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FUZZY KNOWLEDGE BASE CONSTRUCTION THROUGH BELIEF NETWORKS BASED ON LUKASIEWICZ LOGIC

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ABSTRACT.

In this paper a procedure is proposed to build a fuzzy knowledge base founded on fuzzy belief networks and Lukasiewicz logic. Fuzzy procedures are developed to assess the belief values of a consequent in terms of the belief values of its logical antecedents and the belief value of the corresponding logical function and to update belief values when new evidence is available.

INTRODUCTION.

Expert Systems also called Knowledge-based Systems are one of the most fruitful areas of Artificial Intelligence (Graham 1991). A knowledge base is a collection of logical propositions whose relationships model the knowledge about a certain topic.

One of the principal issues in building expert systems is related to the design and construction of knowledge bases capable of modeling real knowledge situations characterized by uncertainty (Yager 1992). This uncertainty may be produced by following factors: (Lara-Rosano 1989)

a) It is impossible to assign the whole truth or the whole falsity to propositions, even to those taken as premises or starting points of a logical discourse.

b) The logical support of a set of premises or conditions for determining a given conclusion or result is uncertain.
c) The premises contain fuzzy terms.

In this paper a procedure to build fuzzy knowledge bases is introduced based on fuzzy belief nertworks and Lukasiewicz logic (Lukasiewicz & Tarski 1930).

BELIEF NETWORKS AND FUZZY KNOWLEDGE BASES

Uncertain knowledge may be represented by a fuzzy knowledge base structured as a fuzzy belief network (Lara-Rosano 1989). Fuzzy belief networks are weighted directed acyclic graphs in which the nodes represent propositions, and the arcs express and quantify in a fuzzy manner the logical dependencies of the consequents in terms of its immediate antecedents, according to present knowledge. The logical belief functions should be drawn from a specific fuzzy logic.

Thus, if the nodes represent the propositions $q_1,q_2,...,q_n$, then each proposition q_i draws arrows from a subset S_i of propositions which are the direct logical antecedents of q_i. Each arrow has a weight that expresses the conditional belief on q_i given the belief of the corresponding logical predecessor.

For instance, consider following fuzzy knowledge, where **q** are propositions and the terms under brackets represent their corresponding belief values:

q_3 = *If John takes a glass water and the water is contaminated with harmful bacteria then John <u>could get</u> sick* [v(q_3)=0.8]
and the following uncertain (fuzzy) facts:

q_1 = *John is thirsty and <u>probably</u> takes a glass water* [v(q_1)=0.7]
q_2 = *The water <u>could be</u> contaminated with harmful bacteria* [v(q2)=0.6]

The question is how to asess the belief value of the hypothesis:

q_4 = *John <u>could get</u> sick.*

In this case, it is obvious that the belief value of q_4 is not independent of the belief values of its antecedents, but a fuzzy function of them. The problem now is to find the most suitable multivalued logic to assess the belief value of an uncertain logical consequence in terms of the belief values of its immediate antecedents and the belief value of the implication.

From the possible multivalued logics it is argued that the most appropriate for use in fuzzy logical networks is the Lukasiewicz logic. (Lukasiewicz & Tarski 1930). In fact:

a) Lukasiewicz logic is the multivalent logic underlying Zadeh's ordinary fuzzy set theory (Giles 1976), having equivalent definitions for union (disjunction), intersection (conjunction) negation and set inclusion. (Zadeh 1965).

b) The fundamental operators & and U are commutative, associative and distributive over one another and idempotent. (Dubois & Prade 1980)

c) Lukasiewicz logic satisfies the De Morgan Laws and is compatible with the Piaget Group of logical transformations (Sinclair 1972), but does not satisfy the Middle-Excluded Law. That is, in this logic a certain proposition could be at the same time more or less true and more or less false, such as is the actual case in uncertain propositions. (Dubois & Prade 1980).

Given propositions **a,b** and their respective belief values **v(a), v(b)** , Lukasiewicz logic defines the following operators:

Conjunction: $\mathbf{v(a \,\&\, b) = min\,[v(a), v(b)]}$
Disjunction: $\mathbf{v(a \,U\, b) = max\,[v(a), v(b)]}$
Negation: $\mathbf{v(-a) = 1 - v(a)}$
Implication: $\mathbf{v(a => b) = min\,[1, 1 - v(a) + v(b)]}$
Modus ponens: $\mathbf{v(a \,\&\, [a => b]) = max\,[0, v(a) + v(a => b) - 1]}$

Therefore, in the former example:

$\mathbf{v(q_1\&q_2) = min[v(q_1), v(q_2)]} = \min(0.7, 0.6) = 0.6$

$\mathbf{v(q_3)} = 0.8$

and the belief value for $\mathbf{q_4}$: *John could get sick* is:

$\mathbf{v(q_4) = max[0, v(q_1\&q_2) + v(q_3) - 1]} = 0.4$

a low belief value, indicating a low possibility for John to get sick.

Due to its mathematical logical foundations, this method is as theoretically sound as the probabilistic methods (Schafer 1976) because it gives belief values for derived propositions with fully logical consistence with respect to the rest of the network.

Adopting the conceptual frame of fuzzy set theory (Zadeh 1965), we may define the uncertain implication as a fuzzy logical function such that:

a) if the premise **x** is true, then the conclusion **y** has a partial belief $\mathbf{v(y) = s(x/y)}$

b) if the premise x is false, then the conclusion y may have any belief value.

The value $s(x/y)$ is on the interval [0,1] and will be called the degree of sufficiency or sufficiency value of x over y , that is the degree of support given by the true proposition x to the uncertain proposition y. It may be further interpreted as the degree of membership of x to the fuzzy set S of sufficient conditions for y to be true.

SEMANTIC CLUSTERS OF PREMISES

Let us suppose a set of n premises $\{x_1, x_2,\ldots,x_n\}$, each one having its own belief value $v(x_i)$ $i=1,2,\ldots,n$ and associated in a conjunctive way to support a conclusion y. Let us call $s(x_i/y)$ the sufficiency value of premise x_i over y. In general, the conjunction $(x_i \;\&\; x_j \;\&\; \ldots)$ of two or more premises will have a specific sufficiency value $s(x_i \;\&\; x_j \ldots/y)$ over a conclusion y according to the synergistic sufficiency of the set, that is, its degree of membership to the fuzzy set of sufficient conditions for y.

In general, $s(x_i \;\&\; x_j \ldots/y)$ will be non-separable in terms of the single values $s(x_i/y)$, $s(x_j/y)$, ... due to the synergistic effect of the conjunction on y. Moreover, the synergy will be more pronounced in certain specific conjunctive sets of premises than in others. These privileged conjunctive sets of premises with higher overall sufficiency are called *semantic clusters* and their identification among all possible conjunctive sets of premises is a matter of expertise and field knowledge.

For instance, a doctor may assign a high belief value to the hypothesis *apendicitis*, based on a semantic cluster defined by a couple of symptoms, none of which taken alone would bring high credibility to the hypothesis.

Every semantic cluster of premises defines a specific implication with its own belief value.

EXAMPLE

For instance, let us have the following reasoning scheme:

x_1 = *It is cloudy*
x_2 = *The barometric pressure is low*
y = *It will rain*

If it is cloudy and the barometric pressure is low, then <u>it is absolutely probable that</u> it will rain.

In this case, the conclusion whose belief value is going to be estimated is y. The supporting premises are x_1 and x_2; $v(x_1)$ and $v(x_2)$ are their belief values and $s(x_1/y)$ and $s(x_2/y)$ are their single sufficiency values. The logical combining function of the premises is the conjunction: *it is cloudy and the barometric pressure is low*, expressed as $(x_1 \& x_2)$. The conjunction has an overall sufficiency value on y represented as $s([x_1 \& x_2]/y)$.

Given the belief values $v(x_1) = 0.8$ $v(x_2) = 0.85$ and the overall sufficiency value $s([x_1 \& x_2]/y) = 0.9$ to estimate v(y), the belief value of y we need to apply the fuzzy expression:

$$\mathbf{v(y) = max[0, v(x_1 \& x_2) + s([x_1 \& x_2]/y) - 1)}$$

but $v(x_1 \& x_2) = \min(0.8, 0.85) = 0.8$ and $s([x_1 \& x_2]/y) = 0.9$. Therefore:

$$v(y) = \max[0, 0.8 + 0.9 - 1] = 0.7$$

EVIDENTIAL BELIEF UPDATING OF FUZZY KNOWLEDGE BASES

In real life situations the initial knowledge base normally is composed of a small set of premises with low belief values, because of lack of evidence. Later on, when evidence arrives, new premises are introduced in the knowledge base, bringing new synergistic support to other premises and modifying the belief value of the uncertain implications.

Thus, we have two different kinds of belief updating of fuzzy knowledge bases:

a) Belief updating of fuzzy implications.
b) Belief updating of premises.

For belief updating of fuzzy implications, the new evidence is joined to the old one, trying to identify new semantic clusters or to reinforce the existing ones. Then the new combined sufficiency values are estimated giving the new belief values for the implications.

In the case of belief updating of premises, because the new evidence, e may confirm or disconfirm the related propositions, it is useful to apply the following Bayesian formula proposed by Pearl (1986) to update the belief values:

$$\mathbf{v'(x) = \alpha\, L\, v(x)}$$

where $v'(x)$ is the new belief value of the proposition x under new evidence e, v(x) is the old belief value, L is a likelihood ratio expressed by:

$$\mathbf{L = P(e|x) / P(e|-x)}$$

where **P(e|x)** is the probability of occurring evidence **e** giving **x**. Thus the meaning of **L** is: how many times more likely would it be for evidence **e** to occur under **x** as opposed to under **not-x**

and α is a normalizing factor:

$$\alpha = 1 / [L\, v(x) + 1 - v(x)]$$

The role of α is to maintain the belief value **v'(x)** less than or equal to one. In order to **v'(x)** to be one the old belief value **v(x)** should be also equal to one and the evidence should support **x**, that is, **L** should be greater than one. If **v(x)** is less than one, then α would be smaller than **L v(x)** and the new belief value **v'(x)** would be also less than one.

The likelihood parameter **L** should be assessed by an expert, taking into account the possible synergy of the new evidence with the old one.

Once the new belief values of the premises and implications are estimated according to the new evidence, the whole belief network is recalculated applying the rules of fuzzy Lukasiewicz logic, to have logical coherent belief values for every proposition in the network.

Suppose in our last example that new evidence comes with certainty:

x_3 = *Tropical wet wind is blowing from the south*; $v(x_3) = 1$

Then we identify a new semantic cluster as $(x_1 \,\&\, x_2 \,\&\, x_3)$ whose sufficiency value we estimate say as 0.95

Further let us suppose that this evidence will bring new confirming support to our premise x_1: *It is cloudy.* Let us estimate the likelihood ratio $L(x_1,x_3) = P(x_1|x_3) / P(x_1|-x_3) = 3$ Then the normalizing factor for x_1 is:

$\alpha = 1 / [L\, v(x_1) + 1 - v(x)] = 1 / [(3 \times 0.8) + 1 - 0.8]$
$\alpha = 1 / 2.6 = 0.3846$

The new belief value $v'(x_1)$ will be then:
$v'(x_1) = \alpha\, L\, v(x_1) = 0.3846 \times 3 \times 0.8 = 0.923$

Therefore, it is more likely that it is cloudy. The belief value of the new conjunction **(x1 & x2 & x3)** is:

min (0.923, 0.85, 1) = 0.85
and the belief value of y = *It will rain* will be:

$v(y) = \max [0, v(x_1 \& x_2 \& x_3) + s([x_1 \& x_2 \& x_3]/y) - 1]$ $= \max [0, 0.85 + 0.95 - 1] = 0.8$

The new incoming evidence has increased the belief value of *It will rain* from 0.7 to 0.8 due to an impact over one of the premises and the definition of a new semantic cluster with a higher sufficiency value.

CONCLUSIONS

In this paper a procedure to build a fuzzy knowledge base founded on fuzzy belief networks and Lukasiewicz logic was proposed. It is based on a knowledge network structure composed by uncertain propositions interconnected by fuzzy logical functions according to their logical dependencies. Under this basis, the belief value of a logical consequent in the knowledge network is defined and fuzzy procedures are developed to assess it in terms of the belief values of its logical antecedents and the belief value of the corresponding logical function.

The procedure permits also updating of fuzzy knowledge bases when new evidence arrives. This updating is then propagated in a logical antecedent-consequent order through the network until the last conclusions are updated. For this updating a Bayesian formula developed by Pearl (1986) is applied, requiring the estimation of only one parameter. Due to the analytical support of a logical mathematical theory, the results have complete logical coherence.

REFERENCES

Dubois D & Prade H, (1980) "Fuzzy Sets and Systems", New York: Academic Press.
Giles R, (1976) "Lukasiewicz Logic and Fuzzy Set Theory", *Int.J. Man-Machine Studies*, 8, 313-327.
Graham, I. (1991) "Fuzzy Logic in Commercial Expert Systems - Results and Prospects", *Fuzzy Sets and Systems*, 40, 451-472.
Lara-Rosano, F. (1989). "Uncertain Knowledge Representation through Fuzzy Knowledge Networks based on Lukasiewicz Logic". in *"Advances in Computer Science"*, Lasker, G.E. (ed). Windsor, Canada: International Institute for Advanced Studies in Systems Research and Cybernetics.
Lukasiewicz J & Tarski A, (1930) "Investigations in the Sentential Calculus". Translation in J. Lukasiewicz *"Selected Works*, Amsterdam: Reidel (1970).
Pearl, J. (1986). "On Evidential Reasoning in a Hierarchy of Hypotheses". *Artificial Intelligence*, 28, 9-25.
Schafer, G. (1976). "Mathematical Theory of Evidence". Princeton Uni. Press. Princeton,

N.J., U.S.A.

Shorliffe E H & Buchanan B G, (1975) "A Model of Inexact Reasoning in Medicine", *Mathematical Biosciences,* 23.

Sinclair H, (1972) "Piaget's Theory of Development: The Main Stages" in Murray F B (ed) *"Critical Features of Piaget's Theory of Development of Thought"* Newark: Univ.of Delaware Press.

Yager R R, (1992) "Expert Systems using Fuzzy Logic" in Yager R R & Zadeh L A (eds) *"An Introduction to Fuzzy Logic Applications in Intelligent Systems"*. Boston: Kluwer Academic Publishers.

Zadeh L A,(1965) "Fuzzy Sets", *Information and Control,*8, 338-353