Abstract: In this paper a hierarchical control structure using a fuzzy system for coordination of the control actions is studied. The architecture involves two levels of control: a coordination level and an execution level. Numerical experiments will be utilized to illustrate the behaviour of the controller when it is applied to a nonlinear plant.

Keywords: fuzzy controller, fuzzy coordinator, hierarchical control.

1 INTRODUCTORY REMARKS: HIERARCHY IN CONTROL SYSTEMS

At its standard conceptual level and almost all the existing real-world applications, fuzzy controllers can be perceived as nonlinear mappings, associating current status of a system under control with an appropriate control action. They are legitimate control structures arising as a result of a certain design methodology. This allows us to emulate control abilities of a human operator. As originally proposed in [8,11,12], the fuzzy controller is a simple-level structure. Despite many algorithmic differences and a vast number of software and hardware implementations available, they are usually homogeneous with respect to handling inference and developing control actions. The design methodology is based on the derivation of control rules from the response of a process. In most of the cases, the process is already being controlled by a general purpose controller supervised by a human operator. This operator can tune the controller based on the knowledge of the status of the systems. We are concerned in this paper on emulating the coordination actions of this operator by a fuzzy system. This coordination action is a natural domain for a fuzzy system, since the decisions are taken according to a set of linguistic rules. However, we are not interested in developing a system that can tune the controller, but in one that can coordinate independent and specialized controllers. The reason for this, is that the undesirable fluctuations in the controlled variables that occur when the controller is retuned for a change in the operating point, can be avoided, by smoothly combining the response of different controllers tuned to operate under different conditions.

In this paper, we consider a control architecture that combines human expertise represented by a fuzzy system, with traditional control algorithms. In this approach the control concepts are organized hierarchically in two levels called the coordination level and the execution level [1,13,14,16]. In the coordination level, the status of the control system is being monitored, in order to decide the best control action that can be applied; while in the execution level, there are different control algorithms, each responsible for a specific control task. The response of all these algorithms is combined by the coordinator, to accomplish the control objective. A good choice for the controllers at the execution level are PID controllers, since they are widely used in practice. In this study, 1. Supported by CONACyT, Mexico, Grant #60558.
we investigate a hierarchical control structure composed of a fuzzy system and different PID controllers applied to the control of a nonlinear system.

The paper is structured as follows: the structure of the control hierarchy is introduced in Section 2; in Section 3, the application of the architecture to the control of a nonlinear system is presented; and, finally conclusions are included in Section 4.

2 STRUCTURE OF THE SYSTEM

The fuzzy controller operates at the higher conceptual level while "local" PID controllers are distributed as the basic components of the execution level. The example of a single input–single output system is shown in Fig. 1.

![Figure 1. Structure of the system.](image)

![Figure 2a. Memberships for each PID.](image)

The fuzzy controller is driven by the fuzzy sets of error E and change of error and ΔE, defined over the universes of discourse UE and UAE, and it infers a fuzzy set for selection of the controllers U, defined over the universe of discourse UL. The defuzzified variable over UL is called λ, and depending on its values a different combination of PID controllers becomes active. Each controller is represented in UL by a membership function. In this way the outputs of the controllers are combined by a center or area method, as shown in the following equation:

\[
    u = \frac{\sum_{i=1}^{n} u_i \mu_i(\lambda)}{\sum_{i=1}^{n} \mu_i(\lambda)}
\]

where \( n \) is the number of PID controllers, \( u_i \) is the output of the \( i \)th PID, \( \mu_i(\lambda) \) represents the degree of membership of the \( i \)th PID controller in UL, and \( u \) is the control output. This final control signal is produced by the aggregation block visualized in Figure 1. The control rules in the fuzzy system are standard rules of the form: IF error is \( E_k \) AND change of error is \( \Delta E_k \) THEN selection is \( U_j \), \( k=1,2, \ldots, N \); where \( N \) stands for the number of rules. \( E_k \) and \( \Delta E_k \) are fuzzy sets defined in the universes of discourse UE and UAE. \( U_j \) is a fuzzy set defined over the universe of discourse UL. The universe of discourse UL is partitioned into \( n \) fuzzy sets representing each of the PID controllers, as shown in Figure 2a. The rules are combined into a three-dimensional fuzzy relation \( R = E_1 \times \Delta E_1 \times U_1 + \cdots + E_N \times \Delta E_N \times U_N \), and the inference procedure utilizes the standard max–min compositional rule.
2.1 Case of 2 PID controllers

Consider the case of 2 PID controllers and 9 rules. The following is an example of the set of control rules:

\[
\begin{array}{c|c|c|c}
\text{error} & \text{N} & \text{Z} & \text{P} \\
\hline
\text{N} & U_1 & U_1 & U_1 \\
\text{Z} & U_1 & U_2 & U_1 \\
\text{P} & U_1 & U_1 & U_1 \\
\end{array}
\]  \tag{2}

The coordination level gives a significant preference to the PID 2 for values of error and its change close to zero, while the PID 1 is used to drive the system close to zero. All the transitions are smooth, guided by the membership functions of the fuzzy sets of error and its change.

In contrast to the coordinator implemented using fuzzy controller, we can also introduce a two-valued relay switch coordinator. It provides a Boolean character of the selection procedure, using rules of the form: \text{IF abs(error) < } \delta_1 \text{ AND abs(change of error) < } \delta_2 \text{ THEN } u = u_1 \text{ ELSE } u = u_2, \text{ where } \delta_1 \text{ and } \delta_2 \text{ are used to specify the point of switching.}

3 APPLICATION TO THE CONTROL OF A WATER TANK

In this section, the hierarchical architecture is applied to the control of a water tank. The control objective is to obtain good dynamical properties, such as a fast transient response free of oscillations. This is accomplished by a fuzzy coordinator in conjunction with 2 discrete-time PID controllers. Simulation results of 2 experiments are presented here. Each individual PID is tested first, then the fuzzy system is introduced to combine both, and its response is compared to that of the relay switch.

3.1 Model of the system.

The water tank is shown in Figure 3. The input is the control command \( u \), that operates the inlet valve in the range from 0 to 100%, and the output is the level \( h \). It is considered that noise applied to system in the outlet valve, represented by \( q_{out} \).

![Figure 3. Water tank.](image-url)
The nonlinear model of the system is given by the following equations

\[ \frac{dh}{dt} = \frac{q_{in} - q_{out}}{area} \]

\[ area = \frac{(h + 1)}{7} \]

\[ q_{in} = q_{max} \cdot cval \]

\[ q_{out} = q_{max} \cdot \sqrt{2gh \cdot \max(h, 0)} \]

\[ cval = \begin{cases} 
0 & u < 0 \\
1 & u \geq 0 \\
1 & u > 1 
\end{cases} \]

where \( q_{max} = 1 \), \( g = 9.81 \text{m/sec}^2 \), and \( a_{out} \) is random noise with a rectangular distribution defined over \([0, 0.125]\). Notice the nonlinearities introduced by the saturation and the equation of area. This model is a modification of that one presented in [3]. The valve has a pure time delay that we model as a part of the controller. The error and change of error of the system are defined to be

\[ e = h_{ref} - h \]

\[ \Delta e = h_t - h_{t-1} \]

3.2 The fuzzy system

The membership functions for error and change of error of the fuzzy controller are considered to be the same. Their values have been selected by experimentation. These membership functions and those for selection of the PID controllers are shown in Figure 4.

3.3 Model of the PID controllers

A discrete-time version of the PID controllers with anti-reset windup [2] is used in the experiments. They have the following structure

\[ w_i = K_i \left[ b_i (h_{ref} - h) \right] + \left[ I_{i+1} + \frac{K_i \Delta t}{T_i} e \right] + \left( \frac{T_i}{T_i + N_i \Delta t} \right) \left[ D_{i+1} - \frac{K_i \Delta e}{N_i} \right] \]

\[ I_i = I_{i+1} + (u_i - w_i) \frac{\Delta t}{T_i} \]

\( z_i = \begin{cases} 
\mu_{\text{min}} & w_i < \mu_{\text{min}} \\
\mu_i & \mu_{\text{min}} \leq w_i \leq \mu_{\text{max}} \\
\mu_{\text{max}} & w_i > \mu_{\text{max}} 
\end{cases} \)

\[ u_i = z_{i+2} \]
where \(i = 1, 2\), \(K_i\), \(Ti\) and \(Td\) are the proportional gain, integration and derivation time respectively, \(N\) is the maximum derivative gain, \(Tt\) is the tracking constant, \(b_i\) is the set point weight factor, \(u_{\text{max}}\) and \(u_{\text{min}}\) are the maximum and minimum values of the control output, and \(\Delta t\) is the sampling period.

It can observe the control output is delayed by 2 sampling periods in order to model the time delay of the valve of the tank. The PID 1 was tuned so that the response is as fast as possible, while the PID 2 was tuned in such a way that the response has good regulation properties. The values of the parameters of the PID controllers are given in the following table:

<table>
<thead>
<tr>
<th>PID 1:</th>
<th>PID 2:</th>
<th>Both:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_1=15)</td>
<td>(K_2=1)</td>
<td>(u_{\text{max}}=1)</td>
</tr>
<tr>
<td>(b_1=1)</td>
<td>(b_2=0)</td>
<td>(u_{\text{min}}=0)</td>
</tr>
<tr>
<td>(Ti_1=0.1)</td>
<td>(Ti_2=15)</td>
<td>(\Delta t=0.1)</td>
</tr>
<tr>
<td>(Td_1=10)</td>
<td>(Td_2=10)</td>
<td></td>
</tr>
<tr>
<td>(N_1=10)</td>
<td>(N_1=0)</td>
<td></td>
</tr>
</tbody>
</table>

It can be observed that the nonlinearities of the plant in closed-loop with saturations and time-delays of the controllers yield an overall nonlinear system difficult to control.

### 3.4 Experiment 1

In this experiment it is considered a constant reference level \(h_{\text{ref}}=4\). The results of the experiments are shown in Figures 5a to 5d. The PID 1 produces a fast response but with some undesirable oscillations (Figure 5a), while the PID 2 produces a slow response with better regulation (Figure 5b). The fuzzy coordinator combines the best features of the controllers, the response is fast with good regulation properties (Figure 5c). Finally, we include the results produced by the induced relay switch (Figure 7d), switching according to the rule: IF \(\text{abs(error)}>0.2\) THEN \(u=u_1\) ELSE \(u=u_2\). Notice that the relay switches in the point in which the two membership functions of selection intersect each other. The response of this system with relay is quite comparable to that of the fuzzy coordinator, except that the control output is changing in an abrupt manner, which is definitely not acceptable for the actuators. In Figures 6a to 6d, it can be observed that the state trajectory of the system with the fuzzy supervisor is again a combination of those of the individual PID controllers. We have achieved a fast response, which is bounded within certain practical limits.

### 3.5 Experiment 2

In this experiment the reference level is changed following a triangular wave. These results are shown in Figure 7. We carry out the simulation in a similar way, taking PID 1 first, then PID 2, next the fuzzy supervisor with both PID controllers, and the last graph is the response with the relay. From the response of the system with PID 1, it can be observed the effect of the nonlinearities and noise of the overall system. The amplitude of the oscillations is larger close to zero than close to the maximum (Figure 7a). From the response of PID 2 we can see that the velocity of response is a factor in the performance of this controller (Figure 7b). Again, the response of the system with the fuzzy supervisor is quite remarkable, the system is able to follow the reference despite the disturbances (Figure 7c). The output of the system with relay is comparable to that of the fuzzy supervisor except that we have a not acceptable control signal, due to the fast changes (Figure 7d). In Figures 8a to 8d, the state trajectories are shown, notice that the response of the system with the fuzzy supervisor is again a combination of those of the individual PID controllers.
Figure 5a. Response with PID 1.

Figure 5b. Response with PID 2.

Figure 5c. Response with fuzzy coordinator.

Figure 5d. Response with relay switch.

Figure 6a. State trajectory, PID 1.

Figure 6b. State trajectory, PID 2.

Figure 6c. State trajectory, fuzzy coordinator.

Figure 6d. State trajectory, relay switch.
Figure 7a. Response with PID 1.

Figure 7b. Response with PID 2.

Figure 7c. Response with fuzzy coordinator.

Figure 7d. Response with relay switch.

Figure 8a. State trajectory, PID 1.

Figure 8b. State trajectory, PID 2.

Figure 8c. State trajectory, fuzzy coordinator.

Figure 8d. State trajectory, relay switch.
4 CONCLUSIONS

We have discussed the hierarchical controller using a fuzzy coordinator. The results are encouraging. The fuzzy controller was found capable of combining control signals of individual PID controllers, so that the overall control characteristics are superior to those obtained for the single PID controller. The advantages of the coordinator over the relay switch were also highlighted. Further studies should lead toward enhancements in expressing control rules and calibrating the fuzzy sets included there.

REFERENCES