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A model for amalgamation in group decision making

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Abstract

In this paper we present a generalization of the model proposed by Montero in [Mon87a, Mon87b, Mon92], by allowing non complete fuzzy binary relations for individuals. A degree of unsatisfaction can be defined in this case, suggesting that any democratic aggregration rule should take into account not only ethical conditions or some degree of rationality in the amalgamating procedure, but also a minimum support for the set of alternatives subject to the group analysis.

Key words: Aggregation rules, fuzzy preferences, group decision making.

1 Introduction

When dealing with the problem of amalgamating individual (or group) opinions, it is usually stated that the set of alternatives is fixed and has been previously (well) defined. Moreover, individuals are assumed to be not only able to judge which alternative is the best between any pair of alternatives, but also in favor of at least one of them. However, we know that these assumptions are not true in practice. Indeed, in any democratic voting process there always is some level of *abstention*. A portion of this abstention can be analyzed through statistical techniques since it can be associated with sampling difficulties. Another portion of this abstention gives instead important information and may become a decisive factor since a too high level of abstention can even make null the whole democratic process. Many can be the causes of abstention, among them:

- low interest: people think that the issues subject to vote are not relevant, so perhaps more information was needed;
- dislike of alternatives: people do not like any of the alternatives presented to them, therefore different alternatives should be proposed.

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No interest (or low information) is usually avoided by means of hard and expensive campaigning. The second situation represents however a key issue in democracy. In fact, abstention is sometimes used by political parties (that are mixing in this way an important democratic opinion with no interest or even non democratic attitudes). This is the case for *referendums* where a particular law $\mathcal{L}$ already existing is subject to vote with three basic ballots: yes (*I want the law to be abolished*), no (*I do not want the law to be abolished*), blank (*I do not care*). Blank votes allow to reach a fixed level of participation (usually 50 %) which is requested in order for the referendum to be legally valid. Therefore, even though blank votes are intended to be indifferent to both alternatives, in reality they are helping both of them (and in particular the winner) and justifying the process itself. For instance, 20 % *yes*, 15 % *no*, 20 % *blank* and 45 % of abstention will cause the law $\mathcal{L}$ to be abolished. Therefore, blank votes must be understood as representing a *positive* indifference to the outcome of the voting process. Thus, this kind of indifference must be distinguished from the *negative* indifference, which represents the fact that both alternatives are rejected. A *red* (rejection) vote can then be included in some democratic voting procedures in order to estimate the real support given by the people to the set of alternatives (the technical vote *null* cannot be understood as a *red* vote in any way). Total participation, *yes, no, blank* and *red* votes, gives information about interest or information level, and no democratic meaning exists if a minimum of votes is not reached. The proportion of red votes over the total number of votes estimates the degree of unsatisfaction with the set of alternatives under analysis and if such a degree is too high the whole set of alternatives is rejected.

Our objective here is to show how fuzzy preferences over the set of alternatives provide us with an easy way to model such a negative indifference. Fuzzy preferences are modeled naturally by means of fuzzy binary relations. The theory of fuzzy relations was originally introduced by Zadeh in his seminal paper [Zad65] and subsequentely developed in [Zad71].

In this paper, we generalize the model initially introduced in [Mon87a, Mon87b] where the set $\mathcal{P}(X)$ of all fuzzy preference relations on X, i.e.

$$\mu : X \times X \rightarrow [0,1]$$

verifying

$$\mu(x,y) + \mu(y,x) \geq 1, \quad \forall x, y \in X$$

were considered in order to model individual and social opinions. Adopting Arrow's crisp model [Arr64], completeness assumption of fuzzy preferences was introduced and postulated to model comparability between alternatives. The following values were introduced

(I) $\mu_I(x,y) = \mu(x,y) + \mu(y,x) - 1$

(B) $\mu_B(x,y) = \mu(x,y) - \mu_I(x,y)$

(W) $\mu_W(x,y) = \mu(y,x) - \mu_I(y,x)$

and were understood as the degree of indifference of the alternatives x, y, the degree of strict preference of alternative x over alternative y and the degree of strict preference of alternative y over alternative x, respectively. In this paper, we drop the completeness hypothesis and therefore, assuming a meaningful level of comparability between alternatives, we also drop the hypothesis that comparability is modeled by completeness. We will also show how intensities of negative indifference can be associated with non complete fuzzy preference relations.

As usual, we will also assume that the set of individuals and the set of alternatives are both finite, with at least two individuals and three alternatives.

2 Incomplete fuzzy preferences

Let us assume from now on that each value $\mu(x,y)$ represents the degree to which comparison between alternatives x and y is in favor of x, i.e. the degree of support of alternative x over alternative y. In case $\mu(x,y)+\mu(y,x)\geq 1$ it can be understood that the comparison between both alternatives generates no problems. We then assign intensities of preference according to the above formulae, associating the exceeding value $\mu(x,y)+\mu(y,x)-1$ with the degree of positive indifference. On the other hand, if $\mu(x,y)+\mu(y,x)<1$, the value $1-\mu(x,y)-\mu(y,x)$ can be understood as the degree to which the comparison has not been accepted. Notice that this remaining intensity value cannot be associated to any distinction between both alternatives, so it is some kind of indifference mainly due to a negative opinion of the comparison itself. Therefore, given an arbitrary fuzzy preference relation $\mu : X\times X \rightarrow [0,1]$, for any fixed pair of alternatives we define

$$\begin{aligned}\mu_{PI}(x,y) &= \max(\mu(x,y)+\mu(y,x)-1,0)\\ \mu_{NI}(x,y) &= \max(1-\mu(x,y)-\mu(y,x),0)\end{aligned}$$

in order to capture the degrees of negative and positive indifference. Notice, in particular that $\mu_{PI}(x,y)$ is the *Lukasiewicz* T-norm representing in this case $x\geq y$ and $y\geq x$. It is, then, obvious that the meaning of the expressions

$$\begin{aligned}\mu_B(x,y) &= \mu(x,y)-\mu_{PI}(x,y)\\ \mu_W(x,y) &= \mu(y,x)-\mu_{PI}(x,y)\end{aligned}$$

is kept. Obviously, this model is based on the assumption that both positive and negative indifference are basically indifferences, so that standardized (complete) preferences can be defined

$$\mu^*(x,y)=\mu(x,y)+\mu_{NI}(x,y)=1-\mu_W(y,x)$$

The μ^* will be called the *completion* of μ.

Moreover, each value

$$\sigma(x,y)=1-\mu_{NI}=\min\{\mu(x,y)+\mu(y,x),1\} \qquad (1)$$

can be associated to the degree to which the comparison between the pair of alternatives x,y is being supported.

According to the above comments, we should be able to evaluate in some way the degree of support for the process itself, and afterwards (assuming a minimum support) obtain the aggregated fuzzy preference relation. Though we will not comment here on how a final decision can be made from such information, we will analyze how to aggregate support and preference intensities.

3 Ethical conditions and rationality

The definition given in [Mon87a, Mon87b] for the measure of acyclity of a fixed chain

$$G=(x_1,x_2,\ldots,x_k,x_{k+1})$$

with $x_{k+1}=x_1$, of different alternatives is obviously still valid for standardized preferences. Indeed, we can define $A^*(\mu)=A(\mu^*)$, where A is defined as $A(\mu^*)=\min_G A_{\mu^*}(G)$, the minimum is evaluated along all chains in X and

$$A_{\mu^*}(G)=1-(\Pi_{i=1}^k\mu^*(x_i,x_{i+1})+\Pi_{i=1}^k\mu^*(x_{i+1},x_i)-2\Pi_{i=1}^k\mu_I^*(x_i,x_{i+1})).$$

Notice that the value

$$\mu_I^*(x,y) = |\mu(x,y) + \mu(y,x) - 1| = \max\{\mu_{PI}(x,y), \mu_{NI}(x,y)\}$$

is understood as a degree of *technical* indifference. However, going back to the referendum example, the opinion of those people against the referendum cannot be used to discriminate between *yes* and *no*. In the following, our final aggregation model will make use of those non complete preferences but as pointed out such aggregated values cannot be properly considered without estimating the support of the global decision problem.

The problem can then be stated as follows: is it possible to find fuzzy aggregation rules that are non (absolutely) irrational ? Therefore, we shall assume that all individual opinions are non irrational in the above sense (i.e. $A^*(\mu) = A(\mu^*) > 0$) and for simplicity we shall also assume that all fuzzy preferences are *reflexive* meaning $\mu(x,x) = 1$ for all $x \in X$. Hence, a non absolutely irrational (NAI) aggregation rule in this general context will be defined as a mapping $S : \mathcal{F}^n(X) \to \mathcal{P}(X)$, where

$$\begin{aligned} \mathcal{F}(X) &= \{\mu | \mu(x,x) = 1 \ \forall x \in X, A(\mu^*) > 0\} \\ \mathcal{P}(X) &= \{\mu | \mu \in \mathcal{F}(X) \wedge \mu(x,y) + \mu(y,x) \geq 1 \forall x,y \in X\} \end{aligned}$$

i.e. $\mathcal{F}(X)$ is the collection of all reflexive, fuzzy binary relations over X and $\mathcal{P}(X)$ is the collection of all complete fuzzy binary relations over X. Notice that social aggregated opinion is assumed to be complete, according to the above comments about usual practive in democracy. The information about people supporting aggregation is a question that we will try to answer later on.

Ethical conditions analogous to those given in [Mon87b, Ovc90, Mon92] or deriving from them can be imposed

(UD) *Unrestricted Domain*: the mapping S is defined over all possible profiles of reflexive fuzzy preferences provided that the support of the set of alternatives is not absolutely *null*. According to the definition given in (1) this means that for all $x,y \in X$ there exists i such that $p^i(x,y) > 0$ or $p^i(y,x) > 0$. This will be subsequentely clarified in section 5.

(NNR) *Non Negative Responsiveness*: for any $(x,y) \in X \times X$ if $p^i(x,y) \geq q^i(x,y)$ and $p^i_W(x,y) \leq q^i_W(x,y)$ then

$$S(p^1,\ldots,p^n)(x,y) \geq S(q^1,\ldots,q^n)(x,y).$$

(IIA) *Independence of Irrelevant Alternatives*: $p^i(x,y) = q^i(x,y)$, $\forall i$ and $\forall x,y \in Y \subseteq X$ implies that

$$S(p^1,\ldots,p^n)(x,y) = S(q^1,\ldots,q^n)(x,y)$$

for any Y *nonempty subset of* X.

(A) *Anonimity*: given any permutation of the set of individuals $\pi : \{1,\ldots,n\} \to \{1,\ldots,n\}$ we have

$$S(p^1,\ldots,p^n) = S(p^{\pi(1)},\ldots,p^{\pi(n)}).$$

(N) *Neutrality*: given any permutation of the set of alternatives $\pi : X \to X$, if $p^i(x,y) = q^i(\pi(x),\pi(y))$ for all i and $x,y \in X$, then

$$S(p^1,\ldots,p^n)(x,y) = S(q^1,\ldots,q^n)(\pi(x),\pi(y)$$

for all $x,y \in X$.

(CS) *Citizen Sovereign*: for any given $p \in \mathcal{P}(X)$ there exists a profile $(p^1, \ldots, p^n)$ such that

$$S(p^1, \ldots, p^n) = p.$$

Notice that A implies the following condition

(ND) *Non Dictatorship*: there is no individual i such that

$$S(p^1, \ldots, p^n) = p^i$$

for any $(p^1, \ldots, p^{i-1}, p^{i+1}, \ldots, p^n)$.

Moreover, notice that condition IIA does not imply that $S(p^1, \ldots, p^n)(x, y)$ depends solely on the values $p^i(x, y)$. In fact, such a conditions implies that in order to get the value $S(p^1, \ldots, p^n)(x, y)$ we need the values $p^1(x, y), \ldots, p^n(x, y)$ along with the values $p^1(y, x), \ldots, p^n(y, x)$. For instance, $p^i(x, y) = 0$ for all i does not necessarily imply that $S(p^1, \ldots, p^n)(x, y) = 0$, even if we assume NNR and CS simultaneously. The condition $p^i(y, x) = 1$ for all i, needs also to be imposed to reach such a conclusion. Condition NNR has also been modified coherently with IIA. Finally, since we want the social aggregation to be complete the Unanimity condition

(U) *Unanimity*: if $p^i = p$ for all i then

$$S(p^1, \ldots, p^n) = p$$

cannot be imposed.

In the next section some particular aggregation rules are proposed in order to show that no Impossibility theorem holds in our context.

4 Aggregation Rules

First we notice that the mean aggregation rule (analyzed in [Mon88b]) which has been shown to be a NAI aggregation rule in the case that all individual preferences are complete (see [Mon87a, Mon87b])

$$M(p^1, \ldots, p^n)(x, y) = \sum_{i=1}^{n} p^i(x, y)/n$$

does not assure rationality when individual preferences are not required to be complete. Indeed, consider the following example.

Example 4.1 Let p^1 and p^2 be two individuals expressing their opinions about three different alternatives $\{x, y, z\}$ in the following way:

$$p^1(x, y) = p^1(x, z) = p^1(y, z) = p^1(y, x) = p^1(z, x) = p^1(z, y) = 1$$
$$p^2(x, y) = p^2(y, x) = 0, \quad p^2(x, z) = p^2(y, z) = p^2(z, x) = p^2(z, y) = 1$$

Intuitively, the individual p^1 is fully satisfied by the set of alternatives. Individual p^2 though not satisfied by alternatives x and y is fully content with the final decision as long as alternative z is taken under consideration.

We then have two NAI individual preferences but the aggregation

$$\begin{aligned} M(p^1, p^2)(x, y) &= M(p^1, p^2)(y, x) = 1/2 \\ M(p^1, p^2)(y, z) &= M(p^1, p^2)(x, z) = M(p^1, p^2)(z, x) = M(p^1, p^2)(z, y) = 1 \end{aligned}$$

is irrational. □

Let us propose now one aggregation rule which is not irrational.

(R1) The rule is based on standardized intensities:

$$T(p^1,\ldots,p^n)(x,y) = \sum_{i=1}^{n}(1 - p^i_W(x,y))/n \quad \forall x,y \in X.$$

It easy to see that the above rule correspond to the mean rule in the case of standardized preferences, i.e.

$$T(p^1,\ldots,p^n)(x,y) = M(p^{*,1},\ldots,p^{*,n})(x,y) = \sum_{i=1}^{n} p^{i,*}(x,y)/n$$

where $p^{i,*}$ is the completion of p^i. Therefore, $T(p^1,\ldots,p^n)$ is not absolutely irrational and verifies all of the ethical conditions.

The following property gives a sufficient condition for NAI aggregation rules that can be easily cheched.

THEOREM 4.1 *Let $S : \mathcal{F}^n(X) \rightarrow \mathcal{P}(X)$ be a mapping verifying condition IIA and such that for any fixed pair of alternatives x, y the following relations hold*

$$(C1)\;\; S(p^1,\ldots,p^n)(x,y) = 1 \;\; \rightarrow \;\; \forall i \;\; p^i_W(x,y) = 0$$
$$(C2)\;\; S(p^1,\ldots,p^n)_I(x,y) = 0 \;\; \rightarrow \;\; \exists i | p^i(x,y) + p^i(y,x) = 1$$

then S is a NAI aggregation rule.

Proof. Let G be a fixed chain. If, on one hand, there is some individual acyclic path with some strict preference, that is $p^i_B(x,y) > 0$ for some i and some (x,y) in G, since $p^i_W(y,x) = p^i_B(x,y)$ we then have $S(p^1,\ldots,p^n)(y,x) < 1$. Hence, in view of the fact that $S(p^1,\ldots,p^n)$ is complete, it must be the case that $S(p^1,\ldots,p^n)_B(x,y) > 0$. Therefore, such acyclic path will have positive weight in the aggregated fuzzy preference and the aggregation will not be irrational. On the other hand, if $p^i(x,y) + p^i(y,x) \neq 1$ for all i and all pairs (x,y) in G then it must be $S(p^1,\ldots,p^n)_I(x,y) > 0$ for all (x,y) in G. Therefore the indifferences acyclic path has a positive weight and in this case we also obtain a rational aggregation. ■

The converse of the above theorem does not hold, as can be easily seen by considering the following rule

$$I(p^1,\ldots,p^n)(x,y) = 1$$

for all $x, y \in X$.

Moreover, consider the following aggregation rule.

Amortized intensities rule :

$$T_0(p^1,\ldots,p^n)(x,y) = \sum_{i=1}^{n} p^i(x,y)/C\,.$$

where $C = \sum_{i=1}^n \min(p^i(x,y) + p^i(y,x), 1) = \sum_{i=1}^n p^i(x,y) + p^i_W(x,y)$.

The above rule verifies all ethical conditions and it is obviously complete but, as we will prove below, verifies only condition $(C1)$ of the theorem and in fact it is not rational.

Let us then prove that $T_0(p^1,\ldots,p^n)(x,y) = 1$ implies that $p^i_W(x,y) = 0 \; \forall i$. Let us first define $H = \{i : p^i(x,y) + p^i(y,x) < 1\}$.

Suppose on one hand that $T_0(p^1, \ldots, p^n)(x, y) = 1$. Then

$$\sum_{i=1}^{n} p^i(x, y) = \sum_{i=1}^{n} \min(p^i(x, y) + p^i(y, x), 1).$$

Since $p^i(x, y) \leq \min(p^i(x, y) + p^i(y, x), 1)$ then it must be the case that $p^i(x, y) = \min(p^i(x, y) + p^i(y, x), 1)$ for all i. Two cases are possible:

(1) if $i \in H$ then $p^i(y, x) = 0$ which implies that $p^i_W(x, y) = 0$.

(2) $i \notin H$ then $p^i(x, y) = 1$ which implies that $p^i_W(x, y) = 0$.

In both cases then $p^i_W(x, y) = 0$ for all i.

To prove that $T_0(p^1, \ldots, p^n)$ is not rational consider the following example. We have two individuals p^1 and p^2 and three alternatives x, y and z. The two individuals have the same opinion p :

$$p(x, y) = p(y, x) = 1$$
$$p(y, z) = p(z, y) = 1$$
$$p(z, x) = p(x, z) = \frac{1}{3}$$

Then, we have

$$T_0(p^1, p^2)(x, y) = T_0(p^1, p^2)(y, x) = 1$$
$$T_0(p^1, p^2)(y, z) = T_0(p^1, p^2)(z, y) = 1$$
$$T_0(p^1, p^2)(z, x) = T_0(p^1, p^2)(x, z) = \frac{1}{2}$$

and it can be seen that $A(T_0(p^1, p^2)) = 0$ (cfr. section 3).

We can modify T_0 in the following way

(R2) ϵ-amortized intensities:

$$T_\epsilon(p^1, \ldots, p^n)(x, y) = \frac{\sum_{i=1}^{n} p^i(x, y) + \epsilon}{C + \epsilon}.$$

It is easy to see that the above rule (R2) gives a NAI aggregation rule for every $\epsilon > 0$: it is complete and verifies conditions $(C1) - (C2)$ of theorem 4.1. About condition $(C2)$ of theorem 4.1 notice that the ϵ-amortized aggregated opinion will never verify $T_\epsilon(p^1, \ldots, p^n)(x, y) + T_\epsilon(p^1, \ldots, p^n)(y, x) = 1$.

5 Support Analysis

Given a fixed pair of alternatives (x, y) and the individual preference values $p^i(x, y)$ and $p^i(y, x)$, the support of such a comparison relative to the individual i, according to (1) will be the value

$$\sigma^i(x, y) = \min(p^i(x, y) + p^i(y, x), 1) = 1 - p^i_{NI}(x, y)$$

i.e. the Lukasiewicz co-T-norm. Our problem is to obtain for each pair of alternatives a social support value to be evaluated from individual preferences. This problem is therefore analogous to the previous aggregation problem, and since both problems are dealing with intensity values, they should be solved in a coherent way.

Let us first describe a representation result related with aggregation rules.

THEOREM 5.1 *Let $S : \mathcal{F}^n(X) \rightarrow \mathcal{P}(X)$. Then S verifies IIA and N conditions if and only if there exists a function $\phi_S : [0,1]^{2n} \rightarrow [0,1]$ such that*

$$S(p^1,\ldots,p^n)(x,y) = \phi_S(p^1(x,y),p^1(y,x);\cdots;p^n(x,y),p^n(y,x))$$

for all $x, y \in X$.

Proof. In view of the condition IIA, it is clear that for each pair (x,y) the value $S(p^1,\ldots,p^n)(x,y)$ is perfectly determined once the values $p^1(x,y),p^1(y,x),\ldots,p^n(x,y),p^n(y,x)$ are given. Therefore S can be defined by a set of mappings $\phi_S^{(x,y)} : [0,1]^{2n} \rightarrow [0,1]$ such that

$$S(p^1,\ldots,p^n)(x,y) = \phi_S^{(x,y)}(p^1(x,y),p^1(y,x);\cdots;p^n(x,y),p^n(y,x))$$

for such a fixed pair of alternatives. However, due to the N condition, these mappings $\phi_S^{(x,y)}$ do not in fact depend on the particuler pair of alternatives.

The converse is trivial. ■

Notice that since $p^i_W(x,y) = p^i(x,y) - \max\{p^i(x,y) + p^i(y,x) - 1, 0\}$ social aggregation values will be also determined if the values $p^i_W(x,y)$ are given instead of the values $p^i(y,x)$.

The characterization given by theorem 5.1 allows us to define the social support σ in a coherent way with respect to the social preference:

$$\sigma(x,y) = \phi_S(\sigma^1(x,y), 1-\sigma^1(y,x);\cdots;\sigma^n(x,y), 1-\sigma^n(y,x))$$

for all $x, y \in X$.

With this definition all ethical conditions imposed on the social preferences aggregation rule are automatically imposed on the social support aggregation rule. The final social opinion will contain an ethical and non irrational fuzzy preference relation (complete in order to be useful in the subsequent decision making process) but also the aggregated support function.

6 Final Remarks

In this paper a welfare oriented approach (similar to Arrow's model) has been developed, but not a decision oriented one. Real democratic problems are more related to decision making problems, and in this case an analysis of the stability of the final decision should also be included (see for example [Mon90]). In any case, by using fuzzy preference relations, we have been able not only to avoid Arrow's paradox but also other similar restrictive results in the fuzzy context (see [FF75] and also [Mon85, Mon88a]).

Moreover, it has been shown how the problem of negative indifference can be modeled within the fuzzy preferences framework, just dropping out the assumption of completeness. In fact, it is suggested a natural way of dealing with dislike of proposed alternatives and therefore a measure of their support. Critical levels of such a support should be previously defined depending on the characteristic of the alternatives and their social significance.

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References

[Arr64] K.J. Arrow. *Social Choice and Individual Values.* John Wiley & Sons, Inc., New York, 1964.

[FF75] L.W. Fung and K.S. Fu. An axiomatic approach to rational decision making in a fuzzy environment. In L.A. Zadeh, K.S. Fu, K. Tanaka, and M. Shimura, editors, *Fuzzy sets and their applications to Cognitive and decision processes*, pages 227–256. Academic Press, 1975.

[Mon85] J. Montero. A note on fung-fu's theorem. *Fuzzy sets and systems*, 17:259–269, 1985.

[Mon87a] J. Montero. Arrow's theorem under fuzzy rationality. *Behavioral Science*, pages 267–273, 1987.

[Mon87b] J. Montero. Social welfare functions in a fuzzy environment. *Kybernetes*, 16:241–245, 1987.

[Mon88a] J. Montero. Aggregation of fuzzy opinions in a non-homogeneous group. *Fuzzy sets and systems*, 25:15–20, 1988.

[Mon88b] J. Montero. An axiomatic approach to fuzzy multicriteria analysis. In M.M. Gupta and T. Yamakawa, editors, *Fuzzy logic in Knowledge-Based Systems, Decision and Control*, pages 259–269. North-Holland, 1988.

[Mon90] J. Montero. Single-peakedness in weighted aggregation of fuzzy opinions in a fuzzy group. In J. Kacprzyk and M. Fedrizzi, editors, *Multiperson decision making using fuzzy sets and possibility theory*, pages 163–171. Kluwer, 1990.

[Mon92] J. Montero. Fuzzy rationality as a basis for group decision making. In *Fuzz-IEEE 1992*, pages 1237–1243, San Diego, CA, 1992. Proceedings of the IEEE International Conference on Fuzzy Systems.

[Ovc90] S. Ovchinnikov. Means and social welfare function in fuzzy binary relation spaces. In J. Kacprzyk and M. Fedrizzi, editors, *Multiperson decision making using fuzzy sets and possibility theory*, pages 143–154. Kluwer, 1990.

[Zad65] L.A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[Zad71] L.A. Zadeh. Similarity relations and fuzzy orderings. *Information Science*, 3:177–200, 1971.