1 Hz.

• Suspension of the structure changes the boundary conditions:
  • On earth, free-free boundary conditions are simulated by suspending the structure with a very flexible suspension system.
  • Effect scales with:

\[
\frac{\omega_{Suspension}}{\omega_{1st}}
\]

• Need suspension frequency 1 order of magnitude lower than 1st natural frequency of structure.
• 0.1 Hz suspension frequency can be achieved with state-of-the-art suspension systems.
Modelling Approach

Analytical Model

Experiment

Finite Element Model

Modal Test

Linear Structural Dynamic Model

Nonlinear Sub-components

Force-State Map

Describing Function of Force-State Map

Nonlinear Structural Model (Equivalent beam representation or Multi-degree-of-freedom Model)

 Forced Response Characteristics using Harmonic Balance Method

MODE Flight and Ground Experimental Results

Compare to Verify Model
Characterization of the Nonlinear Components

- Force transmitted by a nonlinear structural component is:
  \[ F_i(x, \dot{x}) = F - M\ddot{x} = D(x, \dot{x})\dot{x} + K(x, \dot{x})x \]

- Model requires a force-state map (Force transmitted as a function of the states of the component) of nonlinear subcomponents

- Typical measurement of the force-state characteristics
Science Overview (Contained Fluids)

Major sources of nonlinearities in the dynamics of contained fluids

1. Potential energy stored in surface tension is a nonlinear function of the amplitude of motion

![Simplified Nonplanar Model]
Let \( \eta(r, \theta, t) = f(r, \theta) + \eta_0(r, \theta, t) \) be the function that describes the equilibrium free surface. For example, the equilibrium fluid shape of Silicone Oil in a 3.1 cm cylindrical tank.

The surface tension potential energy is given by

\[
U_o = \sigma \int_0^1 \sqrt{1 + \left( \frac{\partial f}{\partial r} \right)^2 + \left( \frac{\partial f}{\partial \theta} \right)^2} \, ds
\]

Effect scales with the Bond number \( B_o = \rho g a^2 / \sigma \).
2 Convection forces at the free surface
\[ \frac{\partial \eta}{\partial t} + \nabla \phi \cdot \nabla \eta \bigg|_{z=\eta} = \frac{\partial \phi}{\partial z} \bigg|_{z=\eta} \]

Dirichlet or Neumann time dependent boundary condition

"The internal fluid must follow the motion of the free surface"

- This boundary condition is also dependent on the equilibrium free surface

\[ \frac{\partial \eta}{\partial t} = \left. \frac{\partial \phi}{\partial z} \right|_{z=\eta} - \nabla \phi \cdot \nabla (\eta_d + f) \bigg|_{z=\eta} \]

- Even when linearized \[ \frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} - \frac{\partial f}{\partial r} \frac{\partial \phi}{\partial r} \]
Modelling Approach

Start out with Two Sets of Assumed Solutions:
One for Fluid Flow Potential and One for FreeSurface Motion

Step 1:
Use fluid boundary conditions expressed as a variational integral to obtain relationship between the two sets of generalized coordinates

Step 2:
Express Kinetic and Potential Energy in Terms of Generalized Coordinates

Step 3:
Formulate Coupled System Lagrangian

Step 4:
Apply Lagrange's Principle to obtain the Governing Differential Equations
Typical Ground Test Experimental and Predicted Results

Measured and Predicted One-Gravity Results for a Cylindrical Tank with Water. Tank Diameter=3.1 cm. $\mu=0.16$, $\nu=0.89$, $\zeta=9.1\%$, $Bo=33$, $fo=7$ Hz.
Experimental Design (Structures)

- Scaled models of prototypical space truss structures
  - Deployable bays with a bay with variable pre-tension and nonlinear joints
  - Erectable bays
  - Scaled Alpha (α) joint
  - Very flexible appendage (1 Hz)
Component Testing
- Bay testing
- Single joint testing

Analytical Model
- Use force-state results to generate nonlinear model
- Use results to verify nonlinear on component level or to build "component" nonlinear model

Ground Modal Testing
- Determine linear modal characteristics
- Determine Nonlinear Modal characteristics
- Understand suspension effects

Space Modal Testing
- Determine linear modal characteristics
- Determine Nonlinear Modal characteristics
- Use to update FEM
- Use to verify analytical model
- Identify limitation of earth testing
Experimental Design (Contained Fluids)

- Scaled tanks of prototypical spacecraft fluid containers
  - Cylindrical tank with a flat bottom
  - Cylindrical tank with a spherical bottom
- Fluids matching the properties of typical cryogenics
  - Silicone oil (Potential stability problem)
  - Water as a backup
  - Both are non-toxic and non-flammable
Experimental Design (Contained Fluids - Cont.)

- Fluid/Spacecraft interaction studied by including an analog simulation of a spacecraft's mode

```
Fex (Excitation Signal) -> Spacecraft Mode Simulator x(t) -> Shaker x(t) -> Model Fluid
                ^                  |                   |
                |                  |                   |
                |                  v                   v
                |                   Reaction Balance
                |                    |
                |  Fxs (Planar Reaction Slosh Force)  |
                v                     |
                Dry Mass Compensator
```
MODE-0 (STS-40)

- Determine stability of equilibrium free surface
- Determine natural frequency 1st Slosh Mode

Select fluid for STS-48 flight

Analytical Model

- Verify micro-gravity linear model
- Verify nonlinear model
- Identify fundamental differences between earth and micro-gravity slosh behavior

Ground Testing

- Determine uncoupled forced response characteristics
- Determine coupled forced response characteristics

Space Testing (STS-48)

- Determine uncoupled forced response characteristics
- Determine coupled forced response characteristics
Progress to Date
Progress to Date (Continued)

*M.O.D.E. Component Tester*

Off-loading Suspension

Top Plate

Bearing Blocks

STA

Stingers

Force Balances

10 lb Actuators

Test Block (2000 lb.)

Air Pucks
INHIBITING MULTIPLE MODE VIBRATION IN
CONTROLLED FLEXIBLE SYSTEMS

James M. Hyde
Kenneth W. Chang
Prof. Warren P. Seering

Massachusetts Institute of Technology

July 1, 1991
OUTLINE

- Input Pre-Shaping Background
- Developing Multiple-Mode Shapers
- The MACE Test Article
- Tests and Results
SHAPER POSITION IN CONTROL SYSTEM

Closed Loop System

Input Command Shaper → Compensator → Plant → Output

Feedback
LINEAR SYSTEM IMPULSE RESPONSE

\[ y_i(t) = A_i e^{-\zeta \omega (t - t_i)} \sin \left( (t - t_i) \omega \sqrt{1 - \zeta^2} \right) \]

- \( y_i \): Response to Impulse \( i \)
- \( A_i \): Magnitude of Impulse \( i \)
- \( t_i \): Time of Impulse \( i \)
- \( \omega \): System Natural Frequency
- \( \zeta \): System Damping Ratio
RESPONSE TO "N" IMPULSES

\[ y_i(t) = \sum_{i=1}^{N} A_i e^{-\zeta \omega (t-t_i)} \sin\left((t-t_i) \omega \sqrt{1-\zeta^2}\right) \]

\( i \) \hspace{1cm} Impulse Counter

\( N \) \hspace{1cm} Number of Impulses
AMPLITUDE OF THE MULTIPLE-IMPULSE RESPONSE ENVELOPE

\[ Amp = \left( \sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \sin(t_i \omega \sqrt{1 - \zeta^2}) \right)^2 + \left( \sum_{i=1}^{N} A_i e^{-\zeta \omega (t_N - t_i)} \cos(t_i \omega \sqrt{1 - \zeta^2}) \right)^2 \right)^{1/2} \]

Expression for envelope amplitude at \( t_N \),
the time of the final impulse.
ELIMINATING RESIDUAL VIBRATION

\[ \sum_{i=1}^{N} A_i e^{-\zeta \omega t_i \sin (t_i \omega \sqrt{1 - \zeta^2})} = 0 \]

\[ \sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t_i \sin (t_i \omega \sqrt{1 - \zeta^2})} = 0 \]

\[ \sum_{i=1}^{N} A_i e^{-\zeta \omega t_i \cos (t_i \omega \sqrt{1 - \zeta^2})} = 0 \]

\[ \sum_{i=1}^{N} A_i t_i e^{-\zeta \omega t_i \cos (t_i \omega \sqrt{1 - \zeta^2})} = 0 \]
USING THE IMPULSE SEQUENCE

---

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RESPONSE TO INPUTS

- Response to Shaped Input
- -- Response to Unshaped Input

Position

Time (sec)
INSENSITIVITY OF THREE-IMPULSE SEQUENCE

![Graph showing vibration amplitude vs frequency (non-dimensional).]
EXTENDING TO MULTIPLE MODE PROBLEMS
CONVOLUTION

1 Hz Shaper

10 Hz Shaper

Convolved Shaper
CONVOLUTION SEQUENCE PROBLEMS

\[ \omega_1 = 0.20 \text{ Hz} \quad \omega_2 = 0.26 \text{ Hz} \]
\[ \omega_3 = 0.45 \text{ Hz} \quad \omega_4 = 0.59 \text{ Hz} \]
DIRECT SOLUTION CONSTRAINT EQUATIONS

\[ \sum_{i=1}^{N} A_i e^{-\zeta_j \omega_j t_i} \sin \left( t_i \omega_j \sqrt{1 - \zeta_j^2} \right) = 0 \]

\[ \sum_{i=1}^{N} A_i e^{-\zeta_j \omega_j t_i} \cos \left( t_i \omega_j \sqrt{1 - \zeta_j^2} \right) = 0 \]

\[ \sum_{i=1}^{N} A_i t_i e^{-\zeta_j \omega_j t_i} \sin \left( t_i \omega_j \sqrt{1 - \zeta_j^2} \right) = 0 \]

\[ \sum_{i=1}^{N} A_i t_i e^{-\zeta_j \omega_j t_i} \cos \left( t_i \omega_j \sqrt{1 - \zeta_j^2} \right) = 0 \]

These four equations are repeated for each mode "j"
LINEARIZING THE EQUATIONS

Define time mesh with $N$ slots

Impulse times ($t_i$): Known

Impulse amplitudes ($A_i$): Unknown
COST FUNCTION

\[ \text{Cost} = \sum_{j=1}^{M} \left( \sum_{i=1}^{N} A_i \left( e^{-\zeta_j \omega_j t_i} \sin(t_i \omega_j \sqrt{1 - \xi_j^2}) \right) \right)^2 + \left( \sum_{i=1}^{N} A_i \left( e^{-\zeta_j \omega_j t_i} \cos(t_i \omega_j \sqrt{1 - \xi_j^2}) \right) \right)^2 \]

Number of modes
Modal index
LINEAR APPROXIMATION SEQUENCE
INTERPRETED LINEAR SEQUENCE

![Graph showing interpreted linear sequence with time in seconds on the x-axis and amplitude (normalized) on the y-axis.]

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EXACT DIRECT SOLUTION SEQUENCE

![Bar chart showing amplitude over time in seconds. The x-axis represents time in seconds (2.0, 4.0, 6.0, 8.0, 10.0, 12.0), and the y-axis represents amplitude normalized. The chart displays peaks at 2.0, 4.0, 6.0, and 8.0 seconds.]
THE MID-DECK ACTIVE CONTROL EXPERIMENT

(MACE)
MACE SYSTEM SCHEMATIC

Pointing/Tracking Payload (2)

Active Segment

Inertial Platform

Approx. 1.5 m
# IDENTIFIED CLOSED LOOP FREQUENCIES (NON-LINEAR DISCOS MODEL)

<table>
<thead>
<tr>
<th>60° Outboard (Hz)</th>
<th>60° Inboard (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Beginning of 120° Slew)</td>
<td>(End of 120° Slew)</td>
</tr>
<tr>
<td>2.18</td>
<td>1.88</td>
</tr>
<tr>
<td>14.25</td>
<td>13.40</td>
</tr>
<tr>
<td>15.25</td>
<td>14.20</td>
</tr>
<tr>
<td>15.90</td>
<td>15.90</td>
</tr>
</tbody>
</table>
SHAPER DESIGNED FOR "BEGINNING FREQUENCIES"
INSENSITIVITY CURVE FOR
"BEGINNING FREQUENCIES SHAPER"

Vibration Amplitude (normalized)

Frequency (Hz)
120° CLOSED LOOP PAYLOAD SLEW
ENDPOINT RESPONSE TO UNSHAPED SLEW

![Graph showing endpoint displacement over time with a line labeled 'Response to Unshaped Input'.]
SHAPED RESPONSE DETAIL

Endpoint Displacement (mm)

Time (sec)

Response to Shaped Input
INSENSITIVITY CURVE FOR
"ENDING FREQUENCIES SHAPER"

Vibration Amplitude (normalized)

Frequency (Hz)
SHAPE FOR ADDITIONAL FREQUENCIES
TO FURTHER REDUCE RESIDUAL VIBRATION
INSENSITIVITY CURVE FOR 1.5, 1.88HZ
SHAPED RESPONSE DETAIL

![Graph showing shaped response detail with endpoint displacement (mm) on the y-axis and time (sec) on the x-axis. The line labeled "Response to Shaped Input."
INSENSITIVITY CURVE FOR 1.5, 1.88, 2.0 HZ
SHAPED RESPONSE DETAIL

![Graph showing the response to shaped input over time](image-url)

- **Y-axis**: Endpoint Displacement (mm)
- **X-axis**: Time (sec)
- **Note**: The graph illustrates the time evolution of endpoint displacement in response to a shaped input.
FINAL UNSHAPE/SHAPE COMPARISON

![Graph showing endpoint displacement over time for unshaped and shaped inputs.]

- --- Response to Unshaped Input
- --- Response to Shaped Input

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CONCLUSIONS

- Input Shaping Can Suppress Residual Vibration Even in Systems with Changing Natural Frequencies.

- Future: Test Shaping Methods on Physical MACE Structure, Coming on Line in Mid-Summer.
OUTLINE

MODE Family of Experiments
  MACE
  Team: Participating Organizations

Science Program
  Objectives and Rationale
  Science Requirements
  Capturing the Essential Physics
  Science Development Approach

Hardware Development
  Development Model Hardware
  Development Model Test Plan
  Flight Hardware and Operations

Schedule

Summary
# THE MODE FAMILY OF EXPERIMENTS

<table>
<thead>
<tr>
<th>Fluid Test Article (FTA)</th>
<th>Structural Test Article (STA)</th>
<th>MACE Test Article</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupled Non-Linear Dynamics of Fluids and Structures in Zero Gravity</td>
<td>Non-Linear Dynamics of Jointed Truss Structures in Zero Gravity</td>
<td>Influence of Gravity on the Active Control of a Multibody Platform</td>
</tr>
</tbody>
</table>

**Flight # 1:**  
September 1991

**Flight # 2:**  
June 1994

MACE is part of a logical sequence of cost-effective flight experiments designed to advance technology of interest to NASA in the area of controlled structures.
THE MIDDECK ACTIVE CONTROL EXPERIMENT (MACE)

- Substantial commonality of ESM hardware/software
- Significant savings in integration/certification process.
# TEAM: PARTICIPATING ORGANIZATIONS

<table>
<thead>
<tr>
<th>Organization</th>
<th>Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIT Space Engineering Research Center</td>
<td>Science, Modelling, EM Fabrication</td>
</tr>
<tr>
<td>Payload Systems Inc.</td>
<td>Exp. Support, Flight Hardware Fab., Integration</td>
</tr>
<tr>
<td>Lockheed Missiles and Space Co.</td>
<td>Science, Modelling, EM Fabrication</td>
</tr>
<tr>
<td>Sonitech International</td>
<td>DSP Development</td>
</tr>
<tr>
<td>MIT Center for Space Research</td>
<td>Financial Oversight</td>
</tr>
</tbody>
</table>

*Space Engineering Research Center*
OBJECTIVES AND RATIONALE

Objective: To develop a well verified set of CST tools that will allow designers of future CST spacecraft, which cannot be dynamically tested on the ground in a sufficiently realistic on-orbit simulation, to have confidence in the eventual orbital performance of such spacecraft.

- Since the model fidelity required for stability and performance robustness is intimately related to the level of applied control authority, closed-loop testing is required.
- Vehicle qualification testing will most likely occur on the ground where suspension and direct gravity effects will cause the 1-g and 0-g dynamics to differ.
- Differences between the ground and on-orbit environment cause perturbations which can substantially alter closed-loop behavior.
- Therefore it is essential to perform on-orbit closed-loop testing for comparison with ground testing and analytical predictions to develop these tools.
MACE SCIENCE REQUIREMENTS

The test article must be representative of a mission or vehicle architecture so that the developed technology has clear application.

- The test article must be representative of expected near-term missions.
- The test article geometry, performance metric and disturbances must be representative of expected near-term missions.
- Improvement in the performance must be representative of that predicted for CST spacecraft under sensor, actuator and processor dynamic range constraints. Assuming unstaged actuator dynamic range to be 40 db and sensor dynamic range to be 60 db, a performance improvement of 40 db is required.
MACE SCIENCE REQUIREMENTS

The test article must exhibit differences in its dynamic behavior between ground based and on-orbit testing.

- The test article must be difficult to test on the ground because of the effect of gravity and suspension on its flexibility.
- Detailed modeling of gravity and suspension effects on the test article flexibility is required to properly predict closed-loop behavior on the ground.
- At a 20 db performance level, gravity/suspension effects on flexibility will distinguish closed-loop behavior between ground and on-orbit tests when the same active controller is applied.
- The test article must have a performance improvement in the selected performance metric of a minimum of 20 db which must be verified by experiment on the ground.
MACE SCIENCE REQUIREMENTS

A comprehensive series of coherent ground and on-orbit tests must be performed which identify current limitations and develop plausible alternative approaches.

- Control algorithms will be implemented on orbit which are identical to those implemented on the ground. These tests identify current limitations.
- Control algorithms will be implemented on orbit which are derived from the ground model with suspension and gravity effects removed. These tests identify predictive ability.
- Control algorithms will be implemented on orbit which are derived from on-orbit dynamic test data. These test identify the ability to fine tune the control once on orbit.
CAPTURING THE ESSENTIAL PHYSICS: TEST ARTICLE REQUIREMENTS

The simulation of a vehicle with payloads and articulating appendages with pointing and positioning requirements, necessitates a test article with the following attributes:

• appropriately scaled to fit in the middeck while preserving the essential performance requirements.

• two gimballing payloads to enable implementation of multiple interacting control systems with independent objectives.

• two rigid payloads and a flexible appendage, representative of compact high mass fraction devices and a robotic servicer.

• flexible bus with resonances within the controller bandwidth and to exhibit suspension coupling, gravity stiffening and droop.

• sufficiently complex geometry such that the test article undergoes full 3-D kinematic and coupled flexible motion.
SCIENCE DEVELOPMENT APPROACH:

CONTROL OBJECTIVES:

Control Objectives:
- Pointing performance of single and multiple payloads.
- Scanning performance of single and multiple payloads.

Performance Metrics:
- Stability-RMS 2-axis angular position about pointing line of sight or scanning reference profile.
- Jitter-RMS 2-axis angular rate about pointing line of sight or scanning reference profile.
- Slew response time-time required to complete maneuver.
- Percent degradation-reduction from single payload performance associated with addition of an interacting, controlled payload.
**SCIENCE DEVELOPMENT APPROACH: GRAVITY INFLUENCES**

Objective: Identify and quantify the magnitude of the perturbation effects of a gravity field and a suspension system on the dynamics of a suspended test article.

<table>
<thead>
<tr>
<th>GRAVITY FIELD EFFECTS</th>
<th>SUSPENSION SYSTEM EFFECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1) GRAVITY STIFFENING/DESTIFFENING</strong></td>
<td><strong>3) STATIC B. C. PERTURBATIONS</strong></td>
</tr>
<tr>
<td>• changes in membrane energy due to small deformations and</td>
<td>• static translational stiffnesses in the horizontal and</td>
</tr>
<tr>
<td>loading of the structure can be modelled as system</td>
<td>vertical directions are prescribed by the suspension</td>
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<td>stiffness perturbations.</td>
<td>system at each attachment point.</td>
</tr>
<tr>
<td><strong>2) FINITE DEFLECTIONS</strong></td>
<td><strong>4) DYNAMIC B.C. PERTURBATIONS</strong></td>
</tr>
<tr>
<td>• finite deflections require a redefinition of the</td>
<td>• modal coupling with the suspension dynamic modes results</td>
</tr>
<tr>
<td>reference structure; stiffness modifications do not</td>
<td>in dynamic impedances at the attachment points.</td>
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<tr>
<td>capture the perturbation to the eigenstructure.</td>
<td></td>
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<tr>
<td>**5) DYNAMIC LOADING DUE TO GRAVITY FIELD AND SUSPENSION</td>
<td></td>
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<tr>
<td>CONSTRANTS</td>
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<tr>
<td>• dynamic torques which result from center of mass axis</td>
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<tr>
<td>offsets with respect to the suspension support plane(s).</td>
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</tbody>
</table>
HARDWARE DEVELOPMENT

• Three sets of hardware are being developed under the MACE program:
  The Development Model for science development
  The Engineering Model for prototyping flight hardware
  The Flight Hardware for actual flight

• The purpose of the Development Model is to develop the science associated with the MACE program by validating theory through experimental implementation.

• The purpose of the Engineering Model is to attempt to redesign the DM and its support equipment to operate within the constraints of the STS middeck.

• The purpose of the Flight Hardware is to provide one unit for crew training and spare parts and one unit for flight.
## DEVELOPMENT MODEL TEST PLAN

<table>
<thead>
<tr>
<th>TESTS</th>
<th>Jun '91</th>
<th>Jul '91</th>
<th>Aug '91</th>
<th>Sept '91</th>
<th>Oct '91</th>
<th>Nov '91</th>
<th>Dec '91</th>
<th>Jan '92</th>
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</tbody>
</table>

**Components**
- gimbal characterization
- attitude control
- w/ glc
- w/ glc
- two axis gimbal control
- strut characterization

**Ensemble**
- prelim sers
- system ID and STAR fit
- prelim
- rigorous
- verify gravity effects
- nonlinear ID
- attitude control
- w/ glc
- w/ glc
- two axis gimbal control
- baseline performance data
- successive loop closure

**CCL**
- other designs
- linear command shaping
- nonlinear command shaping

**Other**
- mass cancellation work

**ANALYSES**
- Components
  - model active strut
  - FE analysis of Lexan strut
  - Ensemble
  - beam FE model w/ STAR
  - add gravity and suspension
  - TA configuration

---

*Space Engineering Research Center*
EXPERIMENT OPERATIONS: DAY ONE

On Orbit

Unstow → Open-Loop Identification → Downlink → Algorithm Set #1 → ? → Algorithm Set #2 → Stow → Downlink

Increase Control Authority

Additional Algorithm Set #1

Ground

Receive → Compare with Predicted → Decide Course of Action → Formulate Algorithm Set #3 → Receive
EXPERIMENT OPERATIONS: DAY TWO

On Orbit

Receive → Open-Loop Identification → Algorithm Set #3 → Additional Configurations → Stow → Downlink

Increase Control Authority

Ground

Uplink Algorithm Set #3 → Receive
SUMMARY

- The MODE family of flight experiments is designed to verify analytical tools developed to predict the gravity dependent behavior of proposed space structures.

- The MODE family of flight experiments uses reusable dynamic and control tests facilities and exploits the unique environment on the STS middeck.

- MACE investigates gravity dependent phenomena pertinent to the closed-loop dynamics of proposed space structures.
  - By comparing performance as a function of control authority between ground and on-orbit testing, perturbations in the dynamics due to the change from 1 to 0-g will be identified.
  - By noting the level of control authority where these performance deviations occur, either analytical predictive capabilities or on-orbit identification procedures can be refined.

- The MACE program consists of a strong consortium of university, government and industry to develop a CST flight experiment which is technically relevant, mission relevant and cost effective.
ROBUST CONTROL FOR UNCERTAIN
STRUCTURES

Joel Douglas  
Michael Athans  
Massachusetts Institute of Technology  

July 1, 1991 (SERC Symposium)
APPRAOH

- Assume full-state feedback
- Try to guarantee stability and performance robustness of classical LQR design
  - Guaranteed stability
  - Reasonable guaranteed robustness (gain and phase margin properties)
- Apply to benchmark problem to see interesting properties

\[ \begin{array}{c}
  u \quad m_1 = 1 \quad k \quad m_2 = 1 \\
  \downarrow x_1 \quad \downarrow x_2 = y \\
  .5 \leq k \leq 2 
\end{array} \]
ROBUST LQR FORMULAS

- Standard LQR design when there is no uncertainty

\[ J = \int_0^\infty (x^T(t)Q_0x(t) + \rho u^T(t)u(t))dt \]

\[ PA_0 + A_0^TP + Q_0 - \frac{1}{\rho}PB\nu^TP = 0 \]

- Apply Petersen-Hollot bounds to derive robust Riccati Equation

\[ A = A_0 + \sum_{i=1}^{\rho} q_iE_i \quad |q_i| \leq 1 \]

\[ E_i = l_in_i^T \quad L = [l_1 \ l_2 \ l_3 \ldots]; \quad N = [n_1 \ n_2 \ n_3 \ldots] \]

\[ PA_0 + A_0^TP + (Q_0 + \gamma N N^T) - P\left(\frac{1}{\rho}BB^T - \frac{1}{\gamma}LL^T\right)P = 0 \]

- Control

\[ G = \frac{1}{\rho}B^TP \quad u = -Gx \]
MISMATCHED LQR DESIGN

\[ \begin{align*}
  k &= 0.5 \\
  y &\quad \text{Time} \\
  &0 \quad 5 \quad 10 \quad 15 \quad 20 \\

  k &= 0.875 \\
  y &\quad \text{Time} \\
  &0 \quad 5 \quad 10 \quad 15 \quad 20 \\

  k &= 1.25 \\
  y &\quad \text{Time} \\
  &0 \quad 5 \quad 10 \quad 15 \quad 20 \\

  k &= 1.625 \\
  y &\quad \text{Time} \\
  &0 \quad 5 \quad 10 \quad 15 \quad 20 \\

  k &= 2 \\
  y &\quad \text{Time} \\
  &0 \quad 5 \quad 10 \quad 15 \quad 20 
\end{align*} \]
RLQR DESIGN

$k = 0.5$

$k = 1.25$

$k = 1.625$

$k = 2$
INTERPRETATIONS OF RLQR DESIGN

- Equivalent to an optimal design where we minimize the cost functional

\[ J = \int_0^\infty (x^T(t)Q_0x(t) + x^T(t)\gamma NN^T x(t) + x^T(t)\frac{1}{\gamma} PLL^T P x(t) + \rho u^T(t)u(t))dt \]

\[-\beta d^T(t)d(t)\]

- \(x^T(t)Q_0x(t)\) is the state weighting
- \(x^T(t)NN^T x(t)\) has been shown to be uncertain potential energy of an uncertain spring (or rate of dissipation for a damper)
- \(x^T(t)PLL^T P x(t)\) is an equivalent \(H_\infty\) term.

- Parameter \(\gamma\) is therefore a tradeoff between minimizing unknown uncertain energy and worst case disturbance arising from forces due to parameter errors.
\[ \begin{align*}
  &m_1 = 1 \\
  &m_2 = 1 \\
  &0.5 \leq k \leq 2 \\
  &k = 1.625
\end{align*} \]
$K = 1.625 \ (RLQR)$
DISTURBANCE REJECTION

- Does the RLQR controller reject disturbances?
- Add a white noise disturbance at the output
- Apply both mismatched LQR and RLQR designs

\[ .5 \leq k \leq 2 \]
MISMATCHED LQR DESIGN

\[ k=0.5 \]

\[ k=0.875 \]

\[ k=1.25 \]

\[ k=1.625 \]

\[ k=2 \]

Disturbance
RLQR DESIGN

\[
\begin{align*}
&\text{Time} \\
&k=0.5 \\
&k=1.25 \\
&k=2.0 \\
&k=0.875 \\
&k=1.625
\end{align*}
\]
10° I
\[ \text{Frequency (rad/s)} \]

- Mismatched LQR
- RLQR, gamma=1
- RLQR, gamma=.5
THREE-MASSES, TWO UNCERTAIN SPRINGS

\[ u_1 \rightarrow m_1 = 1 \rightarrow k_1 \rightarrow m_2 = 1 \rightarrow k_2 \rightarrow m_3 = 1 \rightarrow x_3 = y \]

\[ .5 \leq k_1, k_2 \leq 2 \]
PERFORMANCE COMPARISONS:
RLQR (LEFT) VS MISMATCHED LQR (RIGHT)

RLQR $k_1=1.25$ $k_2=1.25$

Nominal LQR $k_1=1.25$ $k_2=1.25$

RLQR $k_1=5$ $k_2=2$

Nominal LQR $k_1=5$ $k_2=2$

RLQR $k_1=1.625$ $k_2=0.875$

Nominal LQR $k_1=1.625$ $k_2=0.875$

RLQR $k_1=2$ $k_2=2$

Nominal LQR $k_1=2$ $k_2=2$
RLQR TRANSIENTS: 2-SPRING SYSTEM

\[ \kappa_1 = 5, \kappa_2 = 1.2 \]

![Graphs of transient responses for a 2-spring system with \( \kappa_1 = 5 \), \( \kappa_2 = 1.2 \).](image)
MISMATCHED LQR TRANSIENTS: 2-SPRING SYSTEM

\[ \kappa_1 = .5, \kappa_2 = 1.25 \]
CONCLUSIONS

- RLQR design is a full state method
- Guarantees stability as well as some robustness
- Interesting energy interpretations
- Understanding underlying fundamentals will help us when we extend to output feedback
Cost Averaging Techniques for Robust Control of Flexible Structural Systems

Nesbitt W. Hagood
Edward F. Crawley

Third Annual SERC Symposium
July 1, 1991

NASA Space Engineering Research Center
Outline

- Introduction
- Modeling of Parameterized Systems
- Average Cost Analysis
- Reduction of Parameterized Systems
- Static and Dynamic Controller Synthesis
- Examples
Problem Statement

- The problem is to design a controller that provides stability robustness over a set of plants described by real parameter uncertainty.
- This type of uncertainty is common in structural plants.
  - Uncertain stiffness
  - Uncertain damping
  - Uncertain modal parameters
- Stability examined in the context of the performance-robustness trade.
Background in Robust Control Synthesis

Bounding Methods

- Develop robustness analysis test
  - Lyapunov, Kharitonov, Small Gain, $\mu$
- Incorporate analysis test into performance metric
  - modify Lyapunov equation or develop stability index
- Find controller which minimizes modified performance metric subject to constraints.

Sensitivity Methods

- Sensitize the cost to the uncertainties, then minimize.
- Multiplicative White Noise (MEOP): Hyland, Bernstein
- Cost Sensitivity Minimization: Skelton
- Multi-Model Techniques: Bryson, Li
Approach of This Work

- Examine cost averaged over a continuously parameterized set of plants.
- Derive analysis tools to determine approximations and bounds to the average cost.
- Apply these analysis tools to the problem of determining critical components and uncertainties.
- Apply these analysis tools to the problem of nonconservative robust controller design.
Modeling Notation

- A system $G(s)$ can be represented in state space notation as:

$$
\begin{bmatrix}
\dot{x} \\
\frac{z}{w} \\
y
\end{bmatrix} = 
\begin{bmatrix}
A & B_1 & B_2 \\
C_1 & 0 & D_{12} \\
C_2 & D_{21} & 0
\end{bmatrix}
\begin{bmatrix}
x \\
w \\
u
\end{bmatrix}
$$

- The controlled system can be represented schematically:
Sets of Systems

- The set, $\Omega$, of parameters is defined on the compact interval
  \[ \Omega = \{ \alpha : \alpha \in \mathbb{R}^r, -\delta_i^L \leq \alpha_i \leq \delta_i^U \ i = 1, \ldots, r \} \]

- The set $\mathcal{G}$ of systems is parameterized as follows
  \[ \mathcal{G} = \{ G(\alpha) \forall \alpha \in \Omega \} \]
  
  for structured parameter dependence

  \[
  G(\alpha) = \begin{bmatrix}
    A_0 + \sum_{i=1}^{r} \alpha_i A_i & B_1 & B_{20} + \sum_{i=1}^{r} \alpha_i B_{2i} \\
    C_1 & 0 & D_{12} \\
    C_{20} + \sum_{i=1}^{r} \alpha_i C_{2i} & D_{21} & 0
  \end{bmatrix}
  \]
The Average Cost

- The exact average cost is defined as the closed-loop $\mathcal{H}_2$-norm (quadratic cost) averaged over the model set.

$$J = \int_\Omega \| G_{zw}(\alpha) \|_2^2 d\mu(\alpha)$$

- Finite average cost implies:
  - The closed-loop systems are stable $\forall \alpha \in \Omega$ except possibly at isolated points.
  - No closed-loop system can have eigenvalues with positive real parts.

- Controllers based on minimization of the average cost will guarantee stability without necessarily guaranteeing performance.
Average Cost Calculation

- The average cost is calculated

\[ J = \text{tr} \left\{ \langle \bar{Q}(\alpha) \rangle \tilde{C}^T \tilde{C} \right\} \]

- For each \( \alpha \in \Omega \), \( \bar{Q}(\alpha) \) is given by

\[ 0 = \tilde{A}(\alpha)\bar{Q}(\alpha) + \bar{Q}(\alpha) \tilde{A}^T(\alpha) + \tilde{B}\tilde{B}^T \]

- The problem is how to compute the average solution to a parameterized Lyapunov equation.

  - Monte-Carlo
  - Direct Integration
  - Stochastic Operator Methods
Operator Decomposition

- Can utilize techniques from the field of wave propagation in random media and turbulence modeling.

- The parameterized Lyapunov equation can be decomposed into a nominal part and a parameter dependent part.

\[ \tilde{A}(\alpha)\tilde{Q} + \tilde{Q}\tilde{A}^T(\alpha) + \tilde{B}\tilde{B}^T = 0 \]

is equivalent to

\[ L_0 [\tilde{Q}] + L_1 (\alpha) [\tilde{Q}] + \tilde{B}\tilde{B}^T = 0 \]

\[ L_0 [\tilde{Q}] = \tilde{A}_0\tilde{Q} + \tilde{Q}\tilde{A}_0^T \]

\[ L_1 (\alpha) [\tilde{Q}] = \tilde{A}_1(\alpha)\tilde{Q} + \tilde{Q}\tilde{A}_1^T(\alpha) \]

- Two methods of computing the average: Perturbation Expansion and Dyson Equation.
Perturbation Expansion Approximation

- Perturbation expansion series can be approximated by retaining only the first two terms.
  \[ \tilde{Q}^P = \tilde{Q}^0 + A(L_0^{-1}L_1(\alpha))^2\tilde{Q}^0 \]
- The perturbation expansion approximate cost is given by
  \[ J^P = \text{tr} \left\{ \tilde{Q}^P \tilde{C}^T \tilde{C} \right\} \]
  where \( \tilde{Q}^B \) is the solution of
  \[ 0 = \tilde{A}_0\tilde{Q}^P + \tilde{Q}^P \tilde{A}_0^T + \tilde{B} \tilde{B}^T + \sum_{i=1}^r \sigma_i \left( \tilde{A}_i\tilde{Q}^i + \tilde{Q}^i \tilde{A}_i^T \right) \]
  \[ 0 = \tilde{A}_0\tilde{Q}^i + \tilde{Q}^i \tilde{A}_0^T + \sigma_i \left( \tilde{A}_i\tilde{Q}^0 + \tilde{Q}^0 \tilde{A}_i^T \right) \quad i = 1, \ldots, r \]
- The equations for the perturbation expansion are related to those used in Skelton's cost sensitivity controller design method.
- The two sets of Lyapunov equations are coupled hierarchically and easily solved using standard techniques.
Bourret Approximation

- The Dyson Equation can be approximated by retaining only the first term of $M$.

$$\tilde{Q}^B = \tilde{Q}^0 + A(L_0^{-1}L_1(\alpha))^2\tilde{Q}^B$$

- The Bourret approximate cost is given by

$$J^B = \text{tr}\left\{\tilde{Q}^B\tilde{C}^T\tilde{C}\right\}$$

where $\tilde{Q}^B$ is the solution of

$$0 = \tilde{A}_0\tilde{Q}^B + \tilde{Q}^B\tilde{A}_0^T + \tilde{B}\tilde{B}^T + \sum_{i=1}^r \sigma_i \left(\tilde{A}_i\tilde{Q}^i + \tilde{Q}^i\tilde{A}_i^T\right)$$

$$0 = \tilde{A}_0\tilde{Q}^i + \tilde{Q}^i\tilde{A}_0^T + \sigma_i \left(\tilde{A}_i\tilde{Q}^B + \tilde{Q}^B\tilde{A}_i^T\right) \quad i = 1, \ldots, r$$

- The Bourret equation represents an infinite series expansion for the approximate average solution.

- The cross-coupling complicates the solution procedure.

- Positive definite solution the Bourret equation guarantees stability over a set smaller than the design set.
Second Order System Example

- Consider the simple spring-mass-damper represented.

- The system has dynamics

\[ \ddot{x}(t) + 2\zeta \omega \dot{x}(t) + \omega^2 x(t) = bf(t) \]

- Let the natural frequency and damping ratios of the the system be uniformly distributed uncertain parameters of the form

\[ \omega^2 = \omega_0^2 + \bar{\omega}^2 - \delta_{\omega^2} \leq \bar{\omega}^2 \leq \delta_{\omega^2} \]

\[ \zeta = \zeta_0 + \bar{\zeta} - \delta_{\zeta} \leq \bar{\zeta} \leq \delta_{\zeta} \]
Cost vs. Model Uncertainty

- The cost as a function of the uncertain damping bound.
Robust Control Synthesis

- Fixed-form controllers

\[
G_c = \begin{bmatrix} 0 & 0 \\ 0 & D_c \end{bmatrix} \text{ or } \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}
\]

- Controller Parameter Optimization Design Procedure

Step 1: Define cost based on exact average, bounding or approximating cost.

Step 2: Append the appropriate equation to the cost using a matrix of Lagrange multipliers.

Step 3: Determine the necessary conditions for optimality using matrix calculus.

Step 4: Minimize the cost numerically using necessary condition for gradient information.

Step 5: Evaluate the resulting controllers for stability and performance robustness.
Necessary Conditions

- For exact average cost minimization with static output feedback.

\[ D_c = -R^{-1}B_2^T \langle \bar{P} (\alpha) \bar{Q} (\alpha) \rangle \langle \bar{Q} (\alpha) \rangle^{-1} \]

where

\[ 0 = \tilde{A}(\alpha)\bar{Q} (\alpha) + \bar{Q} (\alpha) \tilde{A}^T (\alpha) + \tilde{B} \tilde{B}^T \]

\[ 0 = \tilde{A}^T (\alpha)\bar{P} (\alpha) + \bar{P} (\alpha) \tilde{A}(\alpha) + \tilde{C}^T \tilde{C} \]

- The necessary conditions have a form very similar to the necessary conditions derived for the static output feedback problem.

- Uncertainty couples the two parameterized Lyapunov equations.

- The same form is present with the approximations and bounds but with parameter independent equations.

- For dynamic output feedback the necessary condition comprise

  - three gain equations obtained by taking derivatives with respect to \( A_c, B_c, \) and \( C_c \)
  
  - two Lyapunov-based equations
Example 2: The Cannon-Rosenthal Problem

- Consider the four mass/spring/damper system with uncertain body one mass.

- The uncertain mass is represented

\[ \frac{1}{m_1} = \frac{1}{m_{10}} + \dot{m} \quad m_0 = 0.5 \quad |\dot{m}| \leq \delta_m \]

- Spring and mass uncertainty treated by numerous researchers.
Pole-Zero Flip

- The problem was chosen because the plant zero and second mode change relative positions as the parameter is increased to $\hat{m} = 0.6$

- The open loop $u-y$ transfer functions for the Cannon-Rosenthal problem for $m_1 = 0.5$ (solid) and $m_1 = 0.25$ (dashed)
Achieved vs. Designed-for Robustness

- Achieved closed loop stability bounds as a function of the design bound, $\delta_m$. 

![Achieved vs. Designed-for Robustness Graph]

MIT Space Engineering Research Center
Cost vs. Parameter

- System closed-loop $\mathcal{H}_2$-norm as a function of $\tilde{m}$, the deviation about $1/m_1$, for controllers designed using $\delta_m = 0.1$. 

![Graph showing cost vs. parameter](image-url)
Efficiency Plot

- Nominal cost as a function of the achieved stability bound.
Conclusions

- Have investigated a new class of controllers based on minimizing quantities related to the cost averaged over a parameterized set of plants.
- While possessing useful properties, the average cost based controllers were difficult to compute.
- The perturbation based controllers were easy to compute but performed well only at low uncertainty levels.
- The average bound designs were essentially equivalent to the worst case bound designs.
- The worst case bound performed as predicted but gave low efficiency due to conservatism.
- The Bourret approximation based designs were overall best in computability and efficiency.
Overview

- Embedding electronic components for control of intelligent structures.
- Single-chip microcomputer control experiment
- Structural shape determination
- Distributed sensor systems for structural control
Motivation

- Precision control of flexible structures is more readily achieved with large numbers of sensors and actuators
- Signal quality can be improved by distribution of analog processing circuitry along with transducers
- Connectivity (number of lines) can be greatly reduced by distribution of A/D, D/A conversion with digital bus interface circuitry
- Control loop speed can be substantially elevated by distribution of digital processors in a hierarchic controller

Missing element in fully integrated intelligent structures concept:
  embedded electronics
**Objectives**

Establish the feasibility of physically embedding electronic components for the control of intelligent structures.

Demonstrate structural control using processor with minimal number of chips
Embedding Electronics: Approach

- Select a suitable candidate chip for embedding
- Develop embedding technique
- Test mechanical static and fatigue properties
- Test temperature-humidity-bias reliability
Electrical and Mechanical Compatibility

Issues:

• Manufacturing - autoclave pressures and temperatures

• Operational mechanical stress – brittle Si, delicate SiO₂ and metal structures

• Electrical insulation from graphite fibers

• Ionic contamination – device lifetime is typically limited by corrosion

• Minimal disruption of structural plies
Integrated Circuit Chip Packaged for Embedding

Dielectric integrated circuit sensor (Micromet Instruments, Inc.)

Circuit area consists of two metal-oxide-semiconductor field effect transistors and one diode

Packaging is similar to Tape Automated Bonding (TAB)
Integrated Circuit Chip Packaged for Embedding

- Polyimide
- 8.9 mm
- 0.5 mm
- Sensing region
- Dielectric sensor chip
- Protective epoxy layer
- Circuit region
- RTV layer
- Copper leads (356 mm overall)
Embedding Devices within Composite Structures

Plies of graphite/epoxy composite

Glass felt layer

Device with leads to be embedded

1 Ply with notch for leads

3 Plies with holes for device

Layup is \([0/90/0_2]\_s\)
**Test of Embedded Circuit in G/E Coupon**

- long. far field
- long. side
- long. over chip

With epoxy protective layer

device failure at 750 MPa

---

*Space Engineering Research Center*
Test of Embedded Circuit in G/E Coupon

---

- long. far field
- long. side
- long. over chip

device failure at 1150 MPa

With RTV isolation layer

---

Space Engineering Research Center
Temperature/Humidity/Bias Test

- Test conditions:
  3 chips embedded in 60 mm x 75 mm laminates subjected to 80°C and 80% R. H. for 125 hours
  Transistor drain-source currents while under constant bias continuously monitored; characteristic curves recorded at log time intervals
  Two conventional MOSFETs included as experimental controls

- Results:
  One sensor showed intermittent anomalies (hysteresis, elevated current) as early as 14 hours - possible leakage currents through epoxy
  One sensor showed progressive drop in current during final 4 hours - consistent with lead corrosion
  One sensor showed no anomalies
  Conventional controls showed no anomalies
Single-chip Microcomputer Control Experiment
Performance Achieved

Increased damping:

1st mode   0.36\% OL       31\% CL
2nd mode   0.15\% OL       4\% CL
3rd mode   0.20\% OL       11\% CL
**Major Results**

- **Embedding electronics is feasible**
  Compliant isolation layer allows device function to laminate failure
  Chemical isolation is inferior to commercial devices, requires further work
  All failures were at or near lead-chip bond

- **Embedding local processors are plausible**
  Distribution of processing can be justified, depending on problem size
  Nearly all of the required functions included on currently available monolithic devices
Structural Shape Determination Objectives

Objective is to determine "optimal" type and number of sensors to allow accurate reconstruction of structural shape from discrete curvature measurements.

Issues considered include:

- Accuracy of predicted slope and displacement for various integration rules as a function of the number of gages.
- Static and dynamic mode shape determination with strain averaging sensors.
- Frequency characteristics of a single sensor.
- Frequency characteristics of integrated shape measurement.
**Approach to Integration Rule Study**

- Consider a cantilevered beam subjected to a representative set of simple static loadings.
- Distribute a set of strain gages along the length of the beam.
- Integrate the curvature using a variety of integration rules to obtain an estimate of the shape of the beam.
- Vary gage factor and gage placement to obtain error bounds for experimental uncertainties.
- Compare performance of short and long gages to determine whether a "point" or an averaged strain measurement yields better performance.
Deflection Prediction Error vs Number of Gages

Spline integration rule and long gages

Solid line: No gage factor or placement error.
Dashed line: 1% gage factor error and placement error of 0.2% of the beam length.
Dot-dashed line: 5% gage factor error and placement error of 1% of the beam length.
Functional Requirements for Distributed Sensor Systems

- The sensors should be able to sense static modes and resolve them in detail.
- The integral of the static shape, and therefore output of each sensor should roll off quickly in frequency and not have negatives.
- The sensors must have good observability of the dynamic modes targeted for control in the bandwidth.
- The observability of dynamic modes must roll off quickly beyond the control bandwidth.
- The sensors should be easily implementable and must be finite in length.
- If possible, the sensors should not contain negative regions.

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**Approach**

- Consider a pinned-pinned beam with 9 gages distributed along its length.
- Vary the length and spatial weighting of the gages.
- Integrate the sensor measurements to obtain an estimate of the shape of the beam.
- Increase the frequency of the dynamic mode of the beam, and examine the behavior of the tip deflection estimate.
- Verify that observability rolls quickly and monotonically approaches zero.
- Optimize the roll off characteristics by varying spatial weighting of the sensors.
<table>
<thead>
<tr>
<th>Has small negative regions in k</th>
<th>Can be distributed. Has good roll off (-100 db/decade)</th>
<th>Hannings2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has no negative regions in x</td>
<td>Can be distributed. Has good roll off (-300 db in first decade)</td>
<td>Gauss</td>
</tr>
<tr>
<td>Is of infinite extent in x</td>
<td>Has no negative regions in x or k. Can be distributed easily. Very simple shape.</td>
<td>Rectangle</td>
</tr>
<tr>
<td>Has large negative regions in k</td>
<td>Has no negative regions in x. Only -20 db/decade roll off.</td>
<td></td>
</tr>
<tr>
<td>Has negative regions in x. Is of infinite extent in x. Hard to manufacture and distribute.</td>
<td>Gives perfect roll off with no phase lag.</td>
<td>Since</td>
</tr>
<tr>
<td>Disadvantages</td>
<td>Advantages</td>
<td>Shape</td>
</tr>
</tbody>
</table>

Examples of Single Sensor Characteristics
Fourier Transforms of Rectangular, Gaussian and Hanning Squared Windows

Magnitude

Wavenumber
Error vs. Mode Number (Rolloff at mode 3)

Gage shape: Hanning * Hanning

x/L along beam with 9 gages
Conclusions

- Numerical integration schemes have been found that yield accurate shape prediction with a minimum of sensors.

- Spatial weightings for distributed sensors have been identified that yield quick rolloff in:
  - Individual outputs of each sensor
  - Integrated output of all the sensors (tip displacement)

- The functional requirements can nearly be met by simple sensor shapes that are relatively easy to implement.

- Meaningful experiments must be carried out in order to verify theoretical predictions and determine sensitivity of performance to errors incurred during physical implementation of a distributed sensor system.