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Tuning a Fuzzy Controller using Quadratic Response Surfaces

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Abstract

Response surface methodology, an alternative method to traditional tuning of a fuzzy controller, is described. An example based on a simulated inverted pendulum "plant" shows that with (only) 15 trial runs, the controller can be calibrated using a quadratic form to approximate the response surface.

Introduction

Fuzzy Controller

Fuzzy controllers have received considerable attention in practice and in the literature because fuzzy rules can be framed by domain experts for narrowly defined systems. For example, Sugeno and Yasukawa said, "It supports the idea of a fuzzy model that human being can grasp input-output relations of a system qualitatively."¹ Although the general structure of such rules can be accomplished rather directly because of their linguistic flavor, tuning or calibration of the fuzzy variables can be very challenging. The purpose of this research is to explore an alternative method of calibration based on representing the performance of the system relative to the parameters of the controller by a sequence of quadratic functions.

We consider traditional fuzzy controllers in which the knowledge is encoded as rules comprised of combinations of subrules. The subrule i for rule k is of the form, "If x_i is X_i and y_i is Y_i then z_i is (should be) Z_i ," where lowercase letters x and y signify the names of two antecedent objects; X and Y are values of fuzzy linguistic variables describing their objects; z and Z are a consequent object and its fuzzy variable's value. The k th rule contains subrules $i = 1, \dots, J$, which are fused into rule k by the fuzzy operator minimum or maximum, depending on the multivalued logic employed in the system. The term set for the fuzzy values X , Y , and Z commonly includes LARGE NEGATIVE, NEGATIVE, SMALL NEGATIVE, ZERO, SMALL POSITIVE, POSITIVE, AND LARGE POSITIVE. A typical subrule is, "If the error angle is SMALL NEGATIVE and the angular velocity is SMALL NEGATIVE, then the force of the push should be SMALL POSITIVE."

In operation the fuzzy controller is supplied the actual data values for the antecedent variables x and y , x and y . As is usual in practice, these actual values

are assumed to be crisp numerical singletons, in this research. Also the operational controller defuzzifies rule k 's detached consequent value kZ into a crisp numerical singleton which is employed to control the "plant," the system which is being controlled. The current study uses a system that contains only one rule, with eleven subrules.

Controller Tuning

Tuning a controller involves tweaking the several parameters which define the rules with the intention of optimizing or improving key system performance. Among the controllable parameters are the number of linguistic terms and linguistic hedges and conjunctives considered, the granularity of discretization, the method of defuzzifying, and the shape of the fuzzy variables. Many alternatives are available regarding shape: the width of the support and core; triangular vs. trapezoidal vs. sigmoidal shape; regularity vs irregularity among linguistic terms; and so on. The choices of these parameters are dependent on one another and on other system features. For example, systems based on possibilistic logics (such as Mamdani's popular system) can function well with triangular shaped fuzzy terms with slight gaps between the cores of adjacent terms (in subrules).² But a system based on Lukasiewicz' multivalued logic requires fuzzy terms with broad cores, and there must be no gaps between the cores of adjacent terms.³

Tuning can occur prior to employing the controller and adaptive learning can occur while the controller is in operation. Adaptive learning (re-tuning) is needed when the plant experiences extensive changes during use. In recent literature artificial neural networks have been suggested as tuners by several scientists, both for initial learning (see for example Kosko⁴, Keller & Tahani⁵) and adaptively (see for example Hayashi *et al*⁶ and Berenji⁷). We consider an alternative tuning method based on Box and Wilson's⁸ response surface methodology as explicated by Myers⁹.

Controller Performance

The performance of the controlled system may depend on multiple factors. Common performance variables for mobile systems are fuel economy, smoothness of ride, and speed of recovery. Performance factors of the controller itself include speed, robustness, memory needs, physical dimensions, and cost. We are concerned in this study with performance factors which result from tuning decisions. We attempt to optimize system performance in relation to these criteria, or at least to satisfy the more important ones. The methodology employed assumes that the controllable factors and the performance variable are measured by continuous numeric values.

Quadratic Response Surfaces

In theory neural network systems consider all computable functions compatible with their

$$\eta = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_{ii}^2 + \sum_{i,j:i < j} \beta_{ij} x_i x_j \quad (1)$$

architecture. In contrast, response surface methodology considers only quadratic functions. Although in using response surface methodology we reduce the quantity of alternative functions considered, we hope to take advantage of the well-studied nature of quadratic functions (based on quadratic “forms”) to improve the quality of the analysis. The rationale for using quadratic functions as approximations for unspecified functions is the Taylor series expansion of the function η about the point $x_1 = x_2 = x_3 = \dots = x_k = 0$. The assumed quadratic function is expressed algebraically in equation (1).¹⁰

The estimated quadratic function is expressed matrixly in equation (2). \mathbf{b} and \mathbf{x} are vectors with typical elements b_i and x_i ; \mathbf{B} is a symmetric matrix with typical elements $b_{ij}/2$. Each b in (2) is an estimate of the corresponding β in (1). The right side of equations (1) and (2) are called quadratic forms.

$$\hat{y} = b_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} \quad (2)$$

Experimental Design

Experimental design is a time honored methodology cultivated by theoretical and applied statisticians.* One of the achievements of experimental design methods is economy of sample size for multiple factor phenomena. This economy is of great interest to the tuning of fuzzy controllers, if it can be achieved without sacrificing prediction precision.

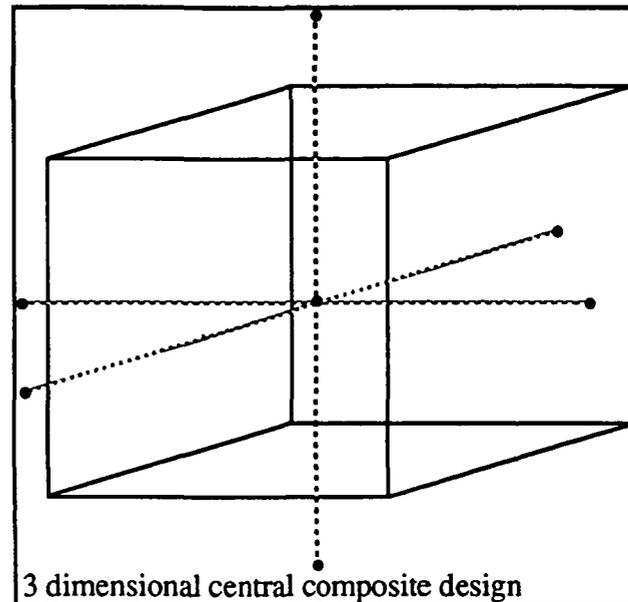
Perhaps the most naive design of a multifactor system is called “one-at-a-time”: each factor’s value is changed one at a time (holding the levels of all other factors constant). In contrast, “full factorial” experimental designs interweave the changes of all factors; if there are k factors and each factor is to be sampled at n levels, then a full factorial experiment requires a sample size of n^k . Full factorial designs are great improvements over the one-at-a-time method in reducing sample size. Even so, in practice n^k can rapidly escalate into a large quantity; the number of factors and levels are usually severely limited.

“Partial factorial” designs trim the sample size of full factorials by upwards of 50% by eliminating carefully selected sample points. But inevitably

*“Control” treatments and randomization of “subjects” to treatments are among the key tenets of experimental design. Many of the desiderata of experimental design are shaded by the stochastic nature of the modelled system. In the present paper we downplay randomness and concentrate on the economical detection of dominant patterns.

partial factorial designs are unable to estimate all terms of the quadratic form; coefficients in pairs of terms are not distinguishable, but are "confounded."

To model a quadratic function every factor must be sampled by at least $n=3$ levels in a full factorial design. But "central composite" designs (ccd) are based on an augmented 2^k (not 3^k) full factorial design. Geometrically the 2^k full factorial design samples all of the vertices of a k -dimensional rectangular solid. In addition to sampling points at the vertices, in the ccd the center point and "axial" points are sampled, thus augmenting the full factorial design. Axial points are found along the orthogonal lines which intersect at the centroid of the rectangular solid. With the ccd we consider, one axial point is selected outside of each face of the rectangular solid. That is, two axial points are selected along each axial line. One point is sampled where all the axial lines coincide in the center of the solid.



In a 3^k full factorial design each factor is tested at three levels and in all combinations. In the ccd each factor is tested at five levels but not all factors are combined. In a 3^k full factorial with $k=3$, the sample size is $3^3=27$. In the ccd the total number of sample points is $2^k + 2k + 1$. With $k=3$, the sample size is only 15. And the relative advantage of the ccd improves as k increases.

Inverted Pendulum Example

Control of an inverted pendulum has become a common testbed among fuzzy researchers. A cart on a straight track is pushed according to the controller's instructions with varying degrees of force. A sensor detects the angle φ in radians that the pole makes with the vertical plumb line. The angular velocity of the pole angle is computed approximately based on the change in φ . Another sensor determines the cart's position ϕ relative to its starting position. A pushing force f is applied to the cart. φ , ϕ and f can take on positive and negative values.

The fuzzy controller was constructed with eleven sub-rules containing φ and $\dot{\varphi}$ as antecedent variables and with f as the consequent variable. Five terms were defined for each variable: NEGATIVE, SMALL NEGATIVE, ZERO, SMALL POSITIVE, AND POSITIVE. All fuzzy (linguistic) variables were represented as symmetrical trapezoids. The scale of the all trapezoids on each universe of

discourse were uniform relative to one another; but, the scales on different universes were independent.

Tuning of this controller was done by calibrating the scale of the axes of the three universes: φ , $\dot{\varphi}$, and f . The criterion variable was the absolute value of the cart position $|\phi|$ at the end of an experimental trial of 5 seconds. If the pole fell during the trial, the ending cart position was a very large number. The farther the cart moved away from its starting position, the less likely that it was in an equilibrium state. Ending cart positions near 0 were considered ideal.

The steps below are referred to as "response surface methodology." RSM is a branch of experimental design which searches for the optimal values of the explanatory variables: values of each factor which together produce the best (maximum or minimum) value of the criterion variable.

Step 1 Select the initial set of sample points

The triads (φ , $\dot{\varphi}$, f) for each of the 15 sample points in this study were set according to the central composite design. Each point corresponds to specifying the scale* values of the 3 variables: pole angle in radians, pole angular velocity in radians per second, and pushing force in newtons. As a practical matter the factor levels were standardized so that the vertices values were expressed as +1 and -1; the centroid value is (0, 0, 0). The standardized values of the axial points were selected to produce an orthogonal design matrix, $\pm\alpha=1.21541$. The initial range for the variables were as follows. Pole angle: 0...0.15625. Angular velocity: 0...2. Pushing force: 0...8. The smaller the scale for φ and $\dot{\varphi}$, the more sensitive is the input sensing of the controller; and the larger the scale for f , the stronger the output of the controller.

Step 2 Perform the experiment

We ran the controller with the simulated** cart-pole "plant" 15 different times. Every experiment was run with a starting angle $\varphi = 0.01$, and all other transient variables set to 0. We recorded the absolute value of the final cart position for each experiment. Time was incremented every 0.02 seconds, cart mass was 1.0 Kg, pole mass was 0.1 Kg, pole length was 0.5 m, and acceleration due to gravity was 9.8 m/s².

*Each (continuous) variable's axis was discretized at 17 equally spaced values, nominally -8, -7, ..., -1, 0, 1, ..., 8. The "scale" value is the distance between adjacent discretization points.

**The simulation was based on equations provided by Hamid Berenji. The differential equations can be found in Berenji's article cited in the references. The simulation assumed a frictionless plant.

Step 3 Fit a quadratic form to the experimental results

We used the least squares criterion to fit a regression surface. In the case of $k=3$, there are 10 regression coefficients to be estimated. The form of the fit regression function is expressed matrically in equation (2).

Step 4 Find the "stationary point" of the quadratic form

The stationary point is $\mathbf{x}_0 = -\mathbf{B}^{-1}\mathbf{b}/2$. The stationary point may be inside or outside of the convex envelope enclosing the experimental region. The stationary point may correspond to a maximum, a minimum, or to a saddle point.

In the example reported on here, the stationary point was typically a saddle point. A typical value of \mathbf{x}_0 is $(-0.129, -0.503, -0.0822)$ and was near the centroid.

Step 5 Reduce the response surface to canonical form

"Canonical analysis" is used to reduce the response surface to canonical form by determining the eigenstructure of the matrix \mathbf{B} . If all of the eigenvalues (characteristic roots) are positive, the stationary point indicates a minimum; if all are negative, the stationary point indicates a maximum; otherwise, a saddle point has been found. A typical case produced eigenvalues 9.71706, -4.90507, and -6.70762. This suggests a saddle point.

The stationary point and the response surface can be interpreted in terms of its canonical form. If, for example, we are seeking a minimum and the stationary point indicates a minimum and the stationary point is inside the experimental region, interpretation of the results are relatively straightforward. If, on the other hand, we are seeking a minimum and the stationary point does not indicate a minimum or the stationary point is outside the experimental region, interpretation of the results is more complex.

The signs and magnitudes of the eigenvalues of matrix \mathbf{B} provide considerable information about the region of the surface in the vicinity of the stationary point. This information is oriented not to the original reference axes, but to the axes described by the eigenvectors. Each eigenvalue has a corresponding eigenvector. If an eigenvalue is negative, then movement in either direction along the corresponding defined axis, produces a decrease in the value of the response variable. An opposite, analogous interpretation applies for a positive eigenvalue. If the magnitude of the eigenvalue is large relative to other eigenvalues, then movement away from the stationary point along the corresponding axis has greater sensitivity than movement along other axes. If one of the eigenvalues is very close to 0, then the stationary point may resemble

more of a near-stationary ridge. This may afford the decision maker considerable latitude in controller tuning.

Although the experiment is supposed to be designed so that the stationary point is inside the experimental region or at least close by, the system may not behave as expected. Evaluation of the eigenstructure may provide import clues regarding the location of additional experimentation.

Step 6 Use ridge analysis to further interpret the response surface

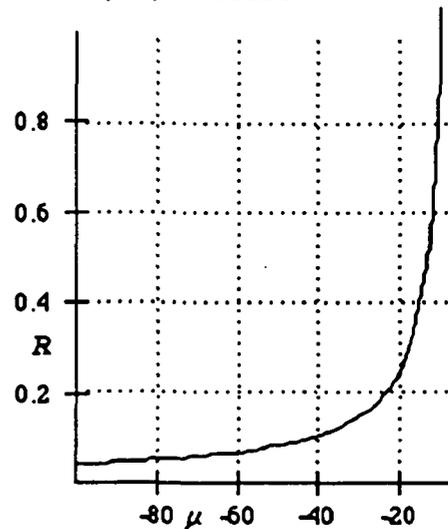
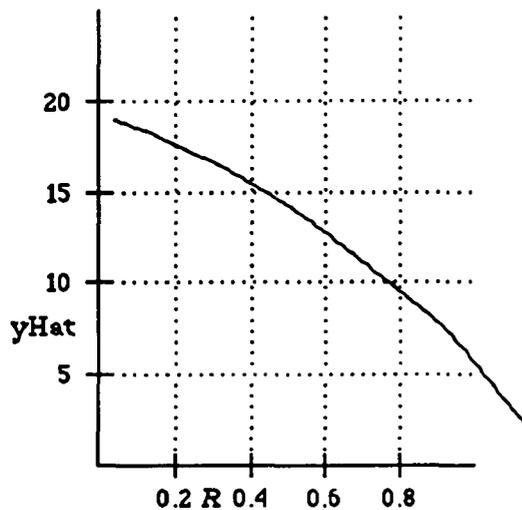
Often analysis of the canonical form suggests that additional experimentation is needed because, for example, the stationary point appears to be a saddle point.* If additional experimentation is indicated, a "ridge analysis" may suggest the direction in which to move in order to select future sampling points. Myers suggests references by Hoerl¹¹ and Draper¹².

To perform a ridge analysis is to perform a constrained optimization; optimize the quadratic function restricting the solutions to being on (hyper)spheres of varying radii. The spheres are centered at the stationary point. To minimize the response, then for each different radius, plot the values of \hat{y} against R . Also plot the values of the x which correspond to each radius. For example, to minimize when the stationary point suggests a saddle point, move in the direction of decreasing response along a "ridge" defined by the series of radii.

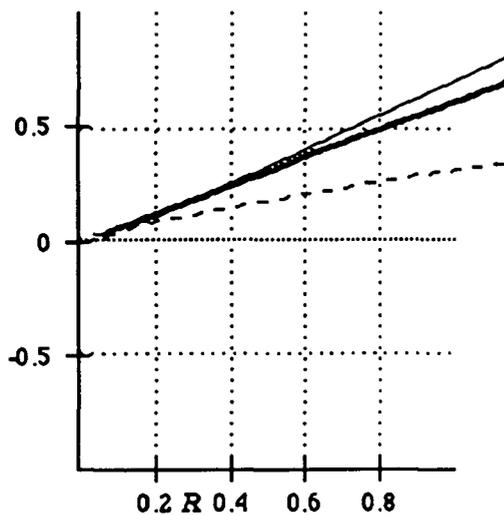
The ridge analysis can be modeled using the method of Lagrangian multipliers. The constraint can be expressed $x'x - R^2 = 0$. The function $F = \hat{y} - \mu(x'x - R^2)$ can be optimized. In practice, the plotting of the solutions of this optimization is a parametric plot. \hat{y} is a function of x , as is R ; in addition R is constrained by (is a function of) μ , $R(\mu)$. Each value of μ determines a radius R , and the optimal value of \hat{y} is determined by that radius. This can be done by selecting values of μ first, then determining the values of $x_j = b_j/2\mu$ which follows from requiring the partial derivatives in the Lagrangian method to equal 0. The range of possible values of μ is determined by whether you wish to maximize or minimize. For maximization, the values of μ must be larger than the largest eigenvalue; for minimization, the values of μ must be smaller than the smallest eigenvalue. With the eigenvalues 9.71706, -4.90507, and -6.70762, μ must be less than -6.70762. The plots below show that the predicted value of the

*A saddle point may be an indication of multiple extrema; such a phenomenon is not consistent with models of the quadratic form.

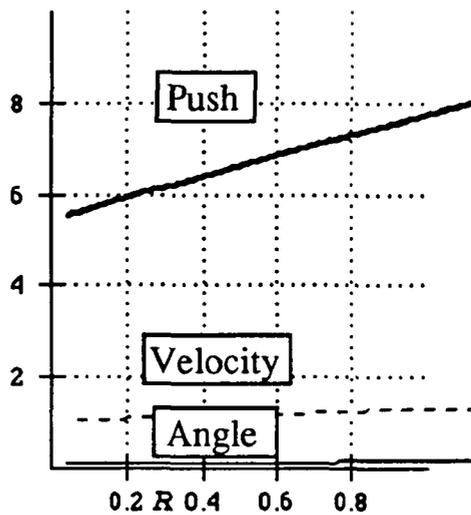
response surface, \hat{y} , reduces relatively steady as the radii, R , increase.*



In relation to R , the plots of the variables angle, velocity, and push show that push, velocity, and push increase slightly. By telescoping in to get more accuracy, the value of the pole angle is found to be between 0.08 and 0.13 radians; angular velocity is between 1 and 1.28 radians per second; and push force is between 5.55 and 7.8 newtons. These ranges provide a narrower range within which to calibrate the three scales.



Standardized angle, velocity, push

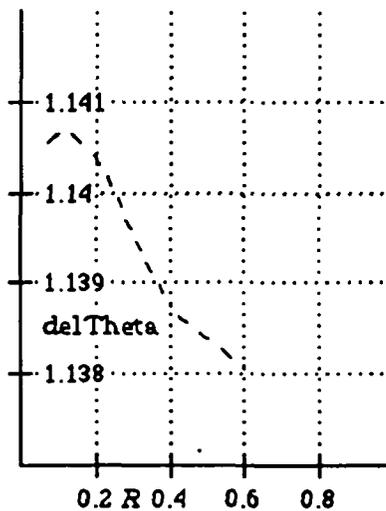
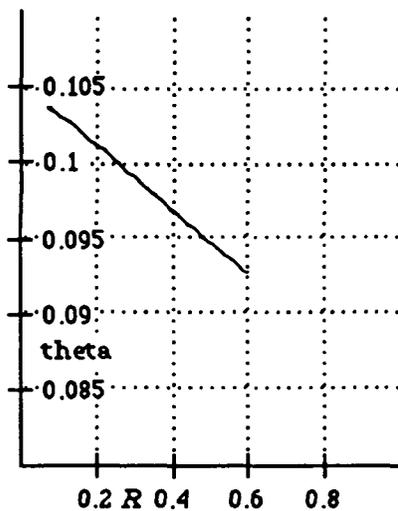
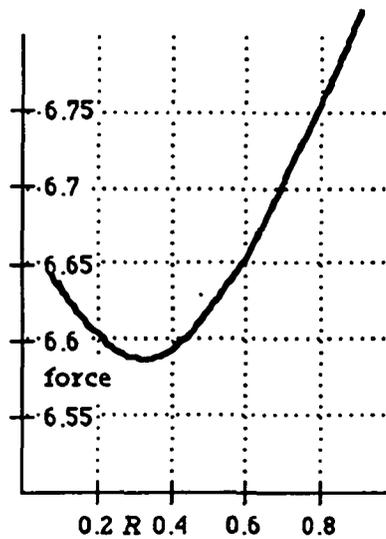
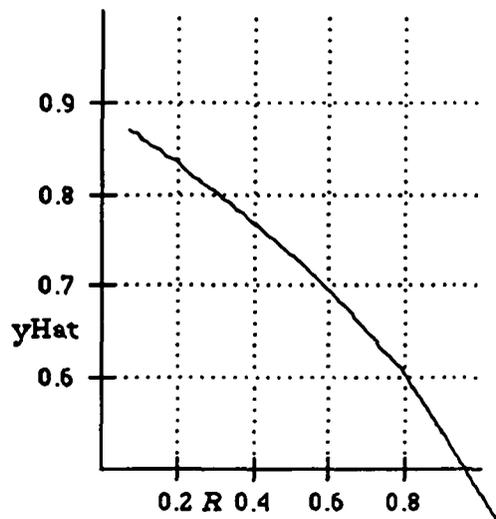


Angle, velocity, push variables

The controller experiments were performed again with the variables limited to these narrower limits. The results of the repeat experiment suggest the controller is able to balance the cart-pole system; the final position of the cart in

*In fact the plot shows \hat{y} becoming negative, which is impossible for the true response value, since only absolute values are considered. But this anomaly is a result of the approximate nature of the fit of the quadratic form, and is not critical.

half of the trials was less than 0.8 m from its starting position and always between 0.29 and 1.42 m. Below we show plots of each key variable relative to the radii R.



By applying a similar analysis to alternative criteria, a fuller assessment of the controller performance can be had. Using plots similar to those for cart position, the alternative criteria's optima can be viewed in relation to the analysis demonstrated here.

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