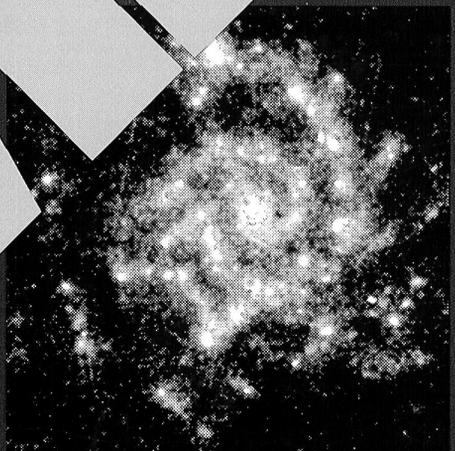


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(NASA-CR-193202) WORKSHOP ON FUZZY
CONTROL SYSTEMS AND SPACE STATION
APPLICATIONS (McDonnell-Douglas
Space Systems Co.) 447 p

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*McDonnell Douglas Space Systems Company
Space Station Division*

MCDONNELL DOUGLAS

NOVEMBER 1990

MDC H6821

**WORKSHOP ON FUZZY CONTROL
SYSTEMS AND SPACE STATION
APPLICATIONS
14-15 NOVEMBER**

CT79

COMPILED BY: E.K. AISAWA, SE&I A&R (HB)
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MDSSC-SSD

APPROVED BY: *Robert A Brown*
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APPROVED BY: *D.C. Wensley*
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MDSSC-SSD

McDonnell Douglas Space Systems Company
5301 Bolsa Avenue, Huntington Beach, CA 92647-2048 (714) 896-3311

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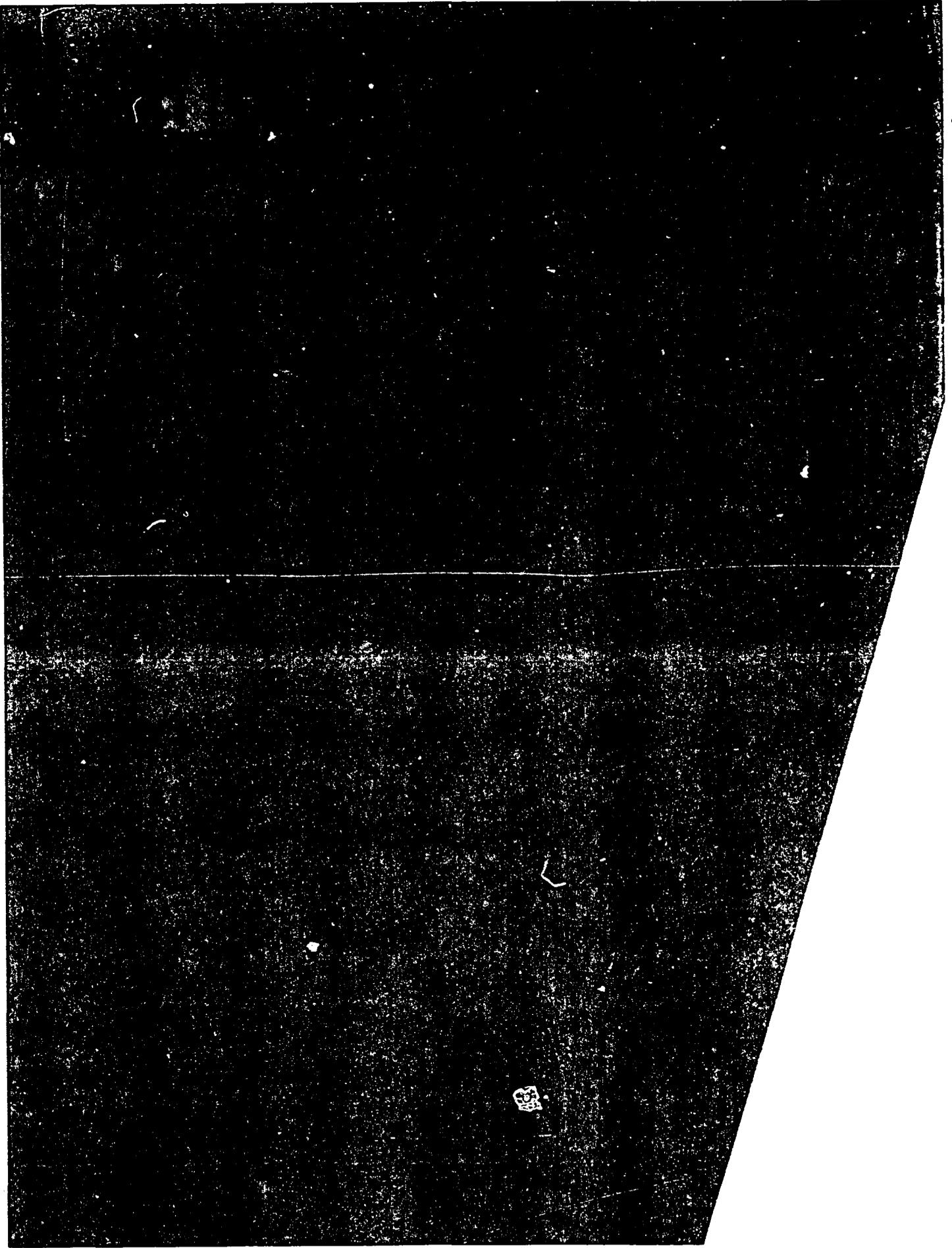
INTRODUCTION

The Workshop on Fuzzy Control Systems and Space Station Applications was held on November 14 and 15, 1990 in room 209A of the McDonnell Douglas Space Systems Company (MDSSC) Space Station Building 17 in Huntington Beach, California. The workshop was co-sponsored by MDSSC-SSD and NASA Ames Research Center. The participants included representatives from academia, industry and government.

These proceedings present the material discussed in an order which closely approximates the actual sequence of the workshop. In addition, these proceedings contain papers by Dr. Bart Kosko on Fuzzy Logic and concluding remarks from Mark Gersh, Bob Brown, Dennis Lawler, Bob Lea and Hamid Berenji.

This document is organized in the following order:

1. Biographies of the fuzzy logic control panel;
2. Opening remarks from Dave Wensley;
3. Presentations by Mark Gersh, Masaki Togai, Michio Sugeno, Jack Aldridge, Hamid Berenji and Xiwen Ma;
4. Papers on Fuzzy Logic theory by Bart Kosko;
5. All MDSSC-SSD presentations including question and answer sessions; and
6. Concluding remarks.



Workshop on Fuzzy Control Systems and Space Station Applications

mit

Biographical Information

Hamid R. Berenji, Ph.D., received his B.S. degree from the University of Science and Technology, Iran and M.S. and Ph.D. degrees from the University of Southern California in 1980 and 1986 in Systems Engineering. From 1984 to 1986, he was an instructor at University of Southern California, Los Angeles. Since 1986, he has worked in the Artificial Intelligence Research Branch of NASA Ames Research Center in Moffett Field, California. He is currently a senior research scientist with Sterling Federal Systems and a principal investigator of the research project on intelligent control. He is a member of the Operation Research Society of America (ORSA), Operation Research Honor Society (Omega Rho), American Association for Artificial Intelligence (AAAI), and Institute of Electrical and Electronic Engineers (IEEE). He is a program committee member of several conferences such as the 1991 IEEE International Symposium on Intelligent Control and is the chairman and the organizer of the intelligent control session at the 1990 Control and Decision Conference in Hawaii. His current research is focused on intelligent control as it relates to neural networks, approximate reasoning and fuzzy control, and uncertainty management in AI.

Bart Kosko, Ph.D., is an Assistant Professor, of Electrical Engineering at the University of Southern California. He is best known for his

fundamental contributions to the field of Fuzzy Estimation Theory. He is Managing Editor of the Springer-Verlag monograph series, Lecture Notes in Neural Computing, and is Associate Editor of Neural Networks, IEEE Transactions on Neural Networks, Journal of Mathematical Biology, and Lecture Notes in Biomathematics.

A much sought after lecturer, Dr. Kosko is an elected governor of the International Neural Network Society and a USC Shell Oil Faculty Fellow. He was program co-chairman of the Summer 1990 International Joint Conference on Neural Networks and Program Chairman of the 1990 International Fuzzy Logic and Neural Networks Conference in Iizuka, Japan. He was Program Chairman of the 1988 IEEE International Conference on Neural Networks (ICNN-88) and Program and Organizing Chairman of the first IEEE ICNN-87.

Dr. Kosko received his master's degree in applied mathematics from the University of California, San Diego and his Ph.D. from the University of California, Irvine.

Dennis Lawler received his B.S. degree in Mathematical Physics from State University of New York in 1980, B.S. degree in Computer Science at the University of Houston and is presently pursuing an advanced degree in Mathematics. Mr. Lawler's work experience in the area of Advanced Automation began

at McDonnell Douglas Corporation where he was a major contributor to the Space Station Freedom advanced automation concepts. His speciality is model based reasoning and as Functional Area Manager for Advanced Automation he has been instrumental in the development of key advanced automation applications including the Thermal Advanced Automation Project, Failure Environment Analysis Tool, Automated Network Monitoring, Fault Analysis Propulsion System and the development of Advanced Automation Methodology for the Space Station.

Robert Lea, Ph.D. received his B.S. and M.S. degrees in Mathematics from Louisiana State University in 1960 and 1962 respectively, and his Ph.D. from the University of Houston in 1972. He joined NASA/JSC in 1962 and has contributed to all of the major space programs. His association with fuzzy logic began in 1985. During this time he has authored and co-authored over 30 papers in this field. He was instrumental in organizing the first and second International Workshops on Neural Networks, and Fuzzy Logic held at the JSC in 1988 and 1990 and the joint video demonstration sessions between Kyushu Institute and JSC, as part of the IFSA Workshop on Fuzzy Logic and Neural Networks held in 1990. He frequently participates in technical conferences and symposiums as an invited speaker or organizer of special sessions. Dr. Lea's specific interests have recently been in the development and application of new technology such as fuzzy logic, neural networks, and Dempster Shafer theory.

Michio Sugeno, Ph.D. is a professor in the Department of Systems Sciences at the Tokyo Institute of Technology. He has authored more than 50 papers and several books. His award winning book on Fuzzy Control was a best seller in Japan. He is an advisor to the Laboratory for International Fuzzy Engineering Research. He is an advisor to the Science and Technology Agency under the Office of the Prime Minister. His current work includes the development of the fuzzy control systems for an unmanned helicopter.

Masaki Togai, Ph.D. received his M.S. and Ph.D. degrees in electrical engineering in 1977 and 1982, respectively, from Duke University. He is president and chief executive officer of Togai InfraLogic, Inc. Dr. Togai has spent the last 10 years leading fuzzy logic development groups at Duke University, AT&T Bell Laboratories, and Rockwell International. He is best known in the industry for developing the world's first fuzzy microchip for real-time approximate reasoning. He is a member of the board of directors of the North American Fuzzy Information Processing Society (NAFIPS), a member of the American Association of Artificial Intelligence, IEEE, International Fuzzy Systems Association (IFSA), and Sigma Xi. In addition, Dr. Togai is the editor-in-chief of the Japan Artificial Intelligence Newsletter; and is an associate editor for the Information Sciences, and the Journal of Approximate Reasoning. He is an author of two books: "Intelligent Robotic Systems", and "Approximate Reasoning in Expert Systems". He has authored and coauthored more that 30 papers.

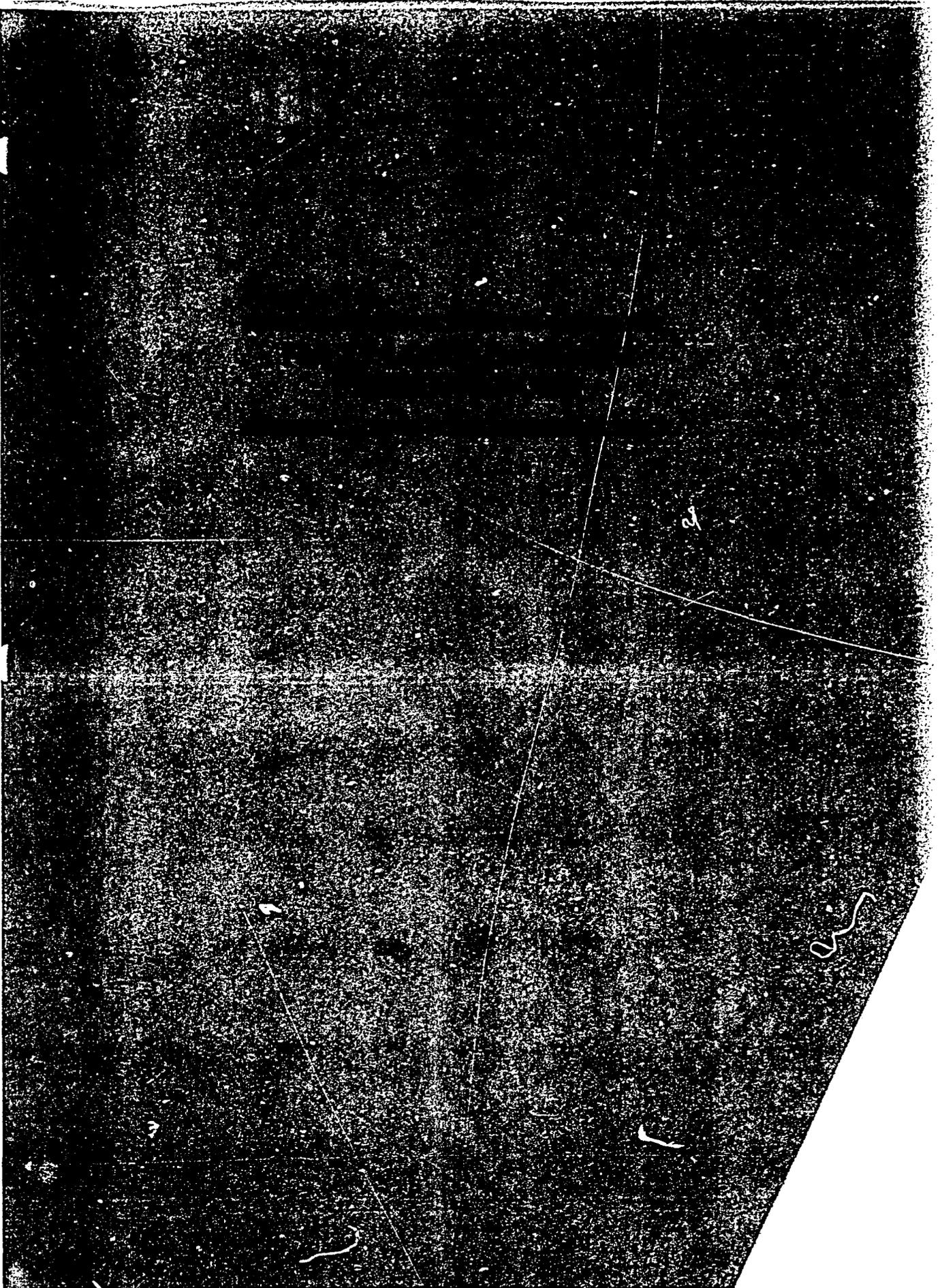
L. A. Zadeh, Ph.D., received his B.S.E.E. from the University of Teheran, Iran, in 1942; M.S.E.E. from the Massachusetts Institute of Technology (MIT), Cambridge, Massachusetts, in 1946; and Ph.D. from Columbia University, New York, New York, in 1949. He has been a professor of electrical engineering and computer sciences at the University of California, Berkeley, California, since 1959. Dr. Zadeh was an instructor in electrical engineering at Columbia University from 1946 to 1950, assistant professor from 1950 to 1953, associate professor from 1953 to 1957, and professor from 1957 to 1959. He has been a member of the Institute for Advanced Study, Princeton, New Jersey, since 1956; was a visiting professor of electrical engineering at MIT in 1962 and 1968; was a visiting scientist at the IBM Research Laboratory in San Jose, California, in 1968, 1973, and 1977; and was a visiting scholar at the Artificial Intelligence Center, SRI International, in Menlo Park, California, in 1981.

Wei Xu, received his B.S. in Management Sciences in 1983 and his M.S. in Statistics in 1985, from Beijing University. From 1987 to 1988 he was involved in research and development work on Fuzzy theory and applications. In 1988 Mr. Xu formed APT Instruments in Japan, where the first programmable fuzzy controller was developed. APT moved its operations to the United States in 1989 and went on to develop the first multi-tasking fuzzy programmable controller. Mr. Xu is a reviewer of "Fuzzy Sets and Systems".

OPENING REMARKS - Dave Wensley

Dave Wensley, Vice President - Deputy General Manager Strategic Development for McDonnell Douglas Space Systems Company-Space Station Division (MDSSC-SSD) welcomed the participants of the Workshop on Fuzzy Control Systems and Space Station Applications with introductory remarks expressing hopes for a successful workshop. Mr Wensley emphasized that McDonnell Douglas is committed to the utilization of advanced technology on the Space Station Freedom Program (SSFP).

Mr. Wensley first discussed the background of the SSFP's status and development environment, including the current budget climate and the technical development . He expressed high expectations for the workshop since it provided an excellent forum for the technical interchange of ideas between the experts in the Fuzzy Logic field and engineers faced with the tasks of producing viable solutions for varied control applications operating within hostile environments.



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INCLAS

**FUZZY CONTROL/SPACE STATION
AUTOMATION**

by

**Mark Gersh
NASA Headquarters**

14 November 1990

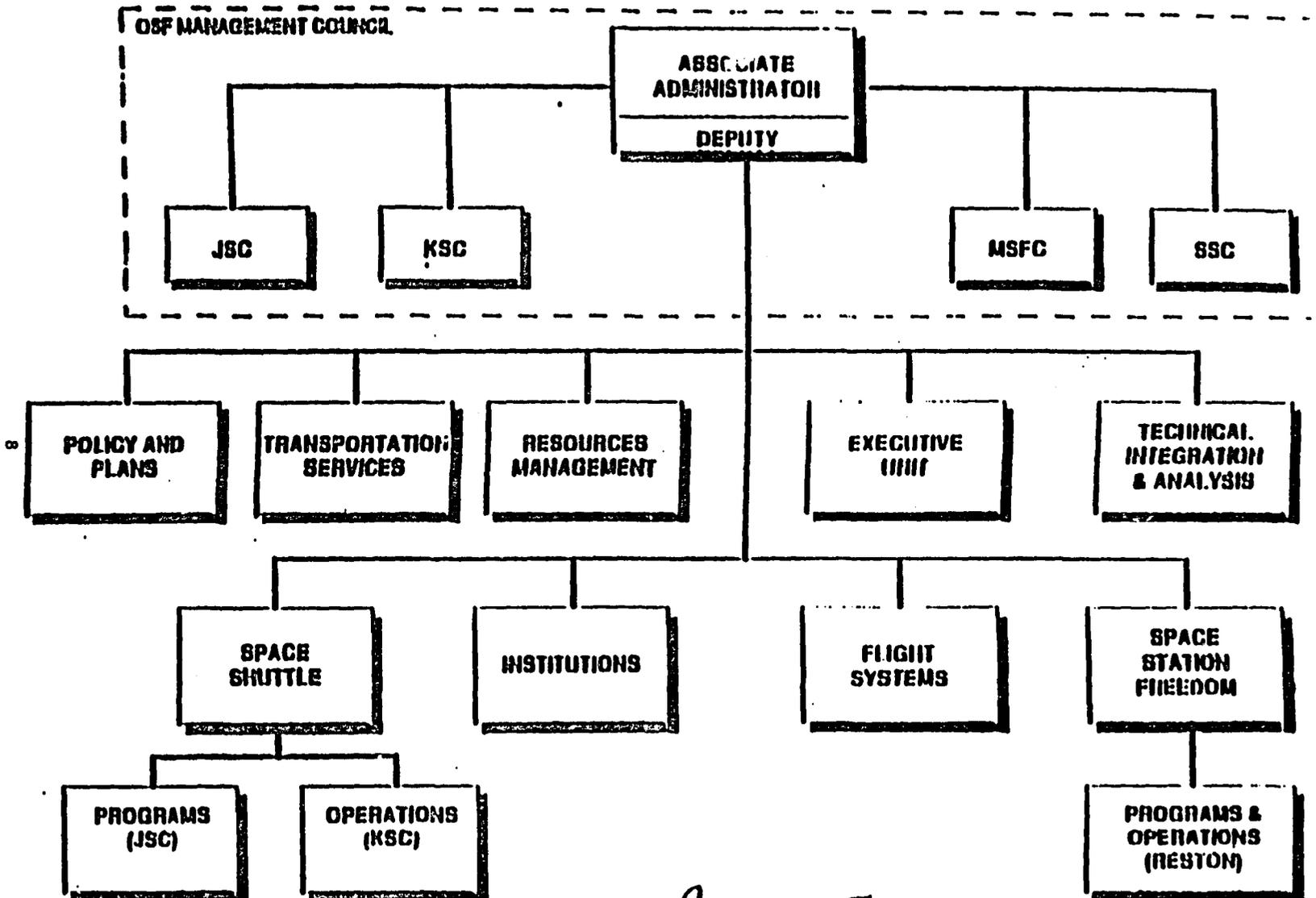
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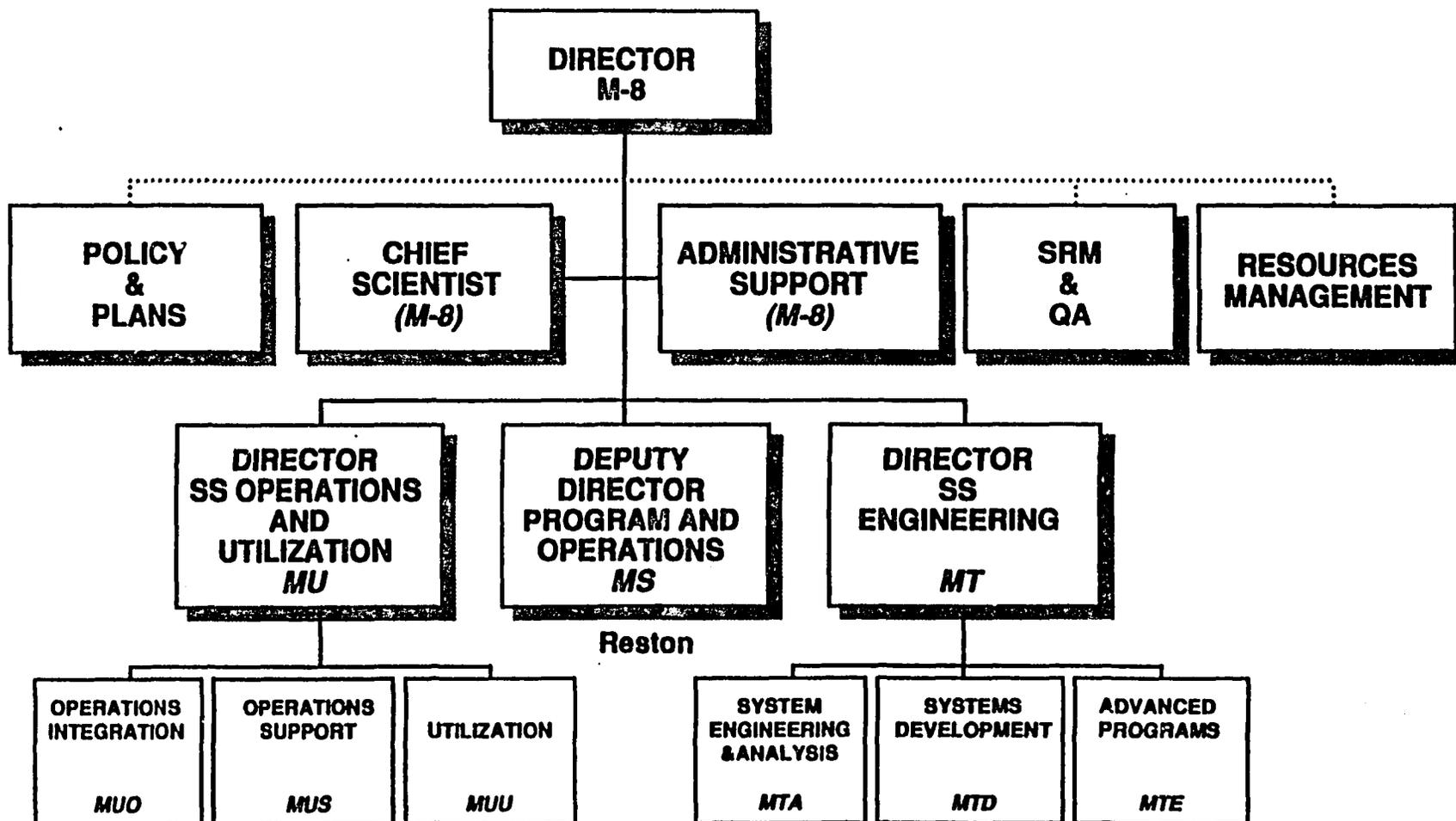
Office of Space Flight



Approved: *Richard F. ...*
 Date: Feb. 9 1969



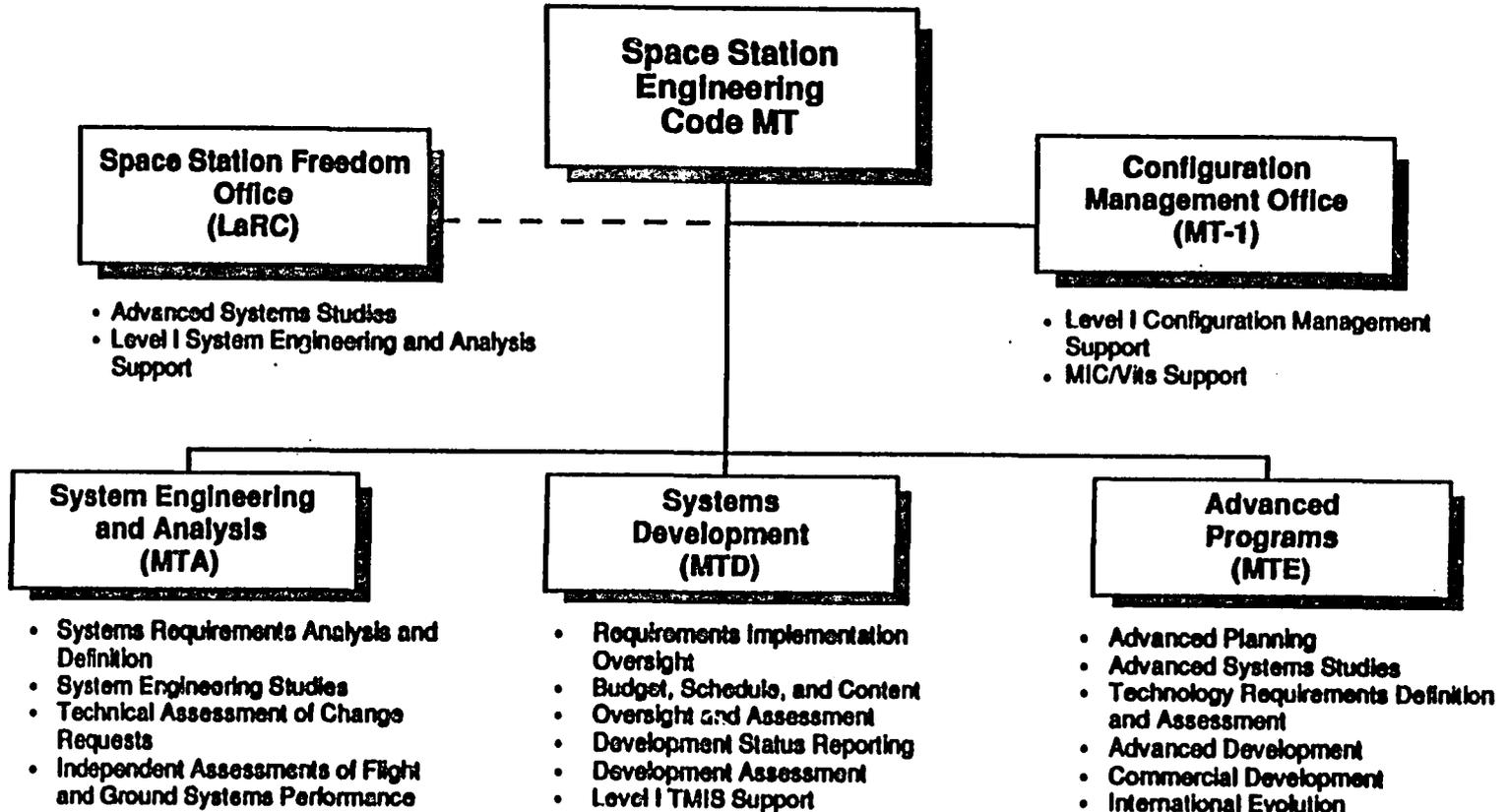
OFFICE OF SPACE FLIGHT SPACE STATION FREEDOM



Code MT Organization and Responsibilities

Space Station Engineering Organization

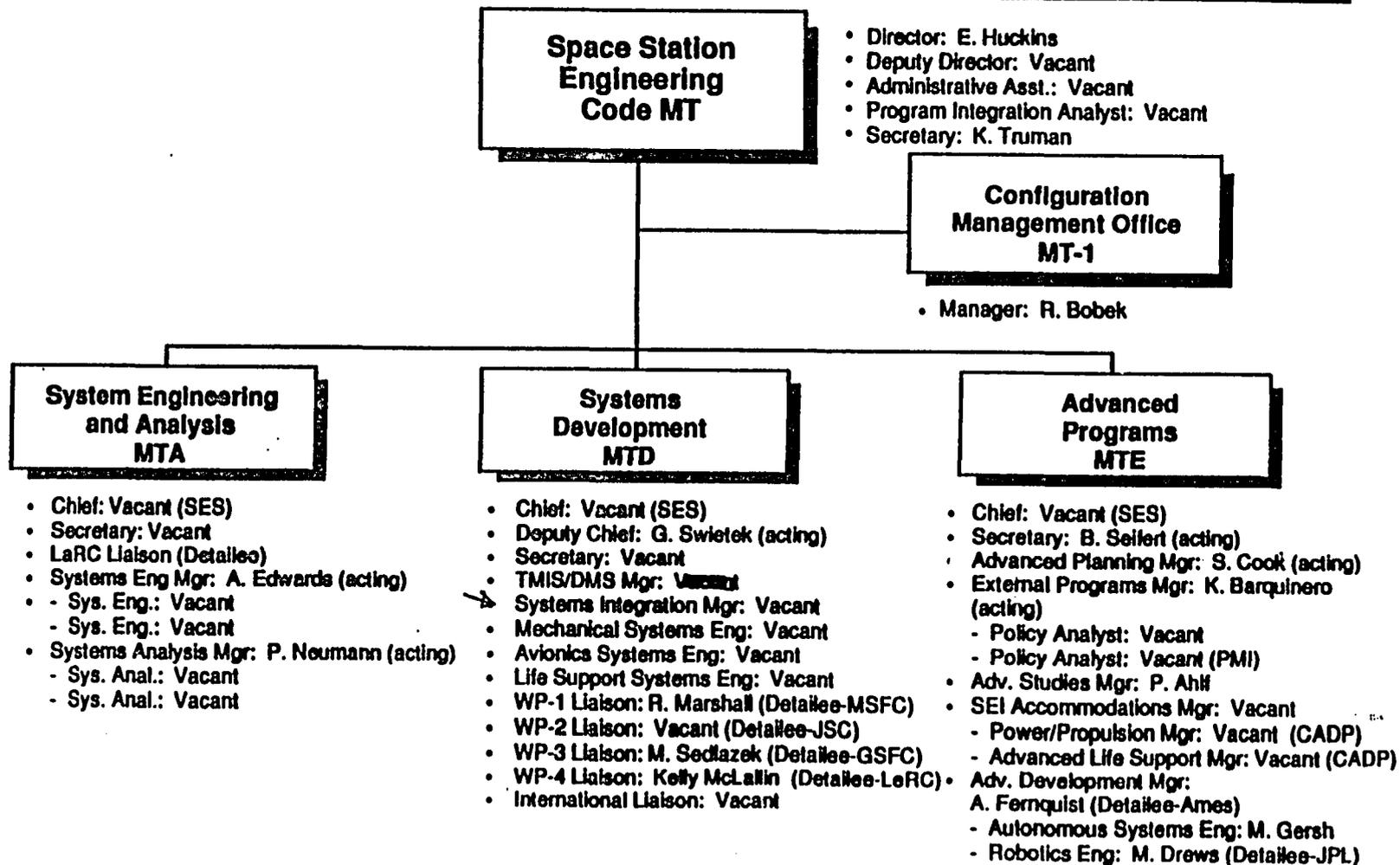
FREEDOM



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Code MT Staffing



SPACE STATION FREEDOM

Objectives



- **Provide a permanently manned presence in space**
- **Enhance capabilities for space science and applications**
- **Stimulate advanced technologies**
- **Promote international cooperation**
- **Encourage private sector participation and utilization**
- **Provide options for future endeavors in space**

SPACE STATION FREEDOM

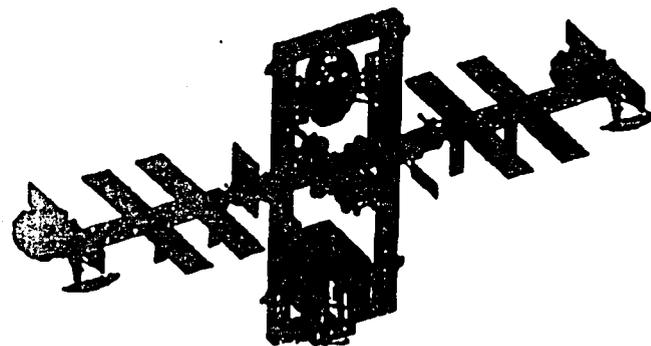
Evolution



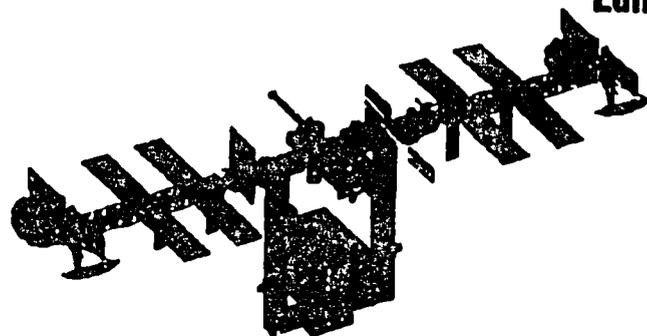
- **Freedom is a permanent facility:**
 - **Upgrades and configuration changes will take place on-orbit**

- **During the operational life of the Space Station:**
 - **National priorities will change**
 - **User needs and mission requirements will change**
 - **Technology will evolve and components will become obsolete**

SPACE STATION FREEDOM EVOLUTION FOR HUMAN EXPLORATION



Lunar & Mars Operations



Lunar Vehicle Operations



Assembly Complete

SPACE STATION FREEDOM

Factors Pointing to Automation & Robotics (A&R)



- **Space Station has a 30 year operational life**
 - **Operations costs, reliability are important concerns**
 - **Incorporation of new technology essential**

- **Crew is most scarce resource**
 - **Productivity is crucial in meeting assembly, user, and servicing requirements**

- **Evolution mission scenarios are crew-intensive**
 - **Science missions will grow and increase demand for crew time**
 - **On-orbit assembly, checkout, launch of Lunar/Mars vehicles**

SPACE STATION FREEDOM

The A&R Promise



- **Increased Mission Safety, Reliability**
 - **Manage system complexity**
 - **Trend analysis, fault detection, isolation, and reconfiguration**
 - **Reduce EVA required**

- **Increased Mission Productivity, Services**
 - **Reduce "housekeeping" overhead**
 - **Reduce experiment overhead**

- **Increased Probability of Mission Success**
 - **Re-planning for contingencies, reactive science**

- **Reduced Operations Costs**
 - **Training, software maintenance, sustaining engineering**

SPACE STATION FREEDOM

Astronaut Office Inputs Concerning A&R



- **Regarding Advanced Automation**
 - **Simple, standardized human interface (idiot proof)**
 - **Provide flexible operations capability**
 - **User (versus technology developer) oriented**
 - **Develop and implement easier applications first**
 - **Help the user do the job easier (don't make it harder)**
 - **Include "What if?" Capability (In-line simulation)**
 - **Backup mode of operation**
 - **System must be able to explain conclusions and actions**
- **Automate tedious and repetitive tasks, time dependent tasks, calibration and alignment tasks, robotic set-up for EVA**

SPACE STATION FREEDOM

Astronaut Office Inputs Concerning A&R



- **Applications supported by crew for improving productivity:**
 - **Automated record keeping and documentation (100%)**
 - **Automated inventory management (96%)**
 - **Automated FDIR (93%)**
 - **Improved human-computer interfaces (92%)**
 - **Robotic construction (92%)**
 - **Exception reporting and alarm filtering (88%)**
 - **External camera and light pointing (87%)**
 - **Robotic external repairs (85%)**
 - **Automated trend analysis (incipient failure detection) (85%)**
 - **Checklist automation (85%)**

SPACE STATION FREEDOM

Astronaut Office Inputs Concerning A&R



- **Applications supported by crew for improving productivity:**
 - **Systems Monitoring and Control (82%)**
 - **EVA retriever robotics (81%)**
 - **Payload-specific automation (79%)**
 - **On-board training systems (72%)**
 - **Internal camera and lighting pointing (58%)**
 - **Speech Recognition (56%)**
 - **Speech Synthesis (54%)**
 - **On-board scheduling/re-scheduling capability (52%)**
 - **IVA rack robot (50%)**
 - **IVA housekeeping robot (46%)**

SPACE STATION FREEDOM

Advanced Development Program



- **Objectives**
 - **Enhanced baseline Space Station Freedom capabilities**
 - **Improve productivity and reliability**
 - **Reduce operations costs**
 - **Prevent technological obsolescence**
 - **Enable Space Station Freedom evolution**

- **Products**
 - **"Engineering" fidelity demonstrations, evaluations**
 - **Detailed requirements, performance specifications**
 - **Mature technology, tools, applications**

SPACE STATION FREEDOM

Flight System Automation and Ground Operations Applications



- **Focused on Automated Status Monitoring, Fault Detection, Isolation, and Recovery (FDIR) using Knowledge-Based System (KBS) techniques**
- **Understand design accommodations ("hooks and scars")**
 - **Instrumentation, control redundancy, interfaces**
- **Identify KBS implementation issues**
 - **Integration with conventional techniques**
 - **Processing, data storage, communication requirements**
 - **Software development, testing, maintenance**
 - **Boundaries of KBS technology (performance, scale, brittleness)**
- **Applications under development for Thermal, Power, Life Support, Data Management, Mission Control**

SPACE STATION FREEDOM

Transition Definition Program Advanced Development - FY 1990

FREEDOM



- **Flight Systems and Ground Operations Automation Tasks**
 - **Focused on automated status monitoring, fault detection, isolation, and recovery (FDIR) using Knowledge-Based System (KBS) techniques**
 - **FDIR KBS applications under development for the Thermal Control System, Power Management and Distribution/Control Systems, Environmental Control and Life Support System, Data Management System, Operations Management System, Mission Control Center (MCC), and the Space Station Control Center (SSCC)**
 - **MCC applications were jointly developed with OAST and OSF and have supported STS-26, STS-29, STS-30, STS-28, STS-34 and STS-32; all will be transitioned to SSCC**

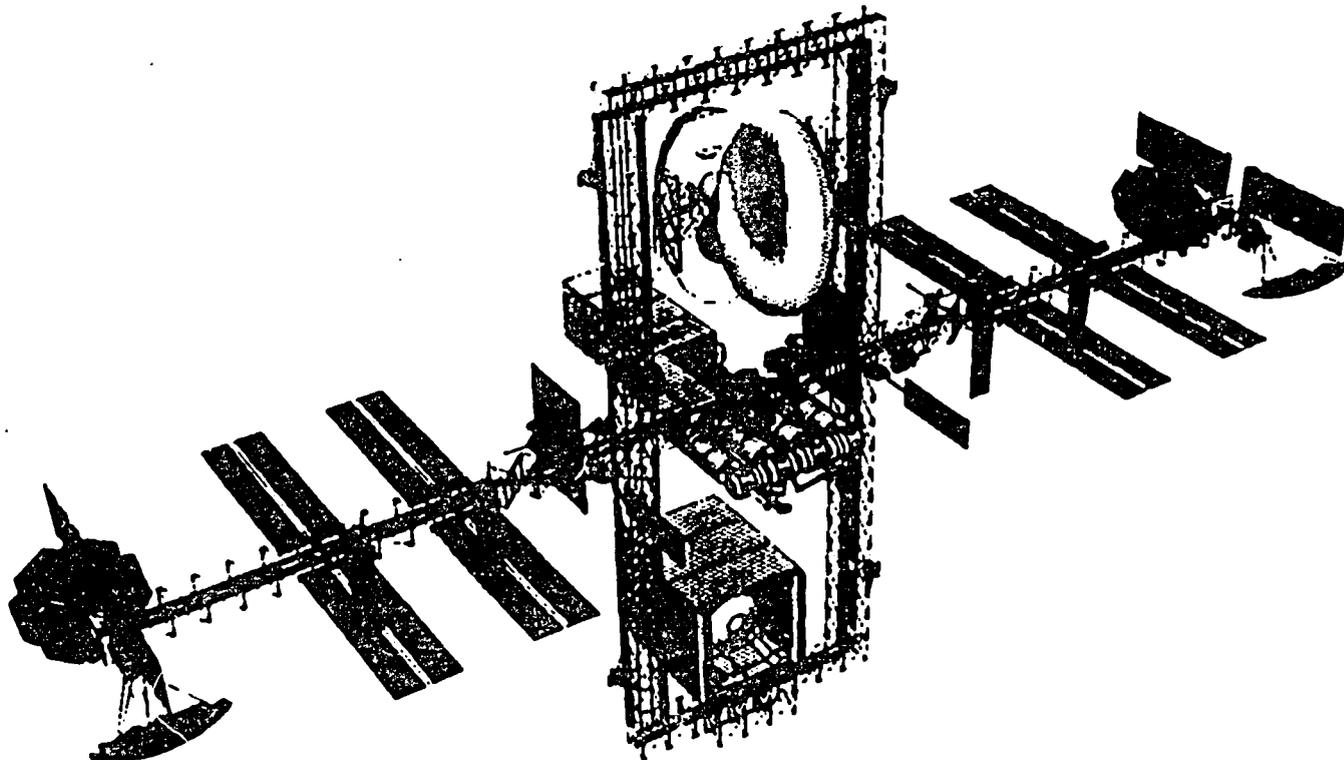
SPACE STATION FREEDOM

Advanced Automation Software Tools



- **Focused on providing programming tools to enable development of integrated KBS applications within the Software Support Environment (SSE)**
 - **KBS programming tools which produce Ada code are under development and evaluation**
- **Develop and demonstrate advanced programming tools which reduce the cost of software development and maintenance for flight and ground systems**
 - **"Programmers Assistant" that uses KBS techniques to aid programmers in Ada software re-use under evaluation**
 - **Programming environment for Intelligent Computer-Aided Training (ICAT) applications under development**

LUNAR/MARS TRANSPORTATION NODE



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SPACE STATION FREEDOM

Some General Thoughts on Technology Transition



- **In an ideal world, technology transition happens when...**
 - **The user is interested and involved in application development**
 - **The application and technology are consistent with operations concepts, procedures, and doctrine**
 - **Implementation is compatible with existing hardware and software and isolated ("firewalled") during initial evaluation period**
 - **"Success" metrics are defined early and guaranteed at some minimal level**
 - **"Bottoms up" and "top down" pressure is simultaneous and consistent**
 - **Post deployment "care and feeding" issues are addressed early**

SPACE STATION FREEDOM

Some General Thoughts on Technology Transition



- **It's not an ideal world...**
 - **Organizational structure creates, encourages insular and myopic view of technology insertion opportunities and operational realities**
 - **Ego and fear of the unknown tends to reinforce status quo**
 - **Personnel and financial resources are limiting factors**
 - **Risk and schedule pressure are harsh realities**

SPACE STATION FREEDOM

Summary

FREEDOM



- **Automation is a key element in meeting Space Station Freedom baseline and evolution requirements**
- **Automation technology is sufficiently mature to warrant early use within the Program**
- **Scope and pace of automation applications will be determined by:**
 - **Success of early testbed prototypes**
 - **Support and acceptance of managers and users**
 - **Consistent implementation methodology and tools**
- **"Technology transfer is a body contact sport." - John Muratore, JSC**
 - **People are a key factor in affecting or preventing technology transfer and utilization**

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CLAS

COMMERCIAL APPLICATIONS

by

**Masaki Togai
Togai InfraLogic, Inc**

N 93-81441

**Fuzzy Logic Workshop
14 November 1990**

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INTERNATIONAL BANK

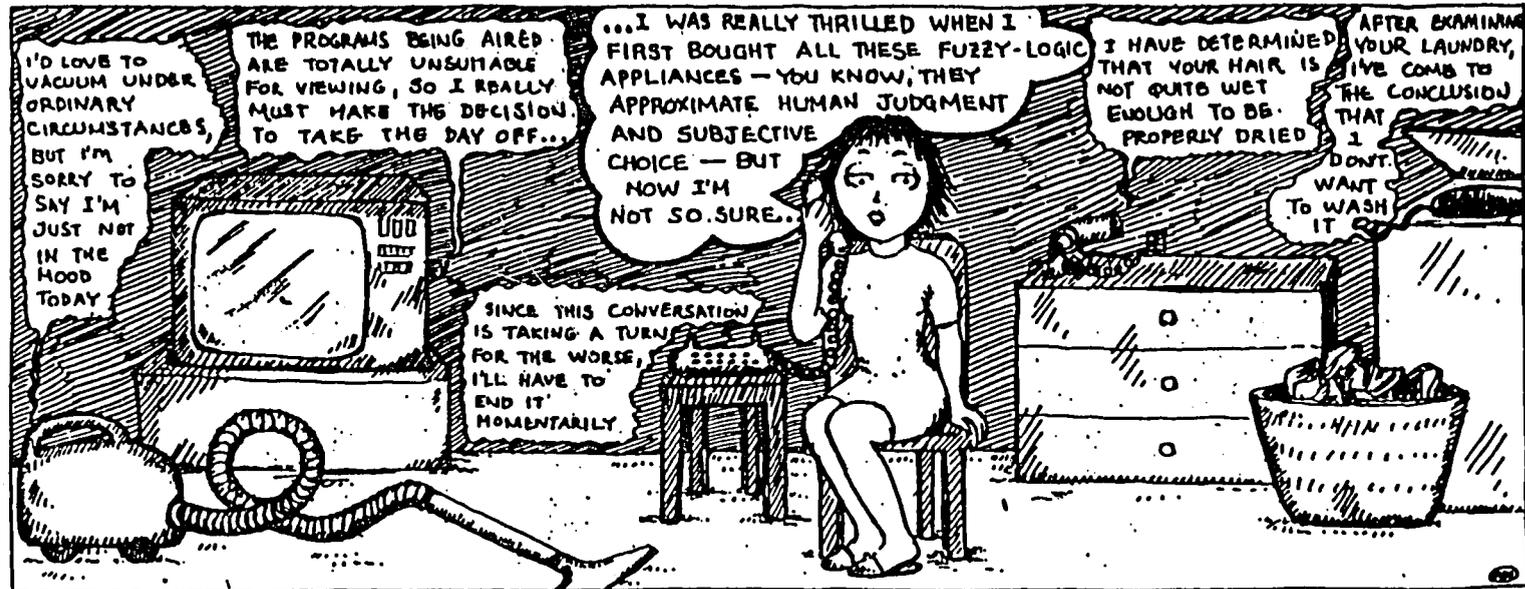
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CARTOONS

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Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

Fuzzy Logic
Is Well Suited For Handling
Non-Linear, Time-Varying, and/or Ill-Defined Problems



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

Japanese Companies Employing Fuzzy

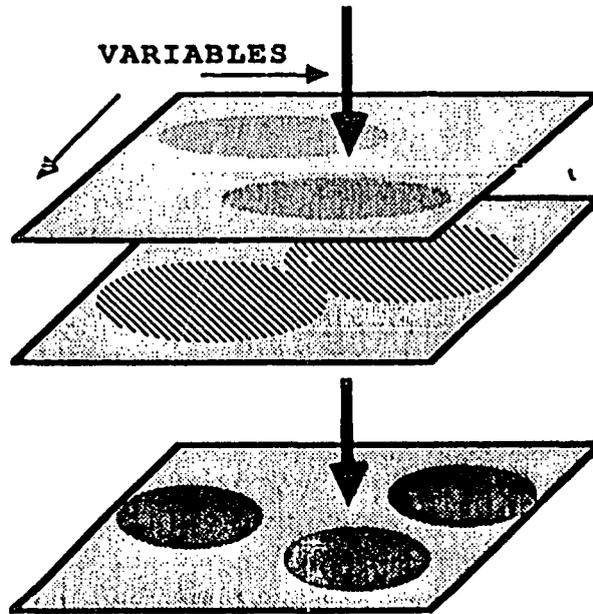
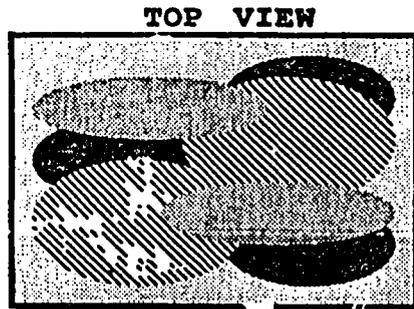
Canon	SLR camera focusing	P	Mitsubishi Heavy	Air-conditioning systems	P
	Stepper control	P	Matsushita (Panasonic)	Temperature controllers	P
Casio	Clean room temp & humidity control	P	Nissan Motor Company	Automatic transmission	D
	Gas cooling plant	P		ABS braking system	D
Daidan	Chemical mixer	P	Nuclear Power Corp	Nuclear power plant control	D
Fuji Electric	Waste burning plant	P	Omron	Factory controllers	P
	Sendai Subway control	P		Robotic controllers	D
Hitachi	Elevator control	P	Ricoh	Camera stabilizers	D
	GaAs crystal growth	D		Camera focusing	D
Idec Izumi	Automatic measuring	P	Sanyo	Voice Recognition	D
Ishida Instruments	Food processing	P	Seiko	Camera iris control	P
Leon Auto Machinery	Iron mill control	P	Subaru	Design expert system	D
Nippon Steel	Golf club selection	P	Toshiba	Automatic transmission	P
Maruman	Robotic controllers	D		Elevator control	P
Mycom	Dredging control	P	Yamaichi Securities	Product design expert system	D
Melden-sha	Machine control	P	Yokogawa Electric	Stock trading	P
	Camera focusing	D		Digital measurement systems	P
Minolta	Cement kiln control	P			
Mitsubishi Chemical	Elevator control	P			
Mitsubishi Electric	Plasma Etching	P			

P - Production
 D - Development

SUITABLE APPLICATION AREA OF FUZZY THEORY

	SUITABLE PROBLEMS		
	Man-Machine Interface Problem	Time-varying dynamics/non-linear problem	Classification problem
Problems of conventional approach	<ul style="list-style-type: none"> •Difficult to express control objectives numerically •Evaluate the control result by human feeling 	<ul style="list-style-type: none"> •Plant dynamics varies in time •Plant is non-linear <li style="text-align: center;">↓ <li style="text-align: center;">overshoot oscillation 	<ul style="list-style-type: none"> •Action to be taken is not clear •Cannot describe all solutions for possible patterns <li style="text-align: center;">↑ <li style="text-align: center;">speed/hardware limitations
Applications	<ul style="list-style-type: none"> Sendai subway Suspension control Crane control Automatic transmission 	<ul style="list-style-type: none"> Temperature control of A/C, plant, etc. Position control of a hard-disk head Auto-cruise 	<ul style="list-style-type: none"> Auto-iris/auto-focus Hand-written character recognition Automatic transmission

CHARACTERISTICS OF FUZZY CONTROL



OBJ 1

OBJ 2

OBJ N

PARALLEL/DISTRIBUTED CONTROL

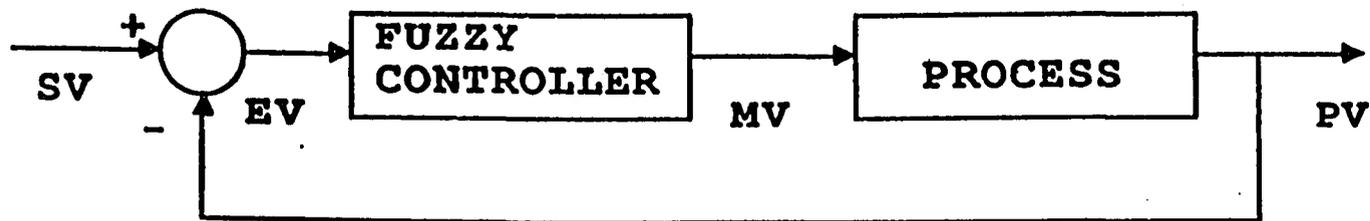
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CHARACTERISTICS OF FUZZY CONTROL

- PARALLEL/DISTRIBUTED CONTROL
- PRODUCTION RULES (IF-THEN)
 - SIMPLE KNOWLEDGE REPRESENTATION
 - MIXED PREMISE EVALUATION
 - EXCEPTION HANDLING
- QUALITATIVE EXPRESSIONS

→ IMPROVEMENT ON QUALITY & ROBUSTNESS

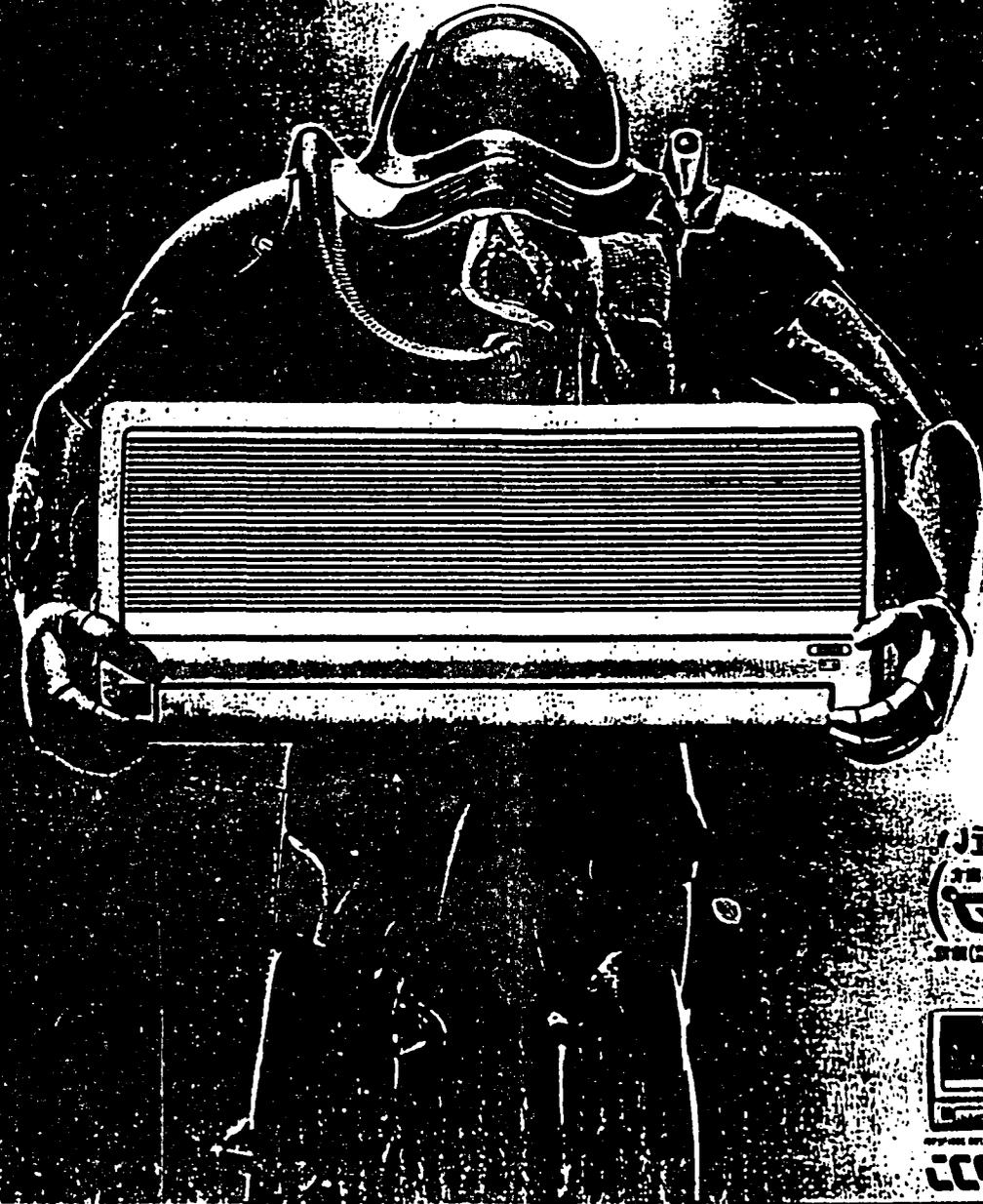
FUZZY CLOSED-LOOP CONTROLLER



8

TOGAI INFRALOGIC, INC.
KYOTO-3

三菱重工



呼吸器。世界初の
エアロネット

三菱重工
（株）三菱重工空調機株式会社
〒100-8555 東京都千代田区千代田1-1-1
三菱ビルディング5F



三菱重工空調機株式会社



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

Mitsubishi Heavy Air Conditioner

April 1988 First Design

Simulation by Summer

Production October 1989



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

Mitsubishi Heavy Air Conditioner

Room Heating and Cooling Times Reduced by 5X

Temperature Stability Increased by 2X

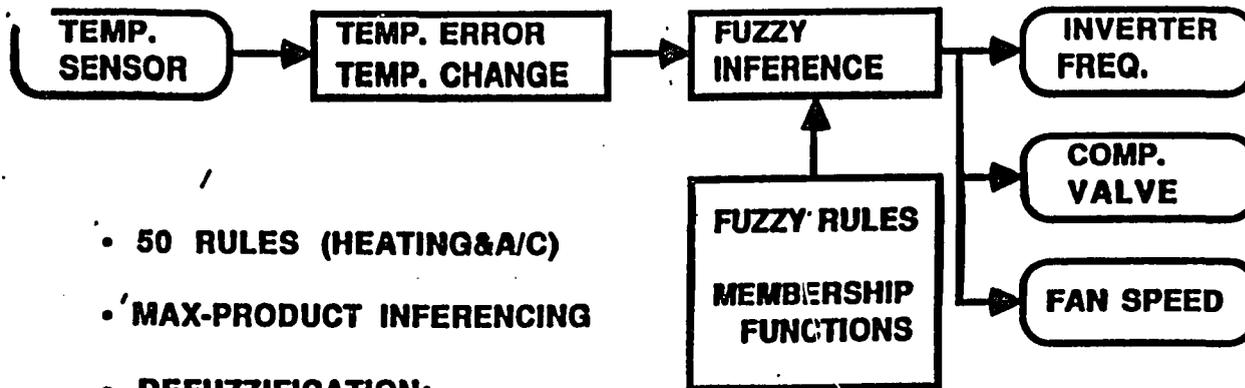
Overall Power Savings of 24%

Reduced the Required Number of Sensors



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

FUZZY INVERTER AIR CONDITIONER SYSTEM



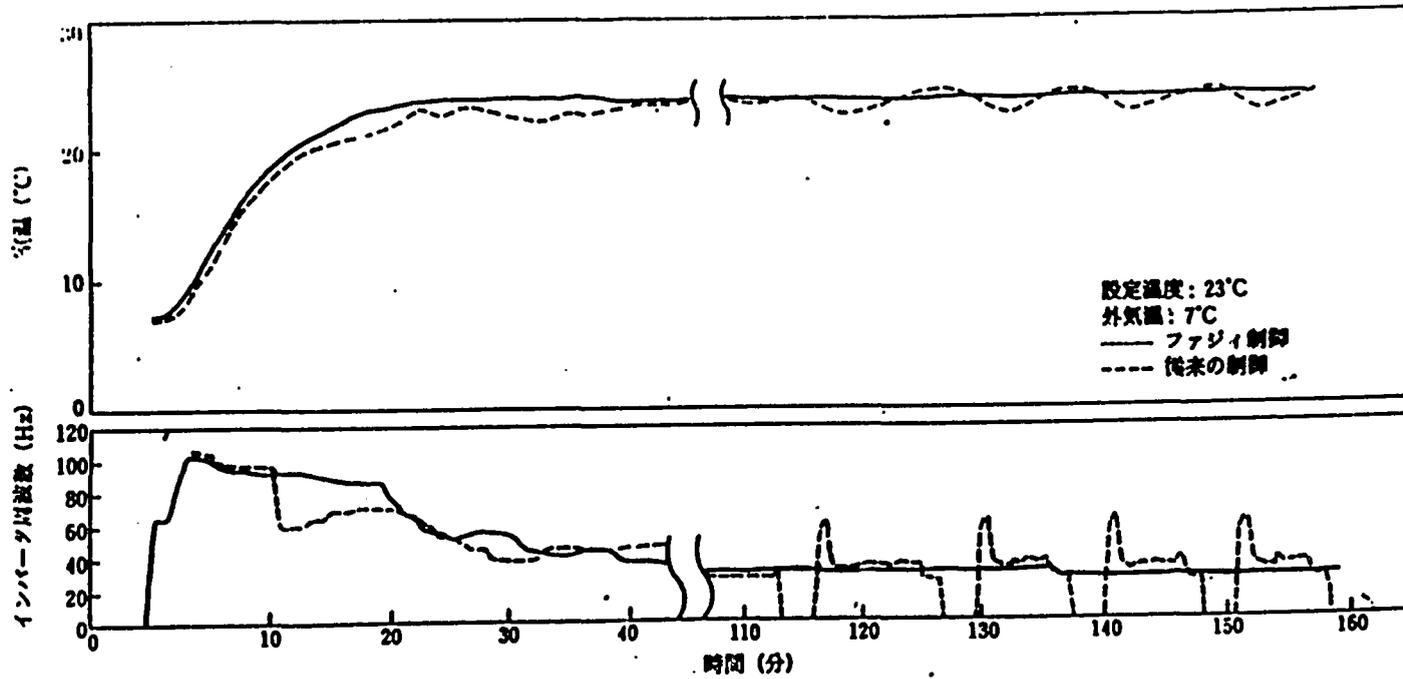
- 50 RULES (HEATING&A/C)
- MAX-PRODUCT INFERENCE
- DEFUZZIFICATION:
CENTROID METHOD

TOGAI INFRALOGIC, INC.
AC/1



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

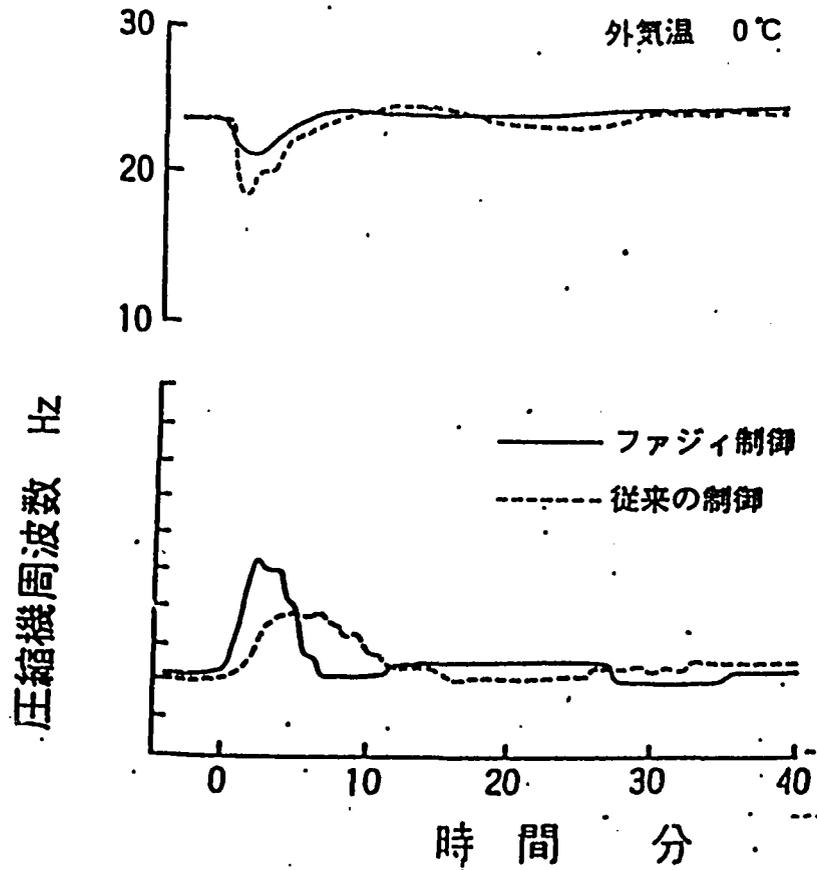
Mitsubishi Heavy Air Conditioner





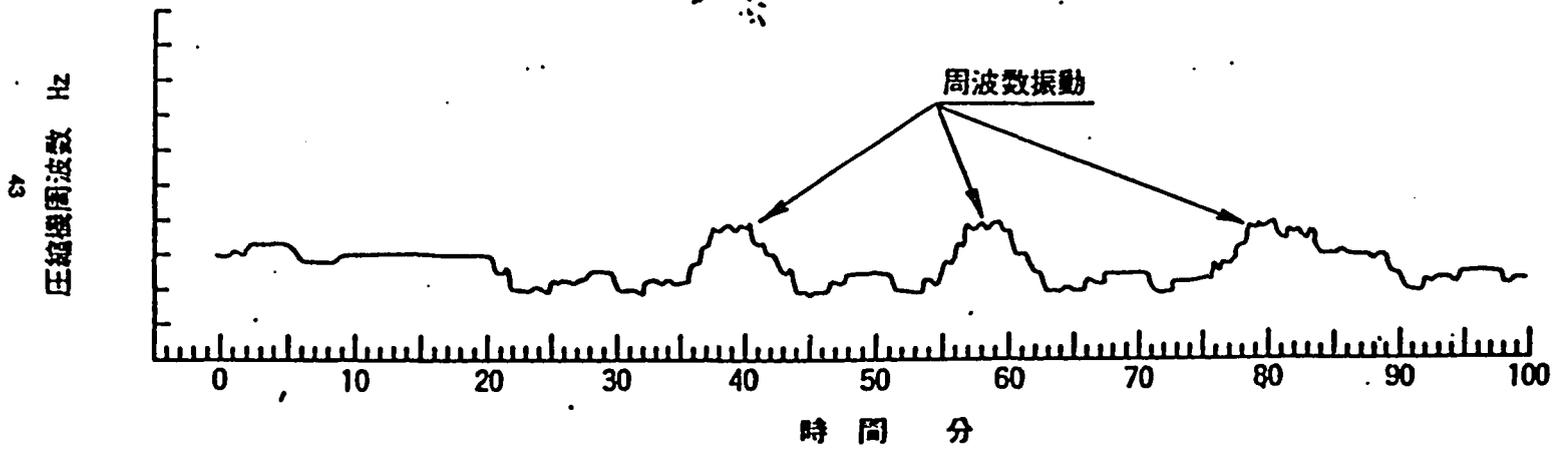
Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

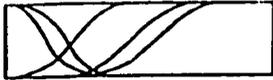
Mitsubishi Air Conditioner





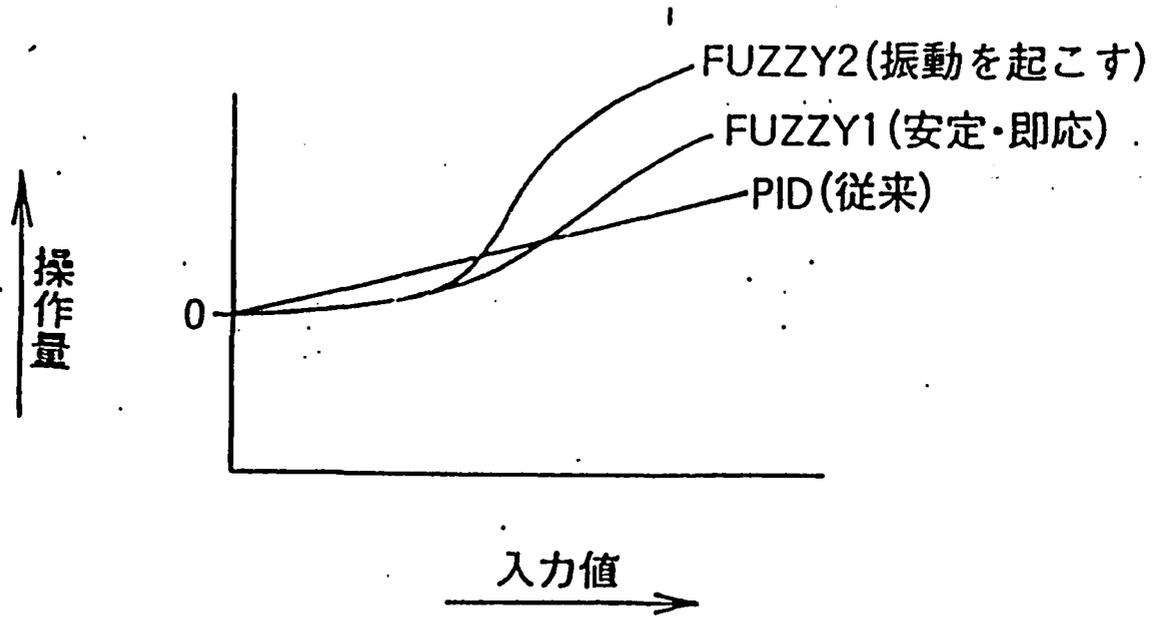
Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

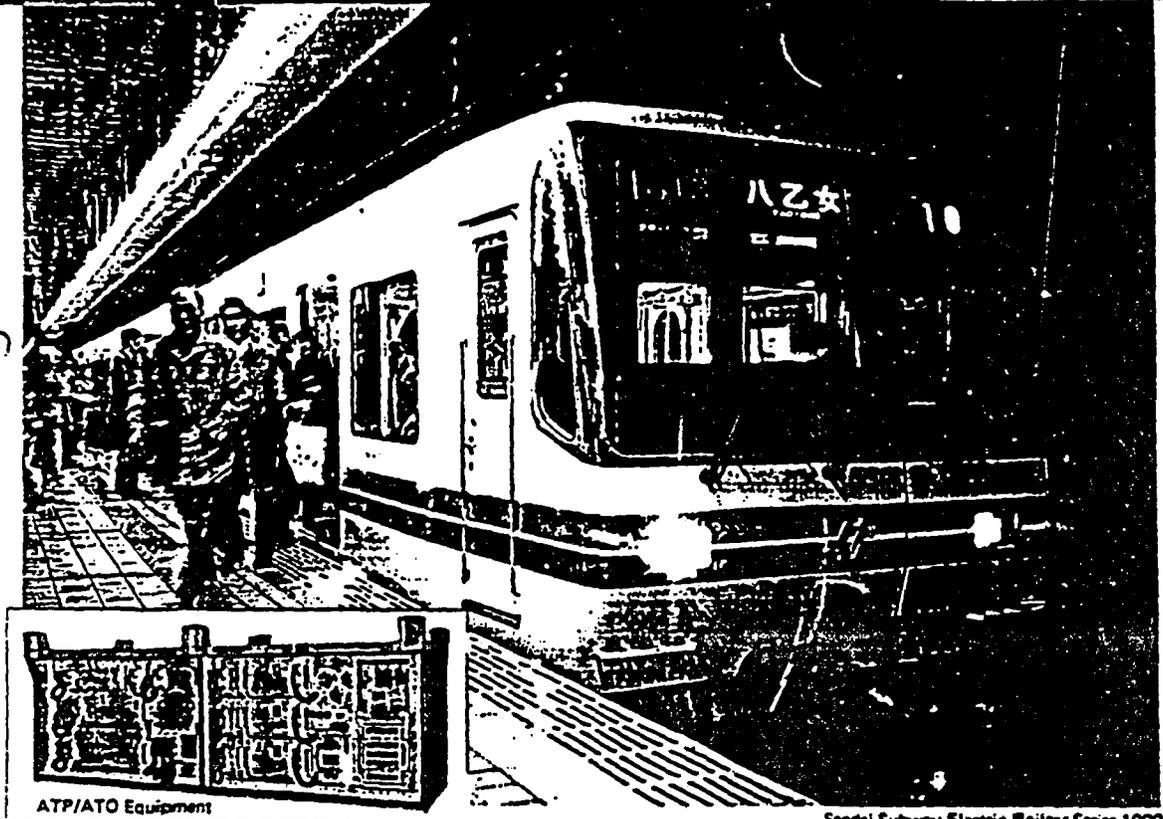




Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

考察





ATP/ATO Equipment

Sendai Subway Electric Railcar Series 1000

Automatic Train Operation System Based on Fuzzy Control

Introduction

Regular research of the automation of train operation began in around 1960 in Japan, and various tests with real cars were conducted for confirming functions of basic elements such as constant speed automatic operation, train automatic stop control at predetermined location, and train interval control.

In the last half of the 1960's, ATO devices were applied to the test cars for the Shinkansen bullet train and the monorail vehicles for the 1970 World Exposition held in Osaka, Japan.

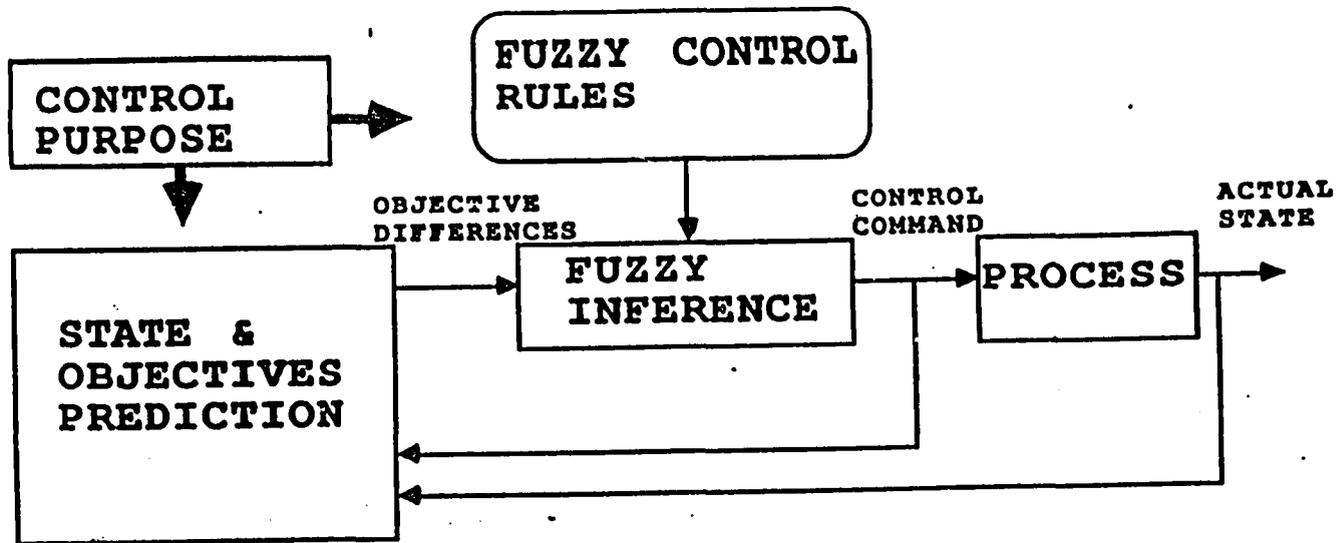
Starting from 1968, ATO devices for remote control were

adopted in diesel hydraulic locomotives operated in steel works, and saving labor and securing safety were realized.

In the 1970's, the ATO devices were used in many subway cars and vehicles of automated guideway transportation systems, and many improvements were made.

Recently, research on application of fuzzy control to automatic operation of subway cars was started. A venue service of the subway line of Sendai Municipal Transportation Bureau was started in July, 1987, and smooth and accurate automatic operation has been realized by employing ATO devices based on fuzzy control.

PREDICTIVE FUZZY CONTROL



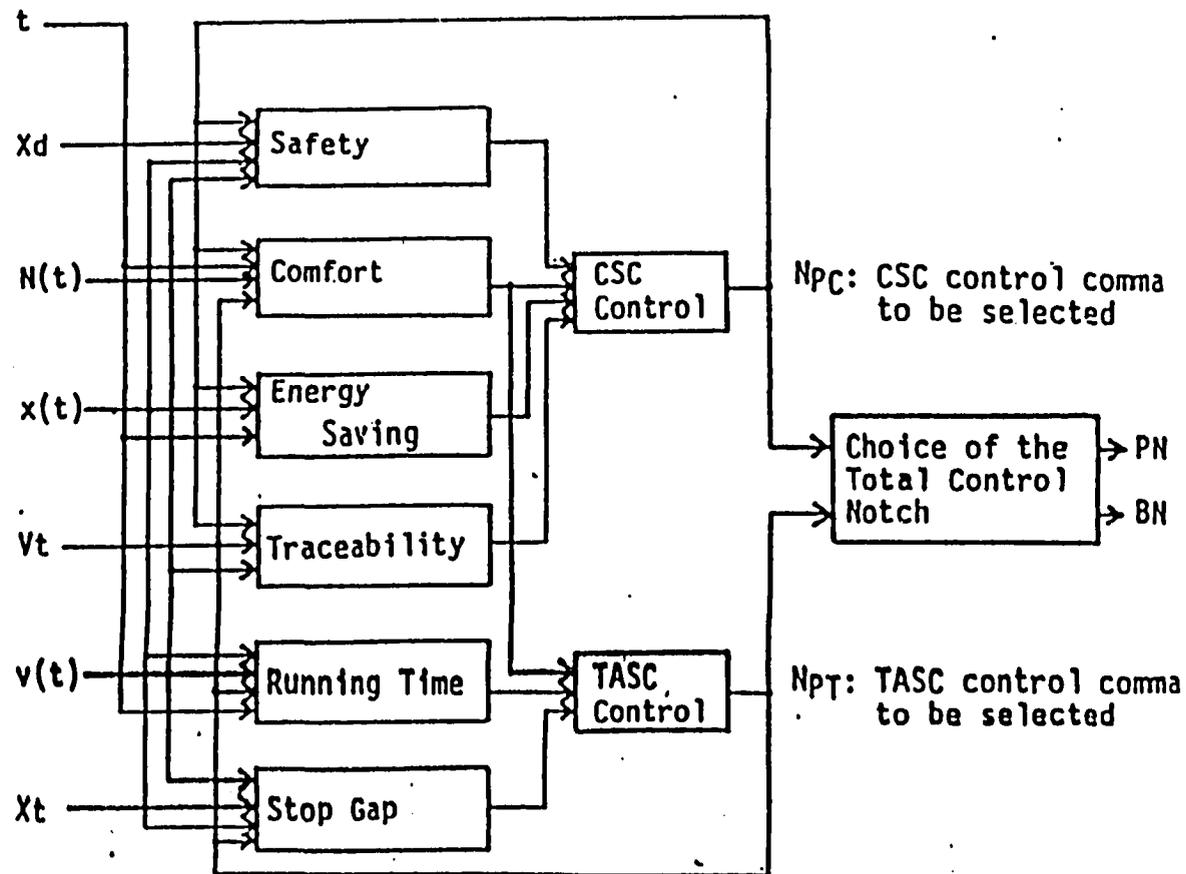
SENDAI SUBWAY CONTROLLER

TOGAI INFRALOGIC, INC.
KYOTO-5



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

The Sendai Subway System





Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

The Sendai Subway System

First Proposed to the Government 1978

Granted Permission to Operate After:
3,000 Empty Subway Runs
300,000 Simulations

Began Operation in 1986

Hitachi Granted Contracts for Tokyo Subway 1991



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

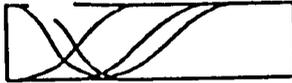
The Sendai Subway System

Performance Improvements

Improvement in Stop Gap by 3X

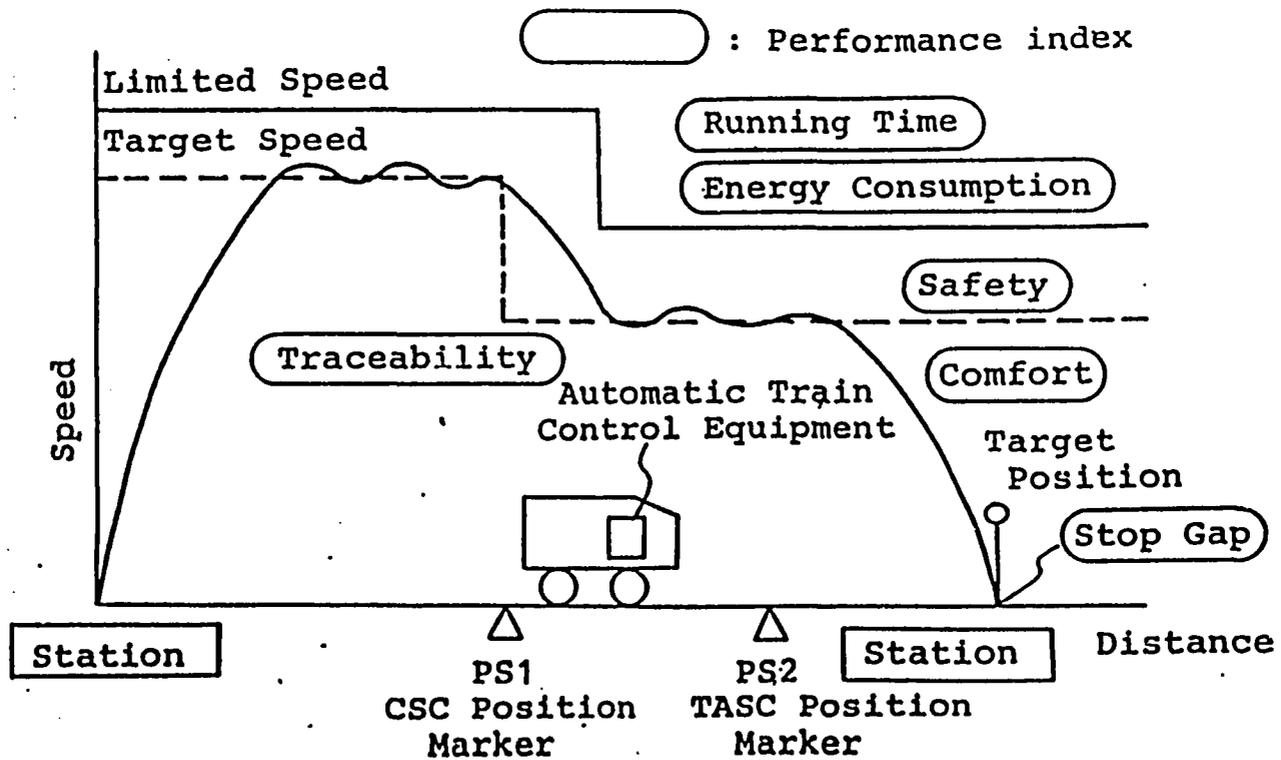
Reduction in Power Settings by 2X

Overall Reduction in Power by 10%



Togai InfraLogic, Inc.
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The Sendai Subway System



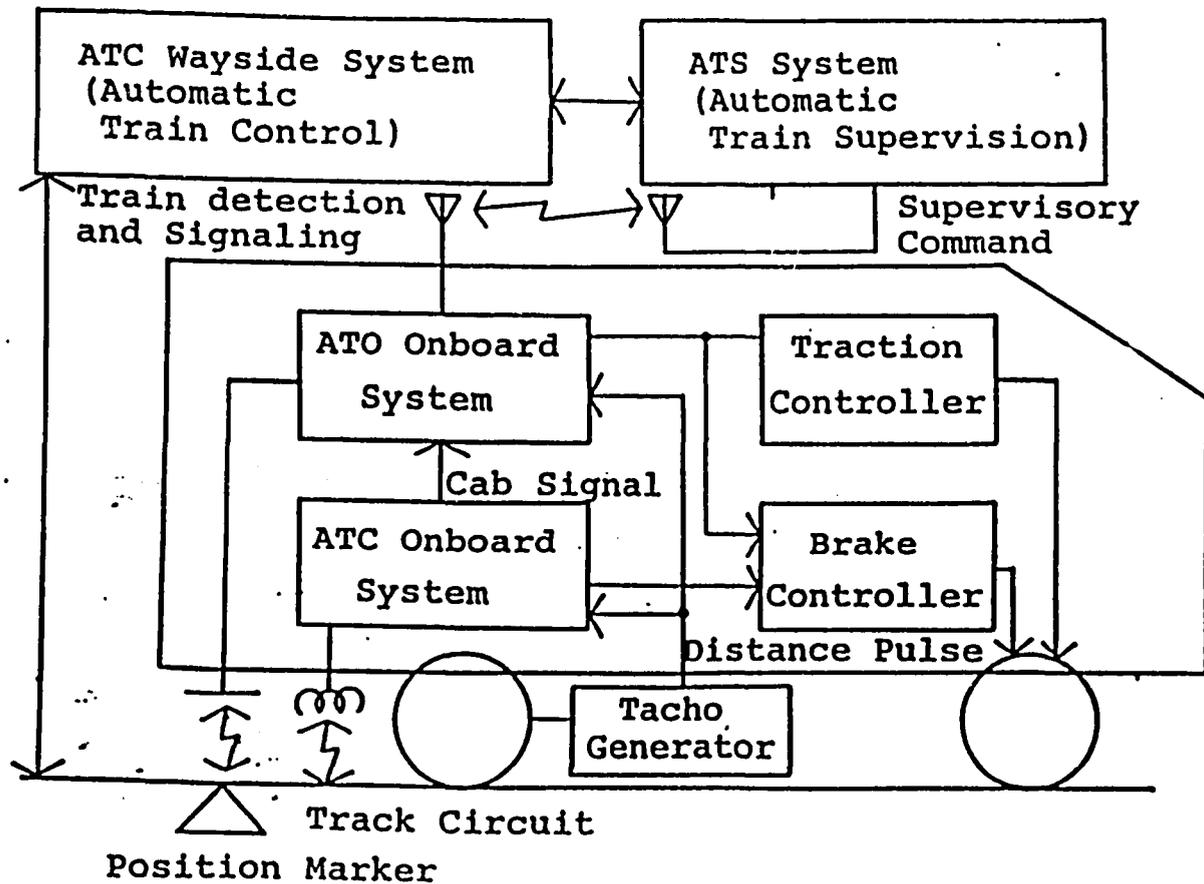
Outline of automatic train operation

8



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The Sendai Subway System



51

Typical configuration of ATO



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

The Sendai Subway System

Table 1 Symbols

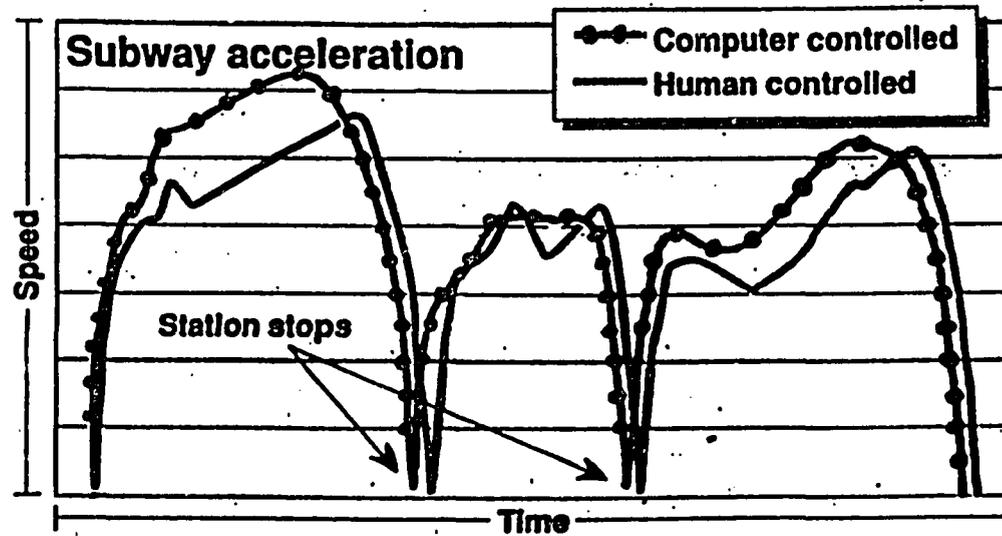
83

t	: time (sec)
$x(t)$: location of train (m)
$v(t)$: velocity of train (km/h)
$N(t)$: control command notch
$X(t)$: target position of next station (m)
V_t	: target speed (km/h)
T_t	: predicted running time (sec)
X_d	: forward location where the maximum speed limit is lower (m)
t_s	: time to reach X_d point (sec)
X_k	: ending location of coasting (m)
$X_z(v)$: beginning point of TASC zone (m)
$t_z = (X_z(v) - x(t)) / v(t)$: time to TASC zone (sec)
t_c	: elapsed time from last notch change (sec)
N_c	: degree of last changed notch
N_p	: control command notch to be selected
$V_p(N_p)$: predicted speed when N_p notch is selected (km/h)
V_e	: velocity allowance range (km/h)
$X_p(N_p)$: predicted stop position if N_p notch is selected (m)
X_e	: allowance of stop gap (m)



Togai InfraLogic, Inc.
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The Sendai Subway System



Source: Togai InfraLogic Inc.

The Register

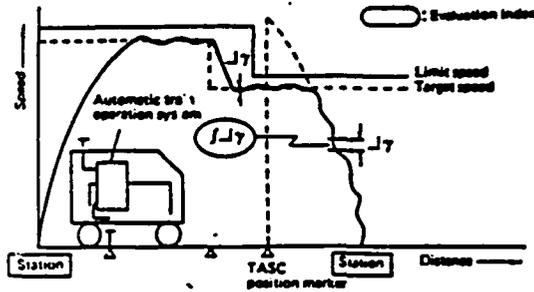


Fig. 1 Outline of automatic train operation by PID control

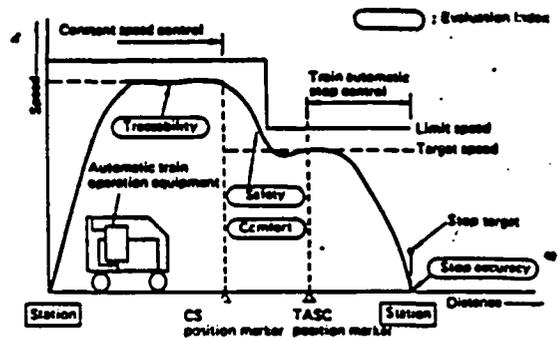


Fig. 2 Outline of automatic train operation by fuzzy control

Theory of Fuzziness & Fuzzy Control

The theory of fuzziness was first proposed in 1965 by professor L.A. Zadeh of the University of California at Berkeley.

The theory of fuzziness deals with a set with ambiguous boundary instead of an ordinary set. In the conventional Boolean set comprising "0" and "1," the boundary of an individual set can be clearly distinguished, but the fuzzy set is characterized by the fact that the boundary between the inside and outside of the set is not obvious.

The fuzzy control is based on the fuzzy set theory which was developed for determining the quantity of subjective fuzziness of human being and for making objective evaluation of the fuzziness possible, and thereby eliminating fuzzy portions as much as possible.

In the conventional automatic train operation system, train operation is performed by a control based on PID Control (Proportional, Integral, and Differential Control) so that target speed pattern is determined for each operating section and can be followed. In this conventional automatic train operation, accurate operation can be achieved in a manner of following the predetermined speed pattern. However, in actual practice, there are many kinds of changes of running conditions such as gradient etc. of

track and the braking force of rolling stock. Therefore, to follow the target speed, it is necessary to send control commands frequently for acceleration and brake application. As a result, smooth operation is apt to become difficult, and riding comfort is likely to be degraded. Moreover, an accuracy of train stopping at predetermined locations of stations cannot be determined through the logic of the control system. Accordingly dispersion should be checked by computer simulation or tests using real cars. This kind of problem occurs because the train operation characteristics as a controlled system are not well adapted to its control system. The characteristics of running train vary complicatedly and non-linearly in response to changes in the external situation. In the conventional control method, complicated controlled systems were dealt with approximating them to simple linear models, and only the follow-up to predetermined speed pattern was taken into account in the evaluation related to control. That is, the conventional control was unable to properly respond to changes in the situation.

On the other hand, in the fuzzy control, the results of certain running operations being considered are predicted in advance as the same as actual decisions made by a

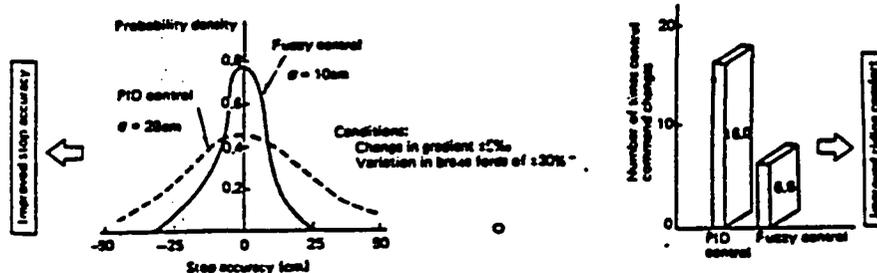


Fig. 3 Comparison of the results between fuzzy control and PID control (stop accuracy and number of times control command changes)



Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

The Sendai Subway System

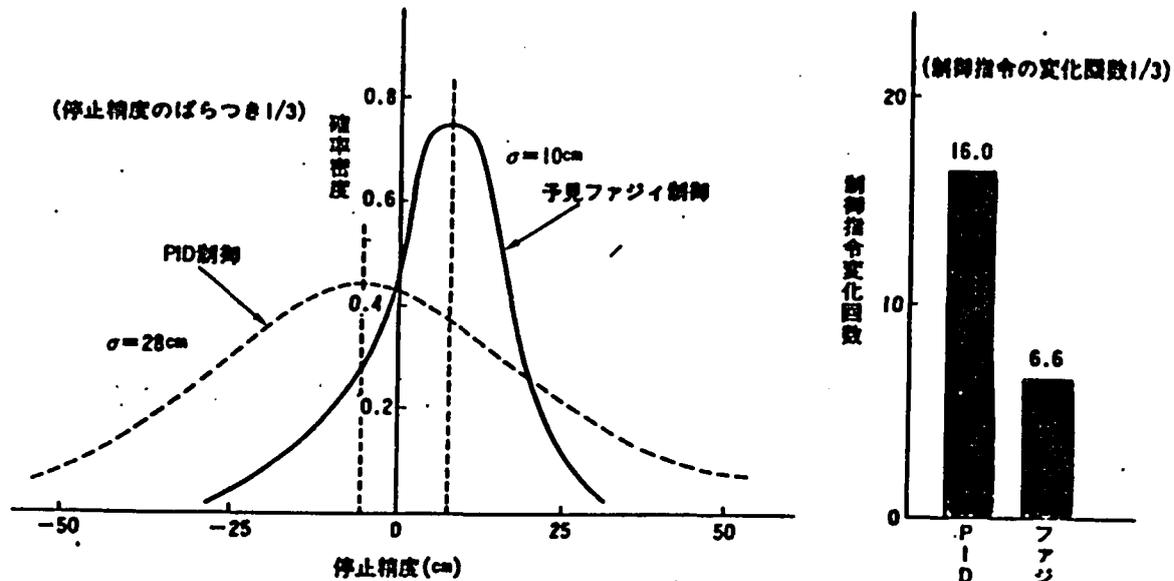


図6 停止精度と乗り心地の評価(菅野道夫著「ビジネスマンのためのファジィ読本」より)

Stop-Gap and Power Settings

AUTOMOTIVE

ELECTRONICS REVIEW

VOL. 2, NO. 15

SEPTEMBER, 1990

\$2.50

Nissan Close To Introducing Fuzzy Logic Transmission Controller

by ANDREA SAXER

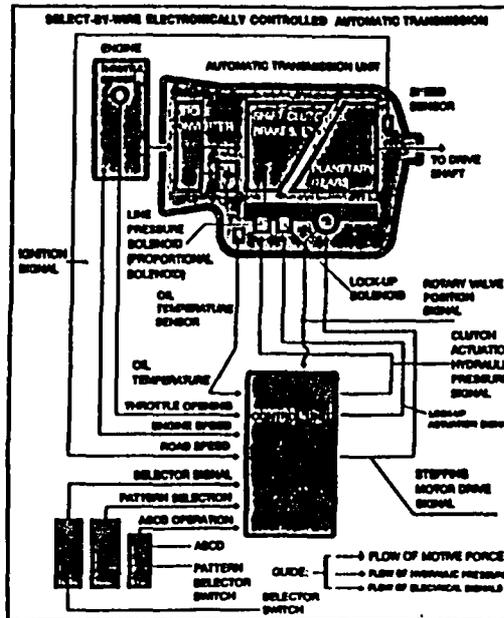
Soon—in the next year or so—Nissan will start selling cars programmed with a new, extra smart transmission control, popularly known as fuzzy logic. Speculation is that inference-based control logic will debut on Nissan's 300 ZX, the company's premier performance car, and perhaps on its Infiniti Q-45 luxury sedan.

Industry Analyst Roger Stociak, San Jose, CA, reported Subaru and Mazda are seriously pursuing the same smart control technique. He predicted a two- to three-year time frame for this to happen, at most.

Like Nissan, Subaru will apply fuzzy logic to transmission control. Mazda is testing it for application in collision avoidance systems. In Mazda's case, Stociak expected a long lead time, as much as ten years, since legal approvals for its systems will require extensive testing.

Hiroshi Takahashi, research engineer at Nissan's Central Engineering Laboratories, Yokosuka, Japan, said Honda and Isuzu are working on fuzzy logic for automotive electronic controls as well. Engine control, with its

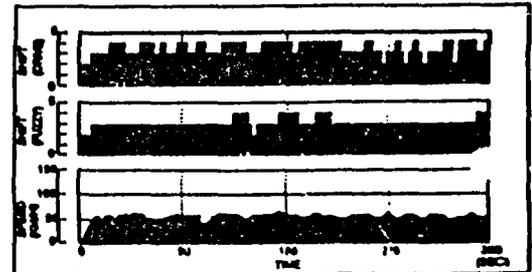
Please, see Fuzzy Logic, p. 12



CONCEPTUALLY SPEAKING: The Nissan Aro-X concept car has a control scheme that could incorporate fuzzy logic, according to Hiroshi Takahashi, a research engineer at Nissan's Central Engineering Laboratories, Yokosuka, Japan.



COULD IT BE THE FIRST? Will the transmission in Nissan's 1991 300 ZX be the first production car to be controlled by fuzzy logic?



WHAT A DIFFERENCE: More shifting, less comfort. Honda studied shift scheduling and found that gears were shifted less when the transmission was controlled by fuzzy logic rather than a traditional program. The comparison chart comes from a paper (#905049) presented by Honda Research and Development Engineers Sakai, Arai, Hasegawa, Sakaguchi and Iwaki at the International Federation of Automotive Engineering Societies in Turin, Italy, in May. Call the Society of Automotive Engineers, Warrendale, PA, 412-775-4811, for a copy.

House Committee OKs
400% IVHS Funding



Toyota J1850
10 Contact



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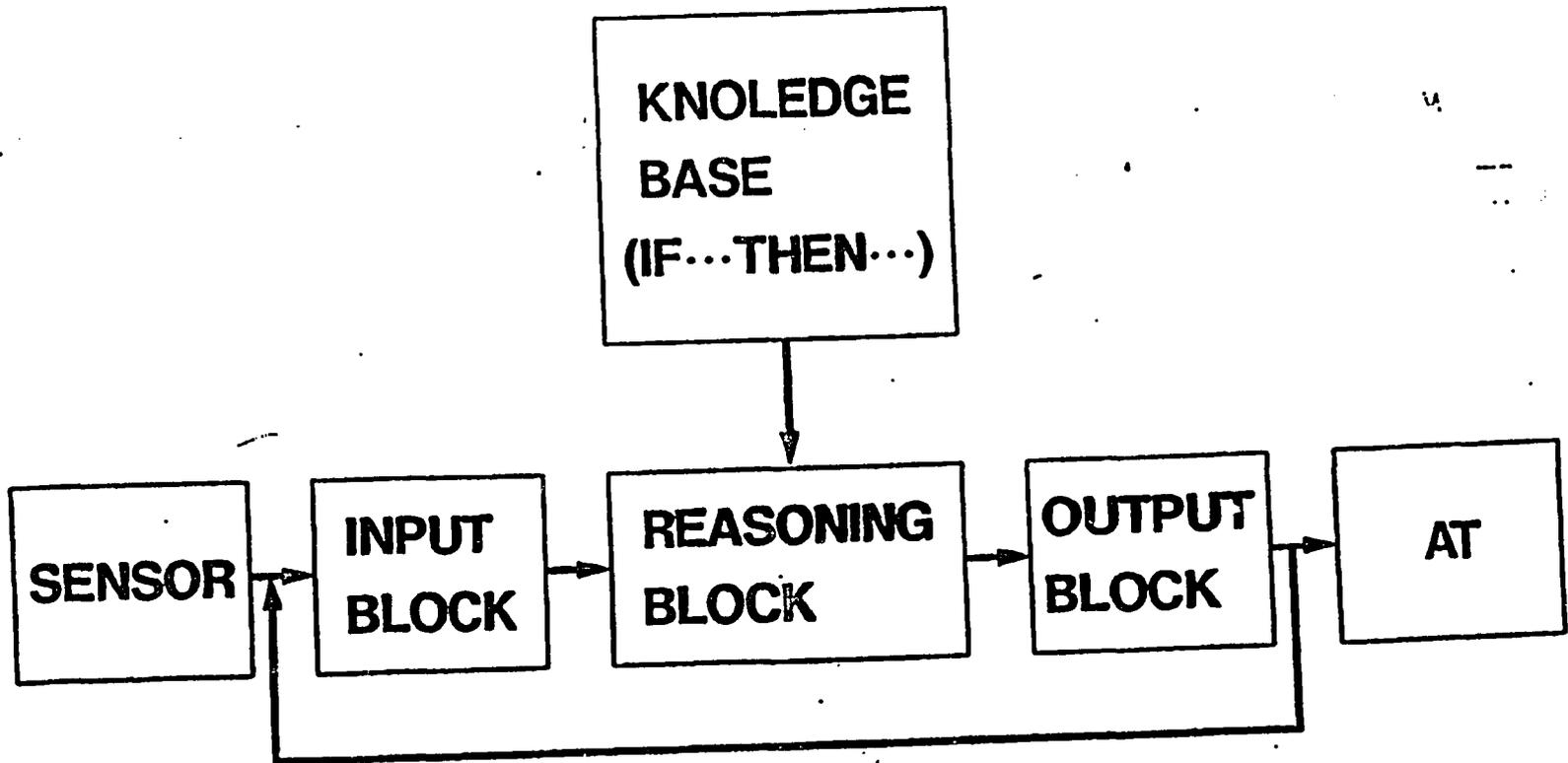
Automatic Transmission

Objectives:

Smoother Ride

Increased Fuel Savings

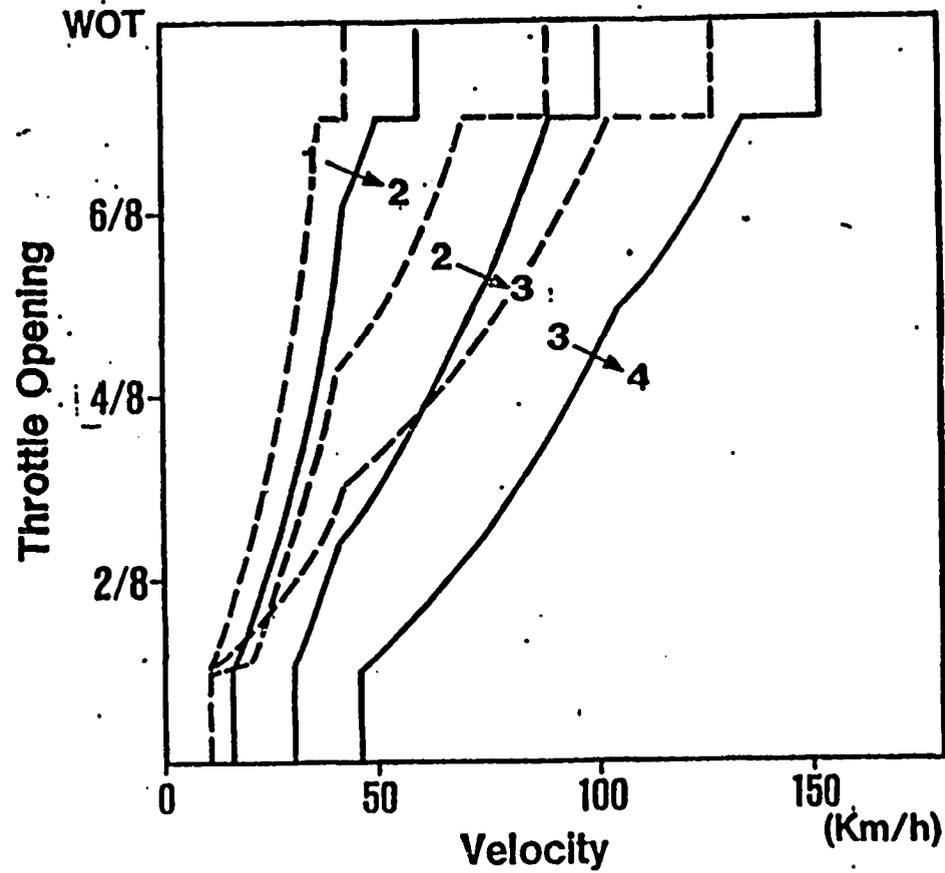
Less Wear





Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

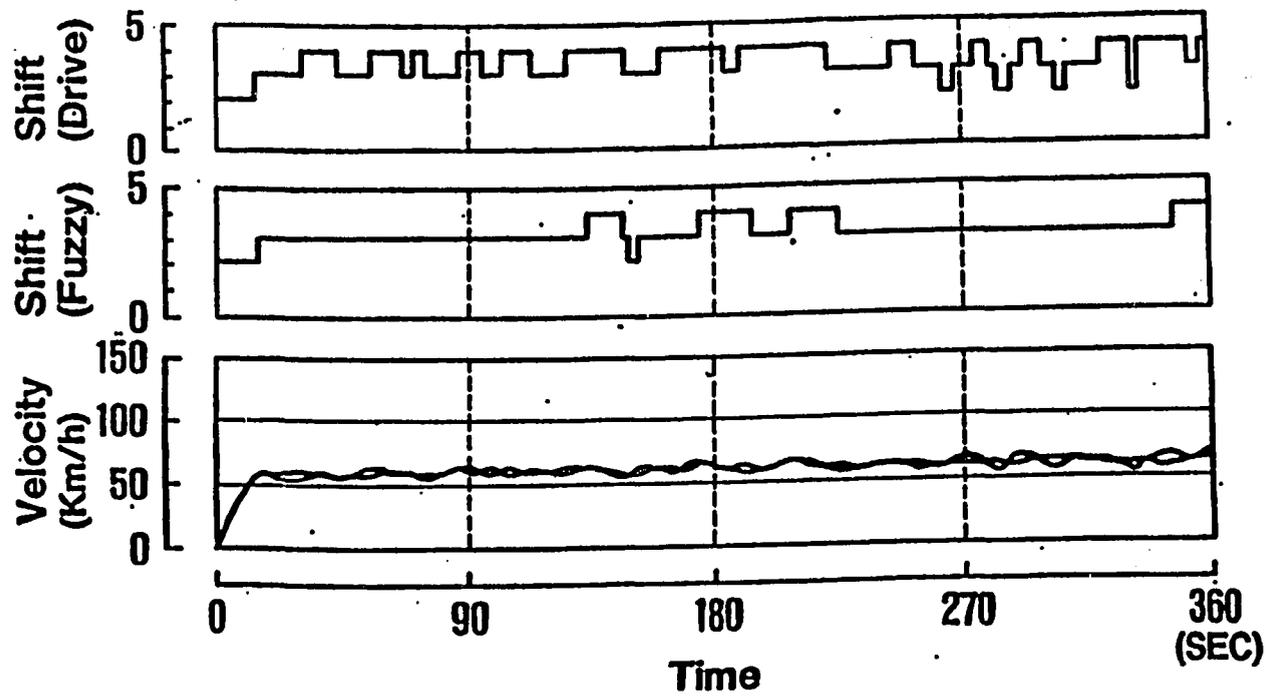
Automatic Transmission





Togai InfraLogic, Inc.
ARTIFICIAL INTELLIGENCE ON A CHIP

Automatic Transmission



Body/Chassis

Nissan Patents 'Fuzzy Logic' ABS, Gearbox

By ANGELA G. KING

TROY, Mich. — Nissan Motor Co. Ltd. has received U.S. patents on an anti-lock brake system and a transmission that incorporate "fuzzy logic" computer programming.

Designed to automate human reasoning, fuzzy logic programming offers various possible solutions, using graded or qualified statements, to a problem rather than the precise yes-or-no solution of strict logic widely used in the electronics industry, according to Lofti A. Zadeh, who first developed the concept of fuzzy reasoning in the mid-1960s.

Mr. Zadeh is currently a Professor of Computer Science in the Electrical Engineering and Computer Science department at the University of California at Berkeley.

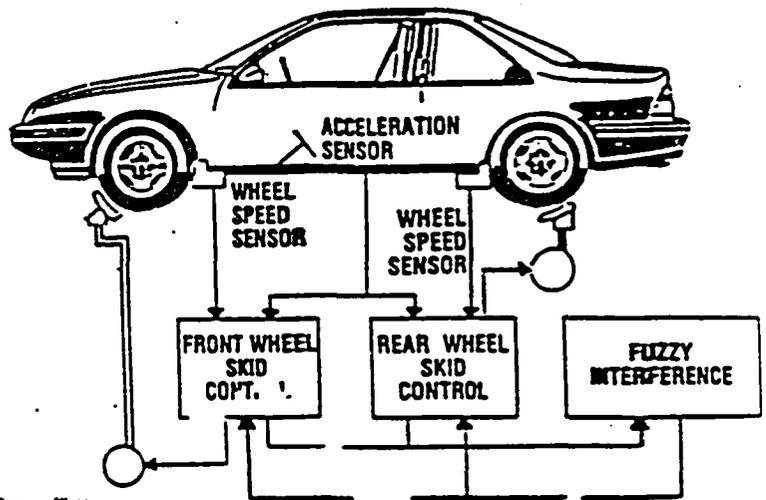
A Nissan spokesman said engineers are still developing the brake and transmissions systems, and no introduction dates have been set. Nissan has developed a fuzzy logic program and is now looking to see if it can be applied to its patented transmission and brake system designs, he explained. Fuzzy logic programming, according to the company, would enhance brake and transmission system performance with improved control flexibility.

Nissan is developing an automatic transmission in which fuzzy logic computer programming is used to electronically shift gears in a manner similar to a driver who weighs different factors to manually shift gears.

WITH A CONVENTIONAL automatic transmission, electronic sensors detect vehicle speed and throttle opening, and gears are shifted based on the predetermined value of these factors. According to Nissan, this type of system is incapable of always providing satisfactory control performance to a driver because it provides at most only about three different shift patterns.

But the Nissan fuzzy control transmission, (patent number 4,841,815), is more flexible and provides a driver with more control performance because it is operated by sensors that assign values to numerous variables, including vehicle speed, throttle opening, acceleration and the rate of change of the throttle opening. Each value is given a weight, and the weights are calculated to make the de-

FUZZY LOGIC CONTROLLED ABS



Source: Nissan

ANTHROPOMORPHIC: Nissan's fuzzy logic program is designed to apply human reasoning characteristics to the control module of its patented ABS design.

cision on whether to shift gears.

Where conventional ABS incorporate sensors that detect vehicle and wheel speed, the Nissan system's (patent number 4,842,342) control unit measures these variables in addition to derivatives of wheel speed with respect to time and derivatives of vehicle speed with respect to time. As in the transmission, certain signals in the brake system are assigned weighted values that determine the frequency of ABS brake actuation.

In a paper entitled "Making Computers Think Like People," Mr. Zadeh explained that fuzzy logic allows computers to handle such imprecise human concepts as "small," "big," "young" and "old" by describing them in ranges of numbers instead of exact terms.

DEVELOPMENT OF FUZZY LOGIC in the early 1970s by Ebrahim Mamdani, a control engineer at Queen Mary College in London, and Seto Assilian, Mr. Mamdani's student at that time, has led to growing interest in the use of this theory in such applications as industrial process control and automo-

bile engine control, said Mr. Zadeh.

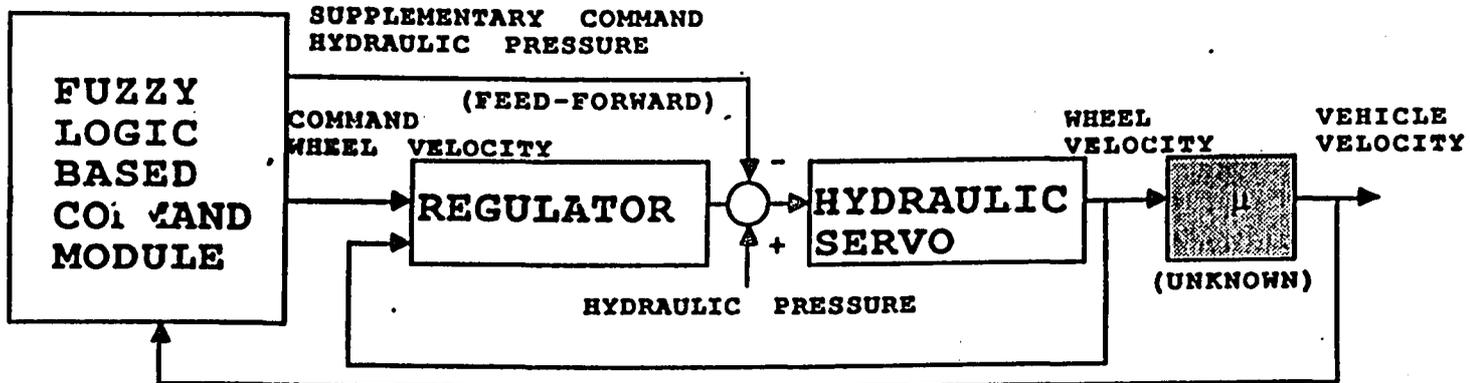
Japan in particular has shown great deal of interest in fuzzy logic. Research is being conducted in Japan in the application of this system in such areas as vehicle control at the Tokyo Institute of Technology's Sugeno Laboratory, and robot control at Hirose University's Hirota Laboratory.

In March, Japan's Ministry of International Trade and Industry established the Laboratory for International Fuzzy Engineering Research (LIFE), a group that consists of 48 member Japanese firms, including Nissan.

A fuzzy system developed by Hitachi, also a member of the new LIFE organization, is already used to control subway trains in Sandai, Japan.

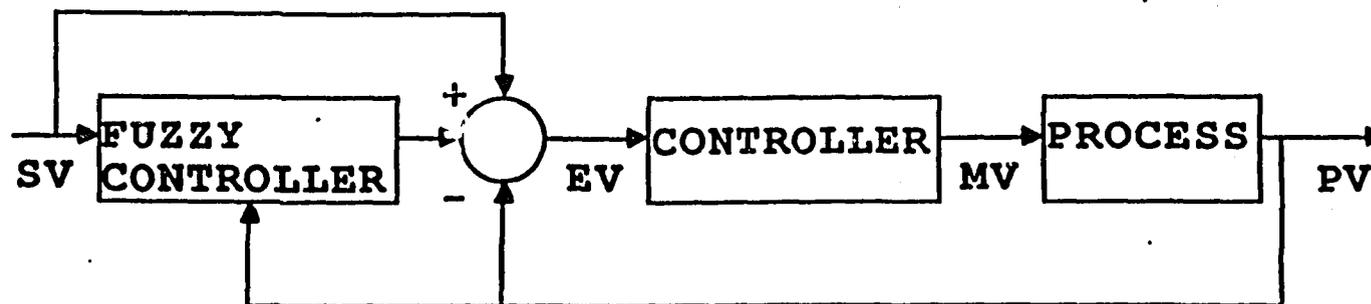
Fuji Heavy Industries Ltd., the maker of Subaru cars, is developing an advanced form of electronic continuously variable transmission, called the ECVT-II, that also uses fuzzy control. The ECVT-II is not in production now and is not expected to appear in an automobile before model year 1991, according to a spokesman at Subaru of America, Cherry Hill, N.J. □

FUZZY LOGIC-BASED COMMAND SYSTEM FOR ABS



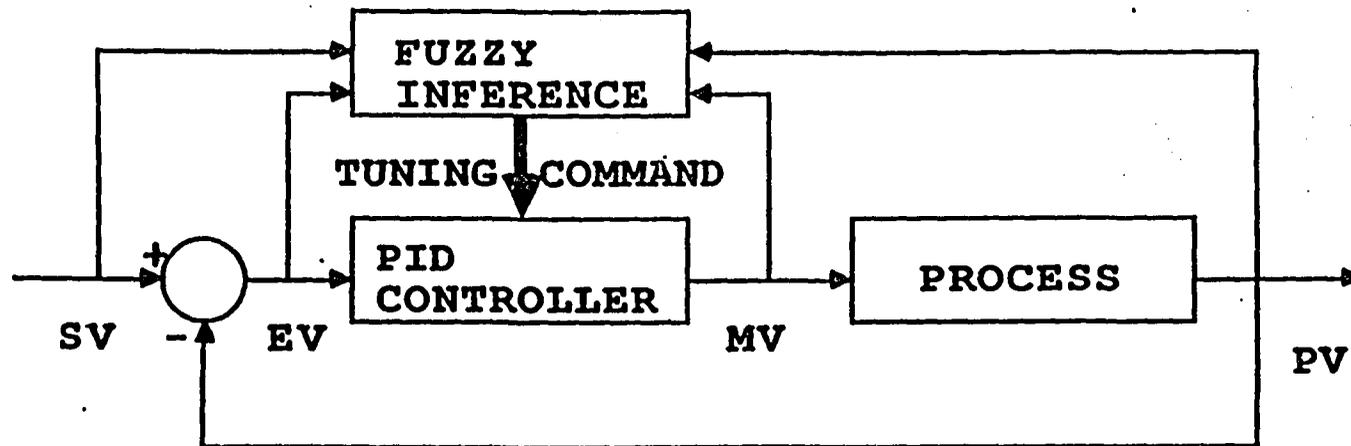
8

FUZZY FEED-FORWARD CONTROLLER



8

FUZZY AUTO-TUNING SYSTEM



TOGAI INFRALOGIC, INC.

KYOTO-3

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CLAS

CURRENT PROJECTS IN FUZZY CONTROL

by

**Michio Sugeno
Tokyo Institute of Technology**

**Fuzzy Logic Workshop
14 November 1990**

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P-13

Projects of Helicopter Flight Control

- o Radio Control by Oral Instructions
(1989 - 1993)

Tokyo Institute of Technology
supported by Science and
Technology Agency

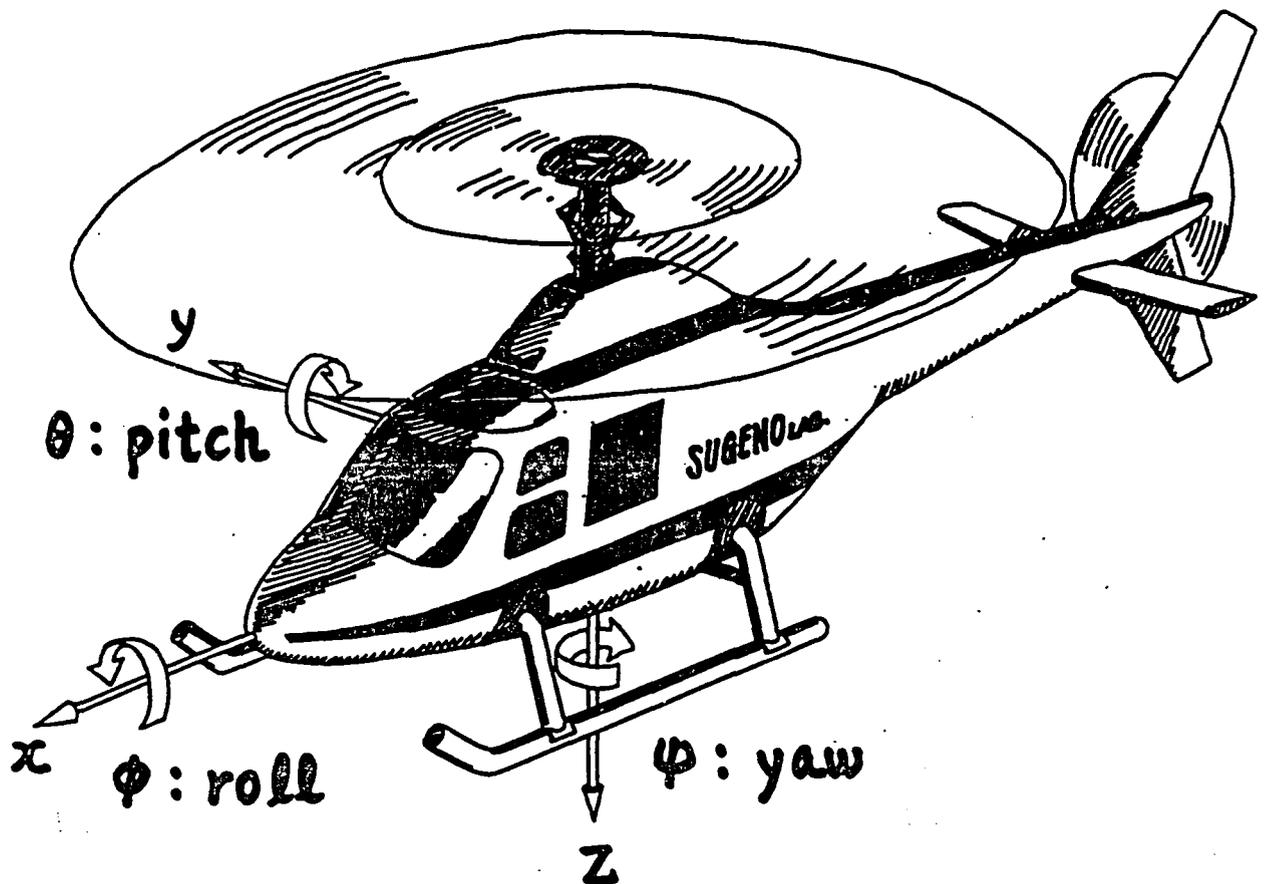
- o Automatic Autorotation Entry
in Engine Failure
(1989 - 1993)

Kawasaki Heavy Industry / TIT
supported by STA

- o Unmanned Helicopter for Sea Rescue
(1990 - 1991)

TOKIMEC / TIT
supported by Ministry of
Transportation

Motion of Helicopter



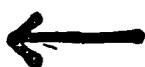
measured variables : \ddot{x} , \ddot{y} , \ddot{z}
 $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$

Control of Movements

Move

Control

up/down



lift of main rotor
(collective pitch level)

forward/backward

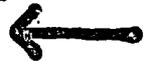
pitching



main rotor revolution
(longitudinal stick)

left/right

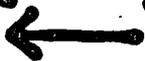
rolling



main rotor revolution
surface
(lateral stick)

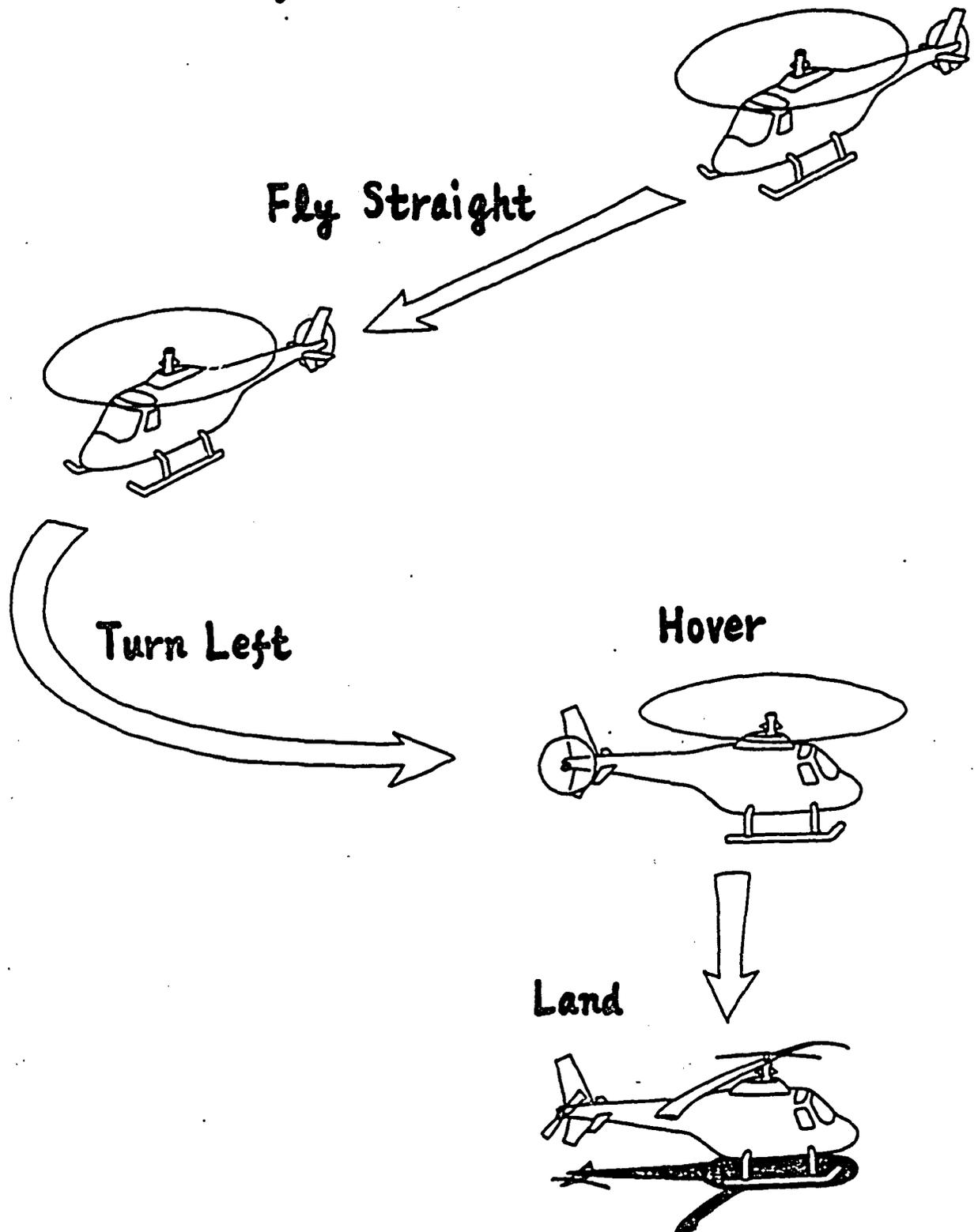
nose direction
(left/right)

yawing

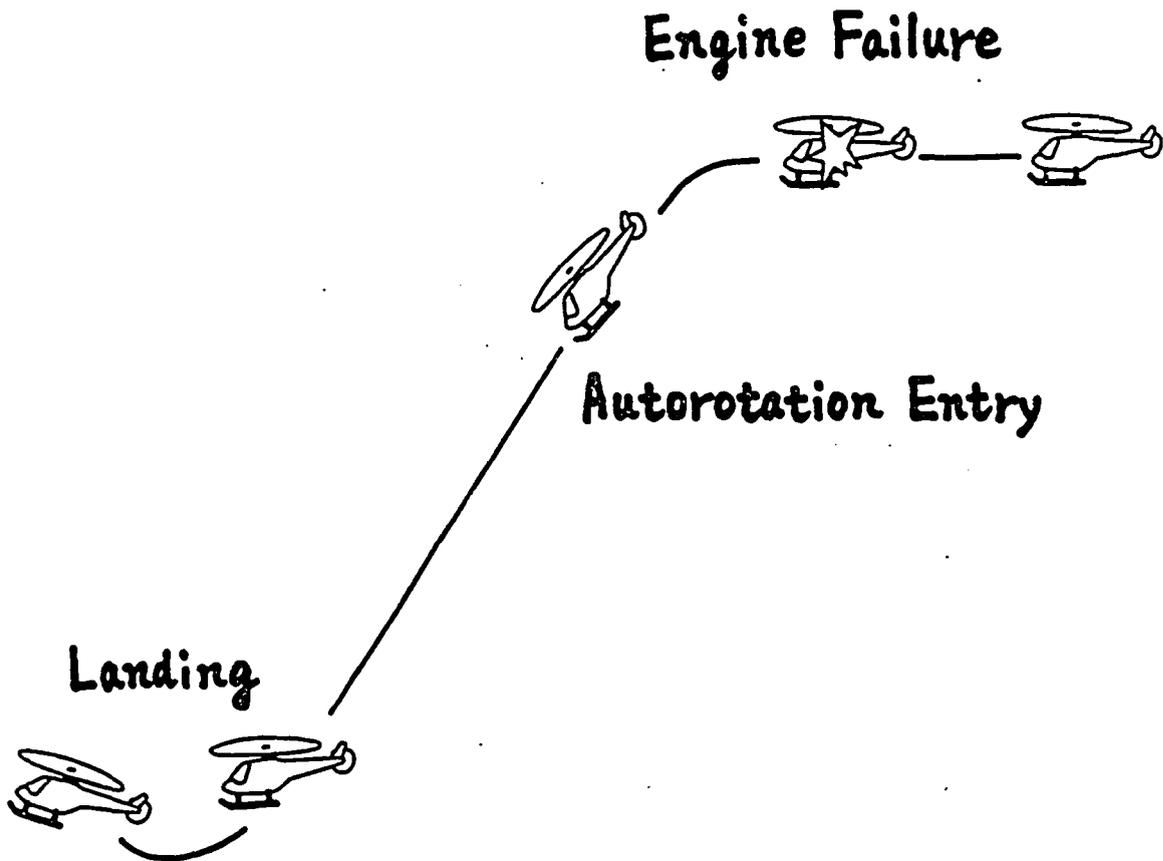


lift of tail rotor
(directional pedal)

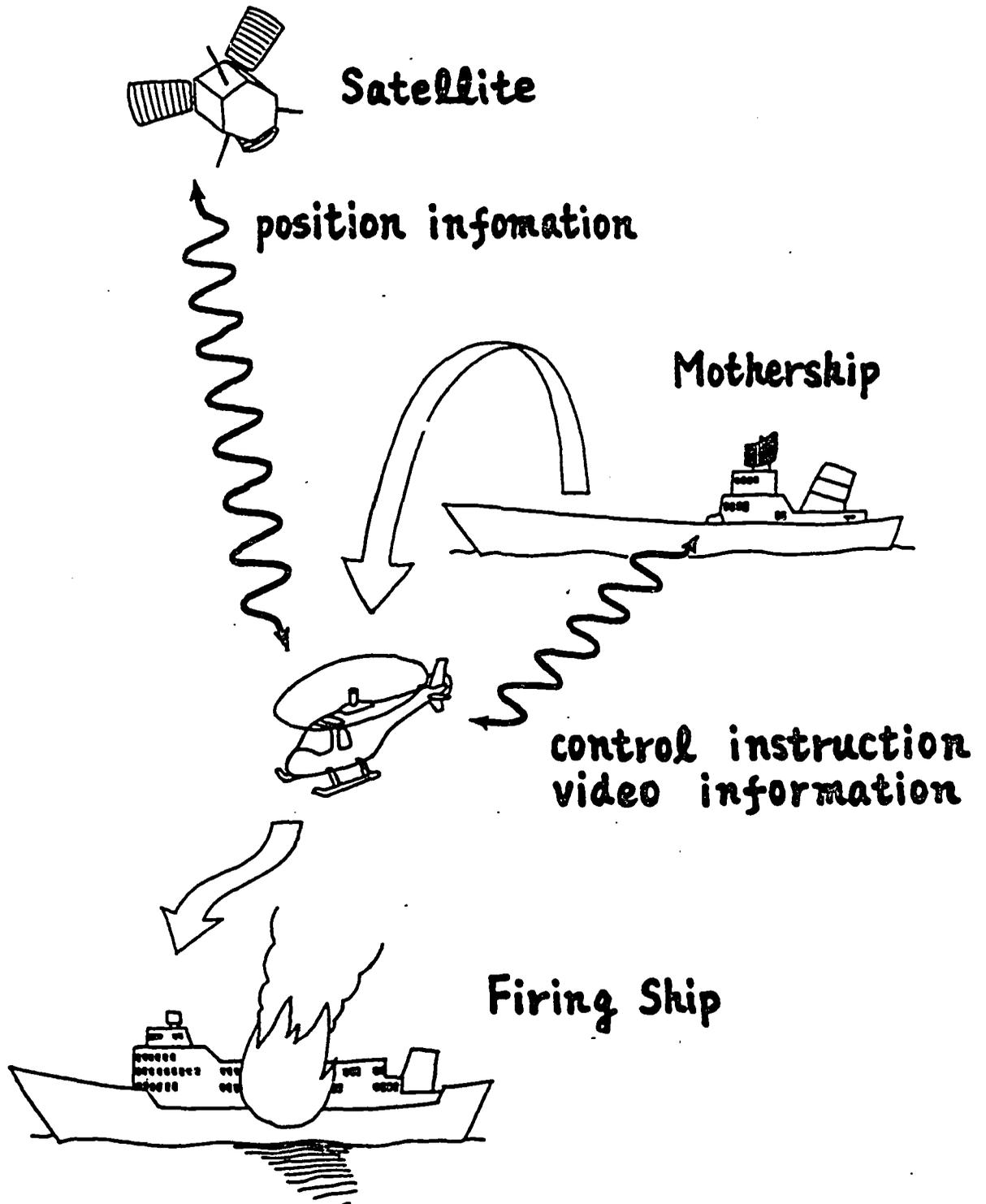
Remote Control of Helicopter by Oral Instructions



Automatic Autorotation Entry



Unmanned Helicopter for Sea Rescue



Linguistic Rules for Hovering

- 1) If the body rolls, then control the lateral in reverse
- 2) If the body pitches, then control the longi. in reverse
- 3) If the nose turns, then control the pedal in reverse
- 4) If the body moves sideways, then control the lateral in reverse
- 5) If the body moves back and forth, then control the longi. in reverse
- 6) If the body moves up and down, then control the collective in reverse

Fuzzy Control Rules for Hovering (longi. stick control)

1) pitch is PO \rightarrow longi. is NE

2) " NE \rightarrow " PO

3) $\frac{d}{dt}$ pitch is PO \rightarrow longi. is NE

4) " NE \rightarrow " PO

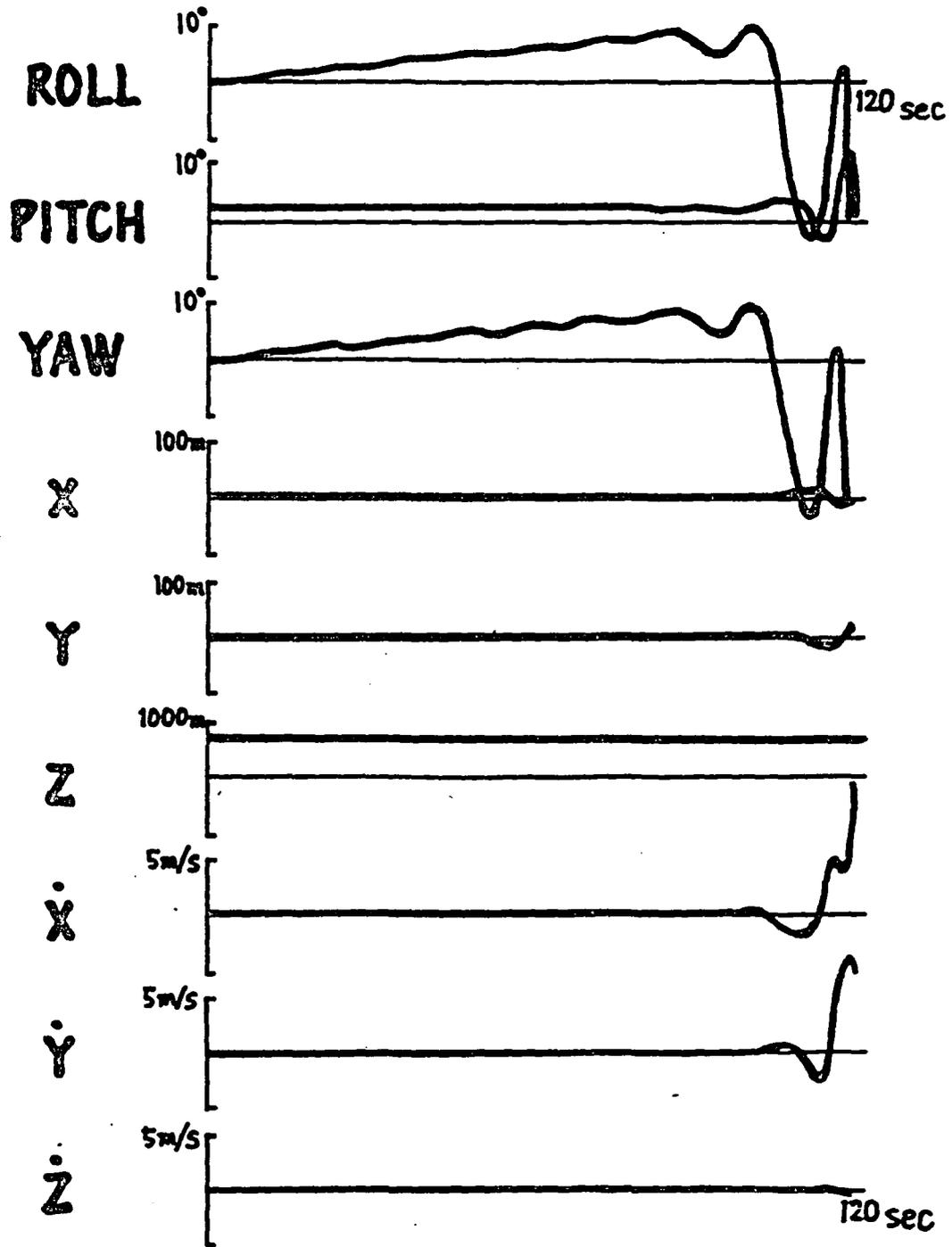
5) x is PO \rightarrow longi. is NE

6) " NE \rightarrow " PO

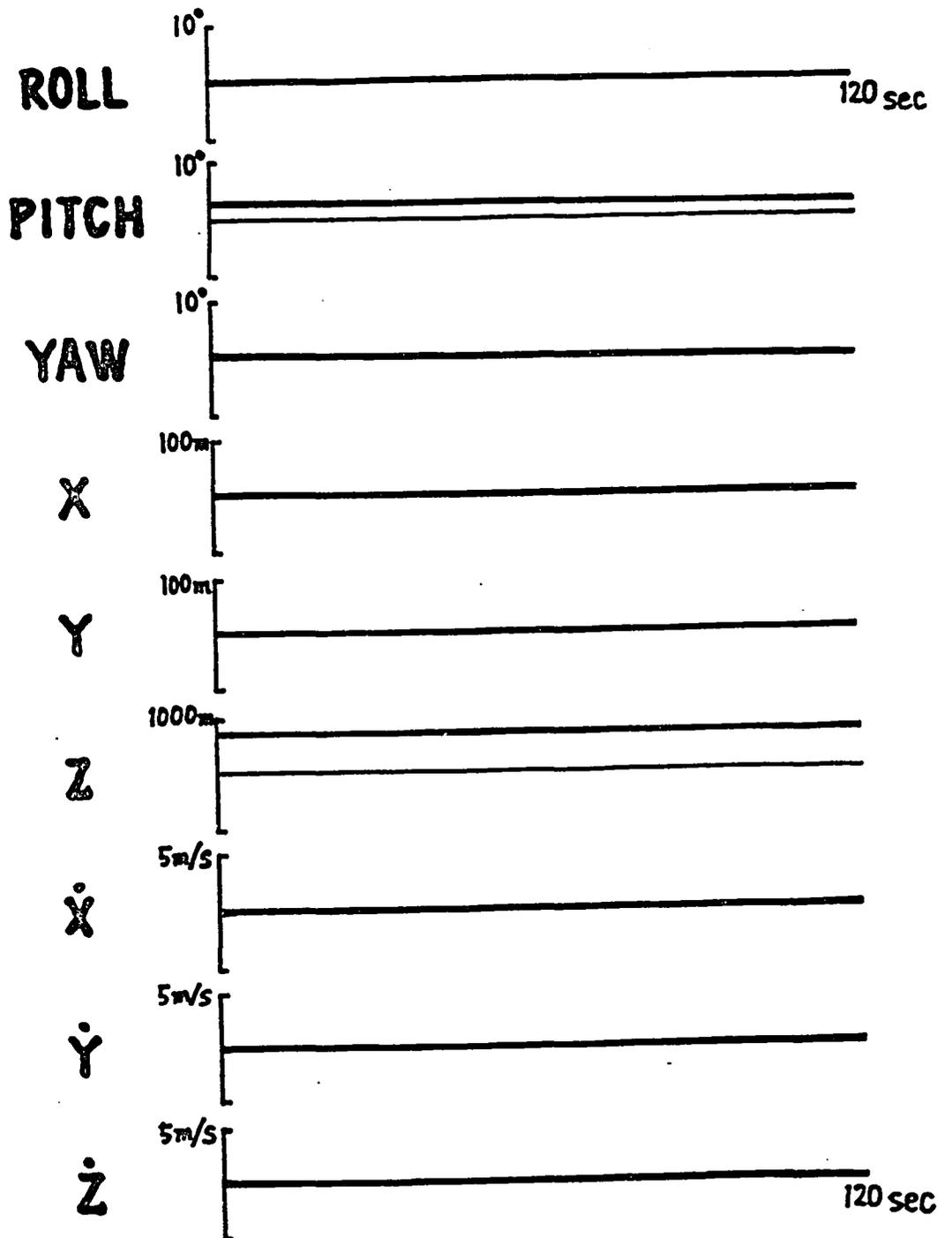
7) $\frac{dx}{dt}$ is PO \rightarrow longi. is NE

8) " NE \rightarrow " PO

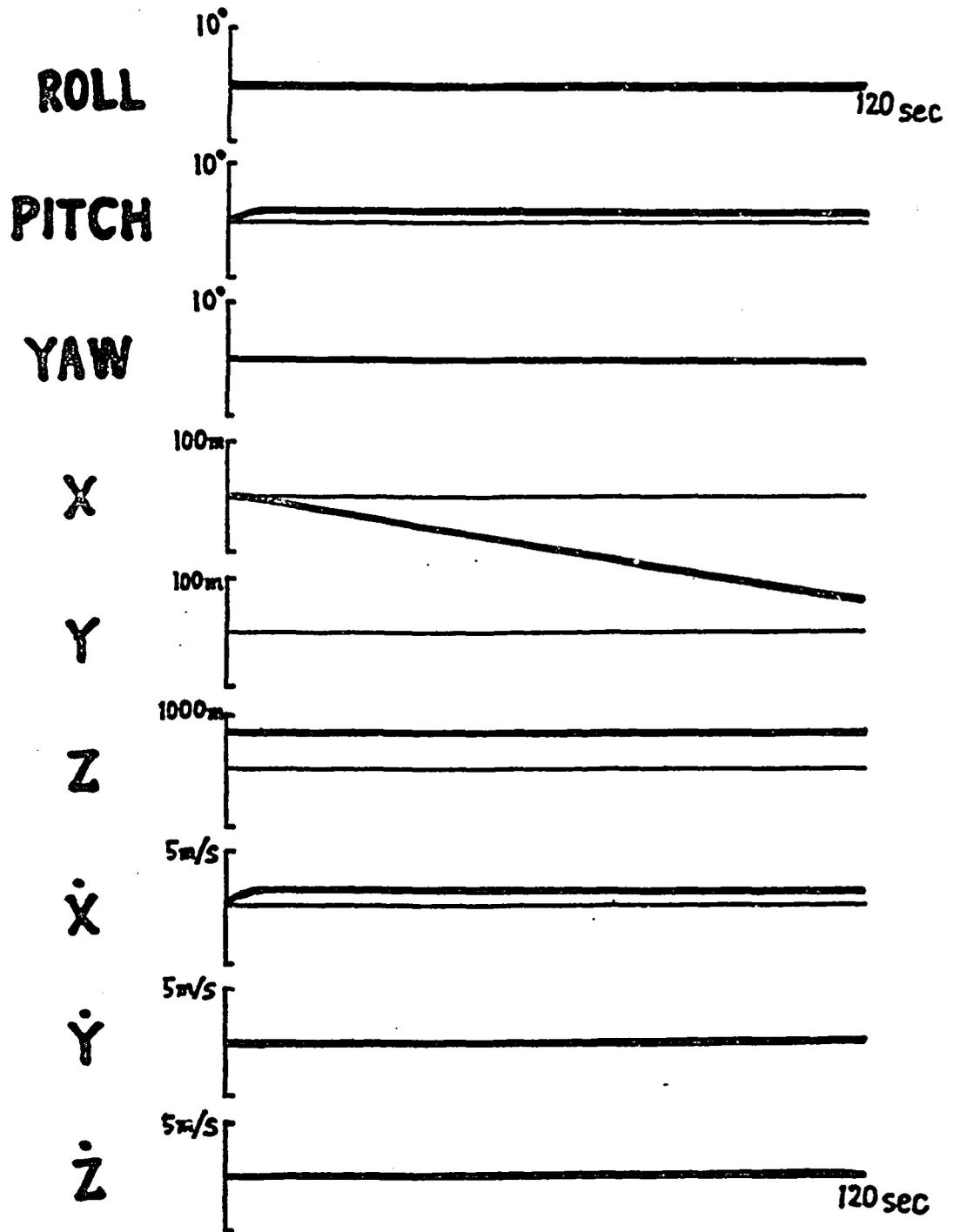
Hovering without Control



Hovering with Fuzzy Control

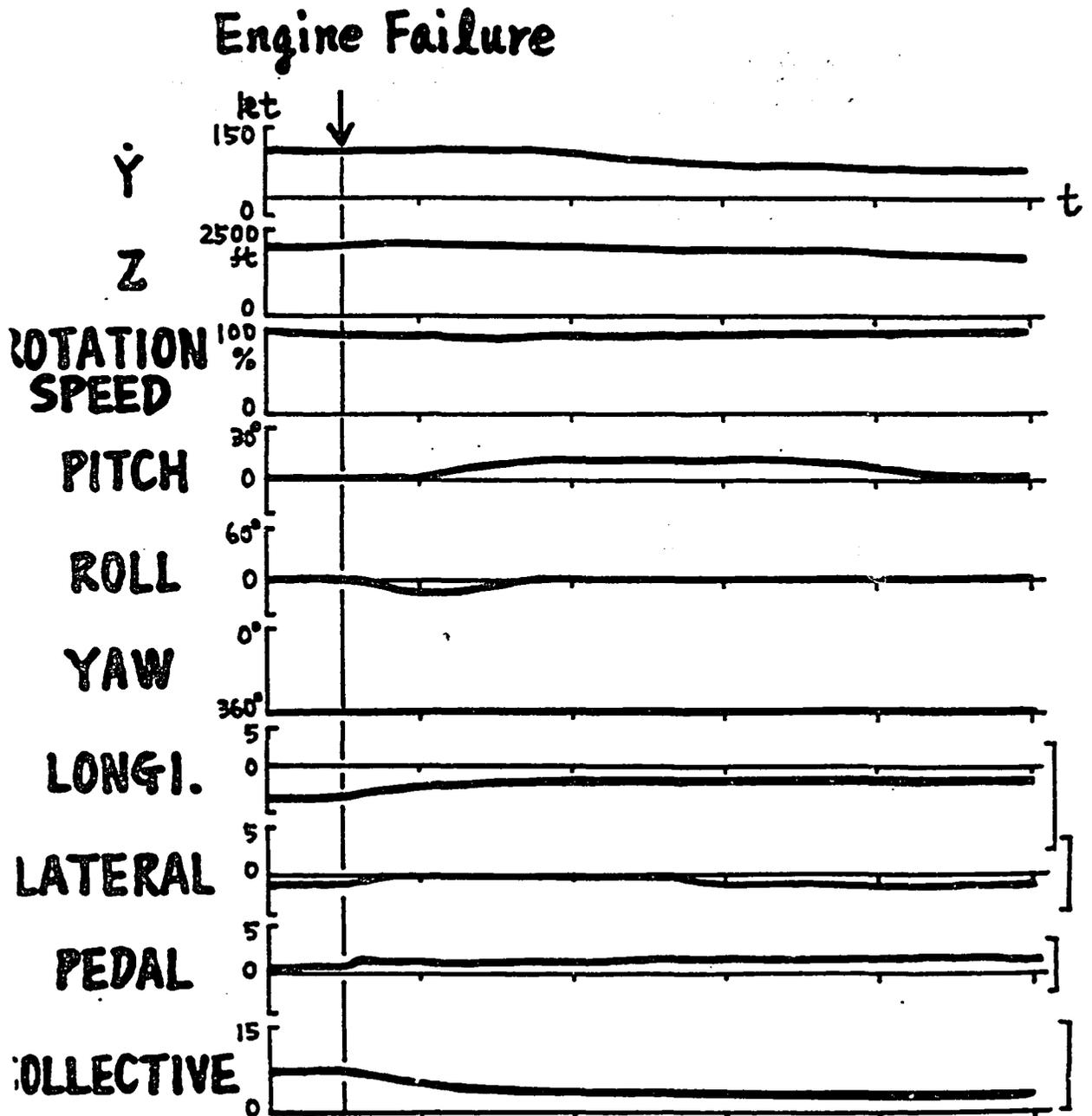


Forward Flying Control in Low Speed



Autorotation Entry by Fuzzy Control

(keep rotation speed of main rotor)



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31 43

LAS

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TERMINOLOGY AND CONCEPTS OF CONTROL AND FUZZY LOGIC

Presented at
A Workshop on Fuzzy Control
Huntington Beach, CA
14 November 1990

Dr. Jack Aldridge, MDSSC-SSD (Houston)
Dr. Robert Lea NASA/JSC
Dr. Yashvant Jani Lincom Corp.
Dr. Jonathan Weiss MDSSC-SSD (Houston)

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P. 20

PURPOSES OF THIS TALK

MCDONNELL DOUGLAS

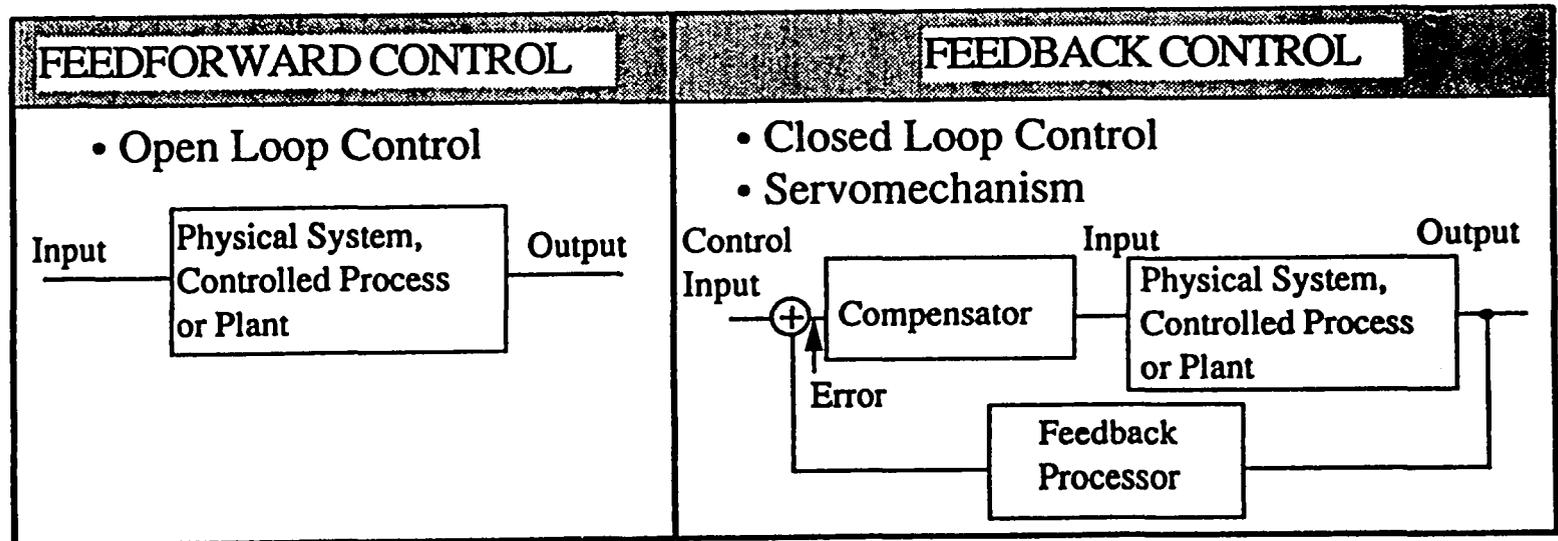
- Briefly review control history - how do ideas "fit together"
- Establish terminology of control theory and fuzzy logic to promote useful discussions
- Establish basic concepts ^{and issues} in both areas for the same purpose

CONTROL SYSTEMS

MCDONNELL DOUGLAS

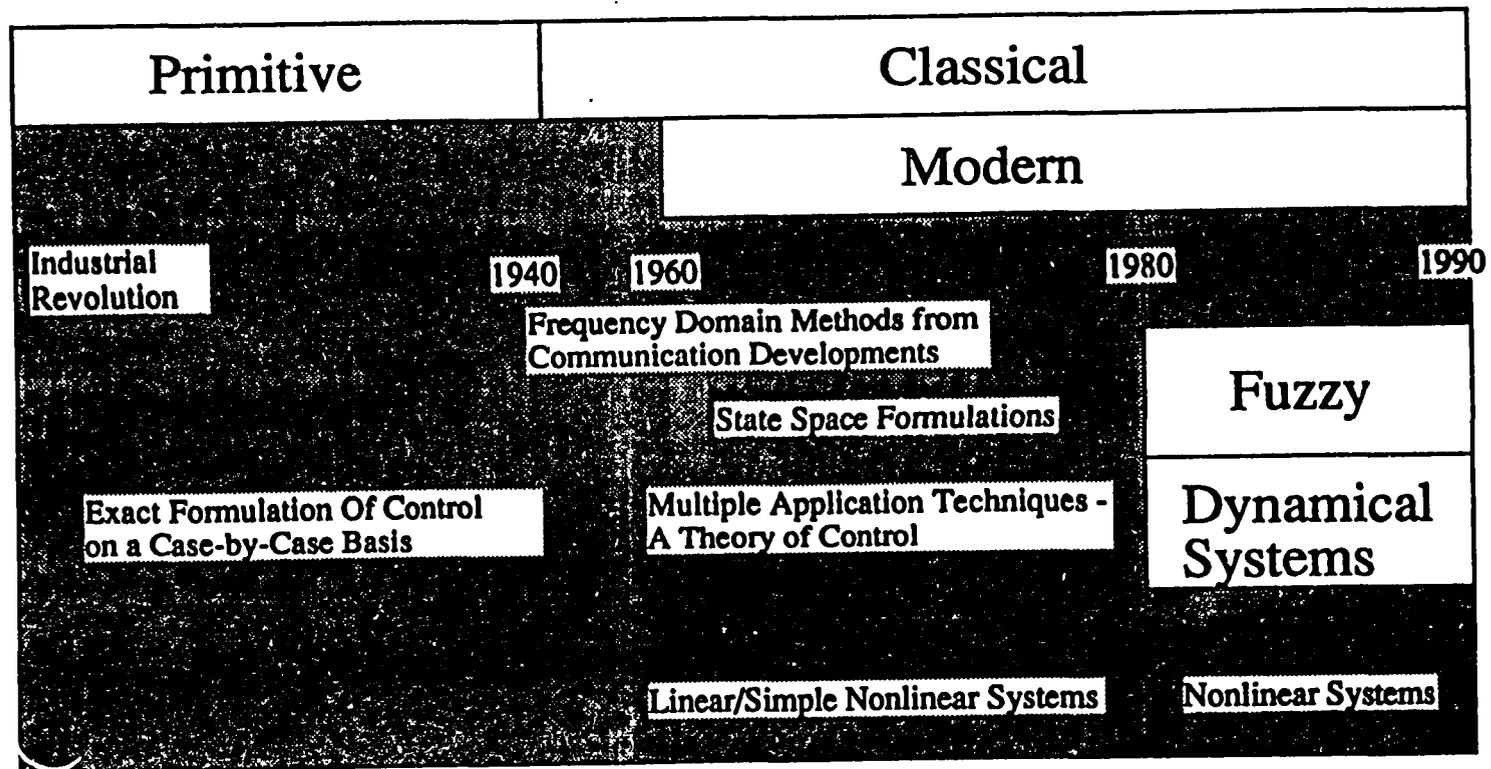
- A means by which a variable quantity or a set of variable quantities is made to conform to a prescribed norm or to vary in a prescribed way
- May be operated by electrical means, mechanical means, hydraulic means, pneumatic means, or a combination

22



CONTROL THEORY WAS FORMULATED IN THREE PHASES

MCDONNELL DOUGLAS



ISSUES IN THE DESIGN OF A CONTROL SYSTEM

MCDONNELL DOUGLAS

Stability and Transient Response

Response Time or Bandwidth

Observability

Controllability

8

Continuous or Sampled Data

Single or Multiple Control Loops

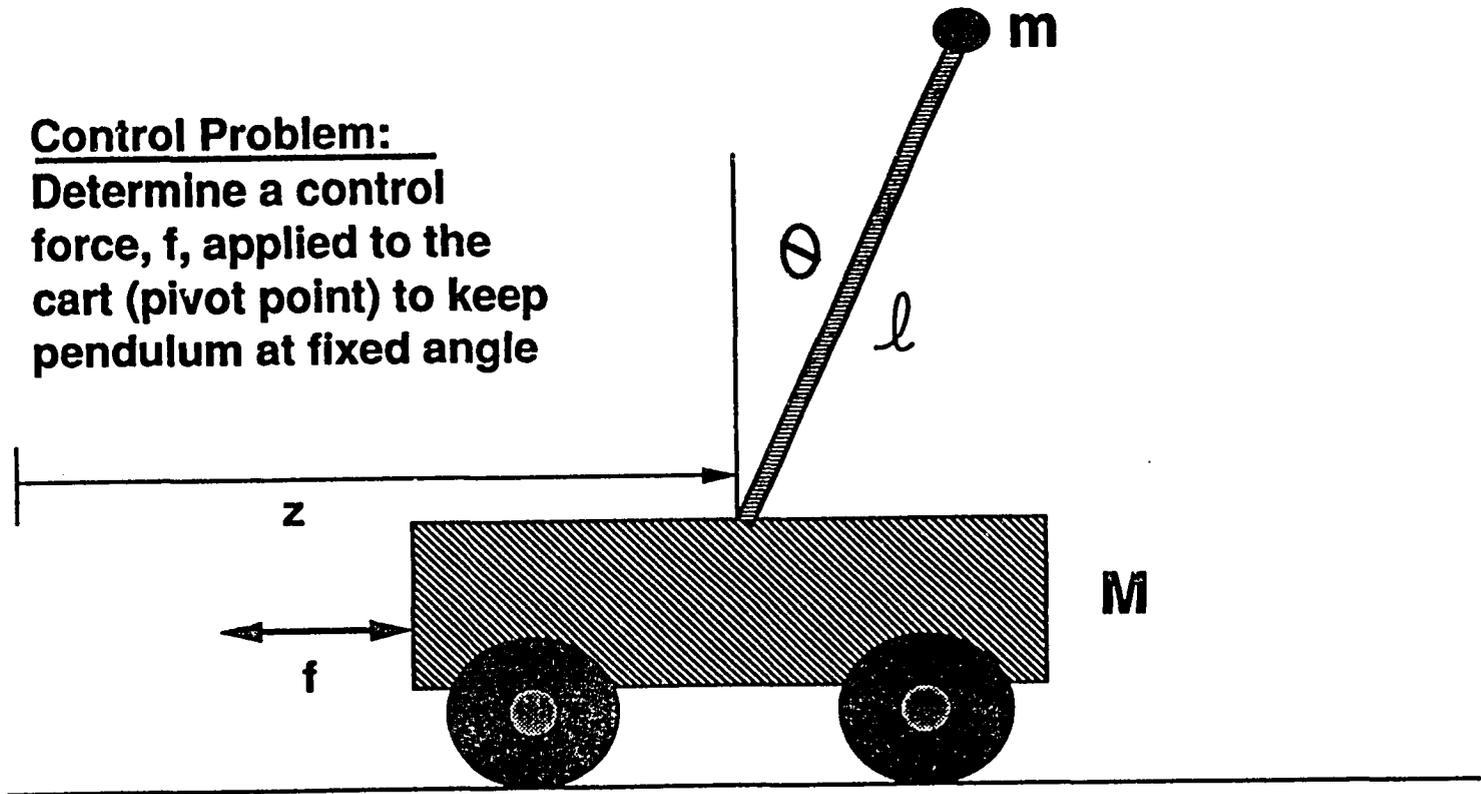
Optimizing or "Near-Optimal" Control

Fixed, Adaptive, or Learning Control

EXAMPLES: INVERTED PENDULUM

MCDONNELL DOUGLAS

Control Problem:
Determine a control force, f , applied to the cart (pivot point) to keep pendulum at fixed angle



STATE SPACE CONTROL FOR INVERTED PENDULUM

MC DONNELL DOUGLAS

Equations of Motion

State

$\begin{bmatrix} z \\ v \\ \theta \\ \omega \end{bmatrix}$

$$(M+m)\ddot{z} + m l \cos\theta \ddot{\theta} - m l \dot{\theta}^2 \sin\theta = f$$

$$m l \cos\theta \ddot{z} + m l^2 \ddot{\theta} - m g l \sin\theta = 0$$

58

State Space Description of Dynamical System

$$\dot{z} = v$$

$$\dot{v} = \frac{m l \omega^2 \sin\theta}{M + m \sin^2\theta} - \frac{m g \cos\theta \sin\theta}{M + m \sin^2\theta} + \frac{f}{M + m \sin^2\theta}$$

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \frac{g \sin\theta (M + m)}{l (M + m \sin^2\theta)} - \frac{m \omega^2 \sin\theta}{M + m \sin^2\theta} - \frac{f \cos\theta}{M + m \sin^2\theta}$$

LINEARIZED STATE SPACE

MCDONNELL DOUGLAS

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$$\dot{z} = v$$

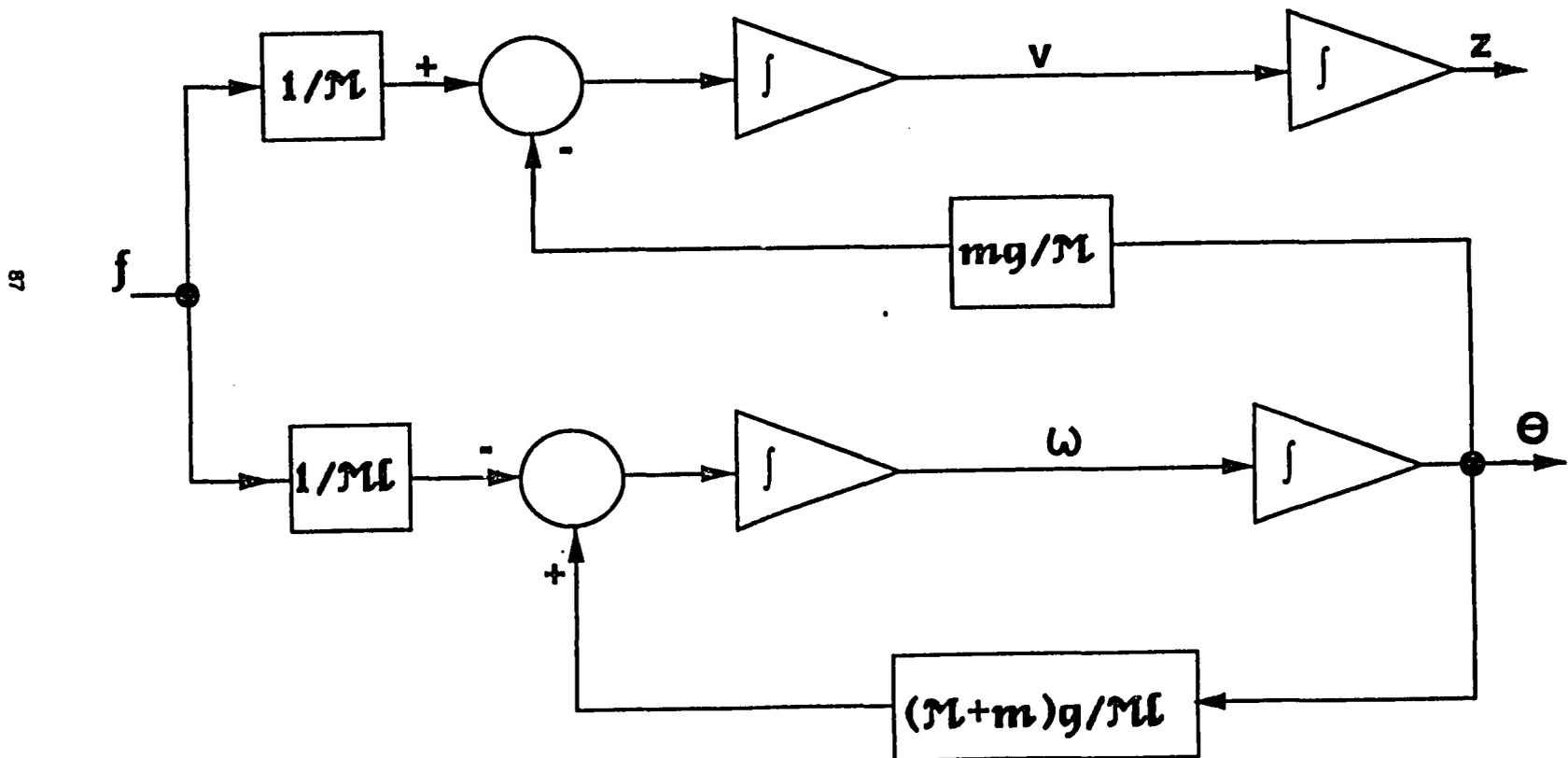
$$\dot{v} = \frac{f}{M} - \frac{mg\Theta}{M}$$

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \frac{(M+m)g\Theta}{Ml} - \frac{f}{Ml}$$

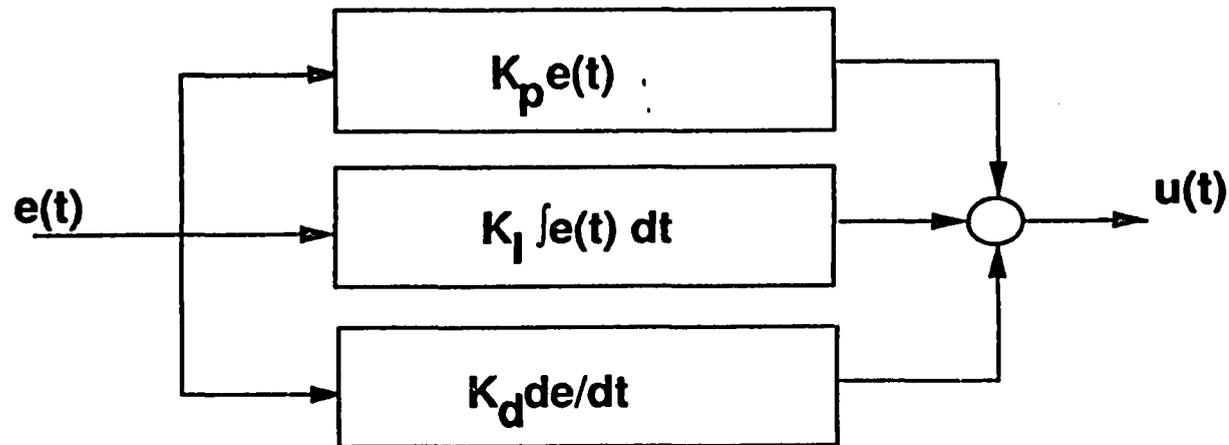
DYNAMICAL SYSTEM MODEL FOR INVERTED PENDULUM ON A CART

MCDONNELL DOUGLAS



PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) CONTROLLER

MCDONNELL DOUGLAS



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Proportional component reduces error
Integral component reduces steady state offset
Derivative component anticipates and reduces overshoots

ADVANTAGES OF USING CONVENTIONAL CONTROL

MCDONNELL DOUGLAS

- **Technology is well established**
- **Many control problems are well approximated by linear plants or can be handled with adaptive systems that perturb controller parameters**
- **Technology is mathematically based allowing general properties of controllers to be explored by a theoretical approach**

8

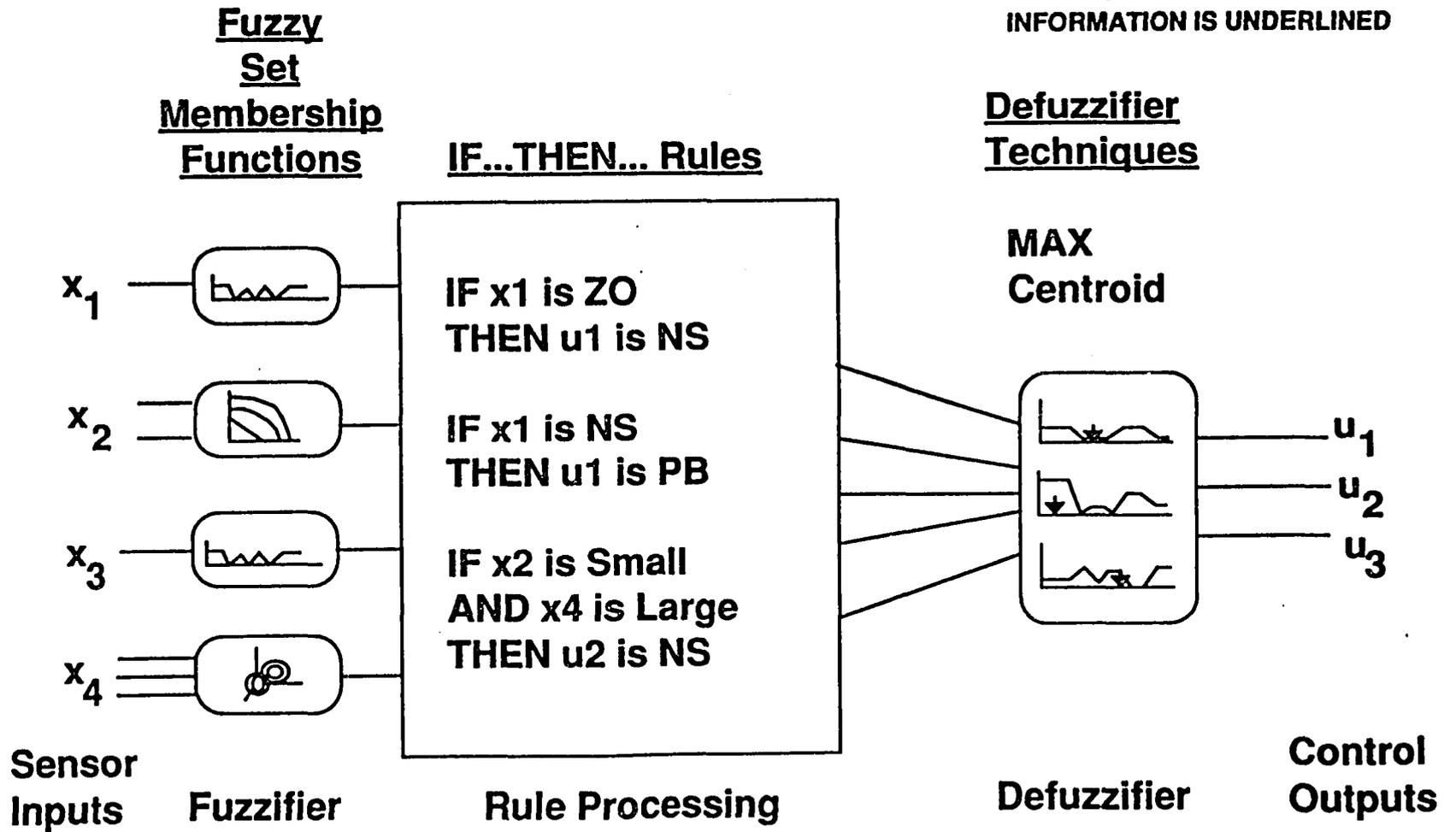
PROBLEMS WITH STATE SPACE CONTROL?

MCDONNELL DOUGLAS

- Model building stage is elaborate, iterative, error-prone, and time consuming
- A performance index that can be used for optimization must be formulated
- Actuators may be nonlinear
- Complex equipment may be poorly described by systems of differential equations but may be best described from experimental data or heuristics (rules of thumb or experience).
- Heuristics may be part of the operating procedure and may be based on mental models other than the physical models

FUZZY CONTROLLER OVERVIEW

DESIGNER-SUPPLIED
INFORMATION IS UNDERLINED



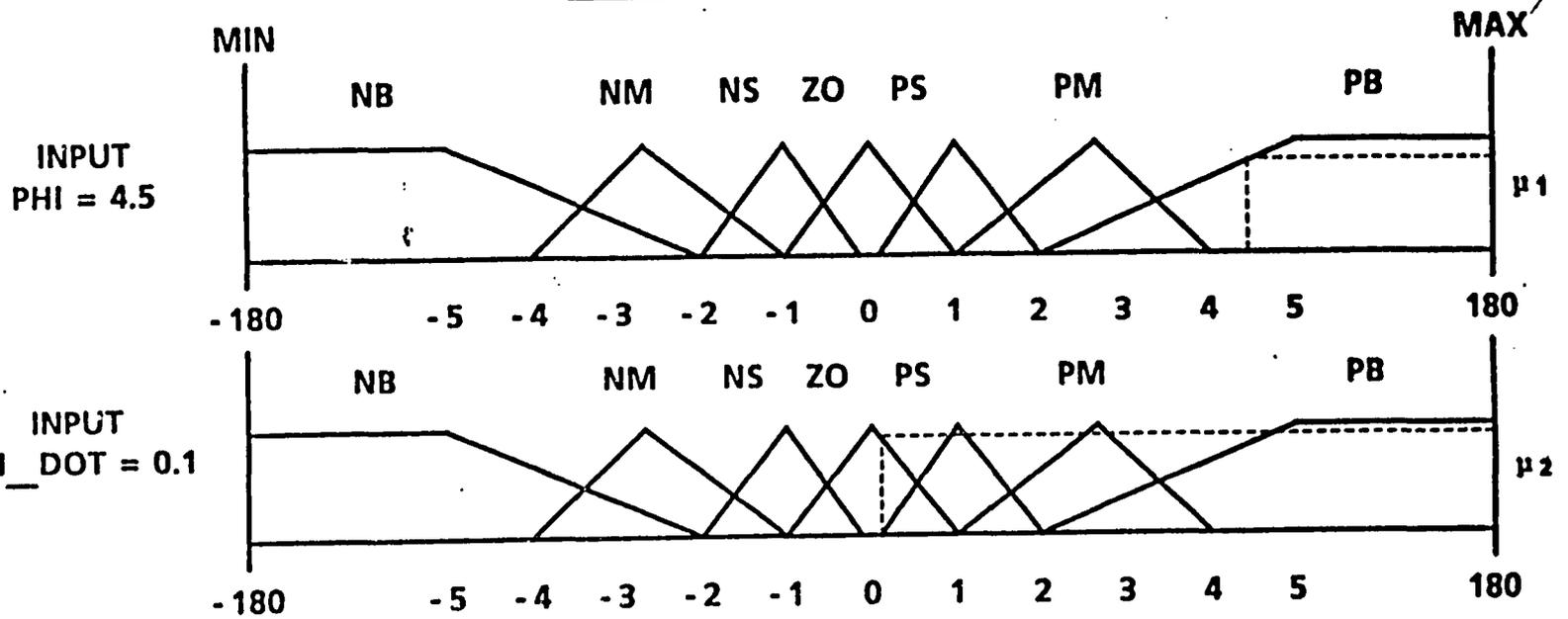
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APPROACH (CONT.)

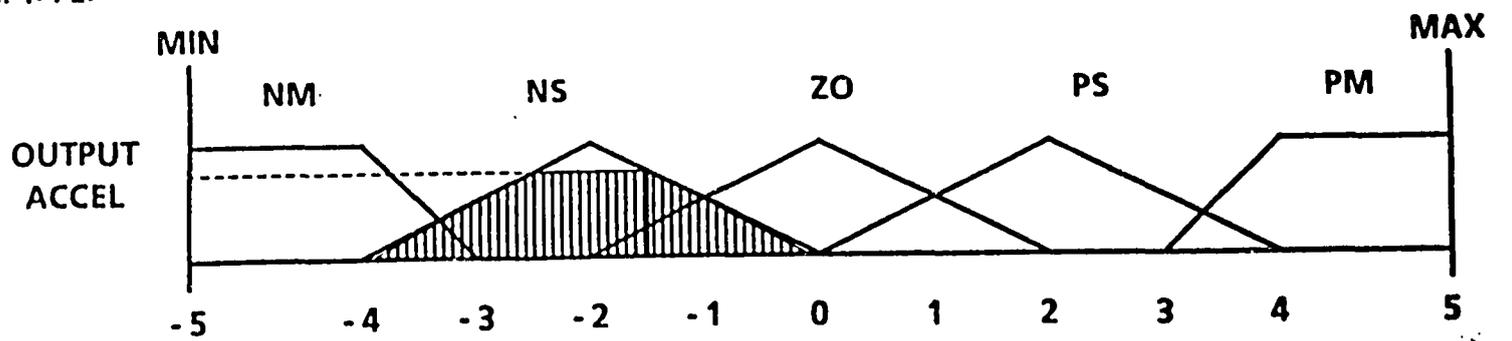
FUZZY PROCESSING



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RULE: IF (PHI EQ PB AND PHI_DOT EQ ZO) THEN ACCELERATION IS NS
 $\text{MIN}(\mu_1, \mu_2)$ IS APPLIED TO NS FUNCTION IN ACCELERATION



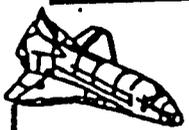
RULE BASE FOR FORCE ON INVERTED PENDULUM CART

MCDONNELL DOUGLAS

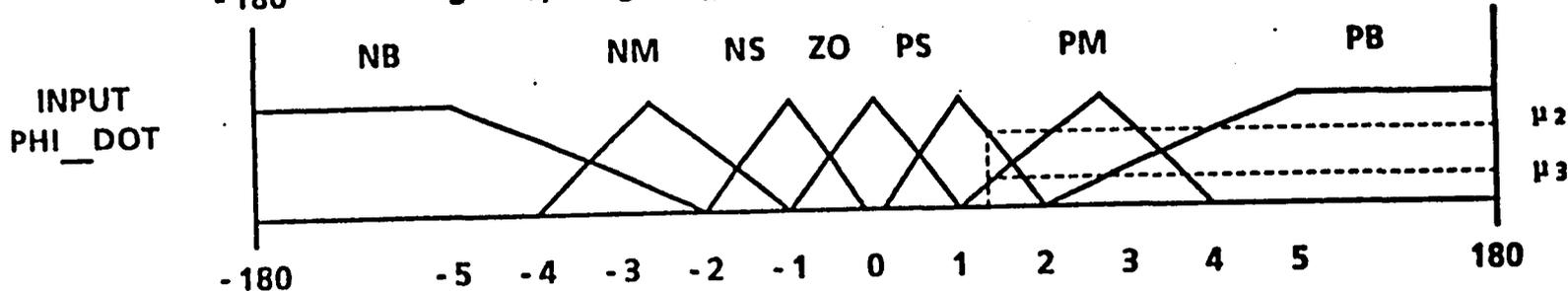
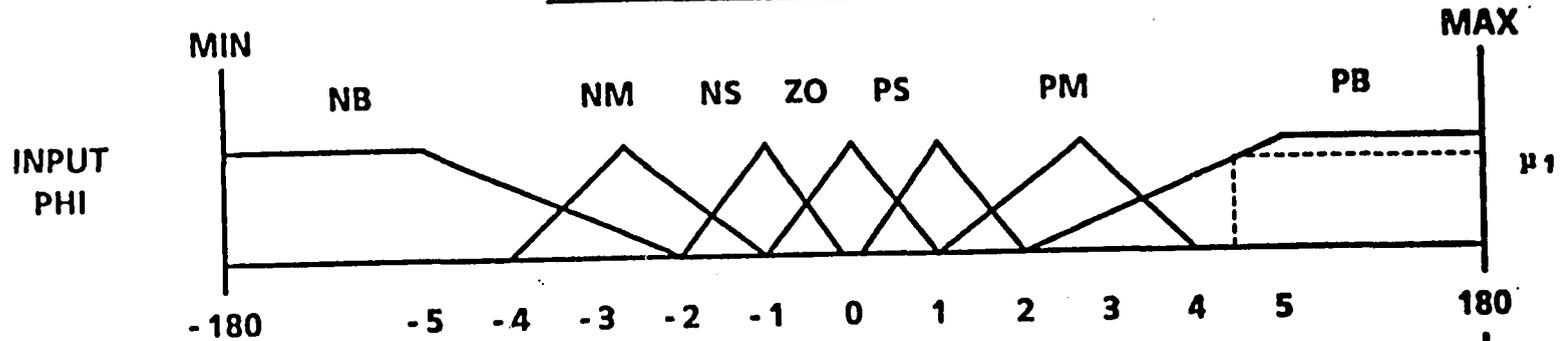
Fuzzy Table Inv Pend

		Angle				
		NB	NS	ZO	PS	PB
Angle Rate	PB	ZO	PB	PB	PB	PB
	PS	NB	ZO	PS	PB	PB
	ZO	NB	NS	ZO	PS	PB
	NS	NB	NB	NS	ZO	PB
	NB	NB	NB	NB	NB	ZO

Example Rule: IF Angle is PS AND Angle Rate is NS THEN Force is ZO

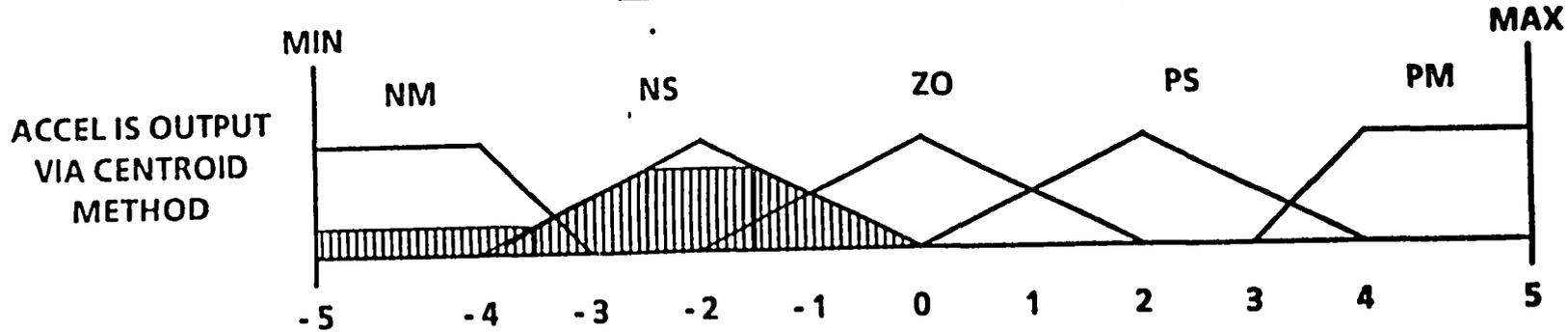


APPROACH (CONT.)
FUZZY PROCESSING



TWO RULES FIRE:

1. IF PHI IS PB AND PHI_DOT IS PS THEN ACCEL IS NS
2. IF PHI IS PB AND PHI_DOT IS PM THEN ACCEL IS NM



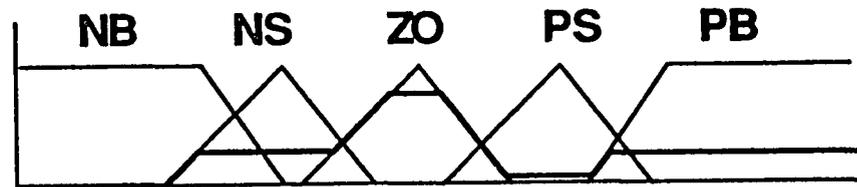
SPACECRAFT SOFTWARE DIVISION

FUZZY RULE PROCESSING

MCDONNELL DOUGLAS

USE MAXIMUM FOR LOGICAL OR

IF ... THEN u is NS .3
IF ... THEN u is ZO .8
IF ... THEN u is PS .1
IF ... THEN u is PB .3



USE MINIMUM FOR LOGICAL AND

8

Rule: IF x1 is NS AND x2 is ZO
THEN u is PS

Facts:

x1 is NS 0.2

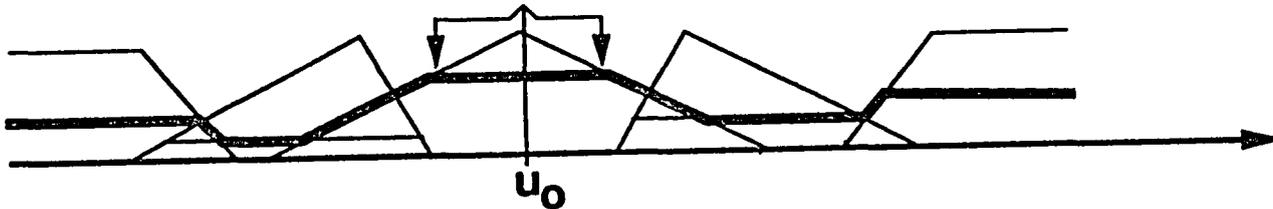
x2 is ZO 0.8 \Rightarrow u is PS 0.2

Other options exist for combining logical connectives but these preserve all results from normal set theory except exclusion law:
 $A \text{ AND NOT } A = \emptyset$

DEFUZZIFICATION

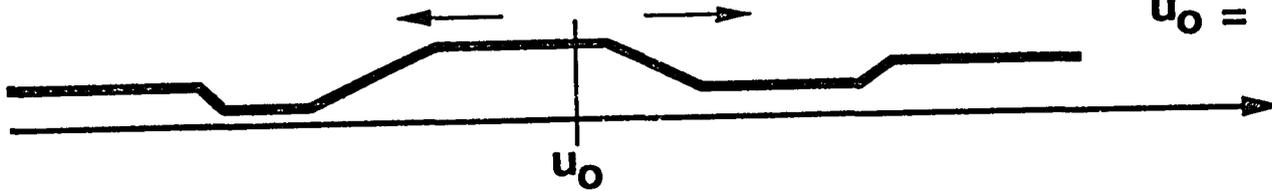
MCDONNELL DOUGLAS

MAX Procedure



8

Centroid Procedure



$$u_0 = \frac{\int u \mu^3 U du}{\int U du}$$

Indexed MAX or Centroid Procedure

Same as above except use only points > threshold value

FUZZY CONTROLLER ADVANTAGES

MCDONNELL DOUGLAS

- **Can exploit heuristic knowledge of operation of controlled systems. This includes physical intuition.**
- **Can accomodate small changes in system or controller parameters. This are the aging effect and nonlinear effects such as flexibility of beams**
- **Experience has been that these techniques seem to handle nonlinearity well**
- **Tools have been developed to assist in studying and building fuzzy controllers in short times**
- **The development of fuzzy chips has provided computationally capable platforms on which to build the controller, independent of general purpose computers used for spacecraft control**

REMAINING ISSUES FOR FUZZY CONTROL

MCDONNELL DOUGLAS

- **Issues such as stability, observability, and controllability raised in servomechanism and state space control are not yet in comparable state of development. This may limit initial applicability to noncritical applications**
- **Definition of membership functions is arbitrary and controller designer dependent.**
- **Procedures for selecting membership functions and defuzzifier options are not firmly established in the control community**
- **There are limited sources for fuzzy control chips**

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CLAS

NASA/ARC Proposed Training In Intelligent Control

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**Hamid R. Berenji
Sterling Federal Systems
Artificial Intelligence Research Branch
NASA Ames Research Center
Moffett Field, CA**

**Workshop on Fuzzy Control Systems
and Space Station Applications, Nov. 14-15, 1990
Huntington Beach, CA**

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Outline

1- General courses:

- Intelligent Control:

- Fuzzy Logic in Control
- Neural Networks in Control
- Artificial Intelligence in control
- Hybrid Approaches

- Uncertainty Management in Artificial Intelligence

2- Hands on Experience

- Experiments with robot arm (simulated and real hardware)
- Cart-pole Balancing

3- Ames Associate Programs

- Spending time at Ames

4- Collaborative work on development of fuzzy controllers

Intelligent Control Course (Fuzzy Logic Control)

- **The basics of Fuzzy Set Theory**
- **Fuzzy Sets Operations**
- **Architecture of Fuzzy Logic Controllers**
 - **Coding the inputs**
 - **Setting up the control knowledge base**
 - **Conflict resolution and decision making**
 - **Decoding the outputs**
- **Successful applications**
 - **Lab Prototypes**
 - **Commercial applications**
- **Advantages and disadvantages**

Intelligent Control Course (Neurocontrol)

- The basics of artificial neural networks
- Artificial Neural Networks:
 - Interactive Activation Model
 - General Error Back-Propagation Method
 - ADALINE and LMS Algorithm
 - Cerebellar Model Arithmetic Computer (CMAC) Model
 - Competitive Learning Models
- Advantages and disadvantages of Neurocontrol
- Applications

Intelligent Control Course (AI-based Approaches)

- The basics of Qualitative Reasoning**
- The basics of rule-based control**
- Applications**
- Advantages and Disadvantages**

Intelligent Control Course (Hybrid Approaches)

- NeuroFuzzy Control**
 - Competitive Learning**
 - Fuzzy Control with reinforcement learning**
- Hierarchical control models**

Hands on Experience

- **Control experiments with**
 - **A simulated model of the robotics arm**
 - **The PUMA robot**
 - **A simulated model of the cart-pole balancing**
 - **The laboratory cart-pole balancing hardware system**
 - **The rendezvous-docking simulator for the Space Shuttle**
- **Computing facilities to use the fuzzy computer chips**
 - **interfaced with a SUN work station**

Ames Associate Program

- Interested participants can spend time at Ames
 - Have to donate their time
- Can utilize the Ames facilities
- From two months to a year

Issues for the Panel Discussion

- Is Fuzzy Logic Control appropriate for this domain?
 - Does an analytical mathematical model exist for this problem or can it be developed within a reasonable time?
 - Who are the experts in this domain? how can their knowledge be modeled?
- What steps (beyond the general methodology) have to be taken in order to develop a fuzzy logic controller for this problem?
- How important are the stability issues? how can we validate the controller?

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INCLAS

OVERVIEW OF FUZZY LOGIC

by

Xiwen Ma
APT Instruments

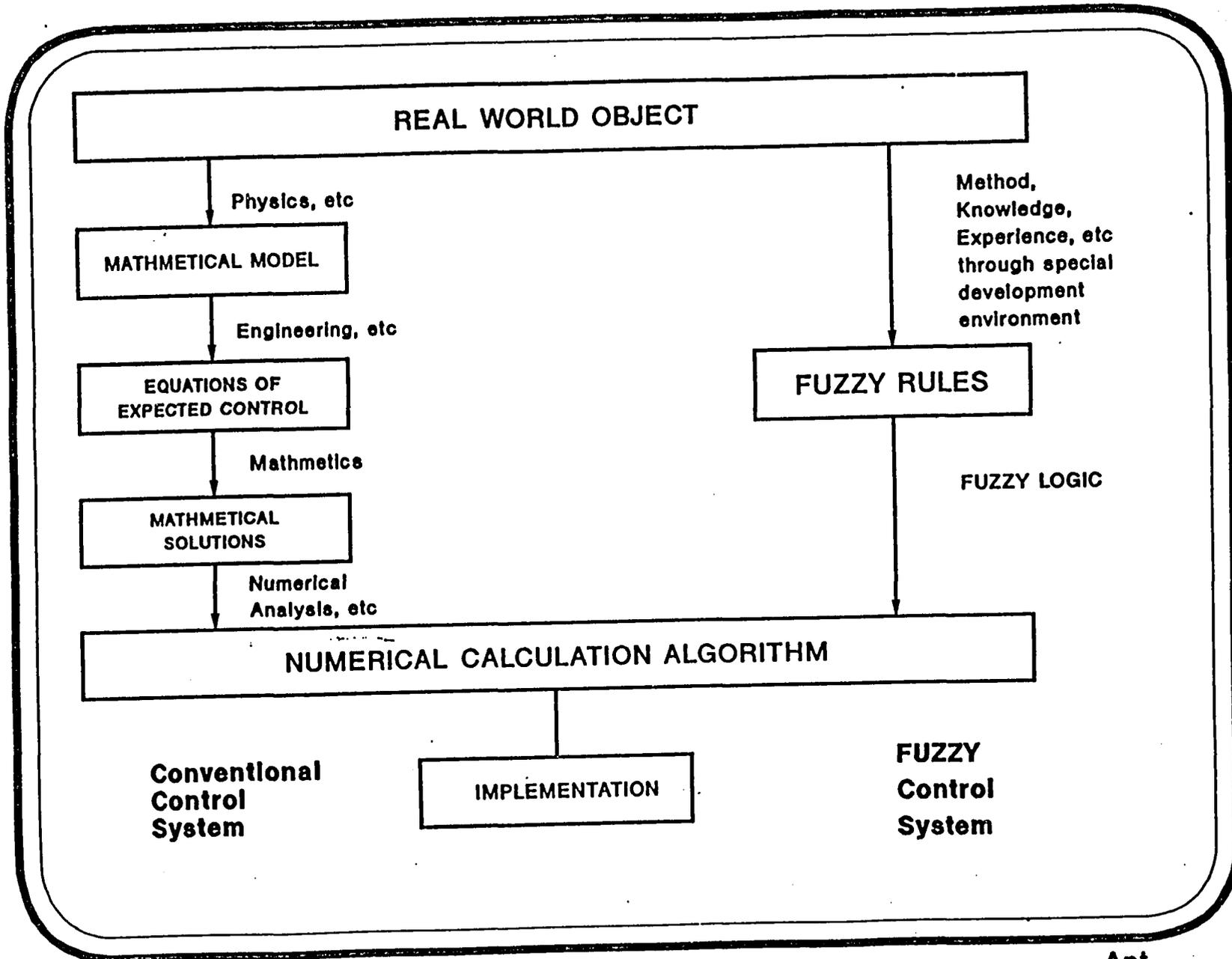
Fuzzy Logic Workshop
14-15 November 1990

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163680,
P-18



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Apt

Overview of FUZZY Logic

DESCRIBE CONTROL FACTORS:

SPEED: stopped/very slow/slow/fast/very fast

Distance to DESTINATION: at/very near/near/far/very far

ACTIONS:

BRAKE: no/slight/medium/full

THROTTLE: no/slight/medium/full

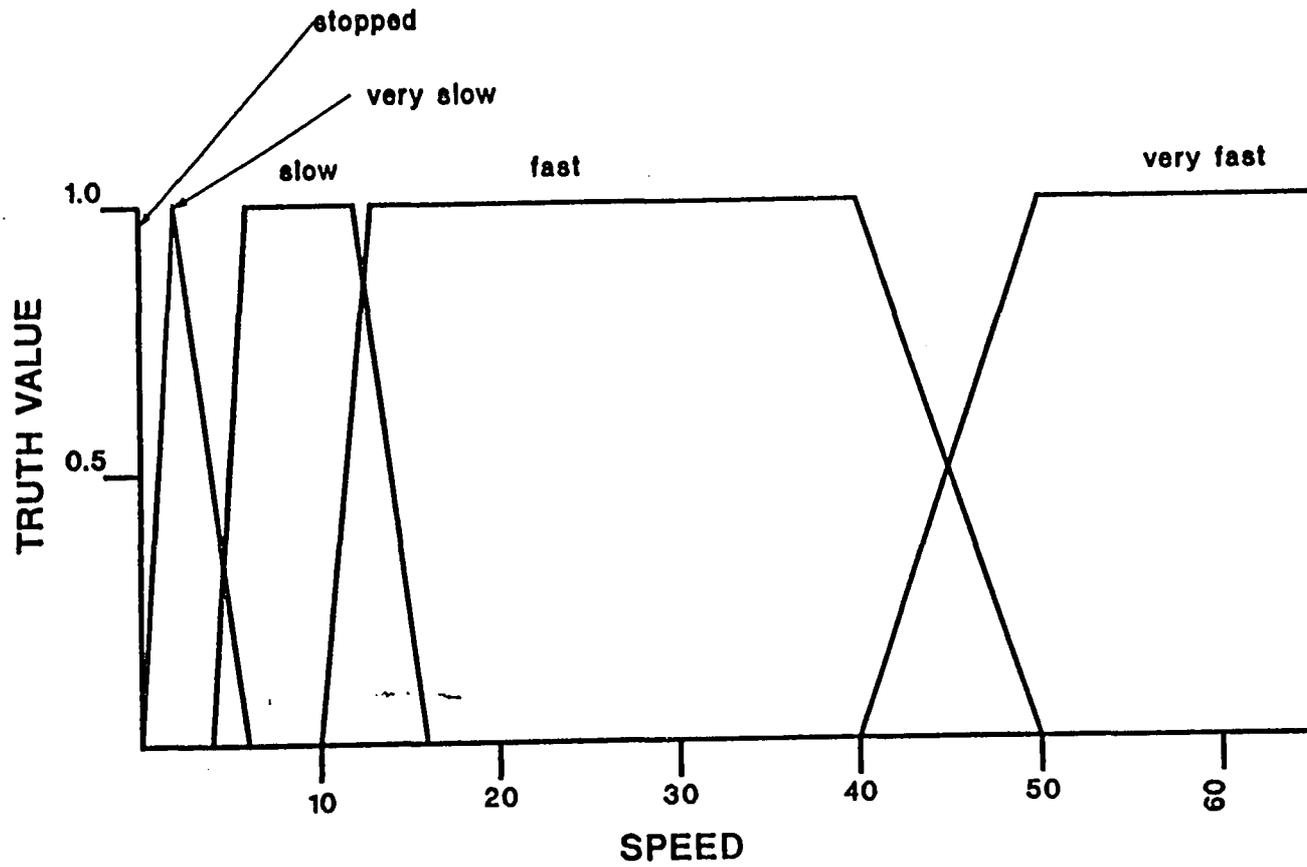
RULES:

1. If **SPEED** is slow and **DESTINATION** is very near
then medium **BRAKE**
2. If **SPEED** is very slow and **DESTINATION** is very near
then medium **BRAKE**
3. If **SPEED** is very slow and **DESTINATION** is at
then full **BRAKE**

....

10. If **SPEED** is slow and **DESTINATION** is very far
then medium **THROTTLE**

Overview of FUZZY Logic



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Overview of FUZZY Logic

if

SPEED is slow and

90%

DESTINATION is near

25%

then

medium BRAKE

25%

$$t(p \& Q) = \min(t(P), t(Q))$$

FUZZY logic: Combining Conditions

Overview of FUZZY Logic

1. If then medium BRAKE 75%

2. If then medium BRAKE 10%

medium BRAKE 75%

$$t(A) = \max(t_{R1}(A), t_{R2}(A))$$

FUZZY logic: Combining Actions

Overview of FUZZY Logic

1. If medium BRAKE 75%
2. Ifmedium BRAKE 10%

.
. .
. .
. .

10. If no BRAKE 50%

slight BRAKE

FUZZY logic: defuzzification ----- averaging

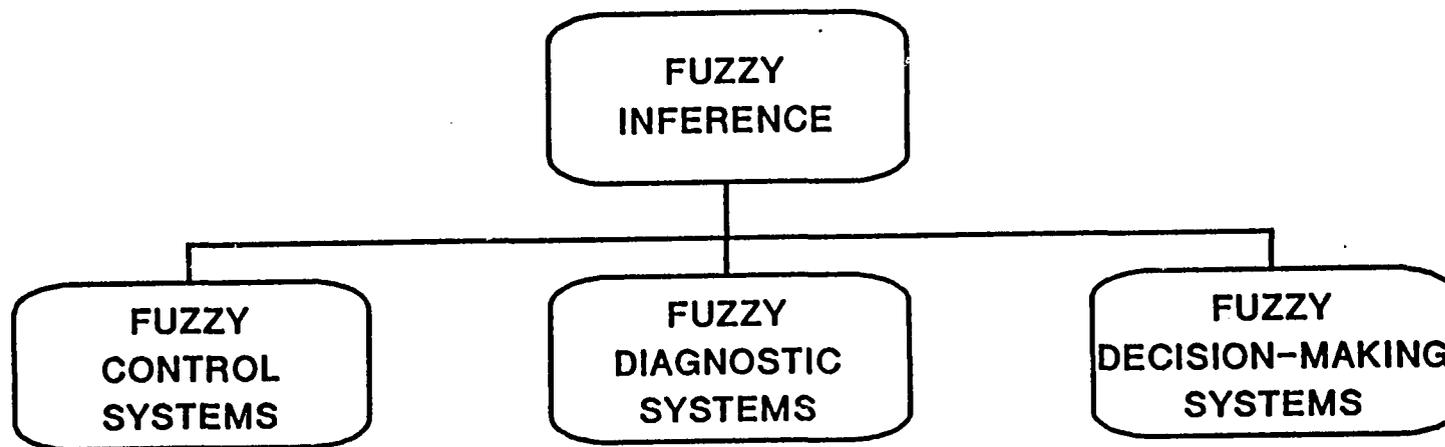
Overview of FUZZY Logic

switch off	75%
switch on	10%

switch off

FUZZY logic: defuzzification ---- most likely

Overview of FUZZY Logic



FUZZY Inference is a general method

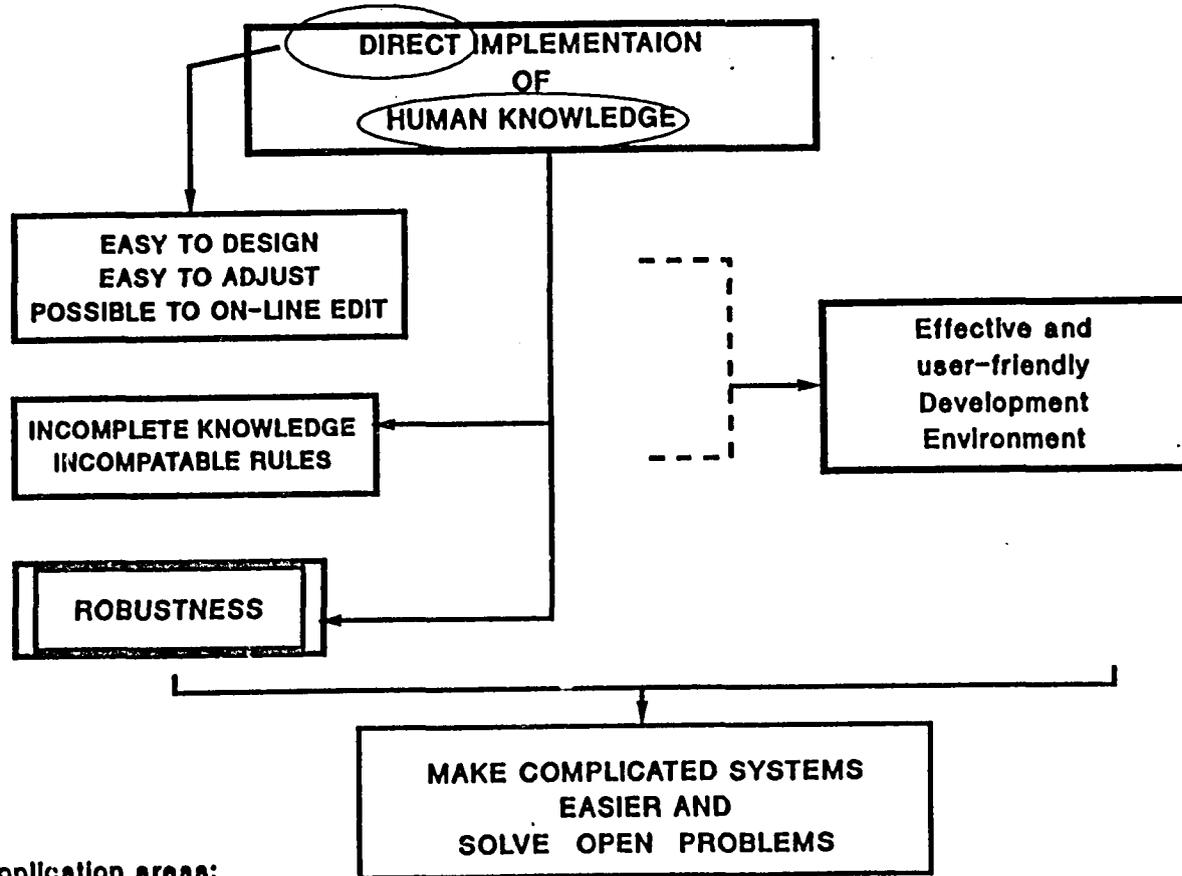
Overview of FUZZY Logic

Example of FUZZY diagnostic system

AIRCRAFT ENGINE-FAULT DIAGNOSTIC SYSTEM (CHINA)

- ✓ 12 subsystems
- ✓ 800 factors
- ✓ 600 faults
- ✓ 89% successful in 3000 test cases
- ✓ 5 minutes computation

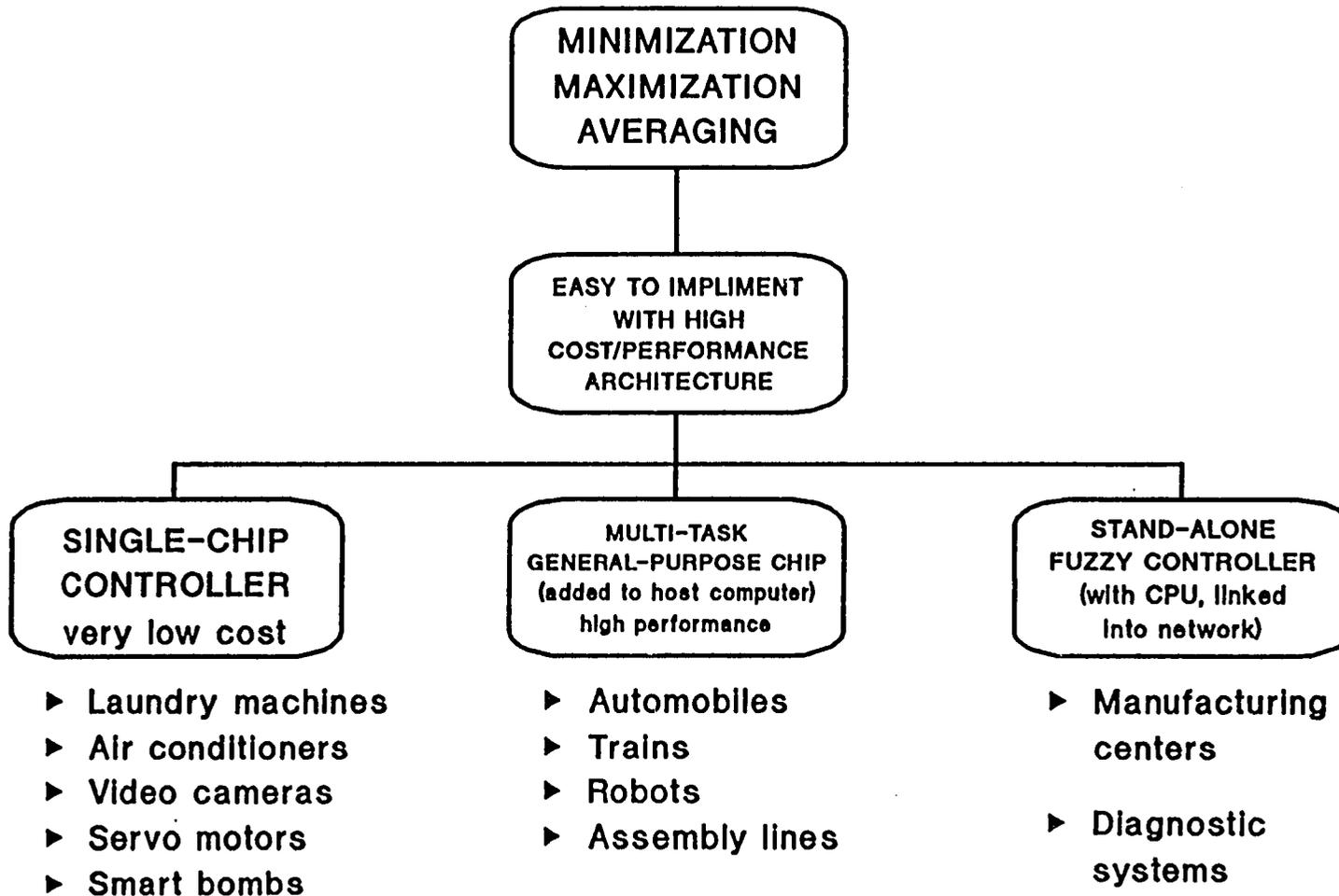
Overview of FUZZY Logic



Application areas:

- ◆ Air/ground traffic control
- ◆ Multi-variate diagnostics
- ◆ Chemical Engineering
- ◆ Aeronautics/Astronautics

Overview of FUZZY Logic



Simplicity of Computation

Overview of FUZZY Logic

UNIFIED FRAMEWORK
FOR
CONVENTIONAL (BINARY) LOGIC
AND
FUZZY LOGIC

EASY TO ADD
PROTECTIVE RULES
(TO AVOID FATAL SITUATIONS)

Overview of FUZZY Logic

AN ALTERNATIVE TO
CONVENTIONAL
NUMERIC METHODS

FUZZY VERSIONS OF:

INTERPOLATION

FINITE ELEMENT METHOD

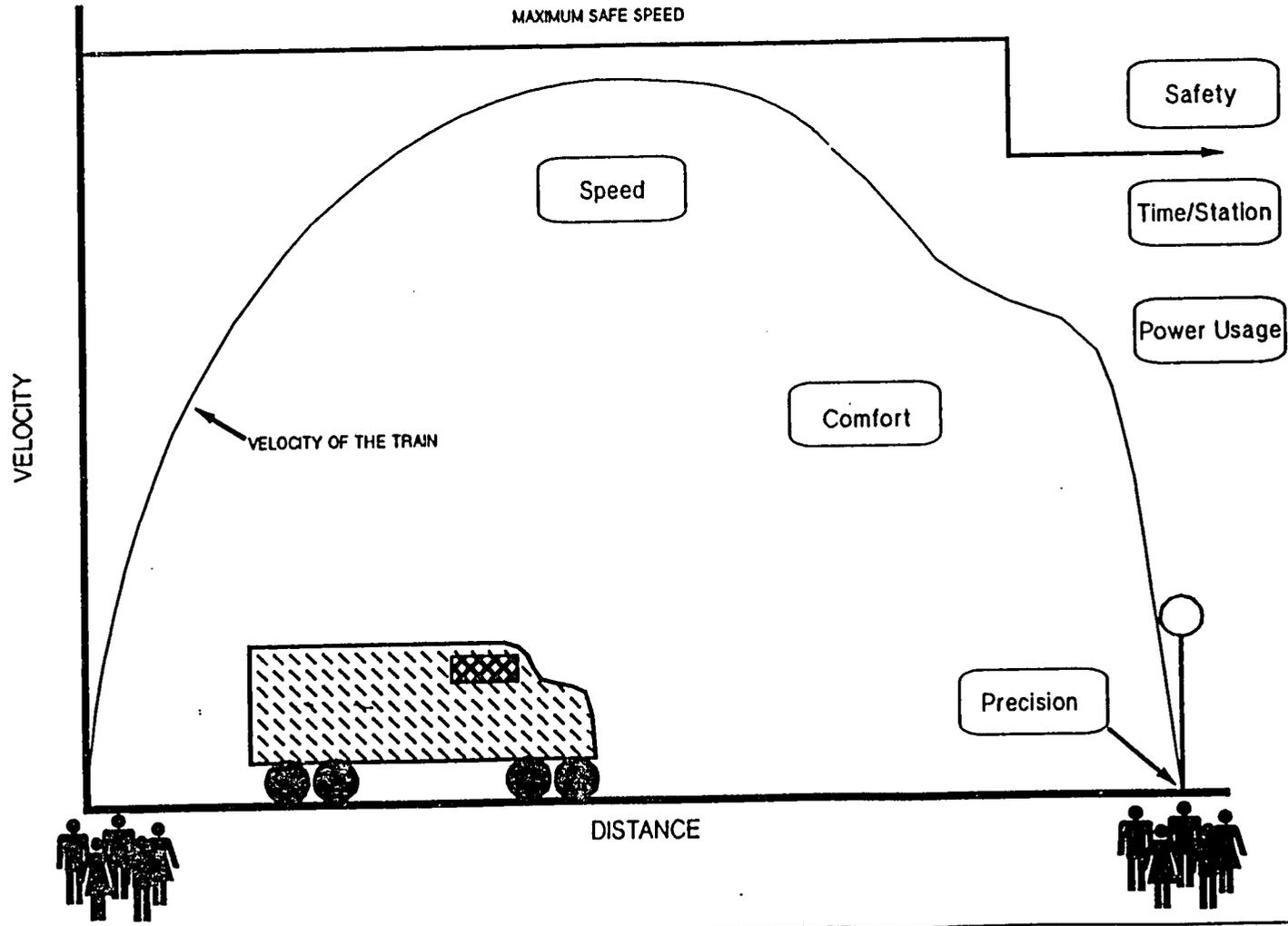
LINEAR/NON-LINEAR
PROGRAMMING

APPLICATIONS:

- ✓ COMPUTER GRAPHICS
- ✓ GRAPHICS PRINTERS
- ✓ CIVIL ENGINEERING

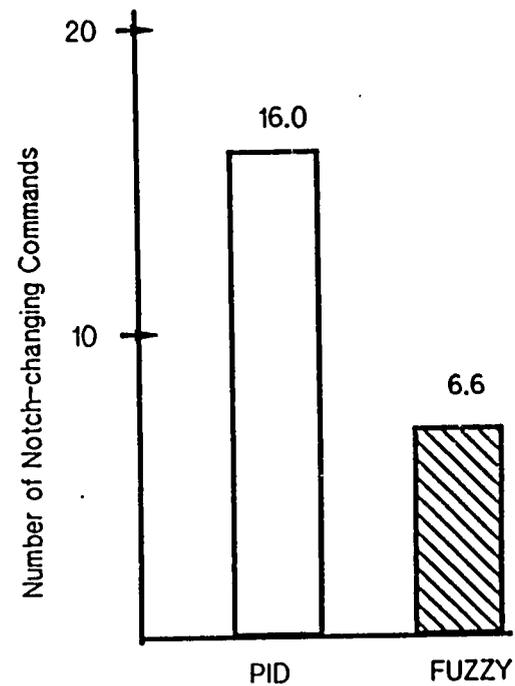
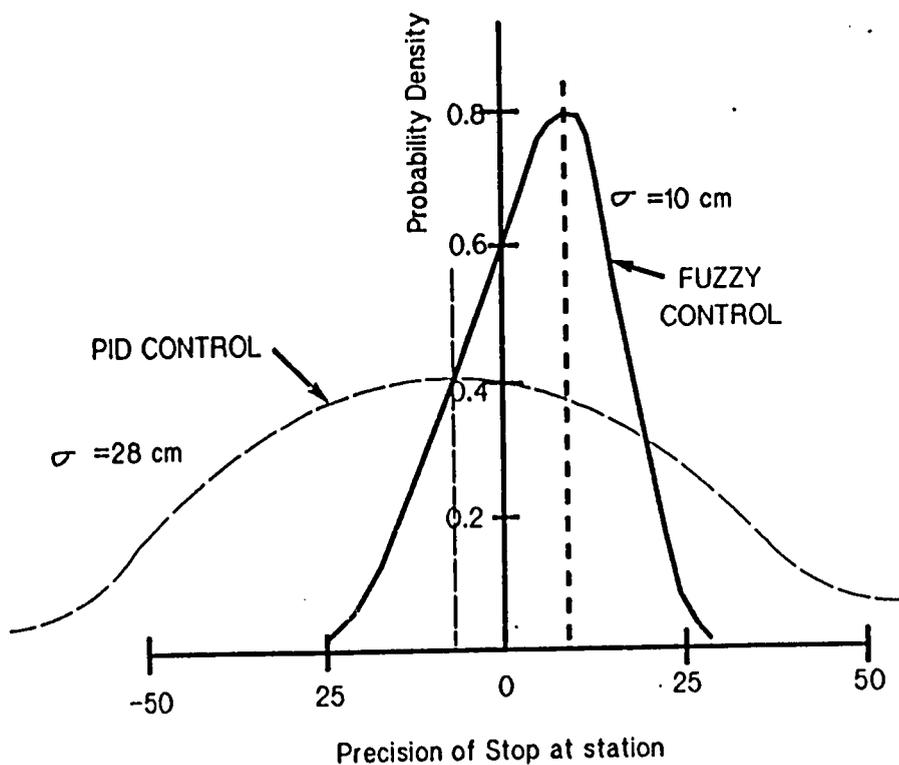
CASE STUDY - SENDAI SUBWAY SYSTEM

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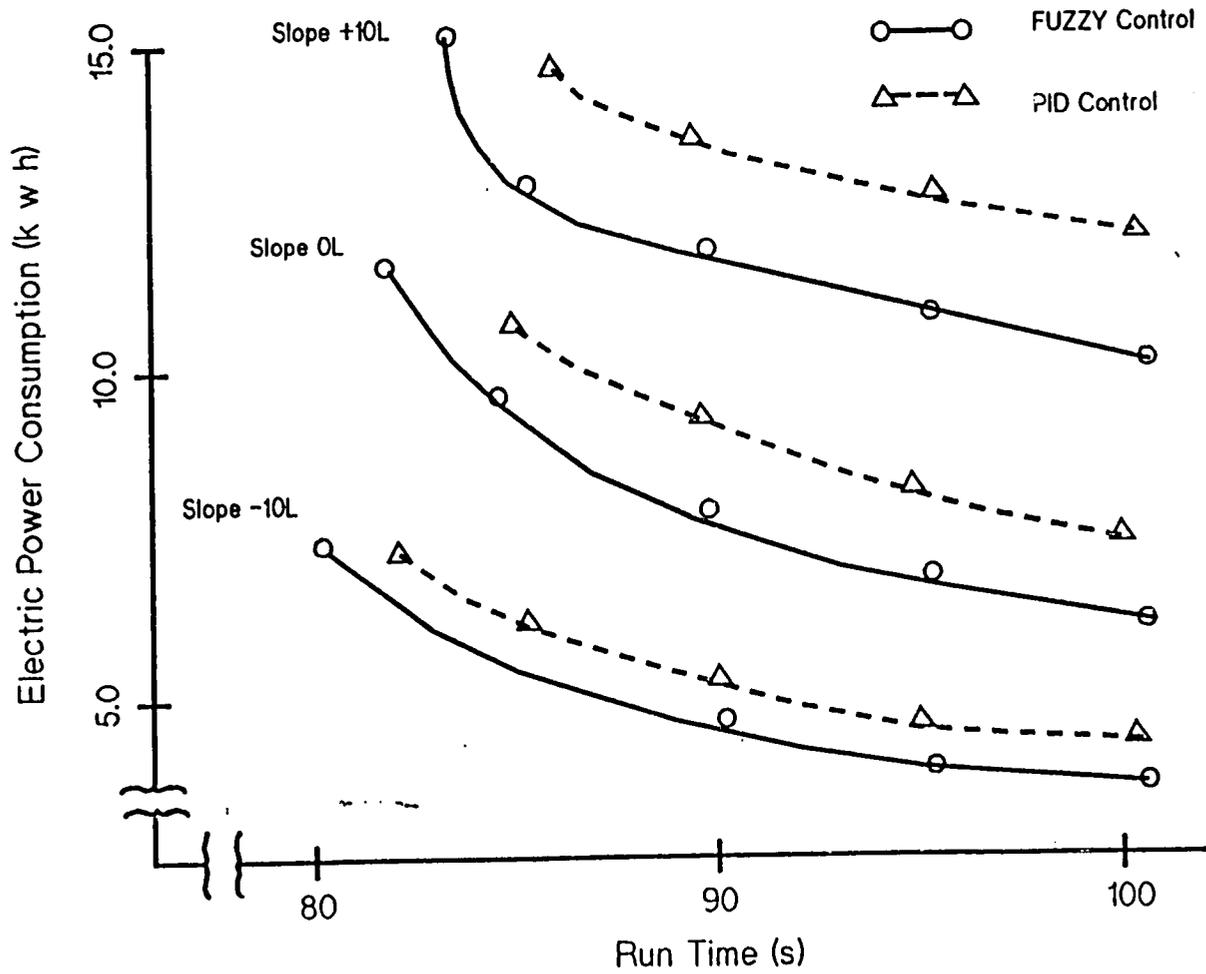
CASE STUDY - SENDAI SUBWAY SYSTEM

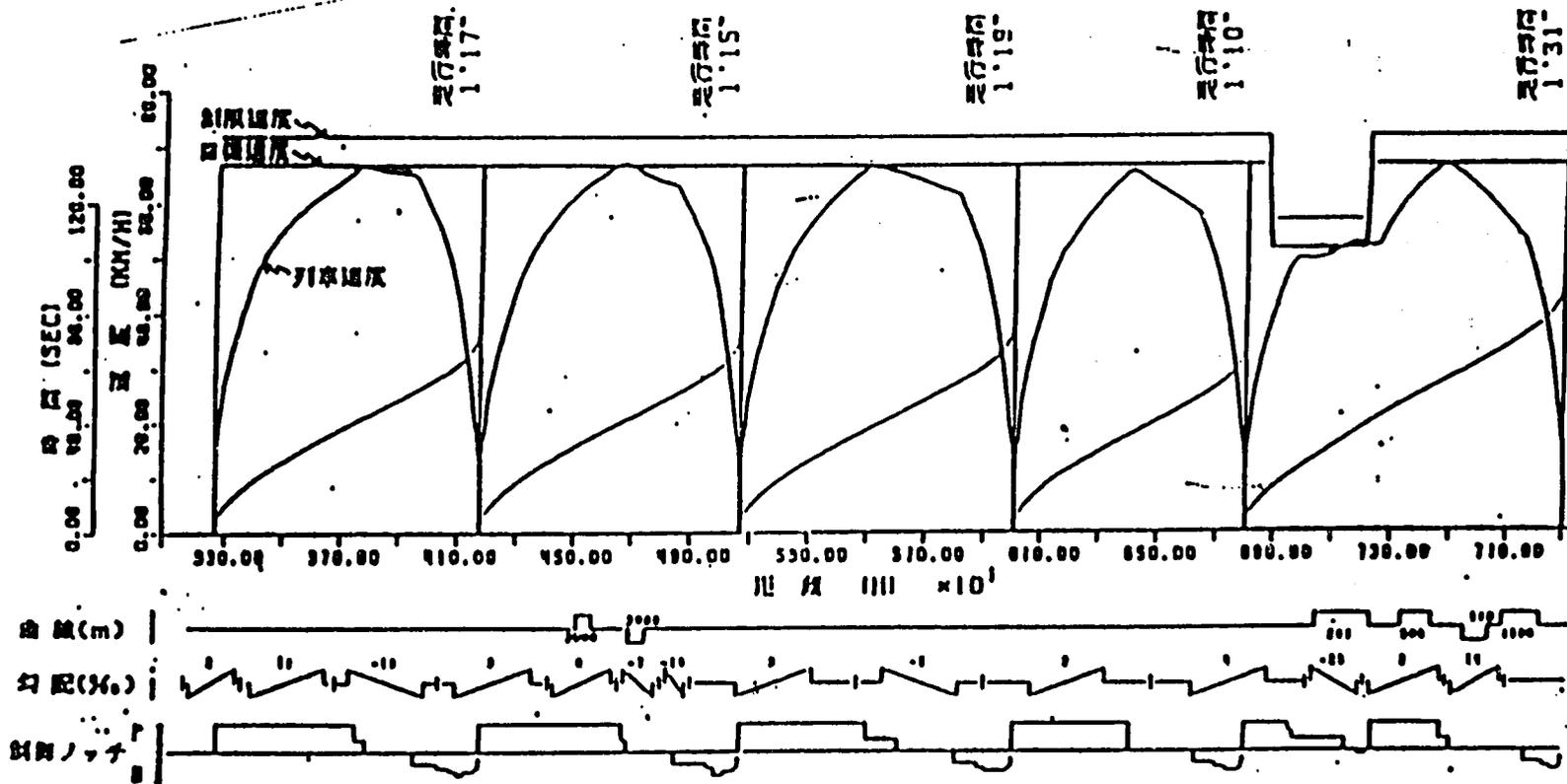
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Evaluation of precision and comfort

Evaluation of Power Consumption and Run Time





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INTRODUCTION TO FUZZY SET THEORY

by

Bart Kosko
University of Southern California

Fuzzy Logic Workshop
14 November 1990

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p. 191

CHAPTER 1

NEURAL NETWORKS AND FUZZY SYSTEMS

From causes which appear similar, we expect similar effects. This is the sum total of all our experimental conclusions.

David Hume

*An Inquiry Concerning Human
Understanding*

A learning machine is any device whose actions are influenced by past experiences.

Nils Nilsson

Learning Machines

Man is a species that invents its own responses. It is out of this unique ability to invent, to improvise, his responses that cultures are born.

Ashley Montagu

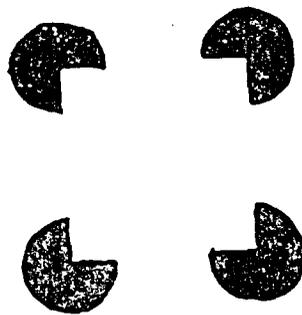
Culture and the Evolution of Man

NEURAL AND FUZZY MACHINE INTELLIGENCE

This book examines how adaptive systems respond to stimuli. Systems map inputs to outputs, stimuli to responses. Adaptation or learning describes how data changes the system, how sample data changes system parameters, how training changes behavior.

Neural Pre-Attentive and Attentive Processing

The human visual system behaves as an adaptive system. Consider how it responds to this stimulus pattern:



What do we see when we look at the Kanizsa [1976] square? We see a square with bright interior. We see illusory boundaries. Or do we? We recognize a bright square. Technically we do not see it, because it is not there.

The Kanizsa square exists in our brain, not "out there" in physical reality on the page. Out there only four symmetric ink patterns stain the page.

In the terminology of eighteenth-century philosopher Immanuel Kant [1783-87], the four ink stains are *noumena*, "things in themselves." Light photons bounce off the noumena and stimulate our surface receptors, retinal neurons in this case. The noumena-induced sensation produces the Kanizsa-square *phenomenon* or perception in our brain. There would be no Kanizsa squares in the spacetime continuum without brains or brainlike systems to perceive them.

Today we understand many of the neural mechanisms of perception that Kant could only guess at. The realtime interaction of millions of competing and cooperating neurons produces the Kanizsa square illusion [Grossberg, 1987], and everything we "see."

We take for granted our high-speed, distributed, nonlinear, massively parallel pre-attentive processing. In our visual processing we pay no attention to how we segment images, enhance contrasts, or discount background luminosity. When we process sound we pay no attention to how our cochleas filter out high-frequency signals [Mead, 1989] or how our auditory cortex breaks continuous speech into syllables and words, compensates for rhythmic changes in speech duration, and detects and often corrects errors in pronunciation, grammar, and meaning. We likewise ignore our realtime pre-attentive processing in the other sense modalities, in smell, taste, touch, and balance.

We experience these pre-attentive phenomena, but we ignore them and cannot control or completely explain them. Natural selection has ensured only that we perform them, ceaselessly and fast.

Attention precedes recognition. We recognize segmented image pieces and parsed speech units. An emergent "search light," perhaps grounded in thalamic neurons [Crick, 1984], seems to selectively focus attention in as few as 70 to 100 milliseconds. We look, see, pay attention, then recognize.

Neural network theory studies both pre-attentive and attentive processing of stimuli. This leaves unaddressed the higher cognitive functions involved in reasoning, decision making, planning, and control. The asynchronous, nonlinear neurons and synapses in our brain perform these functions under uncertainty. We reason with scant evidence, vague concepts, heuristic syllogisms, tentative facts, rules of thumb, principles shot through with exceptions, and an inarticulate pantheon of inexact intuitions, hunches, suspicions, beliefs, estimates, guesses, and the like.

Natural selection evolved this uncertain cognitive calculus. Our cultural conditioning helps refine it. A fashionable trend in the West has been to denigrate this uncertainty calculus as illogical, unscientific, and nonrigorous. We even call it "fuzzy reasoning" or "fuzzy thinking." Modern philosophers [Churchland, 1981] often denigrate the entire cognitive framework as "folk psychology."

Yet we continue to use our fuzzy calculus. With it we run our lives, families, careers, industries, hospitals, courts, armies, and governments. In all these fields we employ the products of exact science, but as tools and decision aids. The final control remains fuzzy.

FUZZINESS AS MULTIVALUEDNESS

Fuzzy theory holds that all things are matters of degree. It mechanizes much of our "folk psychology." Fuzzy theory also reduces black-white logic and mathematics to special limiting cases of gray relationships. Along the way it violates black-white "laws of logic," in particular the law of noncontradiction *not-(A and not-A)* and the law of excluded middle *either A or not-A*, and yet resolves the paradoxes or antinomies [Kline, 1980] that these laws generate. Does the speaker tell the truth when he says he lies? Is set *A* a member of itself if *A* equals the set of all sets that are not members of themselves? Fuzziness also provides a fresh, and deterministic, interpretation of probability and randomness.

Mathematically fuzziness means multivaluedness [Rosser, 1952; Rescher, 1969] and stems from the Heisenberg position-momentum uncertainty principle in quantum mechanics [Birkhoff, 1936]. Three-valued fuzziness corresponds to truth, falsehood, and indeterminacy, or to presence, absence, and ambiguity. Multivalued fuzziness corresponds to degrees of indeterminacy or ambiguity, partial occurrence of events or relations.

Bivalent Paradoxes as Fuzzy Midpoints

Consider the bivalent paradoxes again. A California bumpersticker reads TRUST ME. Suppose instead a bumpersticker reads DON'T TRUST ME. Should we trust the driver? If we do, then, as the bumpersticker instructs, we do not. But if we don't trust the driver, then, again in accord with the bumpersticker, we do trust the driver. The classical liar paradox has the same form. Does the liar from Crete lie when he says that all Cretans are liars? If he lies, he tells the truth. If he tells the truth, he lies. Russell's barber is a man

in a town whose advertises his services with the logo "I shave all, and only, those men who don't shave themselves." Who shaves the barber? If he shaves himself, then according to his logo he does not. If he does not, then according to his logo he does. Consider the card that says on one side "The sentence on the other side is true," and says on the other side "The sentence on the other side is false."

The "paradoxes" have the same form. A statement S and its negation $\text{not-}S$ have the same *truth-value* $t(S)$:

$$t(S) = t(\text{not-}S) \quad (1)$$

The two statements are both TRUE (1) or both FALSE (0). This violates the laws of noncontradiction and excluded middle. For bivalent truth tables remind us that negation reverses truth value:

$$t(\text{not-}S) = 1 - t(S) \quad (2)$$

So (1) reduces to

$$t(S) = 1 - t(S) \quad (3)$$

If S is true, if $t(S) = 1$, then $1 = 0$. $t(S) = 0$ also implies the contradiction $1 = 0$.

The fuzzy or multivalued interpretation accepts the logical relation (3) and, instead of insisting that $t(S) = 0$ or $t(S) = 1$, simply solves for $t(S)$ in (3):

$$2 t(S) = 1 \quad (4)$$

or

$$t(S) = \frac{1}{2} \quad (5)$$

So the "paradoxes" reduce to literal half-truths. They represent in the extreme the uncertainty inherent in every empirical statement and in many mathematical statements. Geometrically, the fuzzy approach places the paradoxes at the midpoint of the 1-dimensional

unit hypercube $[0, 1]$. More general paradoxes reside at the midpoint of n -dimensional hypercubes, the unique point equidistant to all 2^n vertices.

Multivaluedness also resolves the classical *sorites* paradoxes. Consider a heap of sand. Is it still a heap if we remove one grain of sand? How about two grains? Three? If we argue bivalently by induction, we eventually remove all grains and still conclude that a heap remains, or that it has suddenly vanished. No single grain takes us from heap to non-heap. The same holds if we pluck out hairs from a nonbald scalp or remove 5%, 10%, or more of the molecules from a table or brain. We transition gradually, not abruptly, from a thing to its opposite. Physically we experience degrees of occurrence. In terms of statements about the physical processes, we arrive again at degrees of truth.

Suppose there are n grains of sand in the heap. Removing one grain leaves $n - 1$ grains and a truth value $t(S_{n-1})$ of the statement S_{n-1} that the $n - 1$ sand grains are a heap. The truth value $t(S_{n-1})$ obeys $t(S_{n-1}) < 1$ in general. $t(S_{n-1})$ may be close to unity, but we have some nonzero doubt d_{n-1} about the truth of the matter. (The argument still holds if there exist no doubting creatures in the universe.) For instance [Gaines, 1983],

$$t(S_n) = 1 - d_n, \quad (6)$$

where $0 \leq d_n \leq d_{n-1} \leq \dots \leq d_{n-m} \leq \dots \leq 1$. So $t(S_{n-m})$ approaches zero as m increases to n . If we argue inductively, we can interpret the overall inference as the forward chain "(If S_n , then S_{n-1}) and (If S_{n-1} , then S_{n-2}) and ... and (If S_1 , then S_0)." If we multiplicatively interpret the conjunction operator, then

$$t(S_n \rightarrow S_{n-m}) = \prod_{k=0}^m (1 - d_{n-k}). \quad (7)$$

If we interpret the conjunction operator as the minimum operator, as discussed in the homework problems at the end of the chapter, then

$$t(S_n \rightarrow S_{n-m}) = \min(1 - d_n, \dots, 1 - d_{n-m}). \quad (8)$$

$$= 1 - \max(d_n, \dots, d_{n-m}) \quad (9)$$

In both cases the implication truth value $t(S_n \rightarrow S_0)$ equals zero (or some small number). We pay a truth-value fee for each application of *modus ponens*, of concluding B from A and $A \rightarrow B$. The overall inference is vacuous. This reflects the everyday epistemological precept that the longer an explanation, the less we tend to trust it.

Fuzziness in the Twentieth Century

Logical paradoxes and the Heisenberg uncertainty principle led to the development of continuous or "fuzzy" logic in the 1920s and 1930s. Quantum theorists allowed for indeterminacy by including a third or middle truth value in the bivalent logical framework. The next step allowed degrees of indeterminacy, viewing TRUE and FALSE as the two limiting cases of the spectrum of indeterminacy.

Polish logician Jan Lukasiewicz [Rescher, 1969] first formally developed a three-valued logical system in the early 1930s. Lukasiewicz extended the range of truth values from $\{0, 1/2, 1\}$ to all rational numbers in $[0, 1]$, and finally to all numbers in $[0, 1]$ itself. Logics that use the general truth function $t: \{\text{Statements}\} \rightarrow [0, 1]$ define continuous or "fuzzy" logics. Logicians refer to this system as L_1 . The exercises at the end of the chapter develop Lukasiewicz's fuzzy logic.

In the 1930s quantum philosopher Max Black [1937] applied continuous logic componentwise to sets or lists of elements or symbols. Historically, Black drew the first fuzzy-set membership functions. Black called the uncertainty of these structures *vagueness*. Anticipating Zadeh's fuzzy set theory, each element in Black's multivalued sets and lists behaved as a statement in a continuous logic.

In 1965 systems scientist Lotfi Zadeh [1965] published the paper "Fuzzy Sets" that formally developed multivalued set theory, introduced the term fuzzy into the technical literature, and inaugurated a second wave of interest in multivalued mathematical structures, from systems to topologies. The recent emergence of fuzzy commercial products, as

well as new theory, has generated a third wave of interest in multivalued systems.

Zadeh extended the bivalent indicator function I_A of nonfuzzy subset A of X ,

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \quad (10)$$

to a multivalued indicator or membership function $m_A : X \rightarrow [0, 1]$. This allows us to combine such multivalued or fuzzy sets with the pointwise operators of indicator functions:

$$I_{A \cap B}(x) = \min(I_A(x), I_B(x)) \quad , \quad (11)$$

$$I_{A \cup B}(x) = \max(I_A(x), I_B(x)) \quad , \quad (12)$$

$$I_{A^c}(x) = 1 - I_A(x) \quad , \quad (13)$$

$$A \subset B \quad \text{iff} \quad I_A(x) \leq I_B(x) \quad \text{for all } x \text{ in } X \quad . \quad (14)$$

The membership value $m_A(x)$ measures the elementhood or degree to which element x belongs to set A :

$$m_A(x) = \text{Degree}(x \in A) \quad . \quad (15)$$

Just as the individual indicator values $I_A(x)$ behave as statements in bivalent propositional calculus, membership values $m_A(x)$ correspond to statements in a continuous logic. If A defines a fuzzy subset of the real line, as in Figure 1.7 below, then in principle we can graph $m_A : R \rightarrow [0, 1]$ in two dimensions. In practice indicator functions I_A graph as step functions or rectangular pulses on the real line.

Sets as Points in Cubes

Fuzziness prevents logical certainty at the level of black-white axioms. This seems unsettling to some [Quine, 1981] and liberating to others.

At the system level fuzziness allows us to build computer chips and systems that "intelligently" control subways, automobile systems, and numerous consumer electronic and other devices. At this level fuzzy processing may resemble neural processing.

Neural networks and fuzzy systems process inexact information and process it inexactly. Neural networks recognize ill-defined patterns without an explicit set of rules. Fuzzy systems estimate functions and control systems with partial descriptions of system behavior. Experts may provide this heuristic knowledge, or, as we illustrate in Chapters 17 - 19, neural networks may adaptively infer it from sample data.

Neural and fuzzy systems share a more formal mathematical property. They share the same state space. A set of n neurons defines a sequence of n -dimensional continuous or "fuzzy" sets. The neurons emit bounded signals.

The neuronal signals range from some minimum value to some maximum value, say from 0 to 1. At each instant the n -vector of neuronal outputs defines a fuzzy unit or fit vector. Each fit value indicates the degree to which the neuron or element belongs to the n -dimensional fuzzy set.

The neuronal state space, the set of all possible neural outputs, equals the set of all n -dimensional fit vectors, the fuzzy power set. Both equal the unit hypercube $I^n = [0, 1]^n = [0, 1] \times \dots \times [0, 1]$, the set of all vectors of length n and with coordinates in the unit interval $[0, 1]$. Chapter 17 discusses fuzzy systems and associative memories, which map unit cubes to unit cubes, fuzzy sets to fuzzy sets. We shall use this recent geometric view of *sets as points* [Kosko, 1987-90] throughout this book.

The 2^n vertices of I^n represent extremized neuronal-output combinations, as we often find in networks of competitive or laterally inhibitive neurons. Many feedback neural networks [Hopfield, 1984] drive initial states inside the unit cube to nearest vertices. These systems dynamically disambiguate fuzzy input descriptions by minimizing their fuzzy en-

trophy. The midpoint of the cube, where a fuzzy set A equals its own opposite A^c , has maximal fuzzy entropy, as we discuss in Chapter 16. The black-white vertices have minimal fuzzy entropy.

Proper fuzzy sets, nonvertex points, A violate the "laws" of noncontradiction and excluded middle: $A \cap A^c \neq \emptyset$ and $A \cup A^c \neq X$. In Chapter 16 we show that fuzzy entropy, the measure of fuzziness, balances the fuzzy count of the *overlap* $A \cap A^c$ and *underlap* $A \cup A^c$ in a simple ratio: $E(A) = \frac{M(A \cap A^c)}{M(A \cup A^c)}$.

There are 2^n bit vectors of length n . They define the vertices of I^n . So the vertices also represent the nonfuzzy power set of the n elements x_1, \dots, x_n , the set of all nonfuzzy subsets of the n elements. The bit value 0 in the i th slot of a bit vector indicates the absence of element x_i in that subset. The bit value 1 indicates the presence of x_i in the subset. The bit vector (1 0 1 0 0) indicates the subset $\{x_1, x_3\}$ of set $\{x_1, x_2, x_3, x_4, x_5\}$.

Fit values equal the membership values $m_A(x_i)$ discussed above. Fit values measure partial set membership or degrees of *elementhood*. The fit value $1/5$ indicates that element x_i belongs only slightly to the fuzzy subset A . The fit value $1/2$ indicates that x_i belongs to fuzzy set A as much as it does not—as much as it belongs to the complement fuzzy set A^c .

Consider the set X of two elements x_1 and x_2 . The *power set* of X , denoted 2^X , contains the four subsets of X : $2^X = \{\emptyset, \{x_1\}, \{x_2\}, X\}$. These four nonfuzzy sets correspond to four bit vectors:

$$\emptyset = (0 \ 0)$$

$$\{x_1\} = (1 \ 0)$$

$$\{x_2\} = (0 \ 1)$$

$$X = (1 \ 1)$$

The fuzzy power set $F(2^X)$, which contains all continuum-many fuzzy subsets of X , corresponds to unit square. Figure 1.1 displays the fuzzy power set $F(2^X)$.

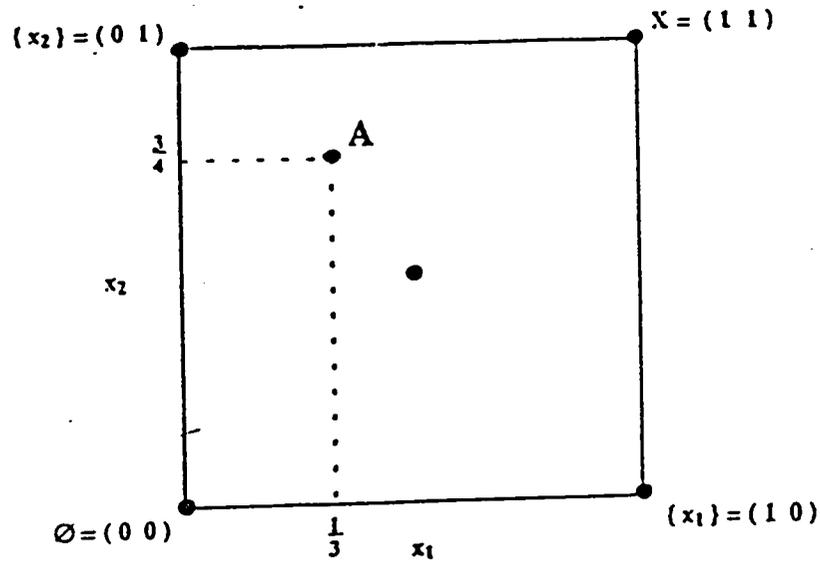


FIGURE 1.1 Fuzzy power set $F(2^X)$ of X corresponds to the unit square when $X = \{x_1, x_2\}$. The four nonfuzzy subsets in the nonfuzzy power set 2^X correspond to the four corners of the 2-cube. The fuzzy subset A corresponds to the fit vector $(1/3, 3/4)$ and to a point inside the 2-cube if $m_A(x_1) = 1/3$ and $m_A(x_2) = 3/4$. The midpoint M of the unit square corresponds to the maximally fuzzy set.

Figure 1.1 represents the fuzzy subset A as a point inside the 2-dimensional unit hypercube. If A has membership degrees or fit values $m_A(x_1) = 1/3$ and $m_A(x_2) = 3/4$ —so x_1 belongs to A less than x_2 does—then A corresponds to the fit vector $(1/3, 3/4)$.

The cube midpoint corresponds to the maximally fuzzy set M . The midpoint set M uniquely obeys the peculiar relation $M = M \cap M^c = M \cup M^c = M^c$, and so maximally violates the bivalent laws of noncontradiction and excluded middle. The classical paradoxes of logic and set theory correspond to midpoint phenomena. Note that the cube midpoint in Figure 1.1 is uniquely equidistant to all 2^2 vertices. The cube midpoint behaves as the black hole of set theory.

Subsethood and Probability

Elementhood represents a special case of *subsethood*. Subsethood measures the degree to which set A belongs to set B , the degree to which A is a subset of B . We denote this subsethood measure as $S(A, B)$:

$$S(A, B) = \text{Degree}(A \subset B) \quad (16)$$

Subsethood provides a unified set-theoretic framework for fuzziness and probability. For instance, in the simplest case A equals the singleton set $\{x_i\}$. Then the subsethood of $\{x_i\}$ in B equals the membership or elementhood value $m_B(x_i)$:

$$S(\{x_i\}, B) = m_B(x_i) \quad (17)$$

(17) follows directly from the Subsethood Theorem (22) below when we interpret $\{x_i\}$ as a bit vector with a 1 in the i th slot and 0s elsewhere.

Subsethood reveals the connection between fuzziness and randomness. Subsethood reduces probability to set theory. Randomness does not depend on the fuzziness or ambiguity of an event. It depends on the uncertainty between certain events. Randomness equals the uncertainty that arises when a nonfuzzy set B is partially contained in one of its own nonfuzzy subsets A . $S(A, B) = 1$ since A is a subset of B . But in general multivaluedness holds. The converse subsethood $S(B, A)$ is less than one but greater than zero:

$$0 < S(B, A) < 1 \quad (18)$$

Classical set theory implicitly forbids the strict inequalities in (18). The law of excluded middle dictates that every set either is or is not a subset of every other set. As a result, for centuries theorists have had to arbitrarily define probability as a frequency ratio or stipulate that it obeyed certain axioms. They could not *derive* probability from more fundamental concepts.

Fuzzy theory derives the axioms of the conditional probability measure $P(B|A)$,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} , \quad (19)$$

the probability that B occurs given that A occurs, from the properties of the subsethood measure $S(A, B)$. If X defines the "sample space" of all elementary outcomes of an experiment, then X is a "sure event" since $P(X) = 1$. Then (19) implies that every probability $P(A)$ equals the conditional probability $P(A|X)$:

$$P(A) = P(A|X) . \quad (20)$$

This identity reflects the general subsethood relationship

$$P(A) = S(X, A) . \quad (21)$$

On the surface the subsethood relation (21) seems absurd. How can superset X belong to one of its own subsets? How can the whole be part of one of its own parts? X cannot *totally* belong to A unless $X = A$. But X can *partially* belong to A . The Subsethood Theorem in Chapter 16 proves that this partial containment depends directly on the overlap between X and A , the intersection $X \cap A$. Figure 1.2 illustrates the Pythagorean geometry of the Subsethood Theorem in three dimensions. The shaded hyper-rectangle defines $F(2^B)$, the fuzzy power set of B .

FUZZY SUBSETHOOD

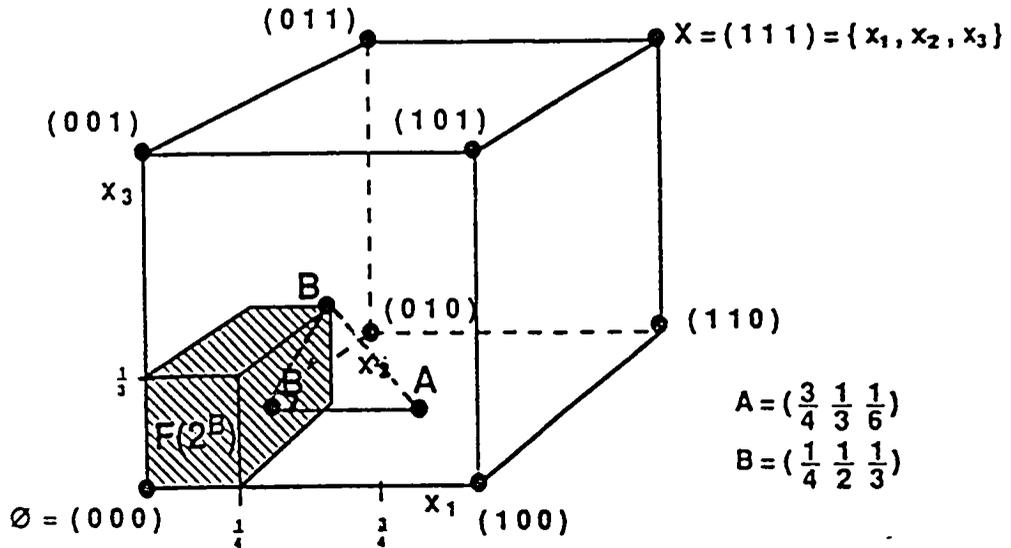


FIGURE 1.2 Subsethood Theorem in R^3 . X contains 3 elements, x_1 , x_2 , and x_3 , and 8 nonfuzzy subsets. Fuzzy subset $B = (1/4, 1/2, 1/3)$ contains infinitely many fuzzy subsets B' such that $S(B', B) = 1$. They define the shaded hyper-rectangle. $S(A, B) < 1$ since A lies outside the hyper-rectangle. The closer A to the hyper-rectangle, the larger the subsethood $S(A, B)$. B^* denotes the subset of B closest to A . B^* equals $A \cap B$ and uniquely defines an orthogonal or Pythagorean relationship between A and B .

The Subsethood Theorem relates $S(A, B)$ to the magnitudes of A , B , and $A \cap B$:

$$S(A, B) = \frac{M(A \cap B)}{M(A)} \quad (22)$$

The ratio in (22) resembles, behaves as, and generalizes the defining ratio (19) of conditional probability. $M(A)$ denotes the fuzzy count of fit vector A :

$$M(A) = a_1 + \dots + a_n \quad (23)$$

$M(A)$ generalizes the classical cardinality count, which sums only 1s and 0s. In the infinite case appropriate integrals replace summations. (22) implies that the fuzzy entropy $E(A)$ of A equals the degree to which $A \cap A^c$ contains its own superset $A \cup A^c$: $E(A) = S(A \cup A^c, A \cap A^c)$.

In Figure 1.2, $A = (3/4, 1/3, 1/6)$ and $B = (1/4, 1/2, 1/3)$. Then the closest subset B^* to A that satisfies the total-subsethood condition

$$b_i^* \leq b_i, \dots, b_n^* \leq b_n \quad (24)$$

corresponds to $B^* = (1/4, 1/3, 1/6)$, which also equals the pairwise minimum of A and B . (24) generalizes (14) above. As discussed in Chapter 16, the Subsethood Theorem ensures this in general:

$$B^* = A \cap B \quad (25)$$

(23) implies that $M(A) = 15/12 = 5/4$, and $M(A \cap B) = 3/4$. Then the Subsethood Theorem gives $S(A, B) = (3/4)/(5/4) = 3/5 = 60\%$.

Relative frequency provides the clearest example of between-set fuzziness. Suppose we flip a coin, draw balls from urns, or shoot at a target. The elementary events in X are trials. Each trial is successful or unsuccessful. So X does not possess fuzzy subsets in its event space (its sigma-algebra). Each coin flip results in a head or a tail, not something in between. Suppose A defines the subset of successful trials. If X contains n trials, then A corresponds to a vertex of I^n and equals a bit vector of 1s and 0s. Suppose n_A successes out of n trials. 1s indicate successes, and 0s indicate failures. The event X equals total success, the bit vector of all 1s. X contains n successes. Then, since $A \cap X = A$, the Subsethood Theorem (22) gives

$$S(X, A) = \frac{n_A}{n} \quad (26)$$

Historically probability theorists have called the subsethood ratio in (26), or its limit, the "probability of success" or $P(A)$. This adds only a cultural tag. The success ratio n_A/n behaves no differently in its deterministic subsethood framework than it did in its "random" framework. The relative-frequency ratio still provides a stable estimate for probability values in our physical, engineering, economic, and gambling models. It still implies all the theorems it has always implied.

But we cannot derive the relative-frequency ratio from between-set relationships if we deny the strict inequality (18) and insist that subsethood is two-valued. Bivalence forces us to assume the ratio as a theoretical primitive.

Whether by design or by accident we have historically followed the bivalent path in mathematics for almost 3,000 years. Bivalence has simplified our formal frameworks but at a cost. It has led to logical paradoxes (bivalent contradictions), unexplained primitives, and "randomness" in a universe that seems to obey physical laws and where every event has causes.

THE DYNAMICAL SYSTEMS APPROACH TO MACHINE INTELLIGENCE: THE BRAIN AS A DYNAMICAL SYSTEM

Several engineering and scientific disciplines study how adaptive systems respond to stimuli. Electrical engineers study the topic as signal processing, nonlinear filtering, coding theory, circuit design, and adaptive control. Computer scientists study it as algorithm and automata theory, computer design, robotics, and artificial intelligence. Mathematicians study it as function approximation, statistical estimation, combinatorial optimization, and dynamical systems. Philosophers study it as epistemology, causality, and action. Biologists study it as neuroscience, biophysics, ecology, evolution, and population biology. Psychologists study it as reinforcement learning, psychometrics, and cognitive science. Economists study it as utility maximization, game theory, econometrics, and market equilibrium theory. Cultural anthropologists study it as culture.

We shall emphasize electrical engineering as we seek general principles of how adaptive

systems process information. We call these principles *machine intelligence* principles. We shall draw freely from the related fields of engineering and science.

The term *artificial intelligence* usually refers to the computer-scientific approach to machine intelligence. This approach emphasizes symbolic processing and tree search. AI has become the emblem for a popular computer-age view of the brain: *brain = computer*. This view ranges from classical science-fiction speculation (the computer HAL in *2001: A Space Odyssey*) to proposed space-based weapons systems.

We shall explore machine intelligence from a *dynamical-systems* viewpoint: *brain = dynamical system*. On this view a maple leaf falling to a potential-energy minimum on the ground better describes brain activity than does a computer executing instructions. The dynamical models we shall study are cast as large systems of differential or difference equations. The principles describe local or global interactions of nonlinear parallel processes.

Some of these machine-intelligence principles and mechanisms may explain natural phenomena and processes. Some already extend our theoretical and mathematical knowledge. But ultimately they should help us build smarter machines. They should give rise to new computational devices—electrical, optical, molecular, plasma, fluid, or other devices.

In this sense machine intelligence becomes an engineering discipline. Nearly a half century ago, Norbert Wiener [1948] outlined the first incarnation of such a machine-intelligence engineering. Wiener called it *cybernetics*.

We shall focus our analysis on artificial neural networks and fuzzy systems. These new, related systems represent broad classes of "machine-intelligent" adaptive systems. Chapters 2 - 6 describe neural network theory. Chapters 7 - 15 describe engineering applications of neural networks. Chapters 16 - 19 present a geometric theory of fuzzy sets and systems and its neural extension to adaptive fuzzy systems.

Neural and Fuzzy Systems as Function Estimators

Neural networks and fuzzy systems estimate input-output functions. Both are trainable

dynamical systems. Sample data shapes and "programs" their time evolution. Unlike statistical estimators, they estimate a function without a mathematical model of how outputs depend on inputs. They are *model-free* estimators. They "learn from experience" with numerical and, sometimes, linguistic sample data.

Neural and fuzzy systems encode sampled information in a parallel-distributed framework. Both frameworks are numerical. We can prove theorems to describe their behavior and limitations. We can implement neural and fuzzy systems in digital or analog VLSI circuitry or in optical-computing media, in spatial-light modulators and holograms.

Artificial neural networks consist of numerous, simple processing units or "neurons" that we can globally program for computation. We can program or train neural networks to store, recognize, and associatively retrieve patterns or database entries; to solve combinatorial optimization problems; to filter noise from measurement data; to control ill-defined problems; in summary, to estimate sampled functions when we do not know the form of the functions.

The human brain contains roughly 10^{11} or 100 billion neurons [Thompson, 1985]. That number approximates the number of stars in the Milky Way Galaxy, and the number of galaxies in the known universe. As many as 10^4 synaptic junctions may abut a single neuron. That gives roughly 10^{15} or 1 quadrillion synapses in the human brain. The brain represents an asynchronous, nonlinear, massively parallel, feedback dynamical system of cosmological proportions.

Artificial neural systems may contain millions of nonlinear neurons and interconnecting synapses. Future artificial neural systems may contain billions of real or virtual model neurons. In general no teacher supervises, stabilizes, or synchronizes these large-scale nonlinear systems.

Many feedback neural networks can learn new patterns and recall old patterns simultaneously, and ceaselessly. Supervised neural networks can learn far more input-output pairs, or stimulus-response associations, than the number of neurons and synapses in the network architecture. Since neural networks do not use a mathematical model of how a system's output depends on its input—since they behave as model-free estimators—we can apply the same neural network architecture, and dynamics, to a wide variety of problems.

Like brains, neural networks recognize patterns we cannot even define. We call this property *recognition without definition*. Who can define a tree, a pillow, or their own face to the satisfaction of a computer pattern-recognition system? These and most concepts we learn *ostensively*, by pointing out examples. We do not learn them as we learn the definition of a circle. We abstract these concepts from sample data, just as a child abstracts the color red from observed red apples, red wagons, and other red things, or as Plato abstracted triangularity from considered sample triangles.

Recognition without definition characterizes much intelligent behavior. It enables systems to generalize. Dogs, lizards, and slugs recognize multitudes of unforeseen, complex patterns without, of course, any ability to define them. Descriptive natural languages developed only yesterday in human evolution. Yet a great deal of modern philosophy, influenced by formal logic and behaviorist psychology, has insisted on concept definition preceding recognition or even discussion. Below we discuss how this insistence has helped shape the field of artificial intelligence and its emblem, the expert system.

Neural networks store pattern or function information with distributed encoding. They superimpose pattern information on the same associative-memory medium—on the many synaptic connections between neurons. Distributed encoding enables neural networks to complete partial patterns and “clean up” noisy patterns. So it helps neural networks estimate continuous functions.

Distributed encoding endows neural networks with fault tolerance and “graceful degradation.” If we successively rip out handfuls of synaptic connections from a neural network, the network tends to smoothly degrade in performance, not abruptly fail. Computers and digital VLSI chips do not gracefully degrade when their components fail. Natural selection seems to have favored distributed encoding in brains, at least in sections of brains.

Neural networks, and brains, pay a price for distributed encoding: crosstalk. Distributed encoding produces crosstalk or interference between stored patterns. Similar patterns may clump together. New patterns may crowd out older learned patterns. Older patterns may distort newer patterns.

Crosstalk limits the neural network's storage capacity. Different learning schemes provide different storage capacities. The number of neurons bounds the number of patterns a

neural network can store reliably with the simplest unsupervised learning schemes. Even for more sophisticated supervised learning schemes, storage capacity ultimately depends on the number of network neurons and synapses, as well as on their function. *Dimensionality limits capacity.*

Biological neurons and synapses motivate the neural network's topology and dynamics. We interpret neurons as simple input-output functions, threshold switches for two-state neurons and asymptotic threshold switches for continuous neurons. We interpret synapses as adjustable weights. In neural analog VLSI chips [Mead, 1989], operational amplifiers model nonlinear neurons, and resistors model synapses.

The overall network behaves as an adaptive function estimator. Indeed commercial adaptive estimators are simple, usually linear, neural networks. These include antennae beam formers, high-speed modems, and echo-cancellers for long-distance telephone calls.

Neural Networks as Trainable Dynamical Systems

Neural networks *geometrize* computation. Network activity burrows a trajectory in a state space of large dimension, say R^n . Each point in the state space defines a snapshot of a possible neural network configuration.

The trajectory begins with a computational problem and ends with a computational solution. The user or the environment specifies the system's initial conditions, which define where the trajectory begins in the state space. In pattern learning, the pattern to be learned defines the initial conditions. In pattern recognition or recall, the pattern to be recognized defines the initial conditions.

Most of the trajectory corresponds to *transient* behavior or computations. Synaptic values gradually change to learn new pattern information. Neuronal outputs fluctuate.

The trajectory ends where the system reaches equilibrium, if it ever reaches equilibrium. In the simplest and rarest case, the equilibrium attractor is a fixed point of the dynamical system. Most popular neural networks converge to fixed points. In more complicated cases the equilibrium attractor is a limit cycle or limit torus. In Chapter 4 we discuss a crude

method for storing discrete time-varying patterns as limit cycles in feedback networks. The equilibrium attractors are *robust* or *structurally stable* if small perturbations do not distort or destroy them.

In general, and in most dynamical systems, the equilibrium attractor is *aperiodic* or chaotic. Once the network enters this region of the state space, it wanders forever without apparent structure or order. Yao and Freeman [1990] have used dynamical neural models and time-series data to argue that rabbit olfactory bulbs process odor information with chaotic attractors. As discussed in the homework problems, the function $x_{k+1} = c x_k (1 - x_k)$ behaves as a chaotic dynamical system for values of c near 4 and x values in the unit interval $[0, 1]$.

In Chapter 3 we discuss global Lyapunov functions for proving that certain feedback neural networks converge to fixed points from any initial conditions. Geometrically we can view the Lyapunov function as a surface sculpted by learned pattern information, as in Figure 1.3.

Figure 1.3 illustrates the geometry of fixed-point stability in feedback neural networks. Patterns behave as rocks on the rubber sheet of learning. The patterns, as well as "spurious" or unlearned patterns, dig out attractor basins in the state space and tend to rest at the local Lyapunov minimum of the attractor. The Lyapunov sheet changes shape as the system learns new patterns. Input patterns Q rapidly classify to nearest stored neighbors as if they were ball bearings rolling into local depressions in a gravity field. In a fixed-point attractor basins the state-trajectory balls stop at the local minima (or hover arbitrarily close to it). In limit-cycle attractors, the ball Q would rotate in an elliptical orbit inside the attractor basin. In limit-tori attractors, Q would cycle toroidally in the attractor basin, as if, in R^3 , winding around the surface of a bagel. In chaotic attractors, Q would wander aperiodically within the attractor region.

In all these cases, the number of attractor basins does not affect the speed of convergence, the rate at which Q falls into the attractor basin. The dimensionality of the state space also does not in principle affect the convergence rate. In practice, Q converges exponentially quickly. This suggests that global stability may underlie our biological neural networks' ability to rapidly recognize patterns, generate answers, and exhibit appropriate

muscle reflexes independent of the amount of pattern information in our brains. Computer-type storage devices tend to slow as the number and complexity of patterns stored in them increases.

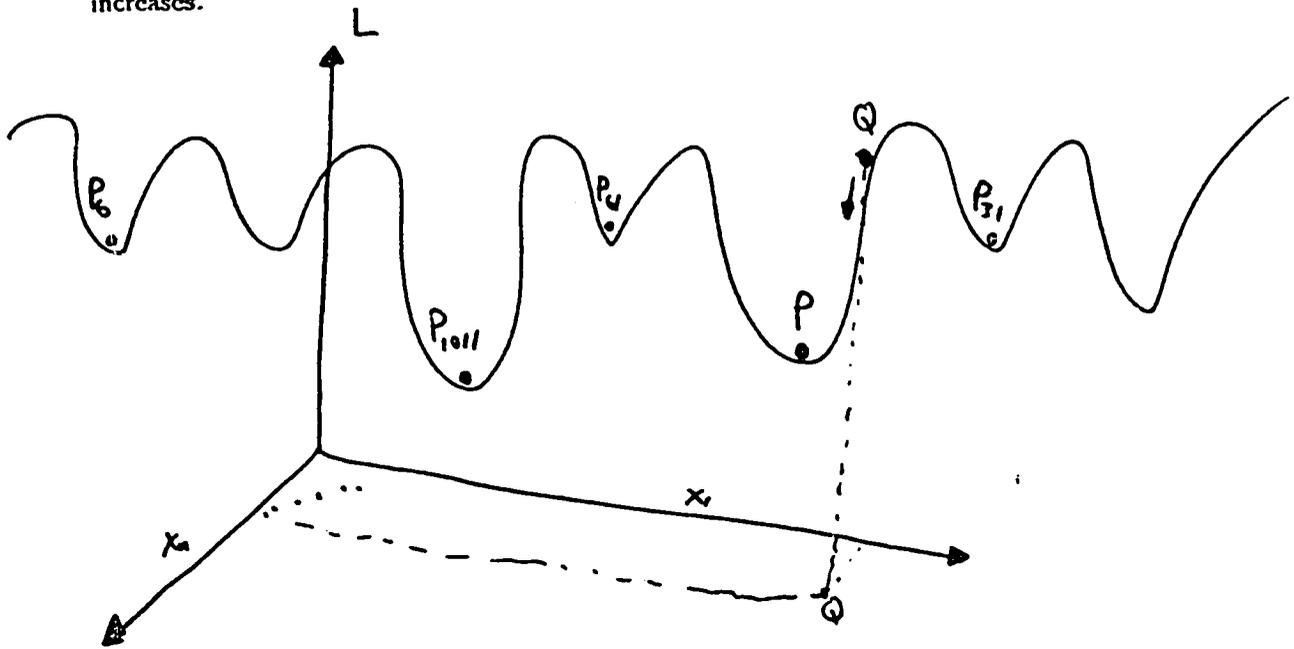


FIGURE 1.3 Global stability of a feedback neural network. Learning encodes the vector patterns P_1, P_2, \dots by gradually sculpting a Lyapunov or "energy" surface in the augmented state space R^{n+1} . Input vector pattern Q rapidly "rolls" into the nearest attractor basin, where the system classifies Q as a learned pattern P or misclassifies Q as a spurious pattern. Q 's descent rate does not depend on the number of stored patterns.

Mathematically we can describe the time evolution of the neural network by the (autonomous) dynamical system equation

$$\dot{x}(t) = f(x) , \tag{27}$$

where the overdot denotes time differentiation. The state vector $x(t)$ describes all neuronal and synaptic values of the neural network at time t . The neural network reaches *steady state* when

$$\dot{x} = 0, \quad (28)$$

holds indefinitely or until new stimuli perturb the system out of equilibrium. Neural computation seeks to identify the steady-state condition (28) with the solution of a computational problem, whether in pattern recognition, image segmentation, optimization, or numerical analysis.

We can locally linearize f by replacing f with its Jacobian matrix of partial derivatives J . The eigenvalues of J describe the system's local behavior about an equilibrium point. For instance, if all eigenvalues have negative real parts, then the local equilibrium is a fixed point and the system converges to it exponentially quickly. More abstractly, generalized eigenvalues or *Lyapunov exponents* describe the underlying dynamical contraction and expansion that may produce chaos.

We can classify neural network models according as they learn with supervision (pattern-class information) and according as they contain closed synaptic loops or feedback. Figure 1.4 provides a rough taxonomy of several popular neural network models.

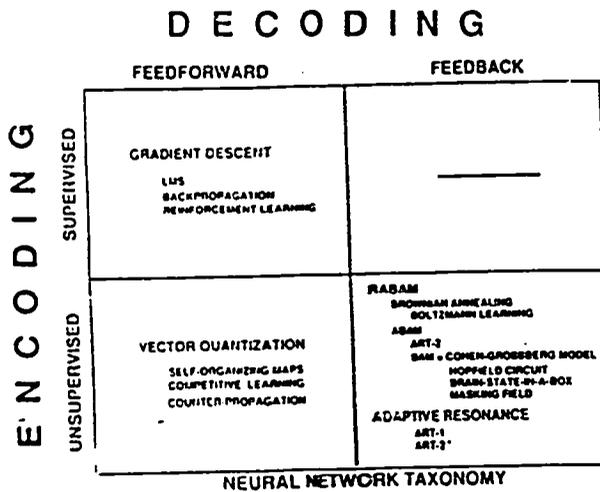


FIGURE 1.4 Taxonomy of neural network models.

Supervised feedforward models provide the most tractable, most applied neural models. We discuss these stochastic gradient systems in Chapter 5, and mention recent attempts to extend these supervised systems into the feedback domain. Unsupervised feedback models provide the most biologically plausible, but mathematically most complicated, models. These networks simultaneously learn and recall patterns. Both neurons and synapses change state when these systems learn and when they recall, recognize, or reconstruct pattern information. Chapter 6 proves global stability for many of these adaptive dynamical systems in the RABAM Theorem. Unsupervised feedforward neural networks tend to converge to locally sampled pattern-class centroids, as discussed in Chapters 4, 6, and 9.

Fuzzy Systems and Applications

Fuzzy systems store banks of fuzzy associations or commonsense "rules." A fuzzy traffic controller might contain the fuzzy association "If traffic is heavy in this direction, then keep

the light green longer." Fuzzy phenomena admit degrees. Some traffic configurations are *heavier* than others. Some green-light durations are *longer* than others. The single fuzzy association (HEAVY, LONGER) encodes all these combinations.

Fuzzy systems are even newer than neural systems. Yet already engineers have successfully applied fuzzy systems in many commercial areas. Fuzzy systems "intelligently" automate subways; focus cameras and camcorders; tune color televisions, control automobile transmissions, cruise controllers, and emergency braking systems; defrost refrigerators and control air conditioners; automate washing machines and vacuum sweepers; guide robot-arm manipulators; invest in securities; control traffic lights, elevators, and cement mixers; recognize Kanji characters; select golf clubs; even arrange flowers.

Most of these applications originated in Japan, though fuzzy products are sold and applied throughout the world. Until very recently, Western scientists, engineers, and mathematicians have overlooked, discounted, or even attacked early versions of fuzzy theory, usually in favor of probability theory. Below, and especially in Chapter 16, we examine this philosophical resistance in more detail and present a new geometrical theory of continuous or "fuzzy" sets and systems.

Fuzzy systems "reason" with parallel associative inference. When asked a question or given an input, a fuzzy system fires each fuzzy rule in parallel, but to different degree, to infer a conclusion or output. Thus fuzzy systems reason with sets, "fuzzy" or continuous sets, instead of bivalent propositions. This generalizes the Aristotelian logical framework that still dominates science and engineering. In one second a digital fuzzy VLSI chip may execute thousands, perhaps millions, of these parallel-associative set inferences. We measure such chip performance in FLIPS, fuzzy logical inferences per second.

Fuzzy systems estimate sampled functions from input to output. They may use linguistic (symbolic) or numeric samples. An expert may articulate linguistic associations such as (HEAVY, LONGER). Or a fuzzy system may adaptively infer and modify its fuzzy associations from representative numerical samples.

In the latter case, neural and fuzzy systems naturally combine. The combination resembles an adaptive system with sensory and cognitive components. Neural parameter estimators embed directly in an overall fuzzy architecture. Neural networks "blindly"

generate and refine fuzzy rules from training data. Chapters 17-19 describe and illustrate these adaptive fuzzy systems.

Adaptive fuzzy systems learn to control complex processes very much as we do. They begin with a few crude rules of thumb that describe the process. Experts may give them the rules. Or they may abstract the rules from observed expert behavior. Successive experience refines the rules and, usually, improves performance.

Chapter 18 applies this adaptive cognitive process to backing up a truck-and-trailer rig to a loading dock. (A supervised neural system can also solve this problem, though at much greater computational cost. So far the truck-and-trailer dynamical system has eluded mathematical characterization.) The fuzzy system quickly learns a set of governing fuzzy rules as it samples actual truck-and-trailer trajectories. Additional training samples improve only marginally the fuzzy system's performance. This property is better experienced than explained. As an exercise for the reader, you might try backing your car into the same parking space five times from five different starting positions.

INTELLIGENT BEHAVIOR AS ADAPTIVE MODEL-FREE ESTIMATION

Below we discuss neural and fuzzy systems in more detail. First we examine the properties neural and fuzzy systems share with us and, more broadly, with all intelligent systems. These properties reduce to the single abstract property of adaptive model-free function estimation: *Intelligent systems adaptively estimate continuous functions from data without specifying mathematically how outputs depend on inputs.* We now elaborate this thesis.

A function f , denoted $f: X \rightarrow Y$, maps an input domain X to an output range Y . For every element x in the input domain X , the function f uniquely assigns the element y in the output range Y . We denote this unique assignment as $y = f(x)$. $f(x) = x^3$ defines a cubic function. $f(x_1, x_2, x_3) = (x_1, x_2, x_1^2 - x_2^2)$ defines a "saddle" or hyperbolic-paraboloid vector function in physical or 3-dimensional space R^3 . Pressure is

a function of temperature, mass of energy ($e = m c^2$), gravity of mass, erosion of gravity, consumption of income. Functions define causal hypotheses. Science and engineering paint our pictures of the universe with functions.

Humans, animals, reptiles, amphibians, and others also estimate functions. We all respond to stimuli. We associate responses with stimuli. We associate actions with scenarios, class labels with patterns, effects with causes. Equivalently, we map stimuli to responses.

Mathematically, all these systems transform inputs to outputs. The transformation defines the input-output function $f : X \rightarrow Y$. Indeed the transformation defines the system. We can operatively characterize any system—atomic, molecular, biological, ecological, economic or legal, geological, galactic—by how it transforms input quantities into output quantities.

We call system behavior "intelligent" if the system emits appropriate, problem-solving responses when faced with problem stimuli. The system may use an associative memory embedded in the resistive network of an analog VLSI chip or embedded in the synaptic webs of its brain. Or the system may use a mathematical algorithm to search a decision tree, as in computer chess programs.

Generalization and Creativity

Intelligent systems also *generalize*. Their behavioral repertoires exceed their experience. Eighteenth-century philosopher David Hume saw why: Intelligent systems associate similar responses with similar stimuli. Small input changes produce small output changes. Hence they estimate *continuous* functions. The pilot lands the airplane at night the same way if only a few of the runway lights are out or if the new runway differs only slightly from more familiar runways. The leopard stalks like prey in like ways in like circumstances. Each minnow in a school smoothly adjusts its swimming behavior to the position of its smoothly moving neighbors.

Function continuity accounts for much novel or creative behavior, if not all of it. We call system behavior "novel" if the system emits appropriate responses when faced with new or

unexpected stimuli. "Novel ideas," says behaviorist psychologist B.F. Skinner [1953], are "responses never made before under the same circumstances.... Novel contingencies generate novel forms of behavior." Usually these new stimuli resemble known or learned stimuli, and our responses usually resemble known responses.

Geometrically, when systems generalize or "create" they map stimulus balls to response balls. Consider a known stimulus-response pair (x, y) . Stimulus x defines a point in the stimulus space S , the set of all possible stimuli for the problem at hand. In practice S often corresponds to the real Euclidean vector space R^n . Response y defines a point in the response space R , which may correspond to R^p .

Now imagine a stimulus ball B_x centered about stimulus x and a response ball B_y centered about response y . All the stimuli x' in B_x resemble stimulus x . The closer stimulus x' is to stimulus x , and hence the smaller the distance $d(x', x)$, the more x' resembles x . The responses y' in B_y behave similarly.

Suppose $y = f(x)$ for some unknown continuous function $f : R^n \rightarrow R^p$. The function f defines the sampled system. Suppose further that f generates the response ball from the stimulus ball: $B_y = f(B_x)$. So for every similar response y' in B_y , we can find some similar stimulus x' in B_x such that $y' = f(x')$. Formally f maps the stimulus ball onto the response ball.

(We use the term "ball" loosely. Technically, $f(B_x)$ need not define an open ball in R^p . Thus we measure B_y with a volume measure below in (29). The *Open Mapping Theorem* in real analysis [Rudin, 1974] implies that all bounded onto linear transformations f map the open ball B_x to some set in R^p that contains the open ball B_y , where $y = f(x)$. At best we can only locally approximate most system transformations f as linear transformations.)

Then we can measure the creativity $C_{B_x}(f)$ of system f , given the stimulus ball B_x , by the volume ratio

$$C_{B_x}(f) = \frac{V(B_y)}{V(B_x)}, \quad (29)$$

where the V operator (Lebesgue measure) measures ball volume in R^n or R^p . C_{B_x} , crude as it is, captures many intuitions. It also resembles a spectral transfer function.

Consider the extreme cases of infinite and zero creativity. For a fixed nondegenerate response ball B_y , as the stimulus ball B_x contracts to x , the creativity measure $C_{B_x}(f)$ increases to infinity. (The point x has zero volume.) $C_{B_x}(f)$ also increases to infinity if the stimulus ball is constant and nondegenerate but the response ball B_y expands without bound as its radius approaches infinity. In both cases an infinitely creative system emits infinitely many responses when presented with, in the first case, a vanishingly small number of stimuli or, in the second case, a fixed set of stimuli.

Infinite creativity need not represent infinite problem solving. The reinforcing environment selects "solutions" from our varied or creative responses. Most creative solutions are impractical. We can emit creative responses without solving problems or contributing to our genetic fitness. Sometimes we call these responses "art" or "play."

At the other extreme, zero creativity occurs when the response ball B_y vanishes or when the stimulus ball expands without bound as its radius grows to infinity. In the first case the system f is a constant function. It maps all stimuli in B_x to a single value y in R^p . Such an f is "dumb" or "dull." In the second case, for an infinite-radius stimulus ball B_x , the stimuli overwhelm the system's response repertoire. Such systems resemble classical pattern-recognition devices that are sensitive only to well-defined, well-centered patterns (faces, zip codes, bar codes).

Small variations in input provide the simplest novel stimuli. The physical or cultural environment may produce these variations. Or we may systematically produce them as grist for our analytical mill. We may vary stimuli to solve a crossword puzzle, to fit physical variables to astronomical data, or to formulate and resolve a mathematical conjecture.

We are all forward-looking creatures. We tend not to see the gradual causal chains that precede our every action, idea, and innovation. Even Beethoven's Fifth Symphony appears less a discontinuity when we examine Beethoven's notebooks and a variety of preceding musical compositions by him and by other composers.

Variation and selection drive biological and cultural evolution. Physical and cultural environments drive the selection process. Function continuity, and other factors, drive variation.

Nature and man experiment with local variations of input parameters. This generates

local variations of output parameters. Then selection processes filter the new outputs. More accurately, they filter the corresponding new systems. We call the new systems "winners" or "fit" if they pass through the selection filters, "losers" or "unfit" if they do not pass through.

Variation and selection rates may vary, especially over long stretches of geological or cultural time. Different perturbed processes unfold at different speeds. So some evolutionary stretches appear more "punctuated" than others [Gould, 1980]. This means some measures of change—ultimately time derivatives—are nonlinear. It does not mean that the underlying input-output functions are discontinuous.

Learning as Change

Intelligent systems also *learn* or *adapt*. They learn new associations, new patterns, new functional dependencies. They sample the flux of experience and encode new information. They compress or quantize the sampled flux into a small, but statistically representative, set of prototypes or exemplars. Sample data changes system parameters.

"Learning" and "adaptation" are linguistic gifts from antiquity. They simply mean *parameter change*. The parameters may be numerical weights in an inner-product sum, average neurotransmitter release rates at synaptic junctions, or gene (allele) frequencies at chromosomal loci in populations.

"Learning" usually applies to synaptic changes in brains or nervous systems, coefficient changes in estimation or control algorithms or devices, or resistor changes in analog VLSI circuitry. Sometimes we synonymously apply "adaptation" to the same changing parameters. In evolutionary theory "adaptation" applies to positive changes in gene frequencies [Wilson, 1975].

In all cases learning means change. Formally, a system learns if and only if the system parameter vector or matrix has a nonzero time derivative. In neural networks we usually represent the synaptic web by an adjacency or connection matrix M of numerical synaptic values. Then learning is *any change in any synapse*:

We can learn well or learn badly. But we cannot learn without changing, and we cannot change without learning.

Learning laws describe the synaptic dynamical system, how the system encodes information. They determine how the synaptic-web process unfolds in time as the system samples new information. This shows one way that neural networks compute with dynamical systems. Neural networks also identify neural activity with dynamical systems. This allows the systems to decode information.

In principle we can harness any dynamical system to encode and decode some information. We can view a kinetic swirl of molecules, a joint population of lynxes and rabbits, and a solar system as systems that transform input states to output states. Initial conditions and perturbations encode questions. Transient behavior computes answers. Equilibrium behavior provides answers. In the extreme case we can even view the universe as a dynamical-system "computer." A godlike entity may choose Big-Bang initial conditions, and there are infinitely many, to encode certain information or to ask certain questions. The dynamical system computes as the universe expands transiently. Universal equilibrium behavior could represent the computational output: a heat-death pattern or perhaps a periodic or chaotic oscillation of expansion and contraction.

Consider mowing a lawn of green grass. The mower "teaches" the lawn the short-grass pattern. The lawn consists of a parallel field of grass blades. Grass blades learn what they are cut. The lawn behaves as a semi-permanent, yet plastic, information storage medium. It tolerates faults and distributes cut patterns over large numbers of parallel units. We can mow our name in the lawn, and read or decode it from a rooftop. In principle we can encode all known information in a sufficiently big lawn. Eventually the lawn will forget this information if we do not resample comparable data, if we do not re-mow the lawn to a similar shape.

Ultimately learning provides only a means to some computational end. Neural networks learn patterns or functions or probability distributions to recognize future patterns, filter

future input streams of data, or solve future combinatorial optimization problems. Fuzzy systems learn associative rules to estimate functions or control systems. We climb the ladder of learning and kick it away when we reach the roof of computation. We care how the learned parameter performs in some computational system, not how it was learned, just as we applaud the piano recital and not the practice sessions.

Neural and fuzzy systems ultimately learn some unknown probability (sub)sethood function $p(x)$. The probability density function $p(x)$ describes a distribution of vector patterns or signals x , a few of which the neural or fuzzy system samples. When a neural or fuzzy system estimates a function $f: X \rightarrow Y$, it in effect estimates the joint probability density $p(x, y)$. Then solution points $(x, f(x))$ should reside in high-probability regions of the input-output product space $X \times Y$.

We do not need to learn if we know $p(x)$. We could proceed directly to our computational task with techniques from numerical analysis, combinatorial optimization, calculus of variations, or any other mathematical discipline. The need to learn varies inversely with the quantity of information or knowledge.

Sometimes the patterns cluster into exhaustive decision classes D_1, \dots, D_k . The decision classes may correspond to high-probability regions or "mountains." (If the pattern vectors are two-dimensional, then $p(x)$ defines a hilly surface in three-dimensional space R^3 .) Then class boundaries correspond to low-probability regions or "valleys" on the probability surface.

Supervised learning uses class-membership information. *Unsupervised* learning does not. An unsupervised learning system processes each sample x but does not "know" that x belongs to class D_i and not to class D_j . Unsupervised learning uses unlabelled samples. Neither supervised nor unsupervised learning systems assume knowledge of the underlying probability density function $p(x)$.

Suppose we want to train a speech-recognition system at an international airport. We want the German lightbulb to light up when someone speaks German to the speech-recognition system, the Hindi lightbulb to light up when someone speaks Hindi, and so on. The system learns as we feed it training waveforms or spectrograms.

We supervise the learning if we label each training sample as German, Hindi, Japanese,

etc. We may do this to compute an error. If the English lightbulb lights up for a German sample, we may algorithmically punish the system for this misclassification.

An unsupervised system learns only from the raw training samples. We do not indicate language class labels. Unsupervised systems adaptively cluster like patterns with like patterns. The speech-recognition system gradually clumps German speech patterns together. In competitive learning, for instance, the system learns class centroids, centers of pattern mass.

Unsupervised learning may seem difficult and unreliable. But most learning is unsupervised, since we do not know accurately the labels of most sample data, especially in realtime processing. Every second our biological synapses learn without supervision on a single pass of noisy data.

SYMBOLS VS. NUMBERS: RULES VS. PRINCIPLES

We all share another property: We cannot articulate the mathematical rules that describe, if not govern, our behavior. We can ask a violinist how she plays, and she can tell us. But her answer will not be a mathematical function. In general her answer will not enable us to reproduce her behavior.

All lifeforms recognize vast numbers of patterns. The most primitive patterns relate to how an organism forages, avoids predators, and reproduces [Wilson, 1975].

On this planet only man articulates rules, and he articulates very few. We articulate some rules in grammar, common law, and science ("physical laws"). Yet all our natural languages, living and dead, and all our systems of law have culturally evolved without conscious design and not in accord with articulated principles [Hayek, 1973]. To some extent this also holds for our accumulated knowledge of medical, biological, and social science.

There have been exceptions, and the exceptions have helped create the field of artificial intelligence. Last century linguists developed the articulated language *Esperanto*. Mathematician Giuseppe Peano similarly devised the language *Interlingua*. A few fans still learn

and speak *Esperanto* and *Interlingua*, but far fewer speak them than speak Latin. This century computer scientists have consciously created the many computer programming languages. Today programmers frequently use C, Pascal, and even Fortran, and infrequently use Algol and Jovial.

Computer scientists developed artificial intelligence in large part around the computer language Lisp, or *list processing*, and more recently around Prolog, or *logic programming*. Lisp and Prolog process symbols and lists of symbols. Symbolic logic, the bivalent propositional and predicate calculi, underlies their processing structure.

Expert System Knowledge as Rule Trees

AI systems store and process propositional rules. The rules are logical implications: IF A , THEN B . They associate actions B with conditions A . The rule antecedents and consequents correspond to step functions defined on their universes of discourse. One part of the input space activates or "fires" A as true, and the other part does not activate A .

Collections of rules define "knowledge bases" or "rulebases." The rule $A \rightarrow B$ locally structures the knowledge of A and B as a logical implication. The knowledge base globally structures the rules as an acyclic tree (or forest). The logical-implication paths $A \rightarrow B \rightarrow C \rightarrow D \rightarrow \dots$ flow from the tree's root nodes or antecedents to its leaf nodes or consequents. The term *knowledge base* stems from the computer-scientific term *database*. Because of the tree structure of knowledge bases, we might more accurately call them *knowledge trees*. Chapter 4 discusses fuzzy cognitive maps, which use feedback and vector-matrix operations to convert knowledge trees to knowledge networks.

Knowledge engineers search the knowledge tree to enumerate logical paths. Path enumeration defines the *inference* process. Forward-chaining inference proceeds from knowledge-tree antecedents to consequents. Backward-chaining inference proceeds from consequents or observations to plausible antecedents or hypotheses. Forward-chaining inference answers what-if question. It derives effects from causes. Backward-chaining inference answers why or how-come questions. It suggests causes for observed effects. Path-

enumeration complexity increases nonlinearly with the number of rules stored. Realtime path enumeration in large knowledge trees may be combinatorially prohibitive, requiring heuristic or approximate search strategies [Pearl, 1984].

Knowledge engineers acquire, store, and process the bivalent rules as symbols, not as numerical entities. This often allows knowledge engineers to rapidly acquire structured knowledge from experts and to efficiently process it. But it forces experts to articulate the propositional rules that approximate their expert behavior, and this they can rarely do.

Symbolic vs. Numeric Processing

Symbolic processing fits naturally in the brain-as-computer framework. Language strings model thoughts or shortterm memory. Rules and relations between language strings model longterm memory. Programming replaces learning. Logical inference replaces time evolution and nonlinear dynamics. Feedforward flow through knowledge trees replaces feedback equilibria.

But we cannot take the derivative of a symbol. We require a sufficiently continuous function. Symbol processing precludes mathematical analysis in the traditional senses of engineering and the physical sciences. The symbolic framework allows us to quickly represent structured knowledge as rules, but prevents us from directly applying the tools of numerical mathematics and from directly implementing AI systems in large-scale integrated circuits.

Figure 1.5 provides a taxonomy of model-free estimators. The taxonomy divides the knowledge type into structured (rule-like) and unstructured types and divides the framework into symbolic or numeric. All entries define model-free estimators because users need not state how outputs mathematically depend on inputs.

FRAMEWORK

		SYMBOLIC	NUMERICAL
KNOWLEDGE	STRUCTURED	AI EXPERT SYSTEMS	FUZZY SYSTEMS
	UNSTRUCTURED	—	NEURAL SYSTEMS

FIGURE 1.5 Taxonomy of model-free estimators. User need not state how system outputs explicitly depend on inputs.

Figure 1.5 outlines the advantages and disadvantages of machine-intelligent systems. AI expert systems exploit structured knowledge, when knowledge engineers can acquire it, but store and process it outside the analytical and computational numerical framework.

Neural networks exploit their numerical framework with theorems, efficient numerical algorithms, and analog and digital VLSI implementations. But neural networks cannot directly encode structured knowledge. They superimpose several input-output samples $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$ on a black-box web of synapses. Unless we check all input-output cases, we do not know what the neural system has learned, and in general we do not know what it will forget when it superimposes new samples (x_k, y_k) atop the old. We cannot directly encode the commonsense traffic-light rule "If traffic is heavy in one direction, keep the light green longer in that direction." Instead we must present the system with a sufficiently large set of input-output pairs, combinations of numerical traffic-density measurements and green-light duration measurements.

Fuzzy Systems as Structured Numerical Estimators

Fuzzy systems directly encode structured knowledge but in a numerical framework. We enter the fuzzy association (HEAVY, LONGER) as a single entry in a FAM-rule matrix. Each entry defines a fuzzy associative memory (FAM) "rule" or input-output transformation. In Chapter 17 we discuss the fuzzy control of an inverted pendulum. Figure 1.6 shows a bank of FAM rules sufficient to control an inverted pendulum.

		θ				
		NM	NS	Z	PS	PM
$\Delta\theta$	NM			PM		
	NS			PS	Z	
	Z	PM	PS	Z	NS	NM
	PS		Z	NS		
	PM			NM		

FIGURE 1.6 Bank of FAM rules to control an inverted pendulum. Each entry in the FAM matrix defines a fuzzy association between output fuzzy sets and paired input fuzzy sets.

θ , $\Delta\theta$, and v define fuzzy variables. Fuzzy variables θ and $\Delta\theta$ define the system's state variables. The angle fuzzy variable θ measures the angle the pendulum shaft makes with the vertical and ranges from -90 to 90 . The angular velocity fuzzy $\Delta\theta$ variable measures

the instantaneous rate of change of angle values. In practice it measures the difference between successive angle values. Output fuzzy variable v measures the current to a motor controller that adjusts the pendulum shaft.

Each fuzzy variable can assume five fuzzy-set values: Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), and Positive Medium (PM). The entry at the center of the FAM matrix defines the steady-state FAM rule: "IF $\theta = ZE$ AND $\Delta\theta = ZE$, THEN $v = ZE$."

We usually define the fuzzy-set values NM, ..., PM as trapezoids or triangles over regions of the real line. For the fuzzy angle variable θ , we can define ZE as a narrow triangle centered at the zero value in the interval $[-90, 90]$. Then the angle value 0 belongs to the fuzzy set ZE to degree 1. The angle values 3 and -3 may belong to ZE only to degree 0.6. Figure 1.7 shows seven trapezoidal fuzzy-set values assumed by fuzzy variable θ .

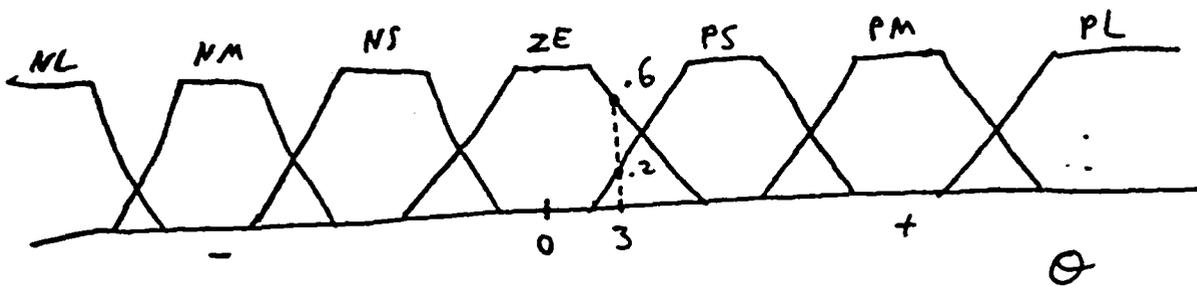


FIGURE 1.7 Seven trapezoidal fuzzy-set values assumed by fuzzy variable θ . Each value of θ belongs to each fuzzy set to some, but usually zero, degree. The exact value 3 belongs to the zero fuzzy number ZE to degree 0.6, to the positive small fuzzy number PS to degree .2, and to positive medium PM to

degree 0.

Fuzzy systems allow users to articulate linguistic FAM rules by entering values in a FAM matrix. Once a fuzzy engineer defines variables and fuzzy sets, the engineer can design a prototype fuzzy system in minutes.

Chapter 17 shows that a large neural-type matrix encodes each FAM rule. When fuzzy variables assume fuzzy subsets of the real line, as when we define ZE as a triangle centered about 0, then these associative matrices have uncountably infinite dimension. This endows each FAM rule with rich structure and "memory capacity." FAM systems do not add these matrices together, which avoids neural-type crosstalk.

A virtual representation scheme allows us to exploit the coding and capacity properties of these infinite matrices without actually writing them down. This holds for binary input-output FAMs (BIOFAMs), which includes all fuzzy systems used in commercial applications. BIOFAMs accept nonfuzzy scalar inputs, such as $\theta = 15$ and $\Delta\theta = -10$, and generate nonfuzzy scalar outputs, such as $v = -3$.

Generating Fuzzy Rules With Product-Space Clustering

Neural networks can adaptively generate the FAM rules in a fuzzy system. We illustrate this in Chapters 17 - 20 with the new technique of unsupervised *product-space clustering*. Synaptic vectors quantize the input-output space. Clustered synaptic vectors track how experts associate appropriate responses with input stimuli. Each synaptic cluster estimates a FAM rule. The experts who generate the input-output data need not articulate the FAM rules. They need only behave as experts. The key geometric idea is *cluster equals rule*.

Consider the input-output product space of the inverted-pendulum system. There are two input variables and one output variable, so the input-output product space equals R^3 (in practice a three-dimensional sub-cube within R^3). Each input-output triple $(\theta, \Delta\theta, v)$ defines a point in R^3 . The time evolution of the inverted-pendulum system defines a smooth curve or trajectory in R^3 . As the fuzzy system stabilizes the inverted pendulum to its vertical position, the trajectory may spiral into the origin of R^3 , where the above

steady-state FAM rule keeps the system in equilibrium until perturbed.

Each fuzzy variable can assume five fuzzy subsets of the x , y , or z coordinate axes of R^3 . The Cartesian product of these fuzzy subsets defines 125 ($5 \times 5 \times 5$) FAM cells in the input-output product space R^3 . Most system trajectories pass through only a few FAM cells. We show in Chapter 17 that these FAM cells equal FAM rules because the FAM cells equal fuzzy cartesian products, and the uncountably infinite entries in the associative matrices correspond to these cartesian products. So FAM rule equals associative (fuzzy Hebb) matrix, which equals fuzzy cartesian product, which equals FAM cell.

Unsupervised neural clustering algorithms efficiently track the density of input-output samples in FAM cells. We need only count the number of synaptic vectors in each FAM cell at any instant to estimate, and to weight, the underlying FAM rules used by the expert or physical process that generates the input-output data. This produces an *adaptive histogram* of FAM-cell occupation. Chapters 17 - 20 apply the adaptive product-space clustering methodology to inverted-pendulum control, backing up a truck-and-trailer in a parking lot, and realtime target tracking.

Suppose a system contains n fuzzy variables, and each fuzzy variable can assume m fuzzy-set values. This defines m^n FAM cells in the input-output product space R^n . Different fuzzy variables can assume different types and different numbers of fuzzy-set variables. So in general there are $m_1 \times \dots \times m_n$ FAM cells. Suppose $n = m = 3$. Suppose the fuzzy sets are low, medium, and high and have bounded extent. Then a Rubik's cube represents the input-output product space partitioned into 27 FAM cells if the fuzzy sets do not overlap. In general FAM cells have nonempty but fuzzy intersection.

If we define n fuzzy variables, each with m fuzzy-set values, then there are 2^{m^n} possible fuzzy systems. Expert articulation, fuzzy engineering, and adaptive estimation produce only a small fraction of the total number 2^{m^n} of possible fuzzy systems. Different fuzzy-set definitions and different encoding or decoding strategies ("inferencing" techniques) produce different classes of 2^{m^n} possible fuzzy systems.

Fuzzy Systems as Parallel Associators

Fuzzy systems store and process FAM rules in parallel. Mathematically a fuzzy system maps points in an input product hypercube (possibly of infinite dimension) to points in an output hypercube. Fuzzy systems associate output fuzzy sets with input fuzzy sets, and so behave as associative memories. Unlike neural associative memories, fuzzy systems do not sum the associative matrices that represent FAM rules. *Neural networks sum throughputs. Fuzzy systems sum outputs.*

Summing outputs avoids crosstalk and achieves modularity. We can meaningfully look inside the black box of fuzzy model-free estimator. Figure 1.8 displays the generic fuzzy system architecture for a single-input, single-output FAM system.

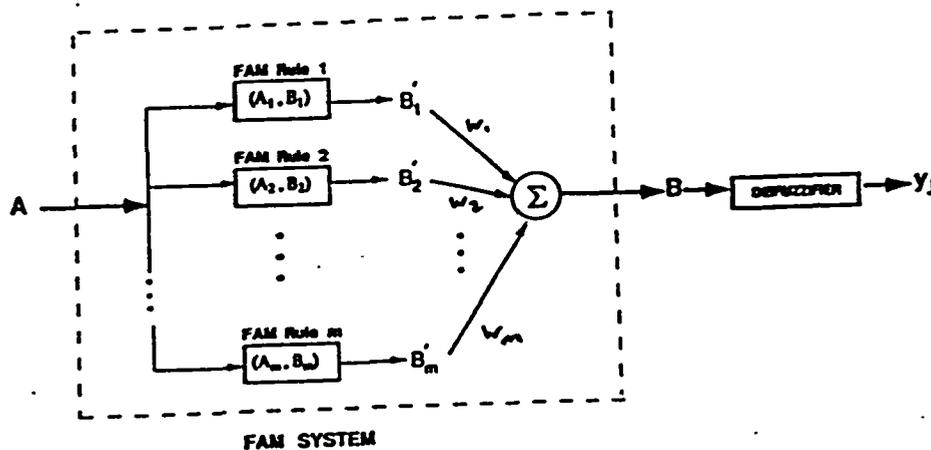


FIGURE 1.8 Fuzzy system architecture. The system maps input fuzzy sets A to output fuzzy sets B . The system stores separate FAM rules and in parallel fires each FAM rule to some degree for each input. Experts or adaptive

algorithms determine the FAM-rule weights w_j . Experts may use only $w_j = 1$ (articulates rule) or $w_j = 0$ (omits rule). Centroidal output converts fuzzy-set vector B to a scalar. In BIOFAM systems A defines a unit binary vector or delta pulse.

Fuzzy inference computes the output fuzzy sets B'_j , weights them with the scalar weights w_j , and sums them to produce the output fuzzy set B :

$$B = \sum_j w_j B'_j \quad (31)$$

In principle in (31) we sum over all m^n possible FAM rules since most rules have weight $w_j = 0$. Chapter 17 discusses the mechanism of the two types of fuzzy inference, correlation-product and correlation-minimum inference.

Adaptive fuzzy systems use sample data and neural or statistical algorithms to choose the coefficients w_j and thus to define the fuzzy system at each time instant. Adaptation changes the system structure. Geometrically, a time-varying between-cube mapping defines an adaptive fuzzy system. In the simplest case, if the input fuzzy sets define points in the unit hypercube I^n , and the output fuzzy sets define points in the unit hypercube I^p , then transformation S defines a fuzzy system if S maps I^n to I^p , $S : I^n \rightarrow I^p$. Then S associates fuzzy subsets of the output space Y with fuzzy subsets of the input space X . So $S(A) = B$. S defines an adaptive fuzzy system if S changes with time:

$$\frac{dS}{dt} \neq 0 \quad (32)$$

BIOFAM systems convert the vector B into a single scalar output value $y \in Y$. We call this process defuzzification, although to defuzzify a fuzzy set formally means to round it off from some point in a unit hypercube to the nearest bit-vector vertex. Fuzzy engineers sometimes compute y as the mode y_{\max} of the B distribution,

$$m_B(y_{\max}) = \sup \{m_B(y) : y \in Y\} \quad (33)$$

m_B denotes the fuzzy membership function $m_B : Y \rightarrow [0, 1]$ that assigns fit values or

occurrence degrees to the elements of Y . If the output space Y equals a finite set of values $\{y_1, \dots, y_p\}$, as in some computer discretizations, then we can replace the supremum in (33) with a maximum:

$$m_B(y_{\max}) = \max_j m_B(y_j) \quad , \quad (34)$$

The more popular centroidal defuzzification technique uses all, and only, the information in the fuzzy distribution B to compute y as the centroid \bar{y} or center of mass of B :

$$\bar{y} = \frac{\int_{-\infty}^{\infty} y m_B(y) dy}{\int_{-\infty}^{\infty} m_B(y) dy} \quad , \quad (35)$$

provided the integrals exist. In practice we restrict fuzzy subsets to finite stretches of the real line. In Chapter 19 we prove that if the fuzzy variables assume only symmetric trapezoid-like fuzzy-set values, then (35) reduces to a simple discrete ratio. The numerator and denominator contain only m products. This discrete centroid trivializes the computational burden of defuzzification and admits direct VLSI implementation.

Figure 1.8 and equation (31) additively combine the weighted fuzzy sets B_j^i . Earlier fuzzy systems [Mamdani, 1977] combined output fuzzy sets with pairwise maxima. Unfortunately, the maximum combination technique,

$$B = \max_j \min(w_j, B_j^i) \quad , \quad (36)$$

based upon the so-called "extension principle" of classical fuzzy theory [Klir, 1988], tends to produce a uniform distribution for B as the number of combined fuzzy sets increases [Kosko, 1987]. A uniform distribution always has the same mode and centroid. So, ironically, as the number of FAM rules increases, system sensitivity decreases.

The additive combination technique (31) tends to invoke the fuzzy version of the Central Limit Theorem. The added fuzzy waveforms pile up to approximate a symmetric unimodal, or bell-shaped, membership function. Different fuzzy waveforms produce simi-

larly *shaped* output distributions B but centered about different places on the real line. We consistently observe this tendency towards a Gaussian membership function after summing only a few fuzzy waveforms. (Technically the CLT requires normalization by the square-root of the number of summed waveforms. Equation (31) does not normalize B because, for defuzzification, we care only about the relative values in B , the relative degrees of occurrence of output values.)

The maximum combination technique (36) forms the envelope of the weighted fuzzy sets B_j . Then B resembles the silhouette of a desert-full of sand dunes. As the number of sand dunes increases, the silhouette becomes flatter. The additive combination technique (31) piles the sand dunes atop one other to form a sand mountain.

Fuzzy inference allows us to reason with sets as if they were propositions. The virtual-representation scheme for FAM rules greatly simplifies the fuzzy inference process if we use exact numerical inputs. Figure 1.9 illustrates the FAM (correlation-minimum) inference procedure derived in Chapter 17. We can apply this inference procedure in parallel to any number of FAM rules with any number of antecedent fuzzy-variable conditions.

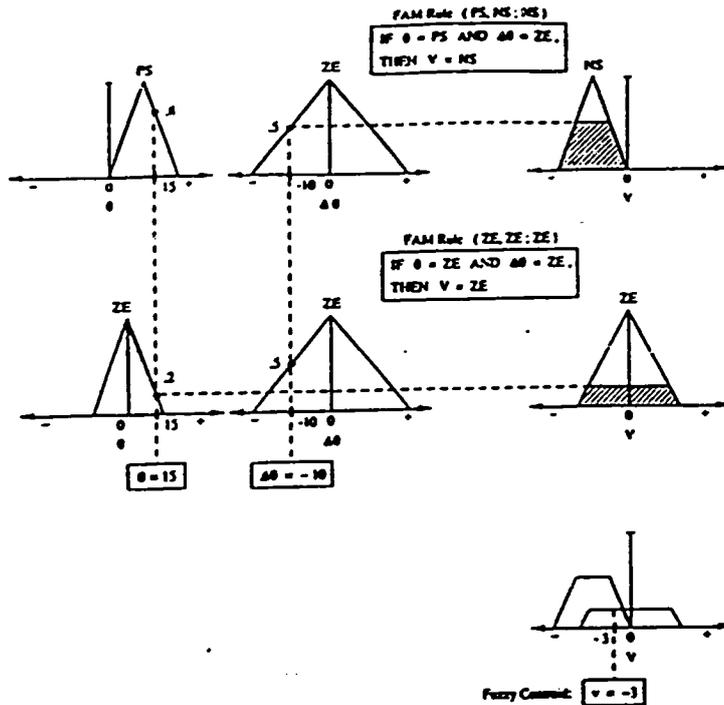


FIGURE 1.9 FAM inference procedure. The fuzzy system converts the numerical inputs, $\theta = 15$ and $\Delta\theta = -10$, into the numerical output $v = -3$. Since the FAM rules combine the antecedent terms with AND, the smaller of the two fit values scales the output fuzzy set. If the FAM rules combined antecedents disjunctively with OR, the larger of the fit values would scale the output fuzzy set.

Fuzzy Systems as Principle-Based Systems

AI expert systems chain through rules. Inference proceeds down, or up, branches of a decision tree. Except for chess trees or other game trees, in practice these search trees

are wider than they are deep. Shallow trees (or forests) can exaggerate the all-or-none effect of bivalent propositional rules. Relative to deeper trees, shallow trees use a smaller proportion of their stored knowledge when they inference. They are noninteractive.

Fuzzy systems are shallow but fully interactive. Every inference fires every FAM rule, itself a fuzzy expert system, to some degree. A similar property holds for the feedback fuzzy cognitive maps discussed in Chapter 4.

Consider an AI judge and a fuzzy judge. Opposing counsel present the same evidence and testimony to both judges. The AI judge rounds off the truth value of every key statement or alleged fact to TRUE or FALSE (1 or 0), opens a rule book, uses the true statements to activate or choose the antecedents of some of the rules, then logically chains through the rule tree to reach a decision. A more sophisticated AI judge may chain through the rule tree with uncertainty-factor algorithms or heuristic search algorithms.

The fuzzy judge weights the evidence to different degrees, say with fractional values in the unit interval [0, 1]. The fuzzy judge does not use a rule book. Instead the fuzzy judge determines to what degree the fuzzy evidence invokes a large set of vague legal principles. The fuzzy judge may cite case precedents to enunciate these principles or to illustrate their relative importance. The fuzzy judge reaches a decision by combining these fuzzy facts and fuzzy principles in an unseen act of intuition or judgement. If pressed, the fuzzy judge may defend or explain the decision by citing the salient facts and relevant legal principles, precedents, and perhaps rules. In general the fuzzy judge cannot articulate an exact legal audit trail of the decision process.

The distinction between the AI judge and the fuzzy judge reduces to the distinction between rules and principles. Recently legal theorists [Dworkin, 1968-77; Hayek, 1973] have focused on this distinction and challenged the earlier "positivist" legal theories of law as articulated rules [Kelsen, 1954; Hart, 1961].

Rules, as Dworkin [1977] says, apply "in an all-or-none fashion." Principles "have a dimension that rules do not—the dimension of weight or importance," and the court "cites principles as its justification for adopting and applying a new rule." Rules greatly outnumber principles. Principles guide while rules specify:

"Only rules dictate results, come what may. When a contrary result has been reached, the rule has been abandoned or changed. Principles do not work that way; they incline a decision one way, though not conclusively, and they survive intact when they do not prevail."

Rules tend to be black or white. They abruptly come into and out of existence. We post rules on signs, vote on them as propositions, and send them in memos: must be 18 to vote, open from 8 am to 5 pm, \$500 fine for littering, office term lasts four years, can take only five sick days a year, and so on. Rules come and go as culture evolves.

Principles evolve as culture evolves. Most legal principles in the United States grew out of medieval British common law. Each year their character changes slightly, adaptively, as we apply them to novel circumstances. These principles range from very abstract principles, such as presumption of innocence or freedom of contract, to more behavioral principles, such as that no one can profit from a crime or you cannot challenge a contract if you acquiesce to it and act on it.

Each principle admits a spectrum of exceptions. In each case a principle holds only to some, often slight, degree. Judges cite case precedents in effect to estimate the current weight of principles. All the principles "hang together" to some degree in each decision, just as all the fuzzy rules (principles) in Figure 1.5 contribute to some degree to the final inference or decision.

We often call AI expert systems rule-based systems because they consist of a bank or forest of propositional rules and an "inference engine" for chaining through the rules. The rule in rule-based emphasizes the articulated, expertly precise nature of the stored knowledge.

The AI precedent and modern legal theory suggest that we should call fuzzy systems principle-based systems. The fuzzy rules or principles indicate how entire clumps of output spaces associate with clumps of input spaces. Indeed FAM rules often behave as partial derivatives. Many applications require only a few FAM rules for smooth system control or estimation. In general AI rule-based systems would require vastly more precise rules to approximate the same system performance.

Adaptive fuzzy systems use neural (or statistical) techniques to abstract fuzzy principles from sampled cases and to gradually refine those principles as the system samples new cases. The process resembles our everyday acquisition and refinement of commonsense knowledge. Future machine-intelligent systems may match, then someday exceed, our ability to learn and apply the fuzzy commonsense knowledge—knowledge we can articulate only rarely and inexactly—that we use to run our lives and run our world.

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PROBLEMS

1. Lukasiewicz's continuous or "fuzzy" logic (L_1 logic) uses a continuous-valued truth function $t : S \rightarrow [0, 1]$ defined on the set S of statements. Lukasiewicz defined the generalized conjunction (AND), disjunction (OR), negation (NOT) operators respectively as

$$t(A \text{ AND } B) = \min(t(A), t(B)) ,$$

$$t(A \text{ OR } B) = \max(t(A), t(B)) ,$$

$$t(\text{NOT-}A) = 1 - t(A) ,$$

for statements A and B . Prove the generalized noncontradiction-excluded-middle law:

$$t(A \text{ AND } \sim A) + t(A \text{ OR } \sim A) = 1 .$$

This equality implies that the classical bivalent law of noncontradiction, $t(A \text{ AND } \sim A) = 0$, holds if and only if the classical bivalent law of excluded middle, $t(A \text{ OR } \sim A) = 1$, holds. Note that in the case of bivalent "paradox," when $t(A) = t(\text{NOT-}A)$, the equality reduces to the equality $1/2 + 1/2 = 1$.

2. Let $t : S \rightarrow [0, 1]$ be a continuous or "fuzzy" truth function on the set S of statements. Define the Lukasiewicz implication operator as the truth function $t_L(A \rightarrow B) = \min(1, 1 - t(A) + t(B))$ for statements A and B . Then prove the following generalized fuzzy *modus ponens* inference rule:

$$\begin{array}{l} t_L(A \rightarrow B) = c \\ t(A) \geq a \end{array}$$

Therefore $t(B) \geq \max(0, a + c - 1)$.

Hence if $t(A) = t_L(A \rightarrow B) = 1$, then $t(B) = 1$, which generalizes classical bivalent *modus ponens*.

3. Use the continuous logic operations in Problem 2 to prove the following generalized fuzzy *modus tollens* inference rule:

$$\begin{array}{l} t_L(A \rightarrow B) = c \\ t(B) \leq b \end{array}$$

Therefore $t(A) \leq \min(1, 1 - c + b)$.

Hence if $t_L(A \rightarrow B) = 1$ and $t(B) = 0$, then $t(A) = 0$, which generalizes classical bivalent *modus tollens*.

4. Define the Gaines implication operator as

$$t_G(A \rightarrow B) = \begin{cases} \min(1, t(B)/t(A)) & \text{if } t(A) > 0 \\ 1 & \text{if } t(A) = 0 \end{cases}$$

Use the Gaines implication operator $t_G(A \rightarrow B)$ to derive generalized fuzzy *modus ponens* and *modus tollens* inference rules. The conclusion of the inference rules should differ from the conclusions of the inference rules in Problems 2 and 3.

5. Exclusive-or (*XOR*) equals negated logically equivalence:

$$t(A \text{ XOR } B) = 1 - t(A = B) .$$

Equivalence equals biconditionality. Bivalent statements are equivalent if and only if the two statements have the same truth values. So the exclusive-or relation holds between two bivalent statements if and only if the two statements have opposite truth values.

Fuzzy statements can be equivalent to different degrees. But equivalence still equals biconditionality:

$$t(A = B) = t((A \rightarrow B) \text{ AND } (B \rightarrow A)) .$$

Prove that if we use the Lukasiewicz implication operator, then exclusive-or equals the absolute difference (or I^1 or fuzzy Hamming distance) of the truth values $t(A)$ and $t(B)$:

$$t_L(A \text{ XOR } B) = |t(A) - t(B)| .$$

6. Set X contains n elements x_1, \dots, x_n . So X contains 2^n nonfuzzy subsets A . Define the bivalent indicator function I_A of nonfuzzy set A as

$$I_A(x_i) = \begin{cases} 1 & \text{if } x_i \in A \\ 0 & \text{if } x_i \notin A \end{cases}$$

So I_A defines the mapping $I_A : X \rightarrow \{0, 1\}$.

Suppose we extend I_A to a multivalued mapping by augmenting its range from $\{0, 1\}$ to $\{y_1, \dots, y_m\}$, where $y_1 = 0$, $y_m = 1$, and $0 < y_j < 1$ if $1 < j < m$. Then I_A defines the mapping $I_A : X \rightarrow \{y_1, \dots, y_m\}$. How many multivalued

subsets does X have? In the 2-dimensional case, $X = \{x_1, x_2\}$, draw the planar lattice that describes the multi-dimensional power set of X , all its multi-dimensional subsets, when $m = 3$, and when $m = 5$.

7. Consider the discrete dynamical system

$$\begin{aligned}x_{k+1} &= f(x_k) \\ &= c x_k (1 - x_k) ,\end{aligned}$$

for x values in $[0, 1]$ and $0 < c \leq 4$. Many dynamical systems transition into chaos as we increase a control or gain parameter, such as c . Select $c = 3.5$ and use the two choices of initial conditions, $x_0 = .5$ and $x_0 = .51$, to generate x_1, \dots, x_{20} . Plot the two trajectories on graph paper. Are they aperiodic (chaotic) or periodic? Does a difference of .01 in initial condition significantly affect the overall shape of the discrete trajectory?

Now repeat the above experiment but use the gain parameter $c = 3.9$ (or $c = 4$). No matter how close two initial conditions, in a chaotic dynamical system they always produce divergent trajectories. Does $c = 3.9$ produce chaos?

CHAPTER 16

FUZZINESS VERSUS PROBABILITY

So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.

. . . Albert Einstein

Fuzzy Sets and Systems

We now explore fuzziness as an alternative to randomness for describing uncertainty. We develop the new *sets-as-points* geometric view of fuzzy sets. This view identifies a fuzzy set with a point in a unit hypercube, a nonfuzzy set with a vertex of the cube, and a fuzzy system as a mapping between hypercubes. Chapter 17 examines fuzzy systems.

Paradoxes of two-valued logic and set theory, such as Russell's paradox, correspond to the midpoint of the fuzzy cube. We geometrically answer the fundamental questions of fuzzy theory—How fuzzy is a fuzzy set? How much is one fuzzy set a subset of another?—with the Fuzzy Entropy Theorem and the Fuzzy Subsethood Theorem.

We develop a new geometric proof of the Subsethood Theorem. A corollary shows that the apparently probabilistic relative frequency $\frac{n_A}{N}$ equals the deterministic subsethood $S(X, A)$, the degree to which the sample space X is contained in its subset A . So the

frequency of successful trials equals the degree to which all trials are successful. We examine recent Bayesian polemics against fuzzy theory in light of the new sets-as-points theorems.

An element belongs to a fuzzy set to some degree in $[0, 1]$. An element belongs to a nonfuzzy set all or none, 1 or 0. More fundamentally, one set is a subset of one of the set to some degree. Sets fuzzily contain subsets as well as elements. Subsethood generalizes elementhood. We shall argue that subsethood generalizes probability as well.

Fuzziness in a Probabilistic World

Is uncertainty the same as randomness? If we are not sure about something, is it only up to chance? Do the notions of likelihood and probability exhaust our notions of uncertainty?

Many people, trained in probability and statistics, believe so. Some even say so, and say so loudly. These voices often arise from the Bayesian camp of statistics, where probabilists view probability not as a frequency or other objective testable quantity, but as a subjective *state of knowledge*.

Bayesian physicist E. T. Jaynes [1979] says that "any method of inference in which we represent degrees of plausibility by real numbers, is necessarily either equivalent to Laplace's [probability], or inconsistent." He claims physicist R. T. Cox [1946] has proven this as a theorem, a claim we examine below.

More recently, Bayesian statistician Dennis Lindley [1987] issued an explicit challenge: "probability is the only sensible description of uncertainty and is adequate for all problems involving uncertainty. All other methods are inadequate."

Lindley directs his challenge in large part at *fuzzy theory*, the theory that *all things admit degrees*, but admit them deterministically. We accept the probabilist's challenge from the fuzzy viewpoint. We will defend fuzziness with new geometric first principles and will question the reasonableness and the axiomatic status of randomness. The new view is the *sets-as-points view* [Kosko, 1987] of fuzzy sets: A fuzzy set defines a point in a unit-hypercube, and a nonfuzzy set defines a corner of the hypercube.

Randomness and fuzziness differ conceptually and theoretically. We can illustrate some differences with examples. Others we can prove with theorems, as we show below.

Randomness and fuzziness also share many similarities. Both systems describe uncertainty with numbers in the unit interval $[0, 1]$. This ultimately means that both systems describe uncertainty numerically. Both systems combine sets and propositions associatively, commutatively, and distributively. The key distinction concerns how the systems jointly treat a thing A and its opposite A^c . Classical set theory demands $A \cap A^c = \emptyset$, and probability theory conforms: $P(A \cap A^c) = P(\emptyset) = 0$. So $A \cap A^c$ represents a probabilistically impossible event. But fuzziness begins when $A \cap A^c \neq \emptyset$.

Questions raise doubt, and doubt suggests room for change. So to commence the exposition, consider the following two questions, one fuzzy and the other probabilistic:

- (i) Is it always and everywhere true that $A \cap A^c = \emptyset$?
- (ii) Do we *derive* or *assume* the conditional probability operator

$$P(B|A) = \frac{P(A \cap B)}{P(A)} ? \quad (1)$$

The second question may appear less fundamental than the first question, which asks whether fuzziness exists. The Entropy-Subsethood Theorem below shows that the first question reduces to the second questions: We measure the fuzziness of fuzzy set A when we measure how much the superset $A \cup A^c$ is a subset of its own subset $A \cap A^c$, a paradoxical relationship unique to fuzzy theory. In contrast, in probability theory the like relationship is impossible (has zero probability): $P(A \cap A^c | A \cup A^c) = P(\emptyset | X) = 0$, where X denotes the sample space or "sure event".

The conditioning or subsethood in the second question lies at the heart of Bayesian probabilistic systems. We may accept the absence of a first-principles derivation of $P(B|A)$. We can simply agree to take the ratio relationship as an axiom. But the new sets-as-points view of fuzzy sets *derives* its conditioning operator as a theorem from first principles. The history of science suggests that systems that hold theorems as axioms continue to evolve.

The first question asks whether we can logically or factually violate the law of noncontra-

diction—one of Aristotle's three "laws of thought" along with the laws of excluded middle, $A \cup A^c = X$, and identity, $A = A$. Set fuzziness occurs when, and only when, it is violated. Classical logic and set theory assume that we cannot violate the law of noncontradiction or, equivalently, the law of excluded middle. This makes the classical theory black or white. Fuzziness begins where Western logic ends—where contradictions begin.

Randomness vs. Ambiguity: Whether vs. How Much

Fuzziness describes *event ambiguity*. It measures the degree to which an event occurs, not whether it occurs. Randomness describes the uncertainty of *event occurrence*. An event occurs or not, and you can bet on it. The issue concerns the occurring event: Is it uncertain in any way? Can we unambiguously distinguish the event from its opposite?

Whether an event occurs is "random". To what degree it occurs is fuzzy. Whether an ambiguous event occurs—as when we say there is 20% chance of light rain tomorrow—involves compound uncertainties, the probability of a fuzzy event.

We regularly apply probabilities to fuzzy events: small errors, satisfied customers, A students, safe investments, developing countries, noisy signals, spiking neurons, dying cells, charged particles, nimbus clouds, planetary atmospheres, galactic clusters. We understand that, at least around the edges, some satisfied customers can be somewhat unsatisfied, some A students might equally be B+ students, some stars are as much in a galactic cluster as out of it. Events can transition more or less smoothly to their opposites, making classification hard near the midpoint of the transition. But in theory—in formal descriptions and in textbooks—the events and their opposites are black and white. A hill is a mountain if it is at least x meters tall, not a mountain if it is one micron less than x in height [Quine, 1981]. Every molecule in the universe either is or is not a pencil molecule, even those that hover about the pencil's surface.

Consider some further examples. The probability that this chapter gets published is one thing. The degree to which it gets published is another. The chapter may be edited

in hundreds of ways. Or the essay may be marred with typographical errors, and so on.

Question: Does quantum mechanics deal with the probability that an unambiguous electron occupies spacetime points? Or does it deal with the degree to which an electron, or an electron smear, occurs at spacetime points? Does $|\psi|^2 dV$ measure the probability that a random-point electron occurs in infinitesimal volume dV ? Or [Kosko, 1990] does it measure the degree to which a deterministic electron cloud occurs in dV ? Different interpretation, different universe. Perhaps even existence admits degrees at the quantum level.

Suppose there is 50% chance that there is an apple in the refrigerator (electron in a cell). That is one state of affairs, perhaps arrived at through frequency calculations or a Bayesian state of knowledge. Now suppose there is half an apple in the refrigerator. That is another state of affairs. Both states of affairs are superficially equivalent in terms of their numerical uncertainty. Yet physically, ontologically, they differ. One is, "random", the other fuzzy.

Consider parking your car in a parking lot with painted parking spaces. You can park in any space with some probability. Your car will totally occupy one space and totally unoccupy all other spaces. The probability number reflects a frequency history or Bayesian brain state that summarizes which parking space your car will totally occupy. Alternatively, you can park in every space to some degree. Your car will partially, and deterministically, occupy every space. In practice your car will occupy most spaces to zero degree. Finally, we can use numbers in $[0, 1]$ to describe, for each parking space, the occurrence probability of each degree of partial occupancy—probabilities of fuzzy events.

If we assume events are unambiguous, as in balls-in-urns experiments, there is no set fuzziness. Only "randomness" remains. But when we discuss the physical universe, every assertion of event ambiguity or nonambiguity is an empirical *hypothesis*. We habitually overlook this when we apply probability theory. Years of such oversight have entrenched the sentiment that uncertainty is randomness, and randomness alone. We systematically assume away event ambiguity. We call the partially empty glass empty and call the small number zero. This silent assumption of universal nonambiguity resembles the pre-relativistic assumption of an uncurved universe. $A \cap A^c = \emptyset$ is the "parallel postulate"

of classical set theory and logic, indeed of Western thought.

If fuzziness is a genuine type of uncertainty, if fuzziness exists, the physical consequences are universal, and the sociological consequence is startling: scientists, especially physicists, have overlooked an entire mode of reality.

Fuzziness is a type of deterministic uncertainty. Ambiguity is a property of physical phenomena. Unlike fuzziness, probability dissipates with increasing information. After the fact "randomness" looks like fiction. Yet many of the laws of science are time reversible, invariant if we replace time t with time $-t$. If we run the universe in reverse as if it were a video tape, where does the "randomness" go? There is as much ambiguity after a sample-space experiment as before. Increasing information specifies the degrees of occurrence. Even if science had run its course and all the facts were in, a platypus would remain only roughly a mammal, a large hill only roughly a mountain, an oval squiggle only roughly an ellipse. Fuzziness does not require that God plays dice.

Consider the inexact oval in Figure 16.1. Does it make more sense to say that the oval is *probably* an ellipse, or that it is a fuzzy ellipse? There seems nothing random about the matter. The situation is deterministic: All the facts are in. Yet uncertainty remains. The uncertainty arises from the simultaneous occurrence of two properties: to some extent the

inexact oval is an ellipse, and to some extent it is not an ellipse.

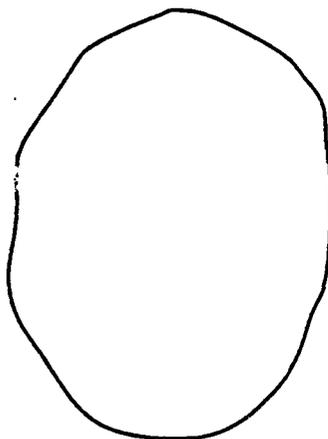


Figure 16.1 Inexact-oval. Which statement better describes the situation: "It is probably an ellipse" or "It is a fuzzy ellipse"?

More formally, does $m_A(x)$, the degree to which element x belongs to fuzzy set A , equal the probability that x belongs to A ? Is $m_A(x) = \text{Prob}\{x \in A\}$ true? Cardinality-wise, sample spaces cannot be too big. Else a positive measure cannot be both countably additive and finite, and thus in general cannot be a probability measure [Chung, 1974]. The space of all possible oval figures is too big, since there are more of these than real numbers. Almost all sets are too big for us to define probability measures on them, yet we can always define fuzzy sets on them.

Probability theory is a chapter in the book of finite measure theory. Many probabilists do not care for this classification, but they fall back upon it when defining terms [Kac, 1959]. How reasonable is it to believe that finite measure theory—ultimately, the summing of nonnegative numbers to unity—exhaustively describes the universe? Does it really describe *any* thing?

Surely from time to time every probabilist wonders whether probability describes anything real. From Democritus to Einstein, there has been the suspicion that, as David Hume [1748] put it, "though there be no such thing as *chance* in the world, our ignorance of the real cause of any event has the same influence on the understanding and begets a like species of belief." When we model noisy processes by extending differential equations to stochastic differential equations, as in Chapters 4-6, we introduce the formalism only as a working approximation to several underlying unspecified processes, processes that presumably obey deterministic differential equations. In this sense conditional expectations and martingale techniques might seem reasonably applied, for example, to stock options or commodity futures phenomena, where the behavior involved consists of aggregates of aggregates of aggregates. The same techniques seem less reasonably applied to quarks, leptons, and void.

The Universe as a Fuzzy Set

The world, as Wittgenstein [1922] observed, is everything that is the case. In this spirit we can summarize the ontological case for fuzziness: *The universe consists of all subsets of the universe.* The only subsets of the universe that are not in principle fuzzy are the constructs of classical mathematics. The integer 2 belongs to the even integers, and does not belong to the odd or negative integers. All other sets—sets of particles, cells, tissues, people, ideas, galaxies—in principle contain elements to different degrees. Their membership is partial, graded, inexact, ambiguous, or uncertain.

The same universal circumstance holds at the level of logic and truth. The only logically true or false statements—statements S with truth value $t(S)$ in $\{0, 1\}$ —are tautologies, theorems, and contradictions. If statement S describes the universe, if S is an *empirical* statement, then $0 < t(S) < 1$ holds by the canons of scientific method and by the lack of a single demonstrated factual statement S with $t(S) = 1$ or $t(S) = 0$. Philosopher Immanuel Kant [1787] wrote volumes in search of factually true logical statements and logically true factual statements.

Logical truth differs in kind from factual truth. " $2 = 1 + 1$ " has truth value 1. "Grass is green" has truth value less than 1 but greater than 0. This produces the math/universe crisis Einstein laments in his quote at the beginning of this chapter. Scientists have imposed a two-valued mathematics, shot through with logical "paradoxes" or antinomies [Kline, 1980], on a multivalued universe. Last century John Stuart Mill [1843] argued that logical truths represent limiting cases of factual truths. This accurately summarized the truth-value distinction between $0 < t(S) < 1$ and $t(S) = 0$ or $t(S) = 1$ but, cast in linguistic form, seems not to have persuaded modern philosophers. The Heisenberg uncertainty principle, with its continuum of indeterminacy, forced multivaluedness on science, though few Western philosophers [Quine, 1981] have accepted multivaluedness. Lukasiewicz, Gödel, and Black [Rescher, 1969] did accept it and developed the first continuous or "fuzzy" logic and set systems.

Fuzziness arises from the ambiguity or vagueness [Black, 1937] between a thing A and its opposite A^c . If we do not know A with certainty, we do not know A^c with certainty either. Else by double negation we would know A with certainty. This ambiguity produces nondegenerate *overlap*: $A \cap A^c \neq \emptyset$, which breaks the "law of noncontradiction." Equivalently, it also produces nondegenerate *underlap* [Kosko, 1986b]: $A \cup A^c \neq X$, which breaks the "law of excluded middle." Here X denotes the ground set or universe of discourse. (Probabilistic or stochastic logics [Gaines, 1983] do not break these laws: $P(A \text{ and not-}A) = 0$ and $P(A \text{ or not-}A) = 1$.) Formally, probability measures cannot take fuzzy sets as arguments. We must first quantize, round off, or defuzzify the fuzzy sets to the nearest nonfuzzy sets.

THE GEOMETRY OF FUZZY SETS: SETS AS POINTS

It helps to see the geometry of fuzzy sets when we discuss fuzziness. To date researchers have overlooked this visualization. Instead they have interpreted fuzzy sets as generalized indicator or membership functions [Zadeh, 1965], mappings m_A from domain X to range $[0, 1]$. But functions are hard to visualize. Fuzzy theorists [Klir, 1988] often picture

membership functions as two-dimensional graphs, with the domain X represented as a one-dimensional axis. The geometry of fuzzy sets involves both the domain $X = \{x_1, \dots, x_n\}$ and the range $[0, 1]$ of mappings $m_A : X \rightarrow [0, 1]$. The geometry of fuzzy sets aids us when we describe fuzziness, define fuzzy concepts, and prove fuzzy theorems. Visualizing this geometry may by itself provide the most powerful argument for fuzziness.

An odd question reveals the geometry of fuzzy sets: What does the fuzzy power set $F(2^X)$, the set of all fuzzy subsets of X , look like? It looks like a cube. What does a fuzzy set look like? A point in a cube. The set of all fuzzy subsets equals the unit hypercube $I^n = [0, 1]^n$. A fuzzy set is any point [Kosko, 1987] in the cube I^n . So (X, I^n) defines the fundamental measurable space of (finite) fuzzy theory. We can teach much of the theory of fuzzy sets—more accurately, the theory of *continuous* sets—on a Rubik's cube.

Vertices of the cube I^n define nonfuzzy sets. So the ordinary power set 2^X , the set of all 2^n nonfuzzy subsets of X , equals the Boolean n -cube $B^n : 2^X = B^n$. Fuzzy sets fill in the lattice B^n to produce the solid cube $I^n : F(2^X) = I^n$.

Consider the set of two elements $X = \{x_1, x_2\}$. The nonfuzzy power set 2^X contains four sets: $2^X = \{\emptyset, X, \{x_1\}, \{x_2\}\}$. These four sets correspond respectively to the four bit vectors (0 0), (1 1), (1 0), and (0 1). The 1s and 0s indicate the presence or absence of the i th element x_i in the subset. More abstractly, we can uniquely define each subset A as one of the two-valued membership functions $m_A : X \rightarrow \{0, 1\}$.

Now consider the fuzzy subsets of X . We can view the fuzzy subset $A = (\frac{1}{3} \frac{3}{4})$ as one of the continuum-many continuous-valued membership functions $m_A : X \rightarrow [0, 1]$. Indeed this corresponds to the classical Zadeh [1965] *sets-as-functions* definition of fuzzy sets. In this example element x_1 belongs to, or fits in, subset A a little bit—to degree $\frac{1}{3}$. Element x_2 has more membership than not at $\frac{3}{4}$. Analogous to the bit vector representation of finite (countable) sets, we say that the *fit vector* $(\frac{1}{3} \frac{3}{4})$ represents A . The element $m_A(x_i)$ equals the i th *fit* [Kosko, 1986b] or *fuzzy unit* value. The sets-as-points view then geometrically represents the fuzzy subset A as a point in I^2 , the unit square, as in Figure 16.2.

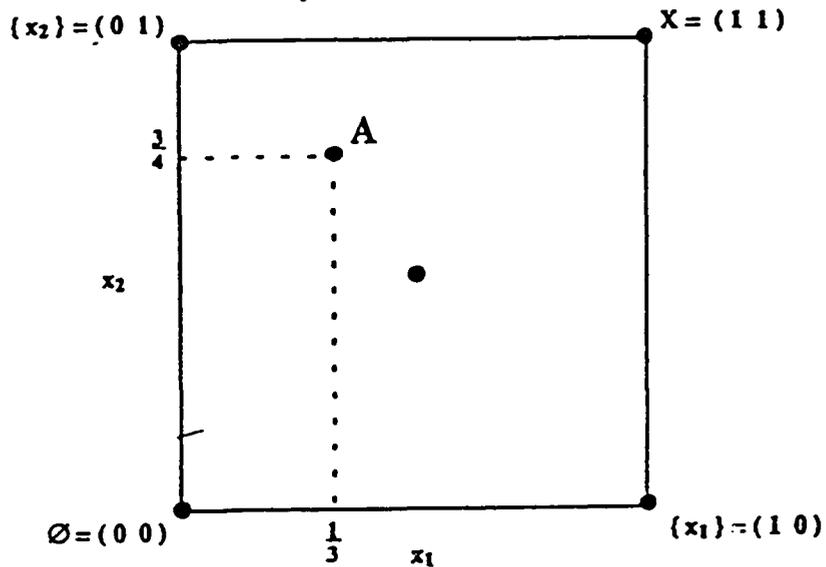


Figure 16.2 Sets as points. The fuzzy subset A is a point in the unit 2-cube with coordinates or fit values $(\frac{1}{3} \frac{3}{4})$. The first element x_1 fits in or belongs to A to degree $\frac{1}{3}$, the element x_2 to degree $\frac{3}{4}$. The cube consists of all possible fuzzy subsets of two elements $\{x_1, x_2\}$. The four corners represent the power set 2^X of $\{x_1, x_2\}$.

The midpoint of the cube I^n is maximally fuzzy. All its membership values equal $\frac{1}{2}$. The midpoint is unique in two respects. First, the midpoint is the only set A that not only equals its own opposite A^c but equals its own overlap and underlap as well:

$$A = A \cap A^c = A \cup A^c = A^c \quad (2)$$

Second, the midpoint is the only point in the cube I^n equidistant to each of the 2^n vertices of the cube. The nearest corners are also the farthest. Figure 16.2 illustrates this

metrical relationship.

We combine fuzzy sets pairwise with minimum, maximum, and order reversal, just as we combine nonfuzzy sets. So we combine set elements with the operators of Lukasiewicz continuous logic [Rescher, 1969]. We define fuzzy set intersection fitwise by pairwise minimum (picking the smaller of the two elements), union by pairwise maximum, and complementation by order reversal:

$$m_{A \cap B} = \min(m_A, m_B) \quad , \quad (3)$$

$$m_{A \cup B} = \max(m_A, m_B) \quad , \quad (4)$$

$$m_{A^c} = 1 - m_A \quad . \quad (5)$$

For example:

$$A = (1 \ .8 \ .4 \ .5)$$

$$B = (.9 \ .4 \ 0 \ .7)$$

$$A \cap B = (.9 \ .4 \ 0 \ .5)$$

$$A \cup B = (1 \ .8 \ .4 \ .7)$$

$$A^c = (0 \ .2 \ .6 \ .5)$$

$$A \cap A^c = (0 \ .2 \ .4 \ .5)$$

$$A \cup A^c = (1 \ .8 \ .6 \ .5)$$

The overlap fit vector $A \cap A^c$ in this example does not equal the vector of all zeroes, and the underlap fit vector $A \cup A^c$ does not equal the vector of all ones. This holds for all properly fuzzy sets, all points in I^n other than vertex points. Indeed the min-max

definitions give at once the following fundamental characterization of fuzziness as nondegenerate overlap and nonexhaustive underlap.

Proposition. A is properly fuzzy iff $A \cap A^c \neq \emptyset$
 iff $A \cup A^c \neq X$.

The proposition says that Aristotle's laws of noncontradiction and excluded middle hold, but they hold only on a set of measure zero. They hold only at the 2^n vertices of I^n . In all other cases, and these are as many of these as there are real numbers, contradictions occur to some degree. In this sense contradictions in generalized set theory and logic represent the rule and not the exception. Fuzzy cubes box Aristotelian sets into corners.

Completing the fuzzy square illustrates this fundamental proposition. Consider again the two-dimensional fuzzy set A defined by the fit vector $(\frac{1}{3} \frac{3}{4})$. We find the corresponding overlap and underlap sets by first finding the complement set A^c and then combining the fit vectors pairwise with minimum and with maximum:

$$\begin{aligned} A &= \left(\frac{1}{3} \frac{3}{4}\right) \\ A^c &= \left(\frac{2}{3} \frac{1}{4}\right) \\ A \cap A^c &= \left(\frac{1}{3} \frac{1}{4}\right) \\ A \cup A^c &= \left(\frac{2}{3} \frac{3}{4}\right) \end{aligned}$$

The sets-as-points view shows that these four points in the unit square hang together,

and move together, in a very natural way. Consider the geometry of Figure 16.3.

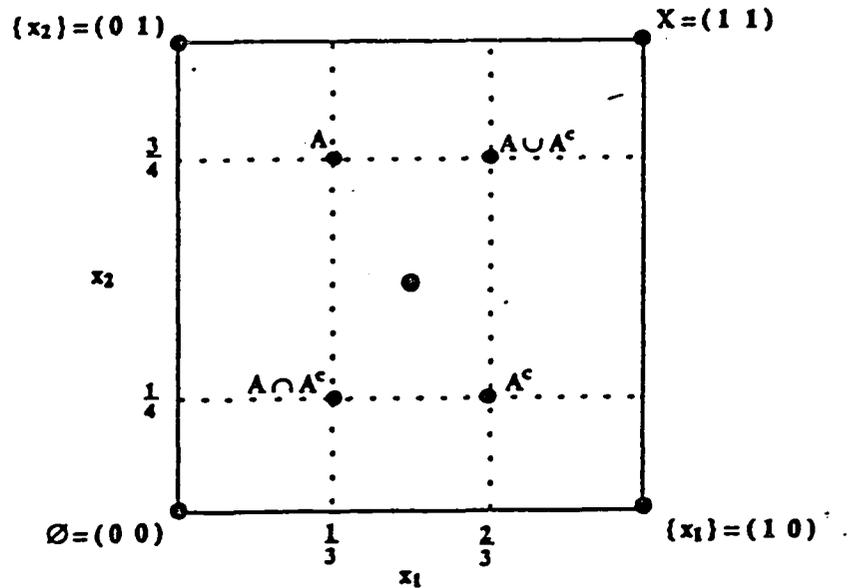


Figure 16.3 Completing the fuzzy square. The fuzzier A is, the closer A is to the midpoint of the fuzzy cube. As A approaches the midpoint, all four points— A , A^c , $A \cap A^c$, and $A \cup A^c$ —contract to the midpoint. The less fuzzy A is, the closer A is to the nearest vertex. As A approaches the vertex, all four points spread out to the four vertices and the bivalent power set 2^X is recovered. In an n -dimensional fuzzy cube, the 2^n fuzzy sets with elements a_i or $1 - a_i$ similarly contract to the midpoint or expand to the 2^n vertices as A approaches total fuzziness or total bivalence.

In Figure 16.3 the four fuzzy sets involved in the fuzziness of set A —the sets A , A^c , $A \cap A^c$, and $A \cup A^c$ —contract to the midpoint as A becomes maximally fuzzy and expand out to the Boolean corners of the cube as A becomes minimally fuzzy. The same contraction and expansion occurs in n dimensions for the 2^n fuzzy sets defined by all combinations

of $m_A(x_1)$ and $m_{A^c}(x_1), \dots, m_A(x_n)$ and $m_{A^c}(x_n)$. The same contraction and expansion occurs in n dimensions for the 2^n fuzzy sets defined by all combinations of $m_A(x_1)$ and $m_{A^c}(x_1), \dots, m_A(x_n)$ and $m_{A^c}(x_n)$.

At the midpoint nothing is distinguishable. At the vertices everything is distinguishable. These extremes represent the two ends of the spectrum of logic and set theory. In this sense the midpoint represents the black hole of set theory.

Paradox at the Midpoint

The midpoint is full of paradox. Classical logic and set theory forbid the midpoint by the same axioms, noncontradiction and excluded middle, that generate the of "paradoxes" or antinomies of bivalent systems. Where midpoint phenomena appear in Western thought, theorists have invariably labeled them "paradoxes" or denied them altogether. Midpoint phenomena include the half-empty and half-full cup, the Taoist Yin-Yang, the liar from Crete who said that all Cretans are liars, Bertrand Russell's set of all sets that are not members of themselves, and Russell's barber.

Russell's barber is a bewhiskered man who lives in a town and who shaves. His barber shop sign says that he shaves a man if and only if he does not shave himself. So who shaves the barber? If he shaves himself, then by definition he does not. But if he does not shave himself, then by definition he does. So he does and he does not—contradiction ("paradox"). Gaines [1983] observed that we can numerically interpret this paradoxical circumstance as follows.

Let S be the proposition that the barber shaves himself and not- S that he does not. Then since S implies not- S and not- S implies S , the two propositions are logically equivalent: $S = \text{not-}S$. Equivalent propositions have the same truth values:

$$t(S) = t(\text{not-}S) \tag{6}$$

$$= 1 - t(S) \tag{7}$$

Solving for $t(S)$ gives the midpoint point of the truth interval (the one-dimensional cube $[0, 1]$): $t(S) = \frac{1}{2}$. The midpoint is equidistant to the vertices 0 and 1. In the bivalent (two-valued) case, roundoff is impossible and paradox occurs. (6) and (7) describe the logical form of the many paradoxes, though different paradoxes involve different descriptions [Quine, 1987].

In bivalent logic both statements S and not- S must have truth value zero or unity. The fuzzy resolution of the paradox uses only the fact that the truth values are equal. It does not constrain their range. The midpoint value $\frac{1}{2}$ emerges from the structure of the problem and the order-reversing effect of negation.

The paradoxes of classical set theory and logic illustrate the price we pay for an arbitrary insistence on bivalence [Quine, 1981]. Scientists often insist on bivalence in the name of science. But in the end this insistence reduces to a mere cultural preference, a reflection of an educational predilection that goes back at least to Aristotle. Fuzziness shows that there are limits to logical certainty. We can no longer assert the laws of noncontradiction and excluded middle *for sure*—and *for free*.

Fuzziness carries with it intellectual responsibility. We must explain how fuzziness fits in bivalent systems, or vice versa. The fuzzy theorist must explain why so many people have been in error for so long. We now have the machinery to offer an explanation: We round off. Rounding off, quantizing, simplifies life and often costs little. We agree to call empty the near empty cup, and present the large pulse and absent the small pulse. We round off points inside the fuzzy cube to the nearest vertex. This roundoff heuristic works fine as a first approximation to describing the universe until we get near the midpoint of the cube. We find these phenomena harder to roundoff. In the logically extreme case, at the midpoint of the cube, the procedure breaks down completely because every vertex is equally close. If we still insist on bivalence, we can only give up and declare paradox.

Faced with midpoint phenomena, the fuzzy skeptic resembles the flat-earthier, who denies that the earth's surface is curved, when she stands at the north pole, looks at her compass, and wants to go south.

Counting with Fuzzy Sets

How big is a fuzzy set? The size or cardinality of A , $M(A)$, equals the sum of the fit values of A :

$$M(A) = \sum_{i=1}^n m_A(x_i) \quad (8)$$

The count of $A = (\frac{1}{3} \frac{3}{4})$ equals $M(A) = \frac{1}{3} + \frac{3}{4} = \frac{13}{12}$. Some fuzzy theorists [Zadeh, 1983] call the cardinality measure M the *sigma-count*. The measure M generalizes [Kosko, 1986a] the classical counting measure of combinatorics and measure theory. (So (X, I^n, M) defines the fundamental measure space of fuzzy theory.) In general the measure M does not yield integer values.

The measure M has a natural geometric interpretation in the sets-as-points framework. $M(A)$ equals the magnitude of the vector drawn from the origin to the fuzzy set A , as Figure 16.4 illustrates.

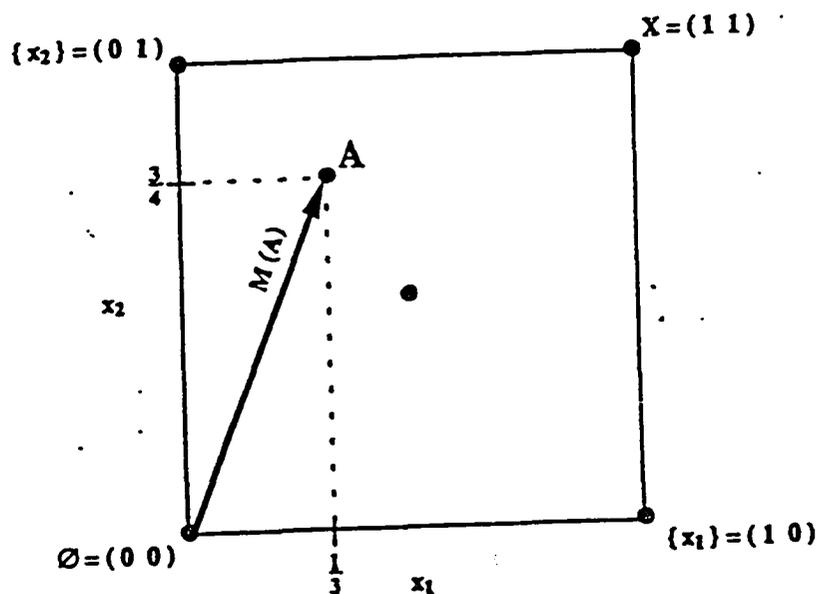


Figure 16.4 The count $M(A)$ of A equals the fuzzy Hamming norm (l^1 norm) of the vector drawn from the origin to A .

Consider the l^p distance between fuzzy sets A and B in I^n :

$$l^p(A, B) = \sqrt[p]{\sum_{i=1}^n |m_A(x_i) - m_B(x_i)|^p}, \quad (9)$$

where $1 \leq p \leq \infty$. The l^2 distance is the physical Euclidean distance actually illustrated in the figures. The simplest distance is the l^1 or fuzzy Hamming distance, the sum of the absolute fit differences. We shall use fuzzy Hamming distance throughout, though all results admit a general l^p formulation. Using the fuzzy Hamming distance we can rewrite the count M as the desired l^1 norm:

$$M(A) = \sum_i m_A(x_i) \quad (10)$$

$$= \sum_i |m_A(x_i) - 0| \quad (11)$$

$$= \sum_i |m_A(x_i) - m_\emptyset(x_i)| \quad (12)$$

$$= l^1(A, \emptyset) \quad (13)$$

THE FUZZY ENTROPY THEOREM

How fuzzy is a fuzzy set? We measure fuzziness with by a *fuzzy entropy* measure. Entropy is a generic notion. It need not be probabilistic. Entropy measures the uncertainty of a system or message. A fuzzy set describes a type of system or message. Its uncertainty equals its fuzziness.

The fuzzy entropy of A , $E(A)$, varies from 0 to 1 on the unit hypercube I^n . Only the

cube vertices have zero entropy, since nonfuzzy sets are unambiguous. The cube midpoint uniquely has unity or maximum entropy. Fuzzy entropy smoothly increases as a set point moves from any vertex to the midpoint. Klir [1988] discusses the algebraic requirements for fuzzy entropy measures.

Simple geometric considerations lead to a ratio form for the fuzzy entropy [Kosko, 1986b]. The closer the fuzzy set A is to the nearest vertex A_{near} , the farther A is from the farthest vertex A_{far} . The farthest vertex A_{far} resides opposite the long diagonal from the nearest vertex A_{near} . Let a denote the distance $l^1(A, A_{near})$ to the nearest vertex, and let b denote the distance $l^1(A, A_{far})$ to the farthest vertex. Then the fuzzy entropy equals the ratio of a to b :

$$E(A) = \frac{a}{b} = \frac{l^1(A, A_{near})}{l^1(A, A_{far})} \quad (14)$$

Figure 16.5 shows the sets-as-points interpretation of the fuzzy entropy, where $A = (\frac{1}{3} \frac{3}{4})$, $A_{near} = (0 \ 1)$, and $A_{far} = (1 \ 0)$. So $a = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ and $b = \frac{2}{3} + \frac{3}{4} = \frac{17}{12}$. So $E(A) = \frac{7}{17}$.

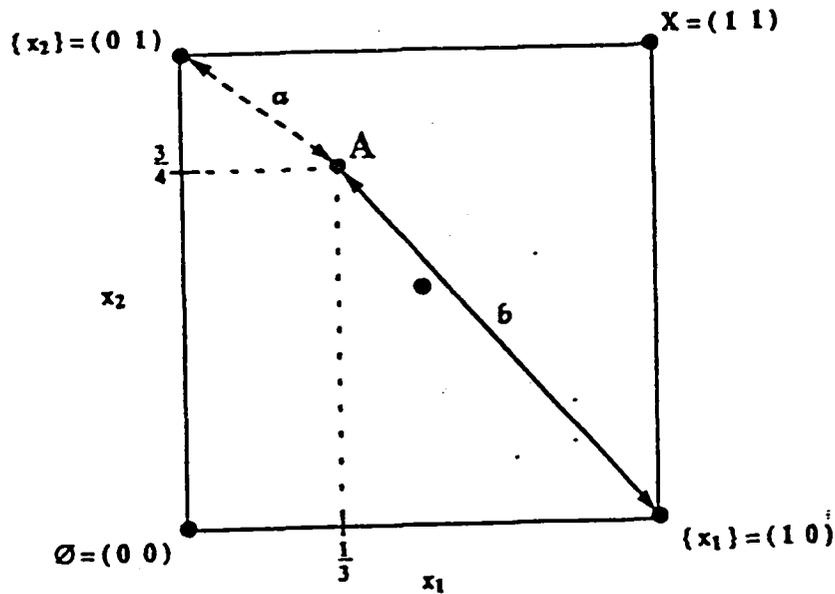


Figure 16.5 Fuzzy entropy, $E(A) = \frac{a}{b}$, balances distance to nearest vertex with distance to farthest vertex.

Alternatively, if you read this in a room, you can imagine the room as the unit cube I^3 and your head as a fuzzy set in it. Once you locate the nearest corner of the room, the farthest corner resides opposite the long diagonal emanating from the nearest corner. If you put your head in a corner, then $a = 0$, and so $E(A) = 0$. If you put your head in the metrical center of the room, every corner is nearest and farthest. So $a = b$, and $E(A) = 1$.

Overlap and underlap characterize set fuzziness. So we can expect them to affect the measure of fuzziness. Figure 16.3 shows the connection. By symmetry, each of the four points A , A^c , $A \cap A^c$, and $A \cup A^c$ is equally close to its nearest vertex. The common distance equals a . Similarly, each point is equally far from its farthest vertex. The common

distance equals b . One of the first four distances is the count $M(A \cap A^c)$. One of the second four distances is the count $M(A \cup A^c)$. This gives a geometric proof of the Fuzzy Entropy Theorem [Kosko, 1986b-87], which states that fuzziness consists of counted violations of the law of noncontradiction balanced with counted violations of the law of excluded middle.

Fuzzy Entropy Theorem:
$$E(A) = \frac{M(A \cap A^c)}{M(A \cup A^c)} \quad (15)$$

An algebraic proof is straightforward. The completed fuzzy square in Figure 16.6, contains a geometric proof (in this special case).

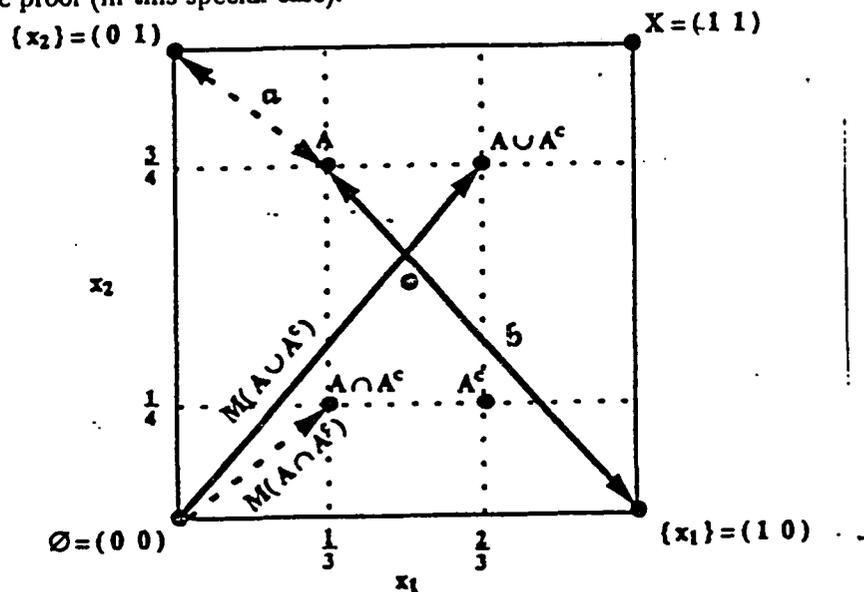


Figure 16.6 Geometry of the Fuzzy Entropy Theorem. By symmetry each of the four points on the completed fuzzy square is equally close to its nearest vertex and equally far from its farthest vertex.

The Fuzzy Entropy Theorem explains why set fuzziness begins where Western logic ends. When sets (or propositions) obey the laws of noncontradiction and excluded middle,

overlap is empty and underlap is exhaustive. So $M(A \cap A^c) = 0$ and $M(A \cup A^c) = n$, and thus $E(A) = 0$.

The Fuzzy Entropy Theorem also provides a first-principles derivation of the basic fuzzy set operations of minimum (intersection), maximum (union), and order reversal (complementation) proposed in 1965 by Zadeh at the inception of fuzzy theory. (Lukasiewicz first proposed these operations for continuous or fuzzy logics in the 1920s [Rescher, 1969].)

For the fuzzy theorist, this result also shows that triangular norms or T -norms [Klir, 1988], which generalize conjunction or intersection, and the dual triangular co-norms C , which generalize disjunction or union, do not have the first-principles status of min and max. For, the triangular-norm inequalities,

$$T(x, y) \leq \min(x, y) \leq \max(x, y) \leq C(x, y), \quad (16)$$

show that replacing min with any T in the numerator term $M(A \cap A^c)$ can only make the numerator smaller. Replacing max with any C in the term $M(A \cup A^c)$ can only make the denominator larger. So any T or C not identically min or max makes the ratio smaller, strictly smaller if A is fuzzy. Then the entropy theorem does not hold, and the resulting pseudo-entropy measure does not equal unity at the midpoint, though it continues to be maximized there. We can see this with the product T -norm [Prade, 1985] $T(x, y) = xy$ and its DeMorgan dual co-norm $C(x, y) = 1 - T(1 - x, 1 - y) = x + y - xy$, or with the bounded sum T -norm $T(x, y) = \max(0, x + y - 1)$ and DeMorgan dual $C(x, y) = \min(1, x + y)$. The Entropy Theorem similarly fails in general if the negation or complementation operator $N(x) = 1 - x$ with a parameterized operator $N_a(x) = \frac{1-x}{1+ax}$ for nonzero $a > -1$.

All probability distributions, all sets A with $M(A) = 1$, in I^n form a $n-1$ dimensional simplex S^n . In the unit square the probability simplex equals the negatively sloped diagonal line. In the unit 3-cube it equals a solid triangle. In the unit 4-cube it equals a tetrahedron, and so on up.

If no probabilistic fit value p_i satisfies $p_i > \frac{1}{2}$, then the Fuzzy Entropy Theorem implies [Kosko, 1987] that the the distribution P has fuzzy entropy $E(P) = \frac{1}{n-1}$. Else

$E(P) < \frac{1}{n-1}$. So the probability simplex S^n is entropically degenerate for large dimensions n . This result also shows that the uniform distribution $(\frac{1}{n}, \dots, \frac{1}{n})$ maximizes fuzzy entropy on S^n but not uniquely. This in turn shows that fuzzy entropy differs from the average-information measure of probabilistic entropy, which the uniform distribution maximizes uniquely.

The Fuzzy Entropy Theorem implies that, analogous to $\log \frac{1}{p}$, a unit of fuzzy information equals $\frac{f}{1-f}$ or $\frac{1-f}{f}$, depending on whether the fit value f obeys $f \leq \frac{1}{2}$ or $f \geq \frac{1}{2}$.

The event x can be ambiguous or clear. It is ambiguous if f equals approximately $\frac{1}{2}$ and clear if f equals approximately 1 or 0. If an ambiguous event occurs, is observed, is disambiguated, etc., then it is maximally informative: $E(f) = E(\frac{1}{2}) = 1$. If a clear event occurs, is observed, etc., it is minimally informative: $E(f) = E(0) = E(1) = 0$. This agrees with the information interpretation of the probabilistic entropy measure $\log \frac{1}{p}$, where the occurrence of a sure event ($p = 1$) is minimally informative (zero entropy) and the occurrence of an impossible event ($p = 0$) is maximally informative (infinite entropy).

THE SUBSETHOOD THEOREM

Sets contain subsets. A is a *subset* of B , denoted $A \subset B$, if and only if every element in A is an element of B . The power set 2^B contains all of B 's subsets. So, alternatively [Bandler-Kohout, 1980], A is a subset of B iff A belongs to B 's power set:

$$A \subset B \text{ if and only if } A \in 2^B. \quad \dots (17)$$

The subset relation corresponds to the implication relation in logic. In classical logic *truth* maps the set of statements $\{S\}$ to two truth values: $t: \{S\} \rightarrow \{0, 1\}$. Consider the truth-tabular definition of implication for bivalent propositions P and Q :

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

The implication is false if and only if the antecedent P is true and the consequent Q is false—when “truth implies falsehood.”

The same holds for subsets. Representing sets as bivalent functions or “indicator” functions $m_A : X \rightarrow \{0, 1\}$, A is a subset of B iff there is no element x that belongs to A but not to B , or $m_A(x) = 1$ but $m_B(x) = 0$. We can rewrite this membership-function definition as

$$A \subset B \quad \text{if and only if} \quad m_A(x) \leq m_B(x) \quad \text{for all } x. \quad (18)$$

Zadeh [1965] proposed the same relation for fuzzy set containment. We refer to this as the *dominated membership function relationship*. If $A = (.3 \ 0 \ .7)$ and $B = (.4 \ .7 \ .9)$, then A is a fuzzy subset of B , but B is not a fuzzy subset of A . Either fuzzy set A is, or is not, a fuzzy subset of B . So the relation of fuzzy subsethood is *not* fuzzy. It is either black or white.

The sets-as-points view asks a geometric question: What do all fuzzy subsets of B look like? What does the *fuzzy power set* of B — $F(2^B)$, the set of all fuzzy subsets of B —look like? The dominated membership function relationship implies that $F(2^B)$ defines the hyper-rectangle snug against the origin in a unit hypercube with side lengths equal to the fit values $m_A(x_i)$. Figure 7 displays the fuzzy power set of the set $B = (\frac{1}{3} \ \frac{2}{3})$. $F(2^B)$ has infinite count if B is not empty. For finite-dimensional sets, we can measure the size of $F(2^B)$ [Kosko, 1987] as the Lebesgue measure or volume $V(B)$, the product of the fit values:

$$V(B) = \prod_{i=1}^n m_B(x_i) . \quad (19)$$

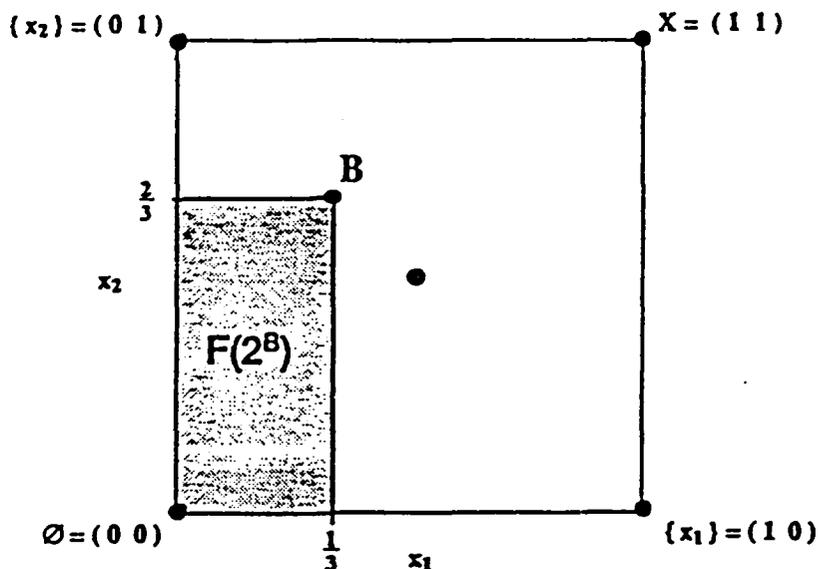


Figure 16.7 Fuzzy power set $F(2^B)$ as a hyper-rectangle in the fuzzy cube. Side lengths equal the fit values $m_B(x_i)$. The size or volume of $F(2^B)$ equals the product of the fit values.

Figure 16.7 illustrates that $F(2^B)$ is not a fuzzy set. Either cube point A is or is not in the hyper-rectangle $F(2^B)$. Different points A outside the hyper-rectangle $F(2^B)$ resemble subsets of B to different degrees. The bivalent definition of subsethood ignores this.

The natural generalization defines fuzzy subsets of $F(2^B)$: Some sets A belong to $F(2^B)$ to different degrees. Then the abstract membership function $m_{F(2^B)}(A)$ can equal any number in $[0, 1]$. This defines degrees of subsethood.

Let $S(A, B)$ denote the degree to which A is a subset of B :

$$S(A, B) = \text{Degree}(A \subset B) \quad (20)$$

$$= m_{F(2^0)}(A) \quad (21)$$

$S(., .)$ denotes the *subsethood measure*. $S(., .)$ takes values in $[0,1]$. We will see that it provides the fundamental, unifying structure in fuzzy theory.

We want to measure $S(A, B)$. We will first present an earlier [Kosko, 1986b-87] algebraic derivation of the subsethood measure $S(A, B)$. We will then present a new, more fundamental, geometric derivation.

We call the algebraic derivation the *fit-violation strategy*. Intuitively we study a law by breaking it. Consider again the dominated membership function relationship: $A \subset B$ if and only if $m_A(x) \leq m_B(x)$ for all x in X .

Suppose element x_v violates the dominated membership function relationship: $m_A(x_v) > m_B(x_v)$. Then A is not a subset of B , at least not totally. Suppose further that the dominated membership inequality holds for all other elements x . Only element x_v violates the relationship. For instance, X may consist of one hundred values: $X = \{x_1, \dots, x_{100}\}$. The violation might occur, say, with the first element: $x_1 = x_v$. Then intuitively A is largely a subset of B . Suppose that X contains a thousand elements, or a trillion elements, and only the first element violates (18). Then it seems A is overwhelmingly a subset of B ; perhaps $S(A, B) = .999999999999$.

This example suggests we should count fit violations in magnitude and frequency. The greater the violations in magnitude, $m_A(x_v) - m_B(x_v)$, and the greater the number of violations relative to the size $M(A)$ of A , the less A is a subset of B or, equivalently, the more A is a *superset* of B . For, both intuitively and by (18), supersetness and subsethood relate as additive opposites:

$$\text{SUPERSETHOOD}(A, B) = 1 - S(A, B) \quad (22)$$

We count violations by adding them. If we sum over all x , the summand should equal

$m_A(x_i) - m_B(x_i)$ when this difference is positive, and equal zero when it is nonpositive. So the summand equals $\max(0, m_A(x) - m_B(x))$. So the unnormalized count equals the sum of these maxima:

$$\sum_{x \in X} \max(0, m_A(x) - m_B(x)). \quad (23)$$

The count $M(A)$ provides a simple, and appropriate, normalization factor. Below we formally arrive at $M(A)$ by examining boundary cases in the geometric approach to subsethood. We can assume $M(A) > 0$, since $M(A) = 0$ if and only if A is empty. The empty set trivially satisfies the dominated membership function relationship (18). So it is a subset of every set. Normalization gives the minimal measure of nonsubsethood, of supersetthood:

$$\text{SUPERSETHOOD}(A, B) = \frac{\sum_x \max(0, m_A(x) - m_B(x))}{M(A)} \quad (24)$$

Then subsethood is the negation of this ratio. This gives the minimal fit-violation measure of subsethood:

$$S(A, B) = 1 - \frac{\sum_x \max(0, m_A(x) - m_B(x))}{M(A)} \quad (25)$$

The subsethood measure may appear ungraceful at first, but it behaves as it should. $S(A, B) = 1$ if and only if (18) holds. For if (18) holds, (23) sums zero violations. Then $S(A, B) = 1 - 0 = 1$. If $S(A, B) = 1$, every numerator summand equals zero. So no violation occurs. At the other extreme, $S(A, B) = 0$ if and only if B is the empty set. The empty set uniquely contains no proper subsets, fuzzy or nonfuzzy. Degrees of subsethood occur between these extremes: $0 < S(A, B) < 1$.

The subsethood measure relates to logical implication. Viewed at the 1-dimensional level of fuzzy logic, and so ignoring the normalizing count ($M(A) = 1$), the subsethood measure reduces to the Lukasiewicz implication operator:

$$S(A, B) = 1 - \max(0, m_A - m_B) \quad (26)$$

$$= 1 - [1 - \min(1 - 0, 1 - (m_A - m_B))] \quad (27)$$

$$= \min(1, 1 - m_A + m_B) \quad (28)$$

$$= t_L(A \rightarrow B) \quad (29)$$

The $\min(\cdot)$ operator in (28) clearly generalizes the above truth-tabular definition of bivalent implication.

Consider the fit vectors $A = (.2 \ 0 \ .4 \ .5)$ and $B = (.7 \ .6 \ .3 \ .7)$. Neither set is a proper subset of the other. A is almost a subset of B but not quite since $m_A(x_3) - m_B(x_3) = .4 - .3 = .1 > 0$. Hence $S(A, B) = 1 - \frac{.1}{1.1} = \frac{10}{11}$. Similarly $S(B, A) = 1 - \frac{1.3}{2.3} = \frac{10}{23}$.

Subsethood applies to nonfuzzy sets. Consider the sets $C = \{x_1, x_2, x_3, x_5, x_7, x_9, x_{10}, x_{12}, x_{14}\}$ and $D = \{x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{12}, x_{13}, x_{14}\}$ with corresponding bit vectors

$$C = (1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1)$$

$$D = (0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1)$$

C and D are not subsets of each other. But C should very nearly be a subset of D since only x_1 violates (18). We find $S(C, D) = 1 - \frac{1}{9} = \frac{8}{9}$ while $S(D, C) = 1 - \frac{4}{12} = \frac{2}{3}$. So D is more a subset of C than it is not. This holds because the two sets are largely equivalent. They have much overlap: $M(C \cap D) = 8$. This observation anticipates the Fuzzy Subsethood Theorem presented below.

We now turn to a new and purely geometric derivation of the subethood operator $S(A, B)$. Consider the sets-as-points geometry of subethood in Figure 16.7. Set A is either in the hyper-rectangle $F(2^B)$ or not in it. Intuitively $S(A, B)$ should approach unity as A approaches the fuzzy power set $F(2^B)$. $S(A, B)$ should decrease, and the supersethood measure $1 - S(A, B)$ should increase, as A moves from $F(2^B)$.

So the key idea is metrical: *How close is A to $F(2^B)$?* Let $d(A, F(2^B))$ denote this I^n distance defined in (9). $d(A, B')$ denotes the distance between A and point B' in the hyper-rectangle, and $B' \subset B$. Distance $d(A, F(2^B))$ equals the smallest such distance. Since the hyper-rectangle $F(2^B)$ is closed and bounded (compact) and convex, some subset B^* of B achieves this minimum distance. So the infimum, the greatest lower bound, equals the distance $d(A, B^*)$:

$$d(A, F(2^B)) = \inf \{d(A, B') : B' \in F(2^B)\} \quad (30)$$

$$= d(A, B^*) \quad (31)$$

We can easily locate the closest set B^* in the hypercube geometry. If A is a subset of B —if A is in the hyper-rectangle $F(2^B)$ —then A equals the closest subset: $A = B^*$. So suppose A is not a proper subset of B . Then A lies outside the hyper-rectangle $F(2^B)$.

We can slice the unit cube I^n into 2^n hyper-rectangles by extending the sides of $F(2^B)$ to hyperplanes. The hyperplanes intersect perpendicularly (orthogonally), at least in the Euclidean case. $F(2^B)$ defines one of the hyper-rectangles. The hyper-rectangle interiors correspond to the 2^n cases whether $m_A(x_i) < m_B(x_i)$ or $m_A(x_i) > m_B(x_i)$ for fixed B and arbitrary A . The edges correspond to the loci of points where some $m_A(x_i) = m_B(x_i)$.

The 2^n hyper-rectangles classify as *mixed* or *pure* membership domination. In the pure case either $m_A < m_B$, or $m_A > m_B$, holds in the hyper-rectangle interior for all x and all interior points A . In the mixed case $m_A(x_i) < m_B(x_i)$ holds for some of the coordinates

x_i , and $m_A(x_j) > m_B(x_j)$ holds for the remaining coordinates x_j in the interior for all interior A . So there are only two pure membership-domination hyper-rectangles, the set of proper subsets $F(2^B)$ and the set of proper supersets, which includes X .

Figure 16.8 illustrates how the fuzzy power set $F(2^B)$ of $B = (\frac{1}{3} \frac{2}{3})$ linearly extends to partition the unit square into 2^2 rectangles. The non-subsets A_1 , A_2 , and A_3 reside in distinct quadrants. The northwest and southeast quadrants define the mixed membership-domination rectangles. The southwest and the northeast quadrants define the pure rectangles.

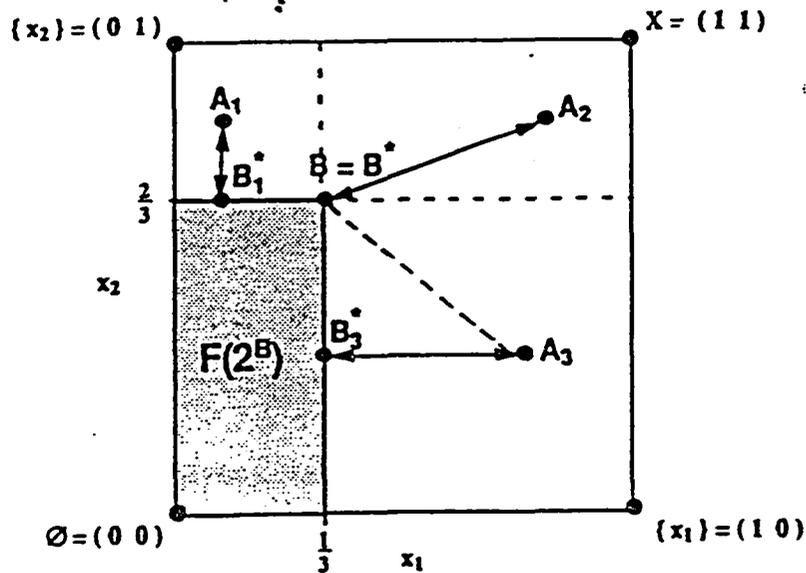


Figure 16.8 Partition of hypercube I^n into 2^n hyper-rectangles by linearly extending the edges of $F(2^B)$. We find the nearest points B_1^* and B_3^* to points A_1 and A_3 in the northwest and southeast quadrants by the normals from $F(2^B)$ to A_1 and A_3 . The nearest point B^* to point A_2 in the northeast quadrant is B itself. This "orthogonal" optimality condition allows $d(A, B)$ by the general Pythagorean Theorem as the hypotenuse in an I^n "right" triangle.

B is the nearest set B^* to A in the pure superset hyper-rectangle. To find the nearest set B^* in the mixed case we draw a perpendicular (orthogonal) line segment from A to $F(2^B)$. Convexity of $F(2^B)$ is ultimately responsible. In Figure 16.8 the perpendicular lines from A_1 and A_3 intersect line edges (1-dimensional linear subspaces) of the rectangle $F(2^B)$. The line from A_2 to B , the corner of $F(2^B)$, is degenerately perpendicular since B is a zero-dimensional linear subspace.

These "orthogonality" conditions also hold in three dimensions. Let your room again be the unit 3-cube. Consider a large dictionary fit snugly against the floor corner corresponding to the origin. Point B equals the dictionary corner farthest from the origin. Extending the three exposed faces of the dictionary partitions the room into 8 octants. The dictionary occupies one octant. We connect points in the other 7 octants to the nearest points on the dictionary by drawing lines, or tying strings, that perpendicularly intersect one of the three exposed faces, or one of the three exposed edges, or the corner B .

The "orthogonality" condition invokes the l^p -version of the Pythagorean Theorem. For our l^1 purposes:

$$d(A, B) = d(A, B^*) + d(B, B^*) \quad (32)$$

The more familiar l^2 -version, actually pictured in Figure 16.8, requires squaring these distances. For the general l^p case:

$$\|A - B\|^p = \|A - B^*\|^p + \|B^* - B\|^p, \quad (33)$$

or equivalently,

$$\sum_{i=1}^n |a_i - b_i|^p = \sum_{i=1}^n |a_i - b_i^*|^p + \sum_{i=1}^n |b_i^* - b_i|^p \quad (34)$$

Equality holds for all $p \geq 1$ since, as is clear from Figure 16.8 or 16.10 and, in general, from the algebraic argument below, either $b_i^* = a_i$ or $b_i^* = b_i$.

This Pythagorean equality is surprising. We have come to think of the Pythagorean

Theorem (and orthogonality) as an ℓ^2 or Hilbert-space property. Yet here it holds in every ℓ^p space—if B^* is the set in $F(2^B)$ closest to A in ℓ^p distance. Of course for other sets strict inequality holds in general if $p \neq 2$. This suggests a special status for the closest set B^* . We shall see below that the Subsethood Theorem confirms this suggestion. We shall use the term “orthogonality” loosely to refer to this ℓ^p Pythagorean relationship, while remembering its customary restriction to ℓ^2 spaces and inner products.

A natural interpretation defines supersethood as the distance $d(A, F(2^B)) = d(A, B^*)$. Supersethood increases with this distance; subsethood decreases with it. To keep supersethood, and thus subsethood, unit-interval valued, we must suitably normalize the distance.

A constant provides the simplest normalization term for $d(A, B^*)$. That constant equals the maximum unit-cube distance, $n^{1/p}$ in the general ℓ^p case and n in our ℓ^1 case. This gives the candidate subsethood measure

$$S(A, B) = 1 - \frac{d(A, B^*)}{n} \quad (35)$$

This candidate subsethood measure fails in the boundary case when B is the empty set. For then $d(A, B^*) = d(A, B) = M(A)$. So the measure in (35) gives $S(A, \emptyset) = 1 - \frac{M(A)}{n} > 0$. Equality holds exactly when $A = X$. But the empty set has no subsets. Only normalization factor $M(A)$ satisfies this boundary condition. Of course $M(A) = n$ when $A = X$. Explicitly we require $S(A, \emptyset) = 0$, as well as $S(\emptyset, A) = 1$.

Normalizing by n also treats all equidistant points the same. Consider points A_1 and A_2 in Figure 16.9. Both points are equidistant to their nearest $F(2^B)$ point: $d(A_1, B_1^*) = d(A_2, B_2^*)$. But A_1 is closer to B than A_2 is. In particular A_1 is closer to the horizontal line defined by the fit value $m_B(x_2) = \frac{2}{3}$. The count $M(A)$ reflects this: $M(A_1) > M(A_2)$. The count gap $M(A_1) - M(A_2)$ arises from the fit gap involving x_1 , and reflects $d(A_1, B) < d(A_2, B)$. In general the count $M(A)$ relates to this distance, as we can see by checking extreme cases of closeness of A to B (and drawing some diamond-shaped ℓ^1 spheres centered at B). Indeed if $m_A > m_B$ everywhere, $d(A, B) = M(A) - M(B)$.

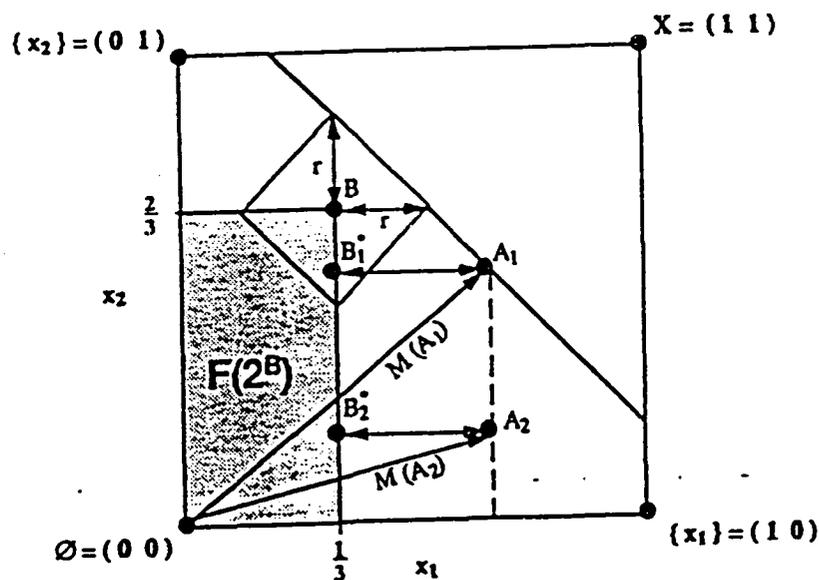


Figure 16.9 Dependence of subsethood on the count $M(A)$. A_1 and A_2 are equidistant to $F(2^B)$ but A_1 is closer to B than A_2 is; correspondingly, $M(A_1) > M(A_2)$. Loci of points A of constant count $M(A)$ define line segments parallel to the negatively sloping long diagonal. l^1 spheres centered at B are diamond shaped.

Since $F(2^B)$ fits snugly against the origin, the count $M(A)$ in any of the other $2^n - 1$ hyper-rectangles can be only larger than the count $M(B^*)$ of the nearest $F(2^B)$ points. The normalization choice of n leaves the candidate subsethood measure indifferent to which of the $2^n - 1$ hyper-rectangles contains A and to where A resides is in the hyper-rectangle. Each point in each hyper-rectangle involves a different combination of fit-violations and satisfactions. The normalization choice of $M(A)$ reflects this fit-violation structure and behaves appropriately in boundary cases.

The normalization choice $M(A)$ leads to the subsethood measure

$$S(A, B) = 1 - \frac{d(A, B^*)}{M(A)} \quad (36)$$

We now show that this measure equals the subsethood measure (25) derived algebraically above.

Let B' be any subset of B . Then by definition the nearest subset B^* obeys the inequality:

$$\sqrt[p]{\sum_{i=1}^n |a_i - b_i^*|^p} \leq \sqrt[p]{\sum_{i=1}^n |a_i - b'_i|^p} \quad (37)$$

where for convenience $a_i = m_A(x_i)$, and $b_i = m_B(x_i)$. We will assume $p = 1$ but the following characterization of b_i^* holds for any $p > 1$.

"Orthogonality" implies $a_i \geq b_i^*$. So first suppose $a_i = b_i^*$. This equality holds if and only if no violation occurs: $a_i \leq b_i$. (If this condition holds for all i , then $A = B^*$.) So $\max(0, a_i - b_i) = 0$. Next suppose $a_i > b_i^*$. This inequality holds if and only if a violation occurs: $a_i > b_i$. (If this holds for all i , then $B = B^*$.) So $b_i^* = b_i$ since B^* is the subset of B nearest to A . Equivalently, $a_i > b_i$ holds if and only if $\max(0, a_i - b_i) = a_i - b_i$. The two cases together prove that $\max(0, a_i - b_i) = |a_i - b_i^*|$. Summing over all x_i gives

$$d(A, B^*) = \sum_{i=1}^n \max(0, m_A(x_i) - m_B(x_i)) \quad (38)$$

So the two subsethood measures (25) and (36) are equivalent.

This proof also proves a deeper characterization of the optimal subset B^* :

$$B^* = A \cap B \quad (39)$$

For if a violation occurs, then $a_i > b_i$, and $b_i = b_i^*$. So $\min(a_i, b_i) = b_i^*$. Otherwise $a_i = b_i^*$, and so $\min(a_i, b_i) = b_i^*$. So $B^* = A \cap B$.

This in turn proves that B^* is a point of double optimality. B^* is both the subset of B nearest A , and A^* , the subset of A nearest to B :

$$d(B, F(2^A)) = d(B, A^*) = d(B, B^*) \quad (40)$$

Figure 16.10 illustrates that $B^* = A \cap B = A^*$ identifies the set within both the hyper-rectangle $F(2^A)$ and the hyper-rectangle $F(2^B)$ that has maximal count $M(A \cap B)$.

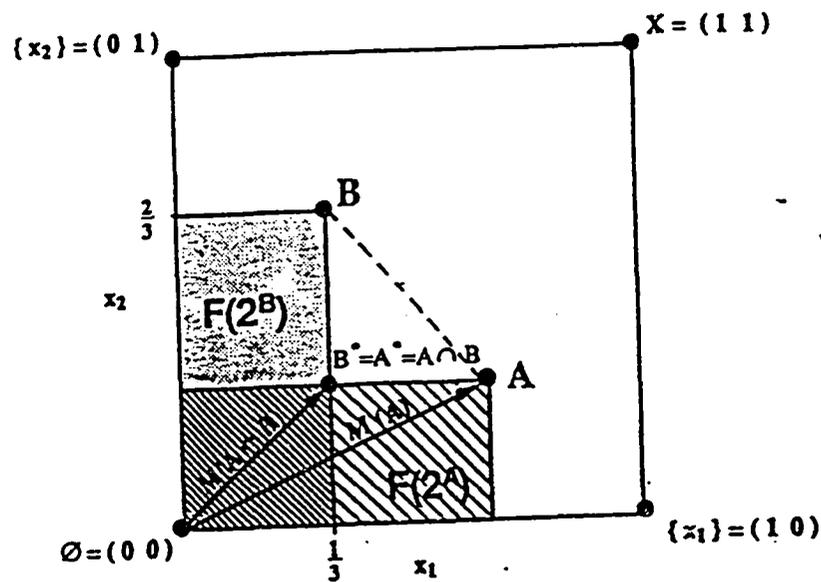


Figure 16.10. B^* as both the subset of B nearest A and the subset A^* of A nearest B : $B^* = A^* = A \cap B$. The distance $d(A, B^*) = M(A) - M(A \cap B)$ illustrates the Subsethood Theorem.

Figure 16.10 also shows that the distance $d(A, B^*)$ equals a vector magnitude difference: $d(A, B^*) = M(A) - M(A \cap B)$. Dividing both sides of this equality by $M(A)$ and rearranging proves a still deeper structural characterization of subsethood, the Subsethood Theorem.

$$\text{Subsethood Theorem.} \quad S(A, B) = \frac{M(A \cap B)}{M(A)} \quad (41)$$

The Subsethood Theorem immediately implies a Bayes Theorem:

Subsethood Theorem.

$$S(A, B) = \frac{M(B) S(B, A)}{M(A)}, \quad (42)$$

since (41) implies $M(A \cap B) = M(B) S(B, A)$.

The ratio form of the subsethood measure $S(A, B)$ has the same ratio form as the conditional probability $P(B|A)$ has in (1). We derived the ratio form for the subsethood measure $S(A, B)$ but assumed it for the conditional probability $P(B|A)$. Since every probability is a conditional probability, $P(A) = P(A|X)$, this suggests we can reduce probability to subsethood. We shall argue that this reduction holds both frequentist or "objective" probability and axiomatic or Bayesian or "subjective" probability.

Consider the physical interpretation of randomness as the relative frequency n_A/n . n_A denotes the number of successes that occur in n trials. Historically probabilists have called the success ratio (or its limit) n_A/n the "probability of success" or $P(A)$. We can now derive the relative-frequency definition of probability as $S(X, A)$, the degree to which a bivalent superset X , the sample space, is a subset of its own subset A . The concept of "randomness" never enters the deterministic set-theoretic framework. This holds equally for flipping coins, drawing balls from urns, or computing Einstein-Bose statistics.

Suppose A is a nonfuzzy subset of X . Then

$$S(X, A) = \frac{M(A \cap X)}{M(X)} \quad (43)$$

$$= \frac{M(A)}{M(X)} \quad (44)$$

$$= \frac{n_A}{n} \quad (45)$$

The n elements of X constitute the *de facto* universe of discourse of the "experiment." (We can take the limit of the ratio $S(X, A)$ if it mathematically makes sense to do so [Kac, 1959].) The "probability" $\frac{n_A}{n}$ has reduced to a degree of subsethood, a purely fuzzy set-theoretical relationship. Perhaps if, centuries ago, scientists had developed set theory before they formalized gambling, the undefined notion of "randomness" might never have culturally prevailed, if even survived, in the age of modern science.

The measure of overlap $M(A \cap X)$ provides the key component of relative frequency. This count does not involve "randomness". $M(A \cap X)$ counts which elements are identical or similar. The phenomena themselves are deterministic and black or white. The same situation gives the same number. We may use the number to place bets or to switch a phone line, but it remains part of the description of a specific state of affairs. We need not invoke an undefined "randomness" to further describe the situation.

Subsethood subsumes elementhood. We can interpret the membership degree $m_A(x_i)$ as the subsethood degree $S(\{x_i\}, A)$, where $\{x_i\}$ denotes a singleton subset or "element" x_i of X . $\{x_i\}$ corresponds to a bit vector with a 1 in the i th slot and 0s elsewhere: $\{x_i\} = (0, \dots, 0, 1, 0, \dots, 0)$. If we view A as the bit vector $(a_1, \dots, a_i, \dots, a_n)$, then $\{x_i\} \cap A = (0, \dots, 0, a_i, 0, \dots, 0)$, the i th coordinate projection. Since the count $M(\{x_i\})$ equals one, the Subsethood Theorem gives

$$S(\{x_i\}, A) = \frac{M(\{x_i\} \cap A)}{M(\{x_i\})} \quad (46)$$

$$= M((0, \dots, a_i, \dots, 0)) \quad (47)$$

$$= a_i \quad (48)$$

$$= m_A(x_i) \quad (49)$$

$$= \text{Degree}(x_i \in A) \quad (50)$$

So subsethood reduces to elementhood if antecedent sets are bivalent singleton sets.

The subsethood orthogonality conditions project A onto the facing side of the hyper-rectangle $F(2^B)$. This projection gives the "normal equations" of least-squares parameter estimation [Sorenson, 1980], a version of which we saw in Chapter 5. In general for two R^n vectors x and y , we project x onto y to give the projection vector $p = cy$. The difference $x - p$ is orthogonal to y : $(x - p) \perp y$. So

$$0 = (x - p) y^T \quad (51)$$

$$= (x - cy) y^T \quad (52)$$

$$= xy^T - cy^T y^T, \quad (53)$$

where column vector y^T denotes the transpose of row vector y . (53) gives the projection coefficient c as the familiar normal equations:

$$c = \frac{xy^T}{yy^T} \quad (54)$$

$$= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2} \quad (55)$$

Consider the unit square in Figure 16.10 with the same A , say $A = (\frac{3}{4} \frac{1}{2})$. But suppose we shift B directly to the left to $B = (0 \frac{2}{3})$. This contracts the rectangle $F(2^B)$ to the line segment $[0 \frac{2}{3}]$ along the vertical axis. These assumptions simplify the correlation

mathematics yet still preserve the least-squares structure. We expect that $B^* = cB$, or $cB = A \cap B$, when we project A onto $F(2^B)$ or, equivalently in this special case, when we project A onto B . The intersection $A \cap B$ equals the minimum fit vector $(0 \frac{1}{2})$, $AB^T = 0 + \frac{2}{6} = \frac{1}{3}$, and $BB^T = 0 + (\frac{2}{3})^2 = \frac{4}{9}$. Then

$$\begin{aligned} c &= \frac{AB^T}{BB^T} \\ &= \frac{\frac{1}{3}}{\frac{4}{9}} = \frac{3}{4} \end{aligned}$$

and

$$\begin{aligned} B^* &= cB \\ &= \frac{3}{4} \begin{pmatrix} 0 & 2 \\ 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & \frac{3}{4} \end{pmatrix} \\ &= A \cap B \end{aligned}$$

as expected. More generally if $B = (b_1 \ b_2)$, $b_1 = 0$, $b_2 > 0$, and $a_2 \leq b_2$, then

$$c = \frac{A B^T}{B B^T} = \frac{a_1 b_1 + a_2 b_2}{b_1^2 + b_2^2} \quad (56)$$

$$= \frac{a_2 b_2}{b_2^2} \quad (57)$$

$$= \frac{a_2}{b_2} \quad (58)$$

Then $c B = (0 \frac{a_2 b_2}{b_2}) = (0 a_2) = A \cap B$ since $a_2 \leq b_2$.

Subsethood has extended the Pythagorean Theorem, relative frequency, and elementhood, and involves the normal equations of least-square estimation. We shall now see how subsethood relates to axiomatic or Bayesian probability and to fuzzy entropy.

Bayesian Polemics

Bayesian probabilists interpret probability as a subjective state of knowledge. In practice they use relative frequencies (subsethood degrees) but only to approximate these "states of knowledge."

Bayesianism is often a polemical doctrine. Some Bayesians claim that they, and only they, use all and only the available uncertainty information in the description of uncertain phenomena. This stems from the Bayes Theorem expansion of the "a posteriori" conditional probability $P(H_i|E)$, the probability that H_i , the i th of k -many disjoint hypotheses $\{H_j\}$, is true given observed evidence E :

$$P(H_i|E) = \frac{P(E \cap H_i)}{P(E)} \quad (59)$$

$$= \frac{P(E|H_i) P(H_i)}{P(E)} \quad (60)$$

$$= \frac{P(E|H_i) P(H_i)}{\sum_{j=1}^k P(E|H_j) P(H_j)} \quad (61)$$

since the hypotheses partition the sample space $X : H_1 \cup H_2 \cup \dots \cup H_k = X$ and $H_i \cap H_j = \emptyset$ if $i \neq j$.

The Bayesian approach uses all available information in computing the posterior distribution $P(H_i|E)$ by using the "a priori" or prior distribution $P(H_i)$ of the hypotheses. The Bayesian approach stems from the ratio form of the conditional probability measure.

The Subsethood Theorem trivially implies Bayes Theorem when the hypotheses $\{H_i\}$ and evidence E are nonfuzzy subsets. More important, the Subsethood Theorem implies the Fuzzy Bayes Theorem in the more interesting case when the observed data E is fuzzy:

$$S(E, H_i) = \frac{S(H_i, E) M(H_i)}{\sum_{j=1}^k S(H_j, E) M(H_j)} \quad (62)$$

$$= \frac{S(H_i, E) f_i}{\sum_{j=1}^k S(H_j, E) f_j} \quad (63)$$

where $f_i = \frac{M(H_i)}{M(X)} = \frac{M(H_i)}{n} = S(X, H_i)$ gives the "relative frequency" of H_i , the degree to which all the hypotheses are H_i .

The Subsethood Theorem implies inequality when the partitioning hypotheses are fuzzy. For instance, if $k = 2$, H^c is the complement of an arbitrary fuzzy set H , and evidence E is fuzzy, then [Kosko, 1986b] the occurrence of nondegenerate hypothesis overlap and underlap gives a lower bound on the posterior subsethood:

$$S(E, H) \geq \frac{S(H, E) f_H}{S(H, E) f_H + S(H^c, E) f_{H^c}} \quad (64)$$

where $f_H = S(X, H)$. The lower bound increases with $M(H)$ and decreases with $M(H^c)$. Since a like lower bound holds for $S(E, H^c)$, adding the two posterior subsethoods gives the additive inequality

$$S(E, H) + S(E, H^c) \geq 1 \quad (65)$$

an inequality Zadeh [1983] arrived at independently by directly defining a "relative sigma-count" as the subsethood measure given by the Subsethood Theorem. If H is nonfuzzy,

equality holds as in the additive law of conditional probability:

$$P(H|E) + P(H^c|E) = 1 \quad (66)$$

The Subsethood Theorem implies a deeper Bayes theorem for arbitrary fuzzy sets, the Odds-Form Fuzzy Bayes Theorem:

$$\frac{S(A_1 \cap H, A_2)}{S(A_1 \cap H, A_2^c)} = \frac{S(A_2 \cap H, A_1)}{S(A_2^c \cap H, A_1)} \frac{S(H, A_2)}{S(H, A_2^c)} \quad (67)$$

We prove this theorem directly by replacing the subsethood terms on the righthand side with their equivalent ratios of counts, canceling like terms three times, multiplying by $\frac{M(A_1 \cap H)}{M(A_1 \cap H)}$, rearranging, and applying the Subsethood Theorem a second time.

We have now developed enough fuzzy theory to examine critically the recent anti-fuzzy polemics of Lindley [1987] and Jaynes [1979] (and thus Cheeseman [1985] who uses Jaynes' arguments). To begin we observe four more corollaries of the Subsethood Theorem:

$$(i) \quad 0 \leq S(H, A) \leq 1, \quad (68)$$

$$(ii) \quad S(H, A) = 1 \text{ if } H \subset A. \quad (69)$$

$$(iii) \quad S(H, A_1 \cup A_2) = S(H, A_1) + S(H, A_2) - S(H, A_1 \cap A_2), \quad (70)$$

$$(iv) \quad S(H, A_1 \cap A_2) = S(H, A_1) S(A_1 \cap H, A_2). \quad (71)$$

Each relationship follows from the ratio form of $S(A, B)$. The third relationship (70) uses the additivity of the count $M(A)$, which follows from $\min(x, y) + \max(x, y) = x + y$.

Suppose we make the notational identification $S(H, A) = P(A|H)$. We then obtain the defining relationships of conditional probability Lindley proposed:

$$\text{Convexity: } 0 \leq P(A|H) \leq 1 \text{ and } P(A|H) = 1 \text{ if } H \text{ implies } A, \quad (72)$$

$$\text{Addition: } P(A_1 \cup A_2|H) = P(A_1|H) + P(A_2|H) - P(A_1 \cap A_2|H) \quad (73)$$

$$\text{Multiplication: } P(A_1 \cap A_2|H) = P(A_1|H) P(A_2|A_1 \cap H). \quad (74)$$

"From these three rules," Lindley tells us, "all of the many, rich and wonderful results of the probability calculus follow. They may be described as the axioms of probability." Lindley takes these as "unassailable" axioms: "We really have no choice about the rules governing our measurement of uncertainty: they are dictated to us by the inexorable laws of logic." Lindley proceeds to build a "coherence" argument around the Odds-Form Bayes Theorem, which he correctly deduces from the axioms as the equality

$$\frac{P(A_2|A_1 \cap H)}{P(A_2^c|A_1 \cap H)} = \frac{P(A_1|A_2 \cap H)}{P(A_1|A_2^c \cap H)} \frac{P(A_2|H)}{P(A_2^c|H)}, \quad (75)$$

where here we interpret A^c as not- A . "Any other procedure," Lindley claims, "is incoherent." This polemic evaporates in the face of the above four subethood corollaries and the Odds-Form Fuzzy Bayes Theorem. Ironically, rather than establish the primacy of axiomatic probability, Lindley seems to argue that it is fuzziness in disguise.

Maximum-entropy estimation provides another source of Bayesian probability polemic [Cheeseman, 1985]. Here the axiomatic argument rests on the so-called Cox's Theorem [1946].

According to physicist E.T. Jaynes [1979]: "Cox proved that any method of inference in which we represent degrees of plausibility by real numbers, is necessarily either equivalent to Laplace's, or inconsistent," where Jaynes cites Laplace as an early Bayesian probabilist. In fact Cox used *bivalent* logic (Boolean algebra) and other assumptions to show that, again according to Jaynes, the "conditions of consistency can be stated in the form of functional equations," namely the probabilistic product and sum rules:

$$P(A \cap B|C) = P(A|B \cap C) P(B|C) , \quad (76)$$

$$P(B|A) + P(B^c|A) = 1 . \quad (77)$$

The Subsethood Theorem implies

$$S(C, A \cap B) = S(B \cap C, A) S(C, B) , \quad (78)$$

$$S(A, B) + S(A, B^c) \geq 1 , \quad (79)$$

with, as we have seen, equality holding for the second subsethood relationship when B is nonfuzzy, which holds in the Cox-Jaynes setting.

In the probabilistic case overlap and underlap are degenerate. So $P(A \cap A^c|B) = P(\emptyset|B) = \frac{P(\emptyset)}{P(B)} = 0$, and $P(B|A \cap A^c) = P(B|\emptyset)$ is undefined. Yet in general $S(B, A \cap A^c) > 0$, and we can define $S(A \cap A^c, B)$ when A and B are fuzzy or nonfuzzy.

Jaynes' claim is either false or concedes that probability is a special case of fuzziness. For strictly speaking, since the subsethood measure $S(A, B)$ satisfies the multiplicative and additive laws specified by Cox and yet differs from the conditional probability $P(B|A)$, Jaynes' claim is false.

Presumably Jaynes was unaware of fuzzy sets. He suggests that the frequency theory of probability provides the only alternative uncertainty theory, and we have reduced relative frequency to the subsethood measure $S(X, A)$. So if we restrict consideration to nonfuzzy sets A and B , equality holds in the above subsethood relations, and Jaynes argues correctly: probability and fuzziness coincide. But fuzziness exists, indeed abounds, outside this restriction and classical probability theory does not. So fuzzy theory extends probability theory. Equivalently, probability represents a special case of fuzziness.

When we examine Cox's actual arguments, we find that Cox assumes that the uncertainty combination operators are continuously *twice differentiable*. Min and max are not twice differentiable. Technically, Cox's theorem does not apply.

THE ENTROPY-SUBSETHOOD THEOREM

We independently derived the Fuzzy Entropy Theorem and the Subsethood Theorem from first principles, from sets-as-points unit-cube geometry. Both theorems involve ratios of cardinalities. So we can suspect a connection.

The Entropy-Subsethood Theorem shows that the connection involves overlap $A \cap A^c$ and underlap $A \cup A^c$. The theorem eliminates fuzzy entropy in favor of subsethood. So subsethood emerges as the fundamental, characterizing quantity of fuzziness—and, arguably, of probability as well.

$$\text{Entropy-Subsethood Theorem: } E(A) = S(A \cup A^c, A \cap A^c) . \quad (80)$$

The theorem follows if we replace B and A in the Subsethood Theorem with respectively overlap $A \cap A^c$ and underlap $A \cup A^c$. Since overlap is a subset of underlap, since $S(A \cap A^c, A \cup A^c) = 1$, the intersection of the two sets equals the overlap.

The Entropy-Subsethood Theorem describes a peculiar relationship. It gives fuzziness or ambiguity as the degree to which the superset $A \cup A^c$ is a subset of its own subset $A \cap A^c$, the extent to which the whole is a part of one of its own parts, a relationship Western logic forbids.

This relationship violates our ingrained Venn-diagram intuitions of unambiguous set inclusion. Only the midpoint of I^* yields total containment of underlap in overlap. The

cube vertices yield zero containment. This parallels in the extreme the relative frequency relationship $S(X, A) = \frac{|A|}{N}$, where nonfuzzy subset A contains to some degree its nonfuzzy superset X .

Figure 16.11 illustrates the Entropy-Subsethood Theorem. It shows that d^* , the shortest distance from underlap $A \cup A^c$ to the hyper-rectangle that defines the fuzzy power set of overlap $A \cap A^c$, equals the distance $d(A \cup A^c, A \cap A^c) = d(A, A^c)$ and equals a difference of vector magnitudes: $d^* = M(A \cup A^c) - M(A \cap A^c)$.

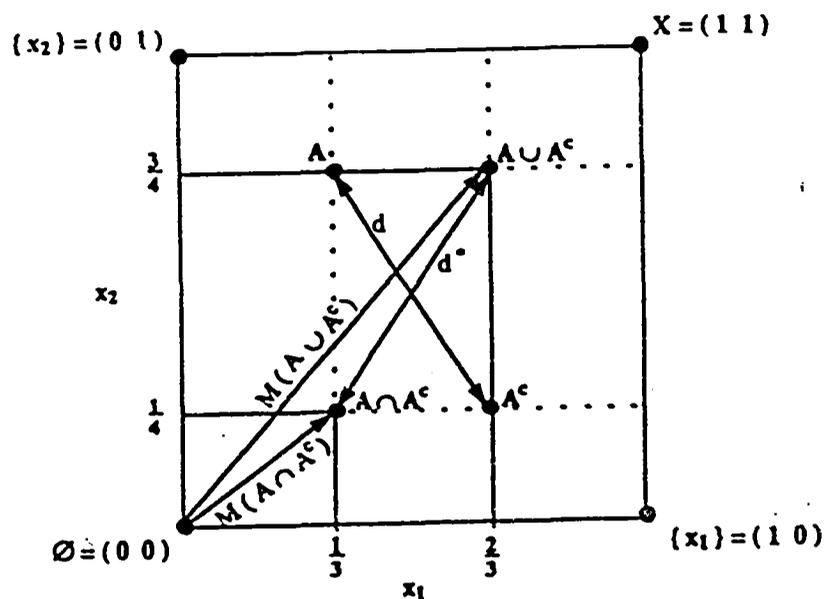


Figure 16.11 Entropy-Subsethood Theorem in two dimensions. Just as the long diagonals have equal length, $d(A, A^c) = d(A \cup A^c, A \cap A^c) = d^* = M(A \cup A^c) - M(A \cap A^c)$, the shortest distance from $A \cup A^c$ to the fuzzy power set of $A \cap A^c$.

The Entropy-Subsethood Theorem implies that no probability measure measures fuzziness. For the moment, suppose not. Suppose fuzzy entropy measures nothing new; fuzziness is simply disguised probability. Suppose, as Lindley [1987] claims, that probability

theory "is adequate for all problems involving uncertainty." Then there exists some probability measure P such that $P = \bar{E}$. P cannot equal zero everywhere because $P(X) = 1$. Then there is some A such that $P(A) = E(A) > 0$. But in a probability space overlap or underlap are degenerate: $A \cap A^c = \emptyset$, and $A \cup A^c = X$.

The Entropy-Subsethood Theorem then implies that $0 < P(A) = E(A) = S(A \cup A^c, A \cap A^c) = S(X, \emptyset)$. X can be a subset to nonzero degree of the empty set only if X itself is empty, and hence only if A is empty: $X = A = \emptyset$. Then the sure event X is impossible: $P(X) = P(\emptyset) = 0$. Or the impossible event is sure: $P(\emptyset) = 1$. Either outcome gives a bivalent contradiction, impervious to normalization. So there exists no probability measure P that measures fuzziness. Fuzziness exists.

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7. Prove the fuzzy De Morgan Laws:

$$(a) A \cap B = (A^c \cup B^c)^c$$

$$(b) A \cup B = (A^c \cap B^c)^c$$

8. Prove:

$$0 \leq \ell^p(A, A_{near}) \leq \frac{n^{1/p}}{2} \leq \ell^p(A, A_{far}) \leq n^{1/p}, \quad p \geq 1.$$

9. Prove the ℓ^1 -version of the Fuzzy Entropy Theorem:

$$E(A) = \frac{\ell^1(A, A_{near})}{\ell^1(A, A_{far})} = \frac{M(A \cap A^c)}{M(A \cup A^c)}$$

10. Prove: $M(A) + M(B) = M(A \cap B) + M(A \cup B)$.

11. Prove: $\frac{1}{n}M(A \cap A^c) + \frac{1}{n}M(A \cup A^c) = 1$.

12. Prove:

$$(a) E(P) = \frac{1}{n-1} \quad \text{if } M(P) = 1 \text{ and all } p_i \leq 1/2,$$

$$(b) E(P) < \frac{1}{n-1} \quad \text{if } M(P) = 1 \text{ and some } p_j > 1/2.$$

13. Prove the fit-violation version of the Subsethood Theorem:

$$S(A, B) = 1 - \frac{\sum_x \max(0, m_A - m_B(x))}{M(A)} = \frac{M(A \cap B)}{M(A)}$$

14. Prove:

$$S(E, H_i) = \frac{S(H_i, E) f_i}{\sum_{j=1}^K S(H_j, E) f_j},$$

where $f_j = S(X, H_j)$, the nonfuzzy sets H_1, \dots, H_K partition X , and E is fuzzy.

15. Prove:

$$S(E, H) \geq \frac{S(H, E) f_H}{S(H, E) f_H + S(H^c, E) f_{H^c}}$$

where $f_H = S(X, H)$ and E and H are arbitrary fuzzy sets.

16. Prove the Odds-Form Bayes Theorem:

$$\frac{S(A_1 \cap H, A_2)}{S(A_1 \cap H, A_2^c)} = \frac{S(A_2 \cap H, A_1)}{S(A_2 \cap H, A_1^c)} \frac{S(H, A_2)}{S(H, A_2^c)},$$

for arbitrary fuzzy sets A_1 , A_2 , and H .

17. Prove directly the additive inequality: $S(A, B) + S(A, B^c) \geq 1$.

18. Prove:

(a) $0 \leq S(H, A) \leq 1$,

(b) $S(H, A) = 1$ if $H \subset A$,

(c) $S(H, A_1 \cup A_2) = S(H, A_1) + S(H, A_2) - S(H, A_1 \cap A_2)$,

(d) $S(H, A_1 \cap A_2) = S(H, A_1) S(A_1 \cap H, A_2)$.

19. Show that $N_a(N_a(x)) = x$ for the generalized negation operator

$$N_a(x) = \frac{1-x}{1+ax}, \quad a > -1, \quad 0 \leq x \leq 1,$$

20. If we define intersection \cap_T pointwise by

$$T(x, y) = 1 - \min(1, [(1-x)^p + (1-y)^p]^{1/p}), \quad p > 0,$$

how should we define the corresponding De Morgan dual union \cup_S ?

21. What De Morgan dual union operator corresponds to the intersection operator

$$\max(0, x + y - 1)?$$

22. Zadeh's consequent conjunction syllogism schematizes as

Q_1 As are Bs

Q_2 As are Cs

Therefore: Q As are Bs and Cs

Show that if $Q_1 = S(A, B)$ and $Q_2 = S(A, C)$, then the fuzzy quantifier Q obeys

$$\max(0, Q_1 + Q_2 - 1) \leq Q \leq \min(Q_1, Q_2).$$

23. Define the *volume subsethood* measure $V(A, B)$ as

$$V(A, B) = \frac{v(A \cap B)}{v(A)},$$

for fit vectors $A = (a_1, \dots, a_n)$ and $B = (b_1, \dots, b_n)$ such that $a_i > 0$. $v(A)$ is the Lebesgue or volume measure of A :

$$v(A) = \prod_{i=1}^n a_i.$$

The volume subsethood measure $V(A, B)$ measures the ratio of the volume of the overlap hyper-rectangle $F(2^{A \cap B})$ to the volume of A 's fuzzy power set $F(2^A)$. Prove that the volume subsethood measure $V(A, B)$ underestimates the subsethood measure $S(A, B)$:

$$V(A, B) \leq S(A, B).$$

ADAPTIVE FUZZY SYSTEMS FOR BACKING UP A TRUCK-AND-TRAILER

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Abstract

We developed fuzzy and neural-network control systems to back up a simulated truck, and truck-and-trailer, to a loading dock in a planar parking lot. The fuzzy systems performed well until we randomly removed over 50 % of their fuzzy-associative-memory (FAM) rules. They also performed well when we replaced key FAM equilibration rules with destructive or "sabotage" rules. We trained the neural network systems with the supervised backpropagation learning algorithm and tested their robustness by removing random subsets of training data in learning sequences. The neural systems performed well but required extensive computation for training. We used unsupervised differential competitive learning (DCL), and product-space clustering, to adaptively generate FAM rules from training data. The original fuzzy and neural control systems generated trajectory data. The DCL system rapidly recovered the underlying FAM rules. Product-space clustering converted the neural truck systems into structured sets of FAM rules that approximated the neural system's behavior.

Fuzzy and Neural Control Systems

We construct fuzzy and neural control systems directly from control data, but from different types of control data. Fuzzy systems use a small number of structured *linguistic* input-output samples from an expert or from some other adaptive estimator. Neural systems use a large number of *numeric* input-output samples from the control process or from some other database. Adaptive fuzzy systems also use numeric control data.

Figure 1 illustrates this difference. The neural system estimates function $f: X \rightarrow Y$ from several numerical point samples (x_i, y_i) . The fuzzy system estimates f from a few fuzzy set samples or fuzzy associations (A_i, B_i) .

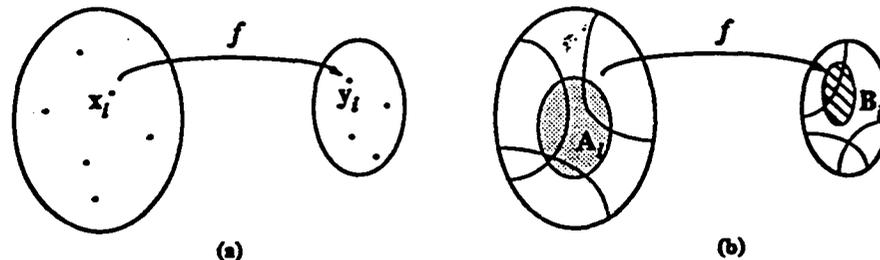


FIGURE 1 Geometry of neural and fuzzy function estimation. The neural approach (a) uses several numerical point samples. The fuzzy approach (b) uses a few fuzzy set samples.

Fuzzy and neural systems offer a key advantage over traditional control approaches. They offer *model-free estimation* of the control system. The user need not specify how the controller's output mathematically depends on its input. Instead the user provides a few common-sense associations of how the control variables behave. Or the user provides a statistically representative set of numerical training samples. Even if a math-model controller is available, fuzzy or neural controllers may prove more robust and easier to modify.

Which system, fuzzy or neural, performs better for which type of control problem de-

depends on the type and availability of sample data. If experts provide structured knowledge of the control process, or if sufficient numerical training samples are unavailable, the fuzzy approach may be preferable. We can construct a fuzzy control system with comparative ease when experts or fuzzy engineers provide accurate structured knowledge. A fuzzy control system seems a reasonable benchmark in such cases, even if we can develop a neural controller or math-model controller.

If we have representative numerical data but not structured expertise, the neural approach may be preferable. Or a statistical regression approach may be more appropriate. The data simply tell their own story—if there is a story to tell. Yet even here we can use a hybrid fuzzy-neural system, an adaptive fuzzy system. We can use the numerical data to generate *fuzzy associative memory* (FAM) rules. The FAM rules can then form the skeleton of a fuzzy control architecture. In short, if structured knowledge is unavailable, estimate it. This may be more practical than it would appear because of the small number of control FAM rules needed to reliably control many realworld processes.

How can we compare fuzzy and neural controllers? Abstract comparison proves difficult because both approaches build a control black box in different ways. That they build black boxes distinguishes them from math-model controllers. It also suggests we can compare them, at least approximately, by their black-box control performance.

Each control system generated an output *control surface* as it ranged over the common input space of parameter values. Figure 5 below shows three-dimensional control surfaces for the fuzzy and neural controllers. For control systems with few input parameters with moderately quantized ranges, we can store both fuzzy and neural controllers—or rather their quantized control surfaces—as decision look-up tables. Then once we specify a system performance criterion, we can in principle quantitatively compare the controllers.

Comparing system trajectories proved more complicated. In the case at hand, we wanted to back up a truck, and truck-and-trailer, to a loading dock. We can measure and compare the quality and quantity of the truck trajectory, perhaps with mean-squared error criteria. Intuitively, we preferred smooth short trajectories to jagged long trajectories. Reaching the loading-dock goal was also important. In practice it is the most important performance requirement. We must balance the trajectory type with the trajectory

destination, and this reduces to the pragmatic issue of balancing means and ends.

Below we develop a simple fuzzy control system and a simple neural control system for backing up a truck, and truck-and-trailer, in an open parking lot. The recent neural network truck backer-upper simulation of Nguyen and Widrow [1989] motivated our choice of control problem.

The fuzzy control system compared favorably with the neural controller in terms of black-box development effort, black-box computational load, smoothness of truck trajectories, and robustness.

We studied robustness of the fuzzy control systems in two ways. We deliberately added confusing FAM rules—"sabotage" rules—to the system, and we randomly removed different subsets of FAM rules. We studied robustness of the neural controller by randomly removing different portions of the training data in learning sequences. We also converted the neural control systems to structured FAM-bank systems.

Backing up a truck

Figure 2 shows the simulated truck and loading zone. The truck corresponds to the cab part of the neural truck in the Nguyen-Widrow neural truck backer-upper system. The three state variables ϕ , x , and y exactly determine the truck position. ϕ specifies the angle of the truck with the horizontal. The coordinate pair (x, y) specifies the position of the rear center of the truck in the plane.

The goal was to make the truck arrive at the loading dock at a right angle ($\phi_f = 90^\circ$) and to align the position (x, y) of the truck with the desired loading dock (x_f, y_f) . We considered only backing up. The truck moved backward by some fixed distance at every stage. The loading zone corresponded to the plane $[0, 100] \times [0, 100]$, and (x_f, y_f) equaled $(50, 100)$.

At every stage the fuzzy and neural controllers should produce the steering angle θ that backs up the truck to the loading dock from any initial position and from any angle in the loading zone.

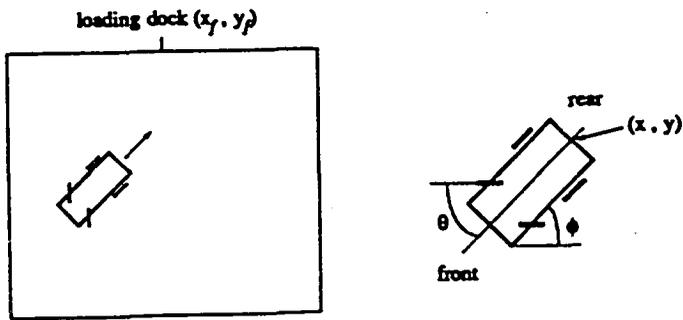


FIGURE 2 Diagram of simulated truck and loading zone.

Fuzzy Truck Backer-Upper System

We first specified each controller's input and output variables. The input variables were the truck angle ϕ and the x -position coordinate x . The output variable was the steering-angle signal θ . We assumed enough clearance between the truck and the loading dock so we could ignore the y -position coordinate. The variable ranges were as follows:

$$\begin{aligned} 0 &\leq x \leq 100 , \\ -90 &\leq \phi \leq 270 , \\ -30 &\leq \theta \leq 30 . \end{aligned}$$

Positive values of θ represented clockwise rotations of the steering wheel. Negative values represented counterclockwise rotations. We discretized all values to reduce computation. The resolution of ϕ and θ was one degree each. The resolution of x was 0.1.

Next we specified the fuzzy-set values of the input and output fuzzy variables. The fuzzy sets numerically represented linguistic terms, the sort of linguistic terms an expert might use to describe the control system's behavior. We chose the fuzzy-set values of the fuzzy variables as follows:

<u>Angle ϕ</u>	<u>z-position z</u>	<u>Steering-angle signal θ</u>
RB: Right Below	LE: Left	NB: Negative Big
RU: Right Upper	LC: Left Center	NM: Negative Medium
RV: Right Vertical	CE: Center	NS: Negative Small
VE: Vertical	RC: Right Center	ZE: Zero
LV: Left Vertical	RI: Right	PS: Positive Small
LU: Left Upper		PM: Positive Medium
LB: Left Below		PB: Positive Big

Fuzzy subsets contain elements with degrees of membership. A fuzzy membership function $m_A : Z \rightarrow [0, 1]$ assigns a real number between 0 and 1 to every element z in the universe of discourse Z . This number $m_A(z)$ indicates the degree to which the object or data z belongs to the fuzzy set A . Equivalently, $m_A(z)$ defines the *fit* (fuzzy unit) value [Kosko, 1986] of element z in A .

Fuzzy membership functions can have different shapes depending on the designer's preference or experience. In practice fuzzy engineers have found triangular and trapezoidal shapes help capture the modeler's sense of fuzzy numbers and simplify computation. Figure 3 shows membership-function graphs of the fuzzy subsets above. In the third graph, for example, $\theta = 20^\circ$ is Positive Medium to degree 0.5, but only Positive Big to degree 0.3.

In Figure 3 the fuzzy sets CE , VE , and ZE are narrower than the other fuzzy sets. These narrow fuzzy sets permit fine control near the loading dock. We used wider fuzzy sets to describe the endpoints of the range of the fuzzy variables ϕ , z , and θ . The wider fuzzy sets permitted rough control far from the loading dock.

Next we specified the fuzzy "rulebase" or bank of *fuzzy associative memory* (FAM) rules. Fuzzy associations or "rules" (A, B) associate output fuzzy sets B of control values with input fuzzy sets A of input-variable values. We can write fuzzy associations as antecedent-consequent pairs or IF-THEN statements.

In the truck backer-upper case, the FAM bank contained the 35 FAM rules in Figure 4. For example, the FAM rule of the left upper block (FAM rule 1) corresponds to the following

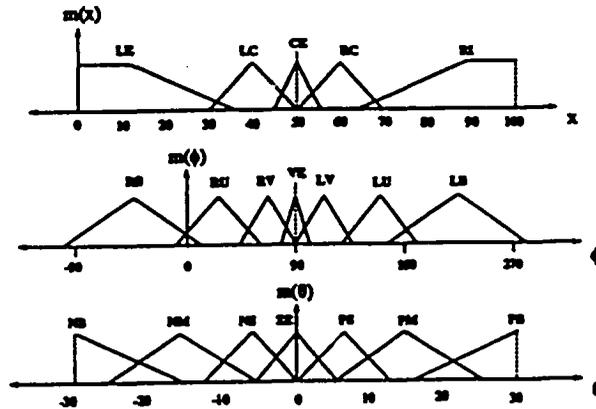


FIGURE 3 Fuzzy membership functions for each linguistic fuzzy-set value. To allow finer control, the fuzzy sets that correspond to near the loading dock are narrower than the fuzzy sets that correspond to far from the loading dock.

fuzzy association:

$$IF \ x = LE \ AND \ \phi = RB, \ THEN \ \theta = PS.$$

FAM rule 18 indicates that if the truck is in near the equilibrium position, then the controller should not produce a positive or negative steering-angle signal. The FAM rules in the FAM-bank matrix reflect the symmetry of the controlled system.

For the initial condition $x = 50$ and $\phi = 270$, the fuzzy truck did not perform well. The symmetry of the FAM rules and the fuzzy sets cancelled the fuzzy controller output in a rare saddle point. For this initial condition, the neural controller (and truck-and-trailer below) also performed poorly. Any perturbation breaks the symmetry. For example, the rule (If $x = 50$ and $\phi = 270$, then $\theta = 5$) corrected the problem.

The three-dimensional control surfaces in Figure 5 show steering-angle signal outputs θ that correspond to all combinations of values of the two input state variables ϕ and x . The control surface defines the fuzzy controller. In this simulation the correlation-minimum FAM inference procedure, discussed in [Kosko, 1990a], determined the fuzzy control surface. If the control surface changes with sampled variable values, the system

		X				
		LE	LC	CE	EC	RI
ϕ	RB ¹	PS	PM	PM	PB	PB
	RU ⁶	NS	PS	PM	PB	PB
	RV	NM	NS	PS	PM	PB
	VE	NM	NM	ZE	PM	PM
	LV	NB	NM	NS	PS	PM
	LU	NB	NB	NM	NS	PS
	LB	NB	NB	NM	NM	NS

FIGURE 4 FAM-bank matrix for the fuzzy truck backer-upper controller.

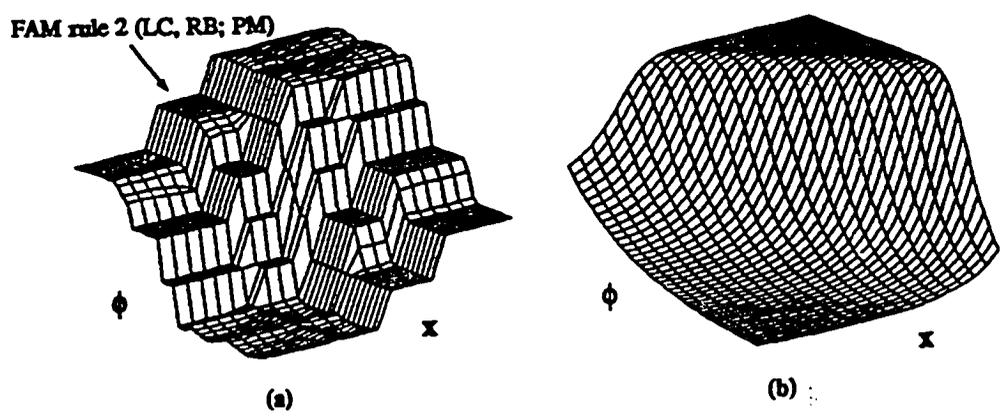


FIGURE 5 (a) Control surface of the fuzzy controller. Fuzzy-set values determined the input and output combination corresponding to FAM rule 2 (IF $x=LC$ AND $\phi=RB$, THEN $\theta=PM$). (b) Corresponding control surface of the neural controller for constant value $y=20$.

behaves as an *adaptive* fuzzy controller. Below we demonstrate unsupervised adaptive control of the truck and the truck-and-trailer systems.

Finally, we determined the output action given the input conditions. We used the correlation-minimum inference method illustrated in Figure 6. Each FAM rule produced the output fuzzy set clipped at the degree of membership determined by the input conditions and the FAM rule. Alternatively, correlation-product inference [Kosko, 1990a] would combine FAM rules multiplicatively. Each FAM rule emitted a fit-weighted output fuzzy set O_i at each iteration. The total output O added these weighted outputs:

$$O = \sum_i O_i \quad (1)$$

$$= \sum_i \min(f_i, S_i) \quad , \quad (2)$$

where f_i denotes the antecedent fit value and S_i represents the consequent fuzzy set of steering-angle values in the i th FAM rule. Earlier fuzzy systems combined the output sets O_i with pairwise maxima. But this tends to produce a uniform output set O as the number of FAM rules increases. Adding the output sets O_i invokes the fuzzy version of the Central Limit Theorem. This tends to produce a symmetric, unimodal output fuzzy set O of steering-angle values.

Fuzzy systems map fuzzy sets to fuzzy sets. The fuzzy control system's output defines the fuzzy set O of steering-angle values at each iteration. We must "defuzzify" the fuzzy set O to produce a numerical (point-estimate) steering-angle output value θ .

As discussed in [Kosko, 1990a], the simplest defuzzification scheme selects the value corresponding to the *maximum fit* value in the fuzzy set. This mode-selection approach ignores most of the information in the output fuzzy set and requires an additional decision algorithm when multiple modes occur.

Centroid defuzzification provides a more effective procedure. This method uses the *fuzzy centroid* $\bar{\theta}$ as output:

$$\bar{\theta} = \frac{\sum_{j=1}^p \theta_j m_O(\theta_j)}{\sum_{j=1}^p m_O(\theta_j)} \quad , \quad (3)$$

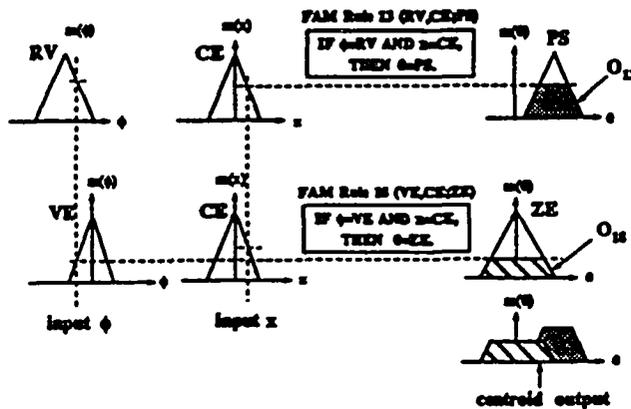


FIGURE 6 Correlation-minimum inference with centroid defuzzification method. Then FAM-rule antecedent's combined with AND use the *minimum* fit value to activate consequents. Those combined with OR would use the maximum fit value.

where O defines a fuzzy subset of the steering-angle universe of discourse $\Theta = \{\theta_1, \dots, \theta_p\}$. The central-limit-theorem effect produced by adding output fuzzy set O_i benefits both max-mode and centroid defuzzification. Figure 6 shows the correlation-minimum inference and centroid defuzzification applied to FAM rules 13 and 18. We used centroid defuzzification in all simulations.

With 35 FAM rules, the fuzzy truck controller produced successful truck backing-up trajectories starting from any initial position. Figure 7 shows typical examples of the fuzzy-controlled truck trajectories from different initial positions. The fuzzy control system did not use ("fire") all FAM rules at each iteration. Equivalently most output consequent sets are empty. In most cases the system used only one or two FAM rules at each iteration. The system used at most 4 FAM rules at once.

Neural Truck Backer-Upper System

The neural truck backer-upper of Nguyen and Widrow [1989] consisted of multilayer

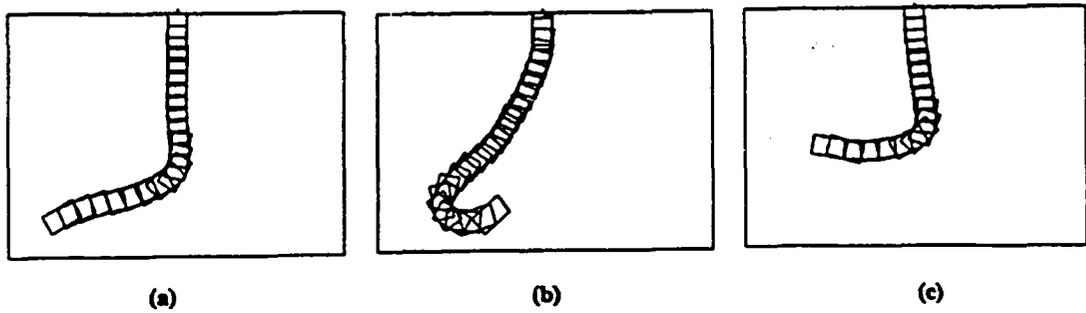


FIGURE 7 Sample truck trajectories of the fuzzy controller for initial positions (x, y, ϕ) : (a) $(20, 20, 30)$, (b) $(30, 10, 220)$, and (c) $(30, 40, -10)$.

feedforward neural networks trained with the backpropagation gradient-descent (stochastic approximation) algorithm. The neural control system consisted of two neural networks: the controller network and the truck emulator network. The controller network produced an appropriate steering-angle signal output given any parking-lot coordinates (x, y) , and the angle ϕ . The emulator network computed the next position of the truck. The emulator network took as input the previous truck position and the current steering-angle output computed by the controller network.

We did not train the emulator network since we could not obtain "universal" synaptic connection weights for the truck emulator network. The backpropagation learning algorithm did not converge for some sets of training samples. The number of training samples for the emulator network might exceed 3000. For example, the combinations of training samples of a given angle ϕ , x -position, y -position, and steering angle signal θ might correspond to 3150 $(18 \times 5 \times 5 \times 7)$ samples depending on the division of the input-output product space. Moreover, the training samples were numerically similar since the neuronal signals assumed scaled values in $[0, 1]$ or $[-1, 1]$. For example, we treated close values, such as 0.40 and 0.41, as distinct sample values.

Simple kinematic equations replaced the truck emulator network. If the truck moved

backward from (x, y) to (x', y') at an iteration, then

$$x' = x + r \cos(\phi') , \quad (4)$$

$$y' = y + r \sin(\phi') , \quad (5)$$

$$\phi' = \phi + \theta . \quad (6)$$

r denotes the fixed driving distance of the truck for all backing movements. We used equations (4)–(6) instead of the emulator network. This did not affect the post-training performance of the neural truck backer-upper since the truck emulator network back-propagated only errors.

We trained only the controller network with backpropagation. The controller network used 24 “hidden” neurons with logistic sigmoid functions. In the training of the truck-controller, we estimated the ideal steering-angle signal at each stage before we trained the controller network. In the simulation, we used the arc-shaped truck trajectory produced by the fuzzy controller as the ideal trajectory. The fuzzy controller generated each training sample (x, y, ϕ, θ) at each iteration of the backing-up process. We used 35 training sample vectors and needed more than 100,000 iterations to train the controller network.

Figure 5b shows the resulting neural control surface for $y = 20$. The neural control surface shows less structure than the corresponding fuzzy control surface. This reflects the unstructured nature of black-box supervised learning. Figure 8 shows the network connection topology for our neural truck backer-upper control system.

Figure 9 shows typical examples of the neural-controlled truck trajectories from several initial positions. Even though we trained the neural network to follow the smooth arc-shaped path, some learned truck trajectories were non-optimal.

Comparison of Fuzzy and Neural Systems

As shown in Figure 7 and 9, the fuzzy controller always smoothly backed up the truck but the neural controller did not. The neural-controlled truck sometimes followed an irregular path.

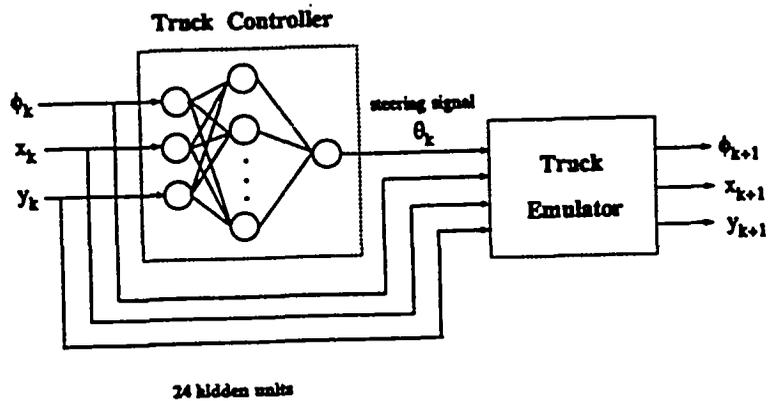


FIGURE 8 Topology of our neural control system.

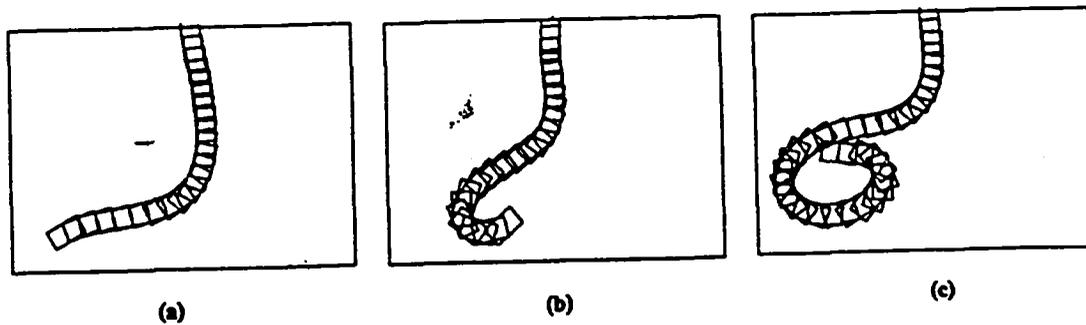


FIGURE 9 Sample truck trajectories of the neural controller for initial positions (x, y, ϕ) : (a) $(20, 20, 30)$, (b) $(30, 10, 220)$, and (c) $(30, 40, -10)$.

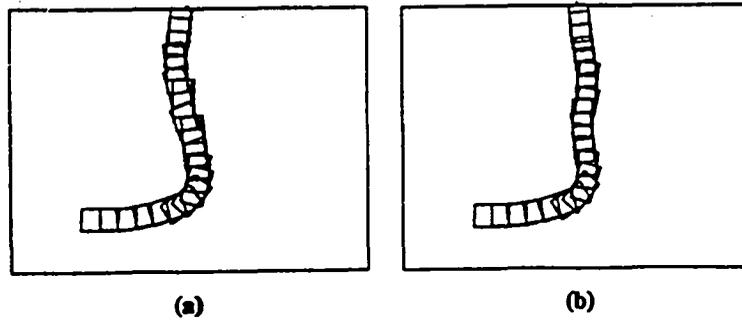


FIGURE 10 The fuzzy truck trajectory after we replaced the key steady-state FAM rule 18 by the two worst rules: (a) IF $x = CE$ AND $\phi = VE$, THEN $\theta = PB$, and (b) IF $x = CE$ AND $\phi = VE$, THEN $\theta = NB$.

Training the neural control system was time-consuming. The backpropagation algorithm required thousands of back-ups to train the controller network. In some cases, the learning algorithm did not converge.

We "trained" the fuzzy controller by encoding our own common sense FAM rules. Once we develop the FAM-rule bank, we can compute control outputs from the resulting FAM-bank matrix or control surface. The fuzzy controller did not need a truck emulator and did not require a math model of how outputs depended on inputs.

The fuzzy controller was computationally lighter than the neural controller. Most computation operations in the neural controller involved the multiplication, addition, or logarithm of two real numbers. In the fuzzy controller, most computational operations involved comparing and adding two real numbers.

Sensitivity Analysis

We studied the sensitivity of the fuzzy controller in two ways. We replaced the FAM rules with destructive or "sabotage" FAM rules, and we randomly removed FAM rules.

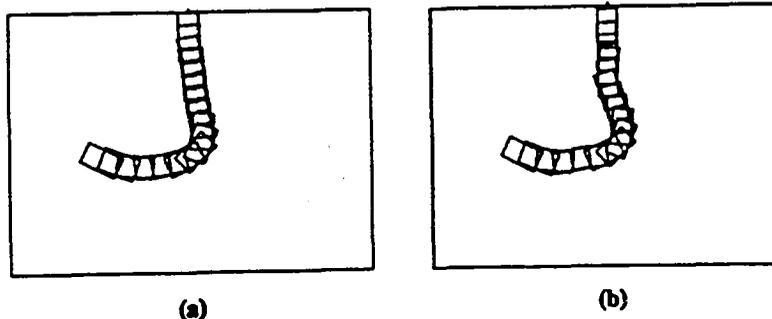


FIGURE 11 Fuzzy truck trajectory when (a) no FAM rules are removed and (b) FAM rules 7, 13, 18 and 23 are removed.

We deliberately chose sabotage FAM rules to confound the system. Figure 10 shows the trajectory when two sabotage FAM rules replaced the important steady-state FAM rule—FAM rule 18: the fuzzy controller should produce zero output when the truck is nearly in the correct parking position. Figure 11 shows the truck trajectory after we removed four randomly chosen FAM rules (7, 13, 18, and 23). These perturbations did not significantly affect the fuzzy controller's performance.

We studied robustness of each controller by examining failure rates. For the fuzzy controller we removed fixed percentages of randomly selected FAM rules from the system. For the neural controller we removed training data. Figure 12 shows performance errors averaged over ten typical back-ups with missing FAM rules for the fuzzy controller and missing training data for the neural controller. The missing FAM rules and training data ranged from 0 % to 100 % of the total. In Figure 12a, the docking error equaled the Euclidean distance from the actual final position (ϕ, x, y) to the desired final position (ϕ_f, x_f, y_f) :

$$\text{Docking Error} = \sqrt{(\phi_f - \phi)^2 + (x_f - x)^2 + (y_f - y)^2} \quad (7)$$

In Figure 12b, the trajectory error equaled the ratio of the actual trajectory length of the

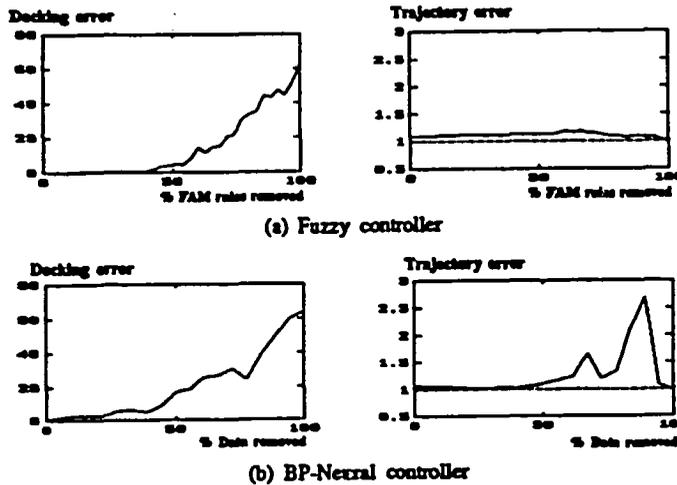


FIGURE 12 Comparison of robustness of the controllers: (a) Docking and Trajectory error of the fuzzy controller, (b) Docking and Trajectory error of the neural controller.

truck divided by the straight line distance to the loading dock:

$$\text{Trajectory Error} = \frac{\text{length of truck trajectory}}{\text{distance}(\text{initial position, desired final position})} \quad (8)$$

Adaptive Fuzzy Truck Backer-Upper

Adaptive FAM (AFAM) systems generate FAM rules directly from training data. A one-dimensional FAM system, $S : I^m \rightarrow I^p$, defines a FAM rule, a single association of the form (A_i, B_i) . In this case the input-output product space equals $I^m \times I^p$. As discussed in [Kosko, 1990a], a FAM rule (A_i, B_i) defines a cluster or ball of points in the product-space cube $I^m \times I^p$ centered at the point (A_i, B_i) . Adaptive clustering algorithms can estimate the

unknown FAM rule (A_i, B_i) from training samples in R^2 . We used differential competitive learning (DCL) to recover the bank of FAM rules that generated the truck training data.

We generated 2230 truck samples from 7 different initial positions and varying angles. We chose the initial positions (20,20), (30,20), (45,20), (50,20), (55,20), (70,20), and (80,20). We changed the angle from -60° to 240° at each initial position. At each step, the fuzzy controller produced output steering angle θ . The training vectors (x, ϕ, θ) defined points in a three-dimensional product-space. x had 5 fuzzy set values: *LE*, *LC*, *CE*, *RC*, and *RI*. ϕ had 7 fuzzy set values: *RB*, *RU*, *RV*, *VE*, *LV*, *LU*, and *LB*. θ had 7 fuzzy set values: *NB*, *NM*, *NS*, *ZE*, *PS*, *PM*, and *PB*. So there were 245 ($5 \times 7 \times 7$) possible FAM cells.

We defined FAM cells by partitioning the effective product-space. FAM cells near the center were smaller than outer FAM cells because we chose narrow membership functions near the steady-state FAM cell. Uniform partitions of the product-space produced poor estimates of the original FAM rules. As in Figure 3, this reflected the need to judiciously define the fuzzy-set values of the system fuzzy variables.

We performed product-space clustering with the version of DCL discussed in [Kosko, 1990a]. If a FAM cell contained at least one of the 245 synaptic quantization vectors, we entered the corresponding FAM rule in the FAM matrix.

Figure 13a shows the input sample distribution of (x, ϕ) . We did not include the variable θ in the figure. Training data clustered near the steady-state position ($x = 50$ and $\phi = 90^\circ$). Figure 13b displays the synaptic-vector histogram after DCL classified 2230 training vectors for 35 FAM rules. Since successful FAM system generated the training samples, most training samples, and thus most synaptic vectors, clustered in the steady-state FAM cell.

DCL product-space clustering estimated 35 new FAM rules. Figure 14 shows the DCL-estimated FAM bank and the corresponding control surface. The DCL-estimated control surface visually resembles the underlying unknown control surface in Figure 5a. The two systems produce nearly equivalent truck-backing behavior. This suggests adaptive product-space clustering can estimate the FAM rules underlying expert behavior in many cases, even when the expert or fuzzy engineer cannot articulate the FAM rules.

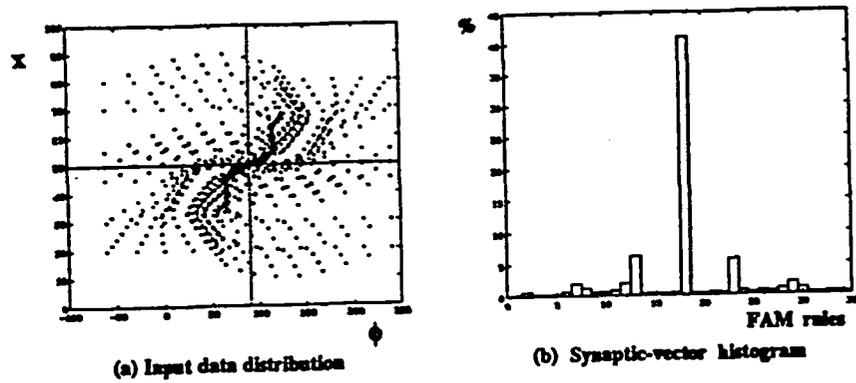


FIGURE 13 (a) Input data distribution, (b) Synaptic-vector histogram. Differential competitive learning allocated synaptic quantization vectors to FAM cells. The steady-state FAM cell (CE, VE; ZE) contained the most synaptic vectors.

	X				
	LE	LC	CE	RC	RI
RB	PS	PM	PM	PE	PE
RU	ZE	PM	PM	PE	PE
RV	NS	ZE	PS	PM	PE
ϕ VE	NM	NM	ZE	PM	PM
LV	NB	NB	NS	PS	PS
LU	NB	NB	NB	NS	PS
LB	NB	NB	NM	NM	NS

(a)

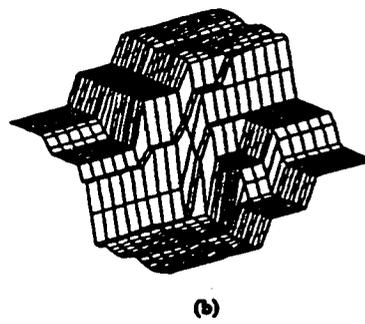
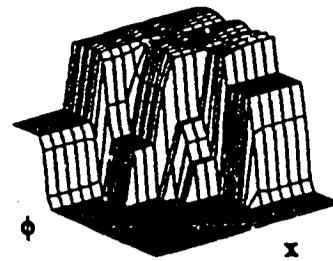


FIGURE 14 (a) DCL-estimated FAM bank. (b) Corresponding control surface.

		x				
		LE	LC	CE	RC	RI
φ	RB	PS	PB	PB	PB	PB
	RU	NM	ZE	PM	PB	PB
	RV	NM	NM	NS	PS	PB
	VE	NM	NM	NM	ZE	PB
	LV	NM	NM	NM	NS	PB
	LU	NM	NM	NM	NM	PM
	LB	NM	NM	NM	NM	NM

(a)



(b)

FIGURE 15 (a) FAM bank generated by the neural control surface in Figure 5b. (b) Control surface of the neural BP-AFAM system in (a).

We also used the neural control surface in Figure 5b to estimate FAM rules. We divided the input-output product-space into FAM cells as in the fuzzy control case. If the neural control surface intersected the FAM cell, we entered the corresponding FAM rule in a FAM bank. We averaged all neural control-surface values in a square region over the two input variables x and ϕ . We assigned the average value to one of 7 output fuzzy sets. Figure 15 shows the resulting FAM bank and corresponding control surface generated by the neural control surface in Figure 5b. This new control surface resembles the original fuzzy control surface in Figure 5a more than it resembles the neural control surface in Figure 5b. Note the absence of a steady-state FAM rule in the FAM matrix in Figure 5a.

Figure 16 compares the DCL-AFAM and BP-AFAM control surfaces with the fuzzy control surface in Figure 5a. Figure 16b shows the absolute difference of the control surfaces. As expected, the DCL-AFAM system produced less absolute error than the BP-AFAM system produced.

Figure 17 shows the docking and trajectory errors of the two AFAM control systems. The DCL-AFAM system produced less docking error than the BP-AFAM system produced for 100 arbitrary backing-up trials. The two AFAM systems generated similar backing-up trajectories. This suggests that black-box neural estimators can define the front-end of

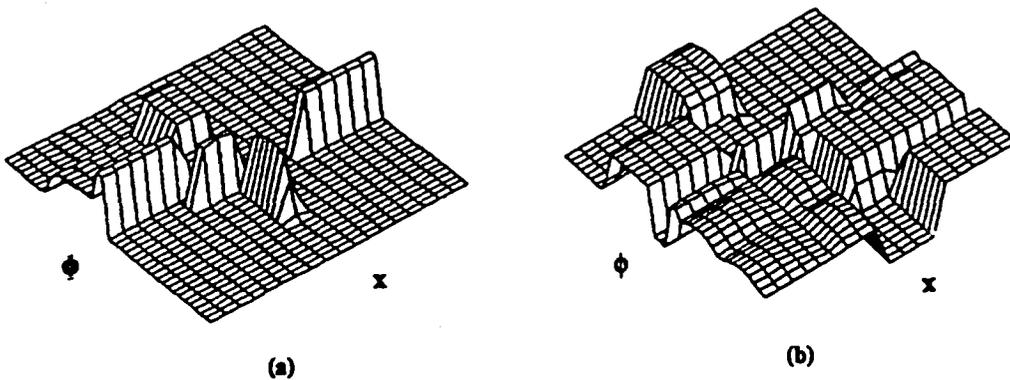


FIGURE 16 (a) Absolute difference of the FAM surface in Figure 5a and the DCL-estimated FAM surface in Figure 14b. (b) Absolute difference of the FAM surface in Figure 5a and the neural-estimated FAM surface in Figure 15b.

FAM-structured systems. In principle we can use this technique to generate structured FAM rules for any neural application. We can then inspect and refine these rules and perhaps replace the original neural system with the tuned FAM system.

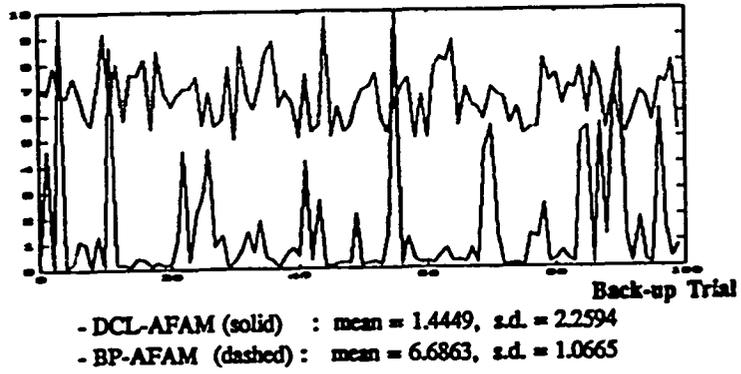
Fuzzy Truck-and-Trailer Controller

We added a trailer to the truck system, as in the original Nguyen-Widrow model. Figure 18 shows the simulated truck-and-trailer system. We added one more variable (cab angle, ϕ_c) to the three state variables of the trailerless truck. In this case a FAM rule takes the form

$$\text{IF } x = \text{LE AND } \phi_t = \text{RB AND } \phi_c = \text{PO, THEN } \beta = \text{NS.}$$

The four state variables x , y , ϕ_t , and ϕ_c determined the position of the truck-and-trailer system in the plane. Fuzzy variable ϕ_t corresponded to ϕ for the trailerless truck. Fuzzy variable ϕ_c specified the relative cab angle with respect to the center line along the trailer. ϕ_c ranged from -90° to 90° . The extreme cab angles 90° and -90° corresponded to two

(a) Docking Error



(b) Trajectory Error

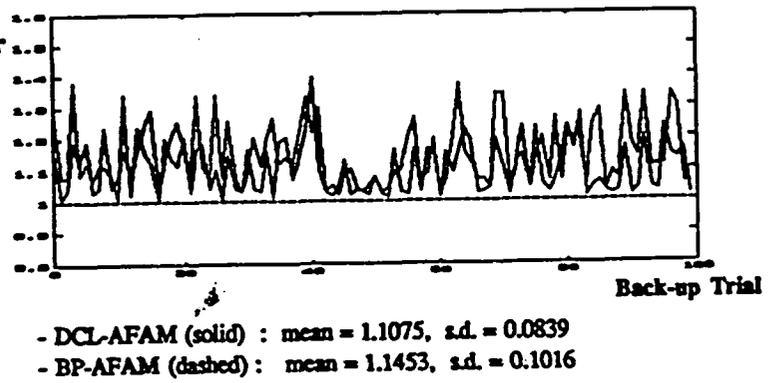


FIGURE 17 (a) Docking errors and (b) Trajectory errors of the DCL-AFAM and BP-AFAM control systems.

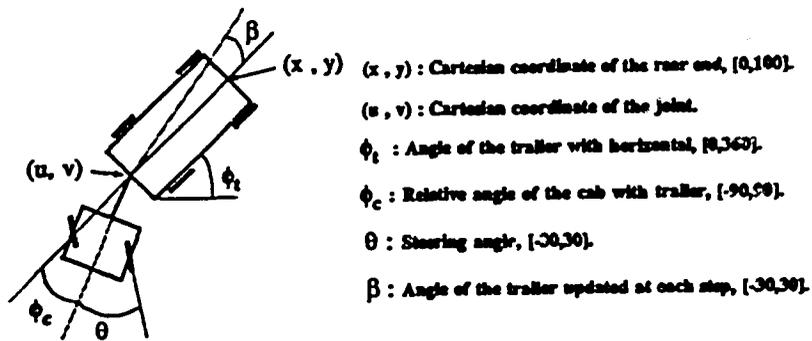


FIGURE 18 Diagram of the simulated truck-and-trailer system.

“jackknife” positions of the cab with respect to the trailer. Positive ϕ_c value indicated that the cab resided on the left-hand side of the trailer. Negative value indicated that it resided on the right-hand side. Figure 18 shows a positive angle value of ϕ_c .

Fuzzy variables x , ϕ_t , and ϕ_c defined the input variables. Fuzzy variable β defined the output variable. β measured the angle that we needed to update the trailer at each iteration. We computed the steering-angle output θ with the following geometric relationship. With the output β value computed, the trailer position (x, y) moved to the new position (x', y') :

$$x' = x + r \cos(\phi_t + \beta), \quad (9)$$

$$y' = y + r \sin(\phi_t + \beta), \quad (10)$$

where r denotes a fixed backing distance. Then the joint of the cab and the trailer (u, v) moved to the new position (u', v') :

$$u' = x' - l \cos(\phi_t + \beta), \quad (11)$$

$$v' = y' - l \sin(\phi_t + \beta), \quad (12)$$

where l denotes the trailer length. We updated the directional vector $(dirU, dirV)$, which defined the cab angle, by

$$dirU' = dirU + \Delta u, \quad (13)$$

$$dirV' = dirV + \Delta v, \quad (14)$$

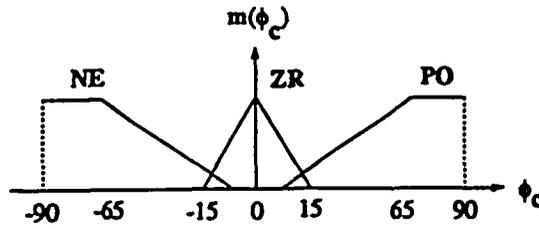


FIGURE 19 Membership graphs of the three fuzzy-set values of fuzzy variable ϕ_c .

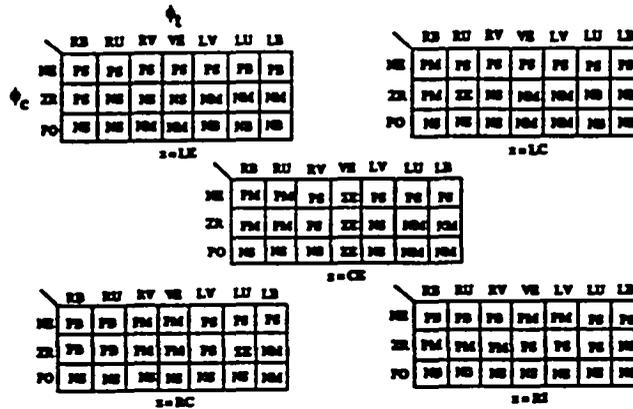


FIGURE 20 FAM bank of the fuzzy truck-and-trailer control system.

where $\Delta u = u' - u$, and $\Delta v = v' - v$. The new directional vector ($dirU'$, $dirV'$) defines the new cab angle ϕ'_c . Then we obtain the steering angle value as $\theta = \phi'_{c,h} - \phi_{c,h}$, where $\phi_{c,h}$ denotes the cab angle with the horizontal. We chose the same fuzzy-set values and membership functions for β as we chose for θ . β ranged from -30° to 30° . We chose the fuzzy-set values of ϕ_c as *NE*, *ZR* and *PO* as in Figure 19.

Figure 20 displays the 5 FAM-rule matrices in the FAM bank of the fuzzy truck-and-trailer system. In Figure 20 we fixed the fuzzy variable x as *LE*, *LC*, *CE*, *RC*, and *RI*. There were 735 ($7 \times 5 \times 7 \times 3$) possible FAM rules and only 105 actual FAM rules.

Figure 21 shows typical backing-up trajectories of the fuzzy truck-and-trailer control

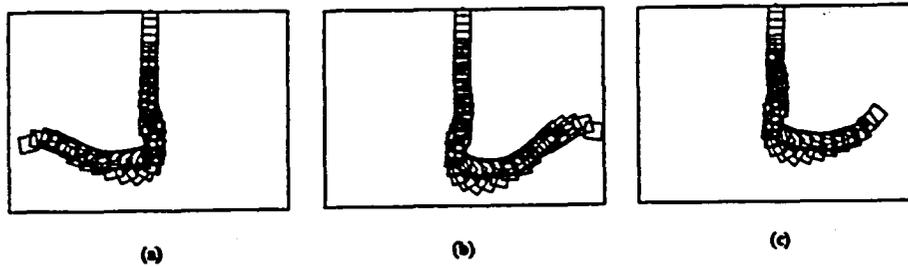


FIGURE 21 Sample truck-and-trailer trajectories from the fuzzy controller for initial positions (x, y, ϕ_t, ϕ_c) : (a) $(25, 30, -20, 30)$, (b) $(80, 30, 210, -40)$, and (c) $(70, 30, 200, 30)$.

system from different initial positions. The truck-and-trailer backed up in different directions depending on the relative position of the cab with respect to the trailer. The fuzzy control systems successfully controlled the truck-and-trailer in jackknife positions.

BP Truck-and-Trailer Control Systems

We added the cab-angle variable ϕ_c as to the backpropagation-trained neural truck controller as an input. The controller network contained 24 hidden neurons with output variable β . The training samples consisted of 5-dimensional space of the form $(x, y, \phi_t, \phi_c, \beta)$. We trained the controller network with 52 training samples from the fuzzy controller: 26 samples for the left half of the plane, 26 samples for the right half of the plane. We used equations (9)–(14) instead of the emulator network. Training required more than 200,000 iterations. Some training sequences did not converge. The BP-trained controller performed well except in a few cases. Figure 22 shows typical backing-up trajectories of the BP truck-and-trailer control system from the same initial positions used in Figure 21.

We performed the same robustness tests for the fuzzy and BP-trained truck-and-trailer controllers as in the trailerless truck case. Figure 23 shows performance errors averaged

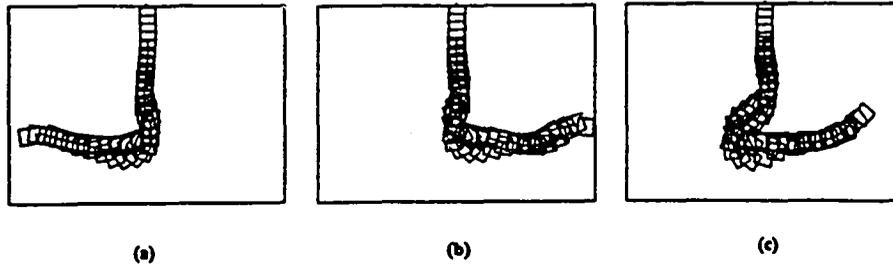


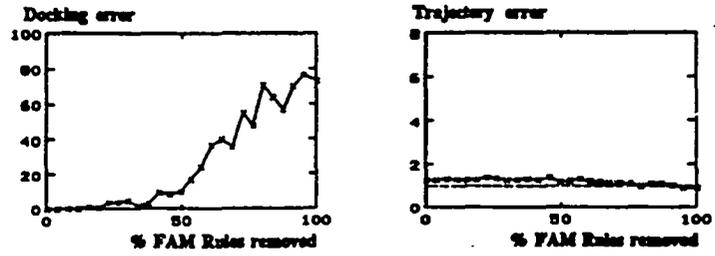
FIGURE 22 Sample truck-and-trailer trajectories of the BP-trained controller for initial positions (x, y, ϕ_t, ϕ_c) : (a) $(25, 30, -20, 30)$, (b) $(80, 30, 210, -40)$, and (c) $(70, 30, 200, 30)$.

over ten typical back-ups from ten different initial positions. These performance graphs resemble closely the performance graphs for the trailerless truck systems in Figure 12.

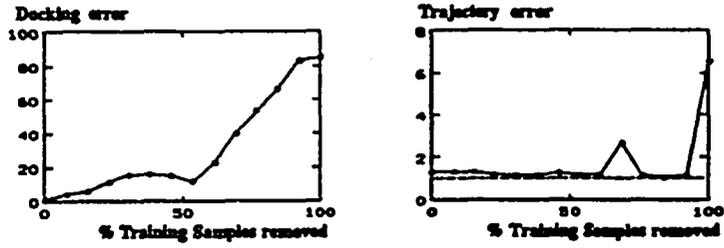
AFAM Truck-and-Trailer Control Systems

We generated 6250 truck-and-trailer data using the original FAM system in Figure 20. We backed up the truck-and-trailer from the same initial positions as in the trailerless truck case. The trailer angle ϕ_t ranged from -60° to 240° , and the cab angle ϕ_c assumed only the three values -45° , 0° , and 45° . The training vectors $(x, \phi_t, \phi_c, \beta)$ defined points in the four-dimensional input-output product-space. We nonuniformly partitioned the product space into FAM cells to allow narrower fuzzy-set values near the steady-state FAM cell.

We used DCL to train the AFAM truck-and-trailer controller. The total number of FAM cells equaled 735 $(7 \times 5 \times 7 \times 3)$. We used 735 synaptic quantization vectors. The DCL algorithm classified the 6250 data into 105 FAM cells. Figure 24 shows the synaptic-vector histogram corresponding to the 105 FAM rules. Figure 25 shows the estimated FAM bank by the DCL algorithm. Figure 26 shows the original and DCL-estimated control surfaces for the fuzzy truck-and-trailer systems.



(a) Fuzzy truck-and-trailer



(b) BP-Neural truck-and-trailer

FIGURE 23 Comparison of robustness of the two truck-and-trailer controllers: (a) Docking and trajectory error of the fuzzy controller, (b) Docking and trajectory error of the BP controller.

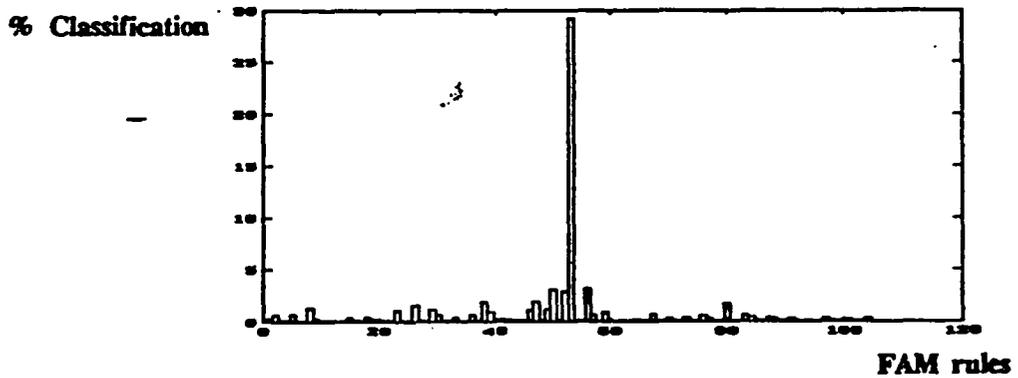


FIGURE 24 Synaptic-vector histogram for the AFAM truck-and-trailer system.

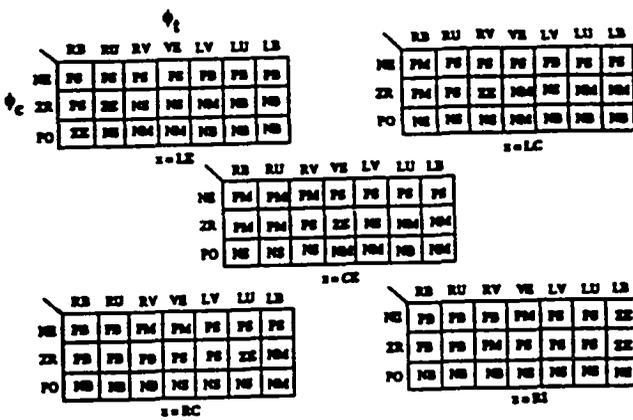
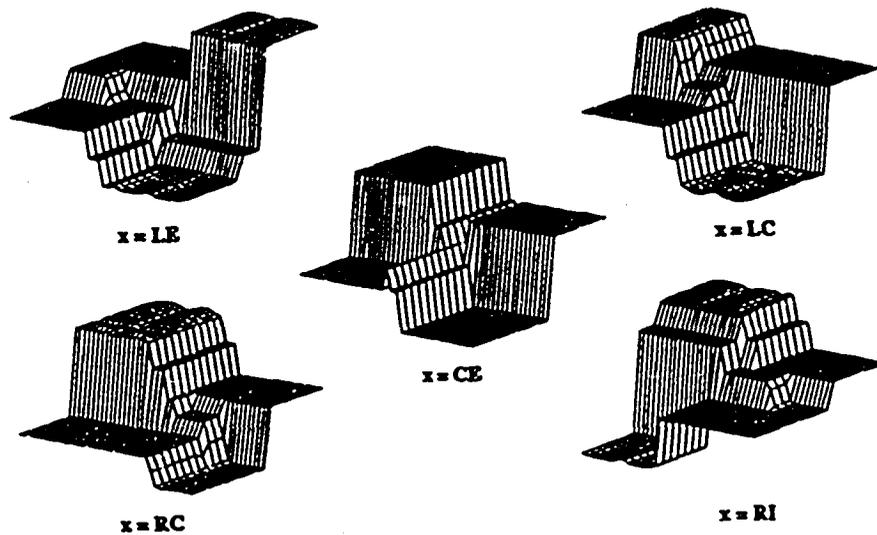
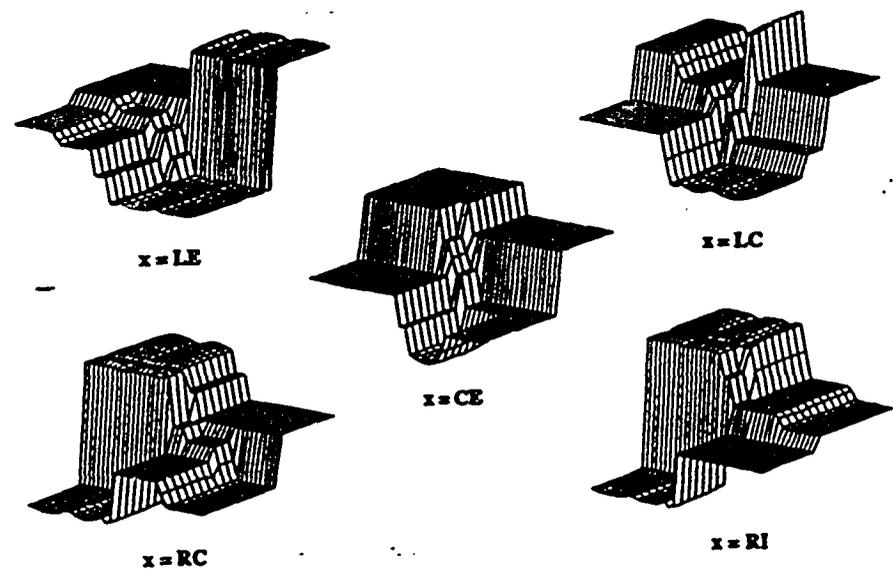


FIGURE 25 DCL-estimated FAM bank for the AFAM truck-and-trailer system.

Figure 27 shows the trajectories of the original FAM and the DCL-estimated AFAM truck-and-trailer controllers. Figure 27a and 27b show the two trajectories from the initial position $(x, y, \phi_t, \phi_e) = (30, 30, 10, 45)$. Figure 27c and 27d show the trajectories from initial position $(60, 30, 210, -60)$. The original FAM and DCL-estimated AFAM systems exhibited comparable truck-and-trailer control performance except in a few cases, where the DCL-estimated AFAM trajectories were irregular.



(a) Original control surfaces for the truck-and-trailer system



(b) DCL-estimated control surfaces for the truck-and-trailer system

FIGURE 28 (a) Original control surface (b) DCL-estimated control surface

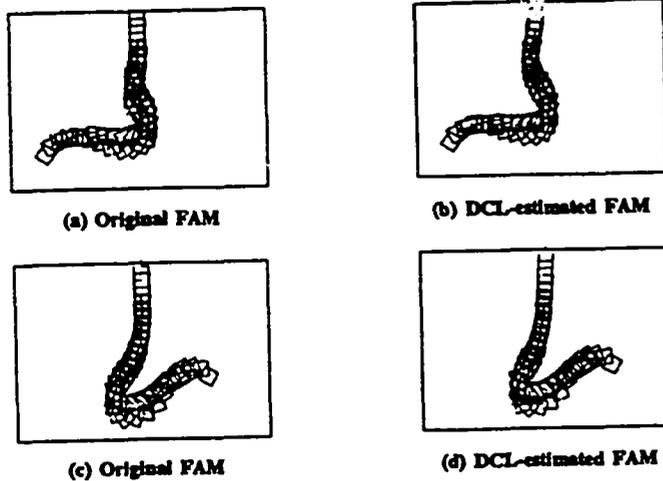


FIGURE 27 Sample truck-and-trailer trajectories from the original and the DCL-estimated FAM systems starting at initial positions $(x, y, \phi_r, \phi_c) = (30, 30, 10, 45)$ and $(60, 30, 210, -60)$.

Conclusion

We quickly engineered fuzzy systems to successfully back up a truck and truck-and-trailer system in a parking lot. We used only common sense and error-nulling intuitions to generate sufficient banks of FAM rules. These systems performed well until we removed over 50 % of the FAM rules. This extreme robustness suggests that, for many estimation and control problems, different fuzzy engineers can rapidly develop prototype fuzzy systems that perform similarly and well.

The speed with which the DCL clustering technique recovers the underlying FAM bank further suggests that we can likewise construct fuzzy systems for more complex, higher-dimensional problems. For these problems we may have access to only incomplete numerical input-output data. Pure neural-network or statistical-process-control approaches may generate systems with comparable performance. But these systems will involve far greater computational effort, will be more difficult to modify, and will not provide a structured

representation of the system's throughput.

Our neural experiments suggests that whenever we model a system with a neural network, for little extra computational cost we can generate a set of structured FAM rules that approximate the neural system's behavior. We can then tune the fuzzy system by refining the FAM-rule bank with fuzzy-engineering rules of thumb and with further training data.

Acknowledgment

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APPENDIX: Product-space Clustering with Differential Competitive Learning

Product-space clustering [Kosko, 1990a] is a form of stochastic adaptive vector quantization. Adaptive vector quantization (AVQ) systems adaptively quantize pattern clusters in R^n . Stochastic competitive learning systems are neural AVQ systems. Neurons compete for the activation induced by randomly sampled patterns. The corresponding synaptic fan-in vectors adaptively quantize the pattern space R^n . The p synaptic vectors m_j define the p columns of the synaptic connection matrix M . M interconnects the n input or linear neurons in the input neuronal field F_X to the p competing nonlinear neurons in the output field F_Y . Figure 28 below illustrates the neural network topology.

Learning algorithms estimate the unknown probability density function $p(x)$, which describes the distribution of patterns in R^n . More synaptic vectors arrive at more probable regions. Where sample vectors x are dense or sparse, synaptic vectors m_j should be dense or sparse. The local count of synaptic vectors then gives a nonparametric estimate of the volume probability $P(V)$ for volume $V \subset R^n$:

$$P(V) = \int_V p(x) dx \quad (15)$$

$$\approx \frac{\text{Number of } m_j \in V}{p} \quad (16)$$

In the extreme case that $V = R^n$, this approximation gives $P(V) = p/p = 1$. For improbable subsets V , $P(V) = 0/p = 0$.

Stochastic Competitive Learning Algorithms

The metaphor of competing neurons reduces to nearest-neighbor classification. The AVQ system compares the current vector random sample $\mathbf{x}(t)$ in Euclidean distance to the p columns of the synaptic connection matrix M , to the p synaptic vectors $\mathbf{m}_1(t), \dots, \mathbf{m}_p(t)$. If the j th synaptic vector $\mathbf{m}_j(t)$ is closest to $\mathbf{x}(t)$, then the j th output neuron "wins" the competition for activation at time t . In practice we sometimes define the nearest N synaptic vectors as winners. Some scaled form of $\mathbf{x}(t) - \mathbf{m}_j(t)$ updates the nearest or "winning" synaptic vectors. "Losers" remain unchanged: $\mathbf{m}_i(t+1) = \mathbf{m}_i(t)$. Competitive synaptic vectors converge to pattern-class centroids exponentially fast [Kosko, 1990b].

The following three-step process describes the competitive AVQ algorithm, where the third step depends on which learning algorithm updates the winning synaptic vectors.

Competitive AVQ Algorithm

1. Initialize synaptic vectors: $\mathbf{m}_i(0) = \mathbf{x}(i)$, $i = 1, \dots, p$.

Sample-dependent initialization avoids many pathologies that can distort nearest-neighbor learning.

2. For random sample $\mathbf{x}(t)$, find the closest or "winning" synaptic vector $\mathbf{m}_j(t)$:

$$\|\mathbf{m}_j(t) - \mathbf{x}(t)\| = \min_i \|\mathbf{m}_i(t) - \mathbf{x}(t)\| \quad , \quad (17)$$

where $\|\mathbf{x}\|^2 = x_1^2 + \dots + x_n^2$ defines the squared Euclidean vector norm of \mathbf{x} . We can define the N synaptic vectors closest to \mathbf{x} as "winners".

3. Update the winning synaptic vector(s) $\mathbf{m}_j(t)$ with an appropriate learning algorithm.

Differential competitive learning (DCL)

Differential competitive "synapses" learn only if the competing "neuron" changes its competitive status [Kosko, 1990c]:

$$\dot{m}_{ij} = \dot{S}_j(y_j) [S_i(x_i) - m_{ij}] \quad , \quad (18)$$

or in vector notation,

$$\dot{\mathbf{m}}_j = \dot{S}_j(y_j) [S(\mathbf{x}) - \mathbf{m}_j] \quad , \quad (19)$$

where $S(\mathbf{x}) = (S_1(x_1), \dots, S_n(x_n))$ and $\mathbf{m}_j = (m_{1j}, \dots, m_{nj})$. m_{ij} denotes the synaptic weight between the i th neuron in input field F_X and the j th neuron in competitive field F_Y . Nonnegative signal functions S_i and S_j transduce the real-valued activations x_i and y_j into bounded monotone nondecreasing signals $S_i(x_i)$ and $S_j(y_j)$. \dot{m}_{ij} and $\dot{S}_j(y_j)$ denote the time derivatives of m_{ij} and $S_j(y_j)$, synaptic and signal velocities. $S_j(y_j)$ measures the competitive status of the j th competing neuron in F_Y . Usually S_j approximates a binary threshold function. For example, S_j may equal a steep binary logistic sigmoid,

$$S_j(y_j) = \frac{1}{1 + e^{-cy_j}} \quad , \quad (20)$$

for some constant $c > 0$. The j th neuron wins the laterally inhibitive competition if $S_j = 1$, loses if $S_j = 0$.

For discrete implementation, we use the DCL algorithm as a stochastic difference equation [Kong, 1991]:

$$\mathbf{m}_j(t+1) = \mathbf{m}_j(t) + c_j \Delta S_j(y_j(t)) [S(\mathbf{x}(t)) - \mathbf{m}_j(t)] \quad \text{if the } j\text{th neuron wins,} \quad (21)$$

$$\mathbf{m}_i(t+1) = \mathbf{m}_i(t) \quad \text{if the } i\text{th neuron loses.} \quad (22)$$

$\Delta S_j(y_j(t))$ denotes the time change of the j th neuron's competition signal $S_j(y_j)$ in the competitive field F_Y :

$$\Delta S_j(y_j(t)) = \text{sgn}[S_j(y_j(t+1)) - S_j(y_j(t))] \quad . \quad (23)$$

We define the signum operator $\text{sgn}(x)$ as

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} . \quad (24)$$

$\{c_t\}$ denotes a slowly decreasing sequence of learning coefficients, such as $c_t = .1(1 - t/2000)$ for 2000 training samples. Stochastic approximation [Huber, 1981] requires a decreasing gain sequence $\{c_t\}$ to suppress random disturbances and to guarantee convergence to local minima of mean-squared performance measures. The learning coefficients should decrease slowly,

$$\sum_{t=1}^{\infty} c_t = \infty , \quad (25)$$

but not too slowly,

$$\sum_{t=1}^{\infty} c_t^2 < \infty . \quad (26)$$

Harmonic-series coefficients, $c_t = 1/t$, satisfy these constraints.

We approximate the competitive signal difference ΔS_j as the activation difference Δy_j :

$$\Delta S_j(y_j(t)) = \text{sgn}[y_j(t+1) - y_j(t)] \quad (27)$$

$$= \Delta y_j(t) . \quad (28)$$

Input neurons in feedforward networks usually behave linearly: $S_i(x_i) = x_i$, or $S(\mathbf{x}(t)) = \mathbf{x}(t)$. Then we update the winning synaptic vector $\mathbf{m}_j(t)$ with

$$\mathbf{m}_j(t+1) = \mathbf{m}_j(t) + c_t \Delta y_j(t) [\mathbf{x}(t) - \mathbf{m}_j(t)] . \quad (29)$$

We update the F_Y neuronal activations y_j with the additive model

$$y_j(t+1) = y_j(t) + \sum_i^n S_i(x_i(t)) m_{ij}(t) + \sum_k^p S_k(y_k(t)) w_{kj} . \quad (30)$$

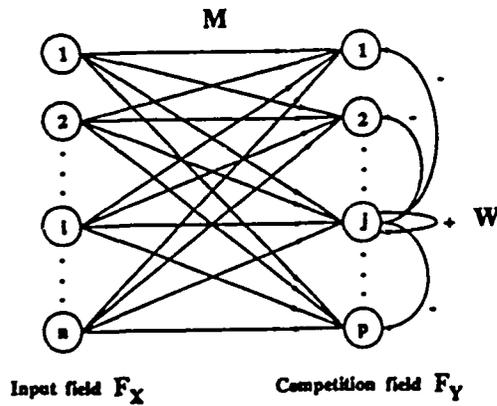


FIGURE 28 Topology of the laterally inhibitive DCL network.

For linear signal functions S_i , the first sum in (30) reduces to an inner product of sample and synaptic vectors:

$$\sum_i^n x_i(t) m_{ij}(t) = \mathbf{x}^T(t) \mathbf{m}_j(t) \quad (31)$$

Then positive learning tends to occur— $\Delta m_{ij} > 0$ —when \mathbf{x} is close to the j th synaptic vector \mathbf{m}_j .

Since a binary threshold function approximates the output signal function $S_k(y_k)$, the second sum in (30) sums over just the winning neurons: $\sum_k w_{kj}$ for all winning neurons y_k .

The $p \times p$ matrix W contains the F_Y within-field synaptic connection strengths. Diagonal elements w_{ii} are positive, off-diagonal elements negative. Winning neurons excite themselves and inhibit all other neurons. Figure 28 shows the connection topology of the laterally inhibitive DCL network.

Product-space clustering

We divided the space $0 \leq x \leq 100$ into five nonuniform intervals $[0, 32.5]$, $[32.5, 47.5]$, $[47.5, 52.5]$, $[52.5, 67.5]$, and $[67.5, 100]$. Each interval represented the five fuzzy-set values LE , LC , CE , RC , and RI . This choice corresponded to the nonoverlapping intervals of the fuzzy membership function graphs $m(x)$ in Figure 3. Similarly, we divided the space $-90 \leq \phi \leq 270$ into seven nonuniform intervals $[-90, 0]$, $[0, 66.5]$, $[66.5, 86]$, $[86, 94]$, $[94, 113.5]$, $[113.5, 182.5]$, and $[182.5, 270]$, which corresponded respectively to RB , RU , RV , VE , LV , LU , and LB . We divided the space $-30 \leq \theta \leq 30$ into seven nonuniform intervals $[-30, -20]$, $[-20, -7.5]$, $[-7.5, -2.5]$, $[-2.5, 2.5]$, $[2.5, 7.5]$, $[7.5, 20]$, and $[20, 30]$, which corresponded to NB , NM , NS , ZE , PS , PM , and PB .

DCL classified each input-output data vector into one of the FAM cells. We added a FAM rule to the FAM bank if the DCL-trained synaptic vector fell in the FAM cell. In case of ties we chose the FAM cell with the most densely clustered data.

For the BP-AFAM generated from the neural control surface in Figure 15, we divided the rectangle $[0, 100] \times [-90, 270]$ into 35 nonuniform squares with the same divisions defined above. Then we added and averaged the control surface values in the square. We added a FAM rule to the FAM bank if the averaged value corresponded to one of the seven FAM cells.

For the truck-and-trailer case, we divided the space $-90 \leq \phi_c \leq 90$ into three intervals $[-90, -12.5]$, $[-12.5, 12.5]$, and $[12.5, 90]$, which corresponded to NE , ZR , and PO . There were 735 FAM cells, and 735 possible FAM rules, of the form $(x, \phi_i, \phi_c; \beta)$.

ADAPTIVE FUZZY SYSTEM FOR TARGET TRACKING

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ABSTRACT

We compared fuzzy and Kalman-filter control systems for realtime target tracking. Both systems performed well, but in the presence of mild process (unmodeled effects) noise the fuzzy system exhibited finer control. We tested the robustness of the fuzzy controller by removing random subsets of fuzzy associations or "rules" and by adding destructive or "sabotage" fuzzy rules to the fuzzy system. We tested the robustness of the Kalman tracking system by increasing the variance of the unmodeled-effects noise process. The fuzzy controller performed well until we removed over 50% of the fuzzy rules. The Kalman controller's performance quickly degraded as the unmodeled-effects variance increased. We used unsupervised neural-network learning to adaptively generate the fuzzy controller's fuzzy-associative-memory structure. The fuzzy systems did not require a mathematical model of how system outputs depended on inputs.

Fuzzy and Math-Model Controllers

Fuzzy controllers differ from classical math-model controllers. Fuzzy controllers do not require a mathematical model of how control outputs functionally depend on control inputs. Fuzzy controllers also differ in the type of uncertainty they represent and how they represent it. The fuzzy approach represents ambiguous or fuzzy system behavior as partial implications or approximate "rules of thumb"—as fuzzy associations (A_i, B_i) .

Fuzzy controllers are fuzzy systems. A finite fuzzy set A is a point [Kosko, 1987] in a unit hypercube $I^n = [0,1]^n$. A fuzzy system $F : I^n \rightarrow I^p$ is a mapping between unit hypercubes. I^n contains all fuzzy subsets of the domain space $X = \{x_1, \dots, x_n\}$. I^n is the fuzzy power set $F(2^X)$ of X . I^p contains all the fuzzy subsets of the range space $Y = \{y_1, \dots, y_p\}$. Element $x_i \in X$ belongs to fuzzy set A to degree $m_A(x_i)$. The 2^n nonfuzzy subsets of X correspond to the 2^n corners of the fuzzy cube I^n . The fuzzy system F maps fuzzy subsets of X to fuzzy subsets of Y . In general, X and Y are continuous not discrete sets.

Math-model controllers usually represent system uncertainty with probability distributions. Probability models describe system behavior with first-order and second-order statistics—with conditional means and covariances. They usually describe unmodeled effects and measurement imperfections with additive "noise" processes.

Mathematical models of the system state and measurement processes facilitate a mean-squared-error analysis of system behavior. In general we cannot accurately articulate such mathematical models. This greatly restricts the range of realworld applications. In practice we often use linear or quasi-linear (Markov) mathematical models.

Mathematical state and measurement models also make it difficult to add non-mathematical knowledge to the system. Experts may articulate such knowledge, or neural networks may adaptively infer it from sample data. In practice, once we have articulated the math model, we use human expertise only to estimate the initial state and covariance conditions.

Fuzzy controllers consist of a bank of fuzzy associative memory (FAM) "rules" or associations (A_i, B_i) operating in parallel, and operating to different degrees. Each FAM

rule is a set-level implication. It represents ambiguous expert knowledge or learned input-output transformations. A FAM rule can also summarize the behavior of a specific mathematical model. The system nonlinearly transforms exact or fuzzy state inputs to a fuzzy set output. This output fuzzy set is usually "defuzzified" with a centroid operation to generate an exact numerical output. In principle the system can use the entire fuzzy distribution as the output. We can easily construct, process, and modify the FAM bank of FAM rules in software or in digital VLSI circuitry.

Fuzzy controllers require that we articulate or estimate the FAM rules. The fuzzy-set framework provides more expressiveness than, say, traditional expert-system approaches, which encode bivalent propositional associations. But the fuzzy framework does not eliminate the burden of knowledge acquisition. We can use neural network systems to estimate the FAM rules. But neural systems also require an accurate (statistically representative) set of articulated input-output numerical samples. Below we use unsupervised competitive learning to adaptively generate target-tracking FAM rules.

Experts can hedge their system descriptions with fuzzy concepts. Although fuzzy controllers are numerical systems, experts can contribute their knowledge in natural language. This is especially important in complex problem domains, such as economics, medicine, and history, where we may not know how to mathematically model system behavior.

Below we compare a fuzzy controller with a Kalman-filter controller for realtime target tracking. This problem admits a simple and reasonably accurate mathematical description of its state and measurement processes. We chose the Kalman filter as a benchmark because of its many optimal linear-systems properties. We wanted to see whether this "optimal" controller remains optimal when compared with a computationally lighter fuzzy controller in different uncertainty environments.

We indirectly compared the sensitivity of the two controllers by varying their system uncertainties. We randomly removed FAM rules from the fuzzy controller. We also added "sabotage" FAM rules to the controller. Both techniques modeled less-structured control environments. For the Kalman filter, we varied the noise variance of the unmodeled-effects noise process.

Both systems performed well for mildly uncertain target environments. They degraded

differently as the system uncertainty increases. The fuzzy controller's performance degraded when we removed more than half the FAM rules. The Kalman-filter controller's performance quickly degraded when the additive state noise process increased in variance.

Realtime Target Tracking

A target tracking system maps azimuth-elevation inputs to motor control outputs. The nominal target moves through azimuth-elevation space. Two motors adjust the position of a platform to continuously point at the target.

The platform can be any directional device that accurately points at the target. The device may be a laser, video camera, or high-gain antenna. We assume we have available a radar or other device that can detect the direction from the platform to the target.

The radar sends azimuth and elevation coordinates to the tracking system at the end of each time interval. We calculate the current *error* e_k in platform position and *change in error* \dot{e}_k . Then a fuzzy or Kalman-filter controller determines the control outputs for the motors, one each for azimuth and elevation. The control outputs reposition the platform.

We can independently control movement along azimuth and elevation if we apply the same algorithm twice. This reduces the problem to matching the target's position and velocity in only one dimension.

Figure 1 shows a block diagram of the target tracking system. The controller's output v_k gives the estimated change in angle required during the next time interval. In principle a hardware system must transduce the angular velocity v_k into a voltage or current.

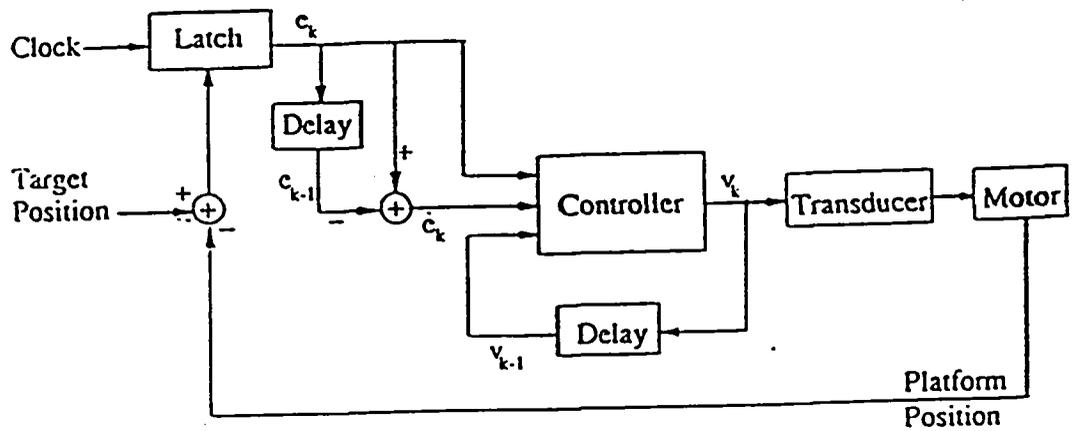


FIGURE 1 Target tracking system.

FUZZY CONTROLLER

We restrict the output angular velocity v_k of the fuzzy controller to the interval $[-6, 6]$. So we must insert a gain element before the voltage transduction. This gain must equal one-sixth the maximum angle through which the platform can turn in one time interval. Similarly, the position error e_k must be scaled so that 6 equals the maximum error. The product of this scale factor and the output gain provides a design parameter—the “gain” of the fuzzy controller.

The fuzzy controller uses heuristic control set-level “rules” or *fuzzy associative memory* (FAM) associations based on quantized values of e_k , \dot{e}_k , and v_{k-1} . We define seven fuzzy levels by the following library of fuzzy-set values of the fuzzy variables e_k , \dot{e}_k , and v_{k-1} :

- LN* : Large Negative
- MN* : Medium Negative
- SN* : Small Negative
- ZE* : Zero
- SP* : Small Positive
- MP* : Medium Positive
- LP* : Large Positive

We do not quantize inputs in the classical sense that we assign each input to exactly one output level. Instead, each linguistic value equals as a fuzzy set that overlaps with adjacent fuzzy sets. The fuzzy controller uses trapezoidal fuzzy-set values, as Figure 2 shows. The lengths of the upper and lower bases provide design parameters that we must calibrate for satisfactory performance. A good rule of thumb is *adjacent fuzzy-set values should overlap approximately 25 percent*. Below we discuss examples of calibrated and uncalibrated systems. The fuzzy controller attained its best performance with upper and lower bases of 1.2 and 3.9—26.2% overlap. Different target scenarios may require more or less overlap.

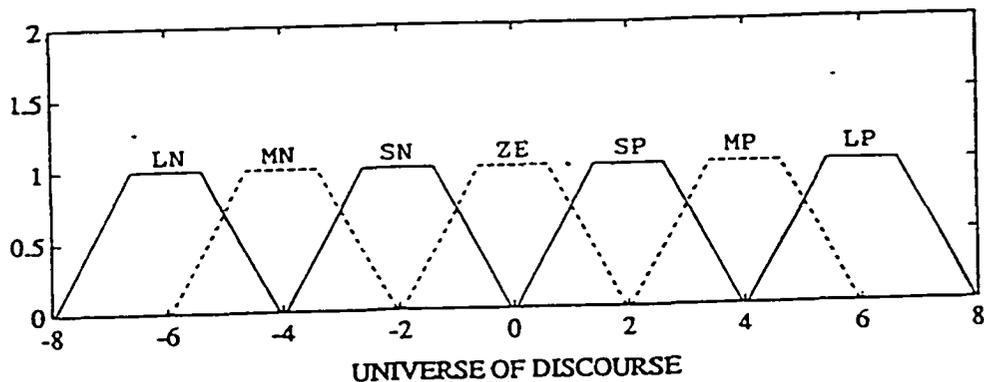


FIGURE 2 Library of overlapping fuzzy-set values defined on a universe

of discourse.

We assign each system input to a fit vector of length 7, where the i th fit, or fuzzy unit [Kosko, 1986], equals the value of the i th fuzzy set at the input value. In other words, the i th fit measures the degree to which the input belongs to the i th fuzzy-set value. For instance, we apply the input values 1, -4, and 3.8 to the seven fuzzy sets in the library to obtain the fit vectors

$$\begin{aligned} 1 &\rightarrow (0 \ 0 \ 0 \ .7 \ .7 \ 0 \ 0) \ , \\ -4 &\rightarrow (0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0) \ , \\ 3.8 &\rightarrow (0 \ 0 \ 0 \ 0 \ .1 \ 1 \ 0) \ . \end{aligned}$$

We determine these fit values above by convolving a Dirac delta function centered at the input value with each of the 7 fuzzy sets:

$$m_{SP}(3.8) = \delta(y - 3.8) * m_{SP}(y) = .1 \ . \quad (1)$$

If we use a discretized universe of discourse, then we use a Kronecker delta function instead. Equivalently, for the discrete case n -dimensional universe of discourse $X = \{x_1, \dots, x_n\}$, a control input corresponds to a bit (binary unit) vector B of length n . A single 1 element in the i th slot represents the "crisp" input value x_i . Similarly, we represent the k th library fuzzy set by an n -dimensional fit vector A_k that contains samples of the fuzzy set at the n discrete points within the universe of discourse X . The degree to which the crisp input x_i activates each fuzzy set equals the inner product $B \cdot A_k$ of the bit vector B and the corresponding fit vector A_k .

We formulate control FAM rules by associating output fuzzy sets with input fuzzy sets. The antecedent of each FAM rule conjoins e_k , \dot{e}_k , and v_{k-1} fuzzy-set values. For example,

$$\text{IF } e_k = MP \text{ AND } \dot{e}_k = SN \text{ AND } v_{k-1} = ZE, \text{ THEN } v_k = SP.$$

We abbreviate this as $(MP, SN, ZE; SP)$.

The scalar activation value w_i of the i th FAM rule's consequent equals the *minimum* of the three antecedent conjuncts' values. If alternatively we combine the antecedents disjunctively with *OR*, the activation degree of the consequent would equal the *maximum* of the three antecedent disjuncts' values. In the following example, $m_A(e_k)$ denotes the degree to which e_k belongs to the fuzzy set A :

		LN	MN	SN	ZE	SP	MP	LP
$e_k = 2.6$	→	(0	0	0	0	1	.4	0)
$\dot{e}_k = -2.0$	→	(0	0	1	0	0	0	0)
$v_{k-1} = 1.8$	→	(0	0	0	.1	1	0	0)
$m_{MP}(e_k) = .4$								
$m_{SN}(\dot{e}_k) = 1$								
$m_{ZE}(v_{k-1}) = .1$								
$w_i = \min(.4, 1, .1) = .1$								

So the system activates the consequent fuzzy set SP to degree $w_i = .1$.

The output fuzzy set's shape depends on the FAM-rule encoding scheme used. With *correlation-minimum* encoding, we clip the consequent fuzzy set L_i in the library of fuzzy-set values to degree w_i with pointwise minimum:

$$m_{O_i}(y) = \min(w_i, m_{L_i}(y)) \quad (2)$$

With *correlation-product* encoding, we multiply L_i by w_i :

$$m_{O_i}(y) = w_i m_{L_i}(y) \quad (3)$$

or equivalently,

$$O_i = w_i L_i \quad (4)$$

Figure 3 illustrates how both inference procedures transform L_i to scaled output O_i . For

the example above, correlation-product inference gives output fuzzy set $O_i = .1SP$, where $L_i = SP$ denotes the fuzzy set of small but positive angular velocity values.

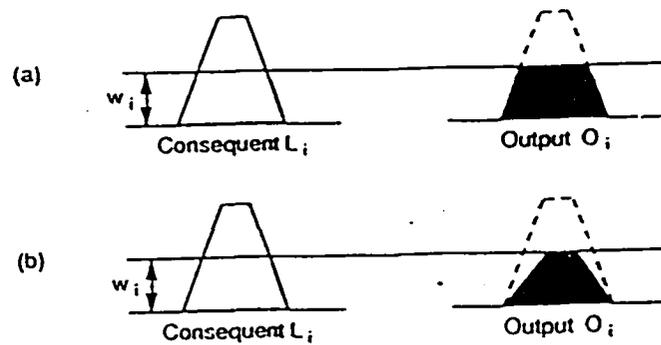


FIGURE 3 FAM inference procedure depends on FAM rule encoding procedure: (a) correlation-minimum encoding, (b) correlation-product encoding.

The fuzzy system activates each FAM rule consequent set to a different degree. For the i th FAM rule this yields the output fuzzy set O_i . The system then sums the O_i to form the combined output fuzzy set O :

$$O = \sum_{i=1}^N O_i, \quad (5)$$

or equivalently,

$$m_O(y) = \sum_{i=1}^N m_{O_i}(y). \quad (6)$$

The control output v_k equals the *fuzzy centroid* of O :

$$v_k = \frac{\int y m_O(y) dy}{\int m_O(y) dy}, \quad (7)$$

where the limits of integration correspond to the entire universe of discourse Y of angular velocity values. Figure 4 shows an example of correlation-product inference for two FAM rules followed by centroid defuzzification of the combined output fuzzy set.

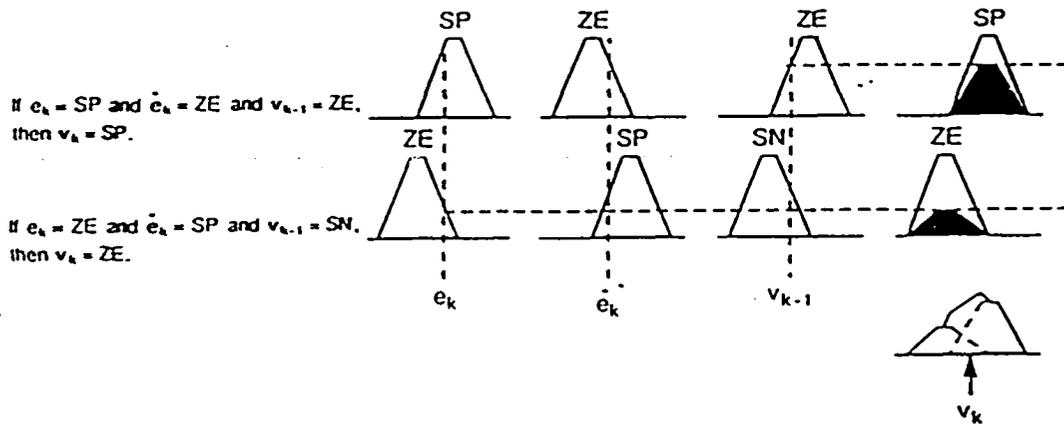


FIGURE 4 Correlation-product inferences followed by centroid defuzzification. FAM rule antecedents combined with AND use the *minimum* fit value to activate consequents. Those combined with OR use the *maximum* fit value.

To reduce computations, we can discretize the output universe of discourse Y to p values, $Y = \{y_1, \dots, y_p\}$, which gives the discrete fuzzy centroid

$$v_k = \frac{\sum_{j=1}^p y_j m_o(y_j)}{\sum_{j=1}^p m_o(y_j)} \quad (8)$$

Fuzzy Centroid Computation

We now develop two discrete methods for computing the fuzzy centroid (7). Theorem 1 states that we can compute the global centroid v_k from local FAM-rule centroids. Theorem 2 states that v_k can be computed from only 7 sample points if all the fuzzy sets

are symmetric and unimodal (in the broad sense of a trapezoid peak), though otherwise arbitrary. Both results reduce computation and favor digital implementation.

Theorem 1: If correlation-product inference determines the output fuzzy sets, then we can compute the global centroid v_k from local FAM-rule centroids:

$$v_k = \frac{\sum_{i=1}^N w_i c_i I_i}{\sum_{i=1}^N w_i I_i} \quad (9)$$

Proof. The consequent fuzzy set of each FAM rule equals one of the fuzzy-set values shown in Figure 2. We assume each fuzzy set includes at least one unity value, $m_A(x) = 1$. Define I_i and c_i as the respective area and centroid of the i th FAM rule's consequent set L_i :

$$I_i = \int m_{L_i}(y) dy \quad (10)$$

$$c_i = \frac{\int y m_{L_i}(y) dy}{\int m_{L_i}(y) dy}$$

$$= \frac{\int y m_{L_i}(y) dy}{I_i}$$

substituting from (10). Hence

$$\int y m_{L_i}(y) dy = c_i I_i \quad (11)$$

Using (3), the result of correlation-product inference, we get

$$\int y m_{O_i}(y) dy = \int y w_i m_{L_i}(y) dy$$

$$\begin{aligned}
&= w_i \int y m_{L_i}(y) dy \\
&= w_i c_i I_i \quad , \quad (12)
\end{aligned}$$

substituting from (11). Similarly,

$$\begin{aligned}
\int m_{O_i}(y) dy &= \int w_i m_{L_i}(y) dy \quad .. \\
&= w_i I_i \quad , \quad (13)
\end{aligned}$$

substituting from (10).

We can use (12) and (13) to derive a discrete expression equivalent to (7):

$$\begin{aligned}
\int y m_O(y) dy &= \int y \left[\sum_{i=1}^N m_{O_i}(y) \right] dy \quad \text{substituting from (6)} \quad , \\
&= \sum_i \int y m_{O_i}(y) dy \\
&= \sum_i w_i c_i I_i \quad , \quad (14)
\end{aligned}$$

from (12). Similarly,

$$\begin{aligned}
\int m_O(y) dy &= \int \sum_{i=1}^N m_{O_i}(y) dy \\
&= \sum_i \int m_{O_i}(y) dy \\
&= \sum_i w_i I_i \quad , \quad (15)
\end{aligned}$$

from (13). Substituting (14) and (15) into (7), we derive a new form for the centroid:

$$v_k = \frac{\sum_{i=1}^N w_i c_i I_i}{\sum_{i=1}^N w_i I_i} , \quad (16)$$

which is equivalent to (9). Each summand in each summation of (16) depends on only a single FAM rule. So we can compute the global output centroid from local FAM-rule centroids. Q.E.D.

Theorem 2: If the 7 library fuzzy sets are symmetric and unimodal (in the trapezoidal sense) and we use correlation-product inference, then we can compute the centroid v_k from only 7 samples of the combined output fuzzy set O :

$$v_k = \frac{\sum_{j=1}^7 m_O(y_j) y_j J_j}{\sum_{j=1}^7 m_O(y_j) J_j} . \quad (17)$$

The 7 sample points are the centroids of the output fuzzy-set values.

Proof. Define \bar{O}_i as a fit vector of length 7, where the fit value corresponding to the i th consequent set has the value w_i , and the other entries equal zero. If all the fuzzy sets are symmetric and unimodal, then the j th fit value of \bar{O}_i is a sample of m_{O_i} at the centroid of the j th fuzzy set. The combined output fit vector is

$$\bar{O} = \sum_{i=1}^N \bar{O}_i . . . \quad (18)$$

Since

$$m_O(y) = \sum_{i=1}^N m_{O_i}(y) ,$$

the j th fit value of \bar{O} is a sample of m_O at the centroid of the j th fuzzy set. Equivalently, the j th fit value of \bar{O} equals the sum of the output activations w_i from the FAM rules with

consequent fuzzy sets equal to the j th library fuzzy-set value.

Define the reduced universe of discourse as $Y = \{y_1, \dots, y_7\}$ such that y_j equals the centroid of the j th output fuzzy set. In vector form

$$\begin{aligned} Y &= (y_1, \dots, y_7) \\ &= (-6, -4, -2, 0, 2, 4, 6) \end{aligned}$$

for the library of fuzzy sets in Figure 2. Also define the diagonal matrix

$$J = \text{diag}(J_1, \dots, J_7) \quad (19)$$

where J_j denotes the area of the j th fuzzy-set value. If the i th FAM rule's consequent fuzzy set equals the j th fuzzy-set value, then the j th fit value of \bar{O} increases by w_i , $c_i = y_j$, and $I_i = J_j$. So

$$\bar{O}JY^T = \sum_{j=1}^7 m_O(y_j) y_j J_j = \sum_{i=1}^N w_i c_i I_i \quad (20)$$

Also,

$$\bar{O}J1^T = \sum_{j=1}^7 m_O(y_j) J_j = \sum_{i=1}^N w_i I_i \quad (21)$$

where $1 = (1, \dots, 1)$. Substituting (20) and (21) into (16) gives

$$v_k = \frac{\sum_{j=1}^7 m_O(y_j) y_j J_j}{\sum_{j=1}^7 m_O(y_j) J_j} \quad (22)$$

which is equivalent to (17). Therefore, (22) gives a simpler, but equivalent form of the centroid (7) if all the fuzzy sets are symmetric and unimodal, and if we use correlation-product inference to form the output fuzzy sets O_i . Q.E.D.

Consider a fuzzy controller with the fuzzy sets defined in Figure 2, and 7 FAM rules with the following outputs:

i	w_i	Consequent
1	0.0	MP
2	0.2	SP
3	1.0	ZE
4	0.4	SN
5	0.1	SP
6	0.8	ZE
7	0.6	SN

Figure 5 shows the combined output fuzzy set O , with the SN , ZE , and SP components displayed with dotted lines. Using (7) we get a velocity output of -0.452 . Alternatively, the combined output fit vector \bar{O} equals $(0, 0, 1.0, 1.8, 0.3, 0, 0)$. From (22) we get

$$v_k = \frac{-2 \times 1 + 0 \times 1.8 + 2 \times 0.3}{1 + 1.8 + 0.3} = -0.452 .$$

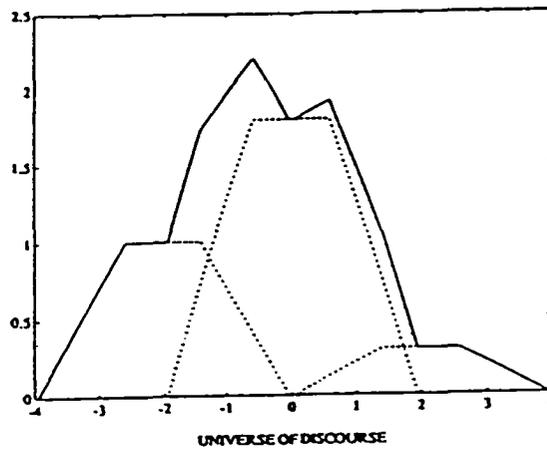


FIGURE 5 Output fuzzy set O .

Fuzzy Controller Implementation

A FAM bank or "rulebase" of FAM rules defines the fuzzy controller. Each FAM rule associates one consequent fuzzy set with three antecedent fuzzy-set conjuncts.

Suppose the i th FAM rule is $(MP, SN, ZE; SP)$. Suppose the inputs at time k are $e_k = 2.6$, $\dot{e}_k = -2.0$, and $v_{k-1} = 1.8$. Then

$$\begin{aligned}
 w_i &= \min(m_{MP}(e_k), m_{SN}(\dot{e}_k), m_{ZE}(v_{k-1})) \\
 &= \min(.4, 1, .1) \\
 &= .1
 \end{aligned}$$

If all the fuzzy sets have the same shape, then they correspond to shifted versions of a

single fuzzy set Z_E :

$$m_{SF}(y) = m_{ze}(y - 2) .$$

Define e^i , \hat{e}^i , and v^i as the centroids of the corresponding antecedent fuzzy sets in the example above. So $e^i = 4$, $\hat{e}^i = -2$, and $v^i = 0$. Then the output activation equals

$$\begin{aligned} w_i &= \min(m_{ZE}(e_k - e^i), m_{ZE}(\hat{e}_k - \hat{e}^i), m_{ZE}(v_{k-1} - v^i)) \\ &= \min(m_{ZE}(-1.4), m_{ZE}(0), m_{ZE}(1.8)) \\ &= \min(.4, 1, .1) \\ &= .1 , \end{aligned}$$

as computed above. Figure 6 schematizes such a FAM rule when presented with crisp inputs.

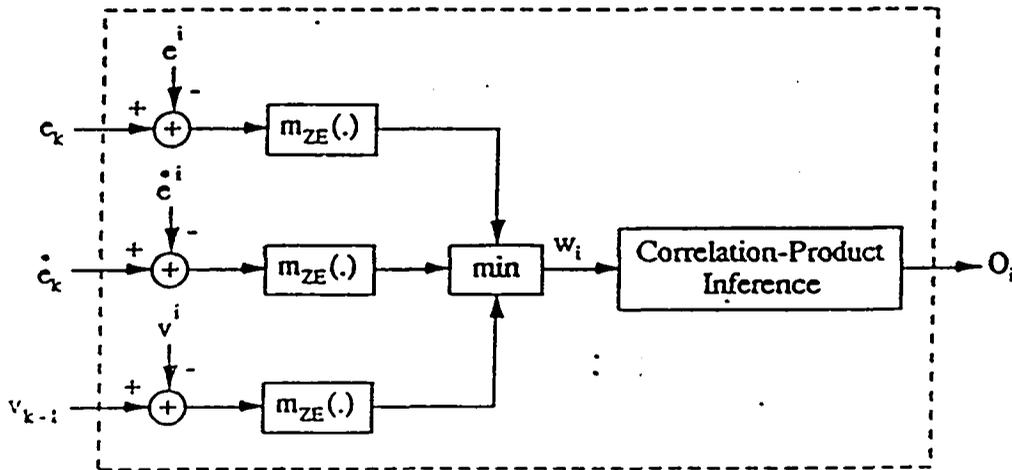


FIGURE 6 Algorithmic structure of a FAM rule for the special case of identically-shaped fuzzy sets and correlation-product inference.

The output fuzzy set O_i in Figure 6 equals the fuzzy set ZE scaled by w_i and shifted by c_i :

$$m_{O_i}(y) = w_i m_{ZE}(y - c_i) \quad (23)$$

Figure 7 illustrates O_i .

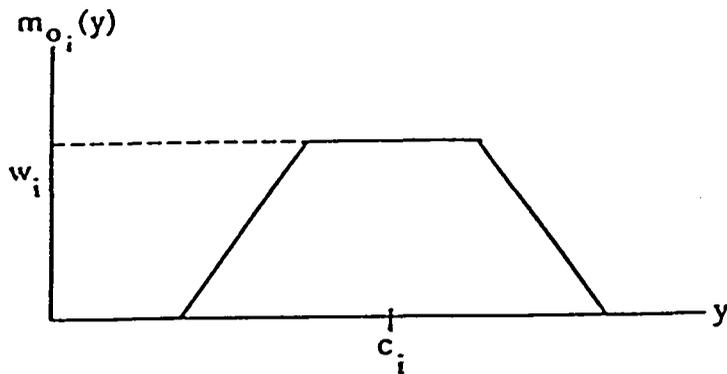


FIGURE 7 Trapezoidal output fuzzy set O_i .

The fuzzy control system activates a bank of FAM rules operated in parallel, as shown in Figure 8. The system sums the output fuzzy sets to form the total output set O , which the system converts to a "defuzzified" scalar output by computing its fuzzy centroid.

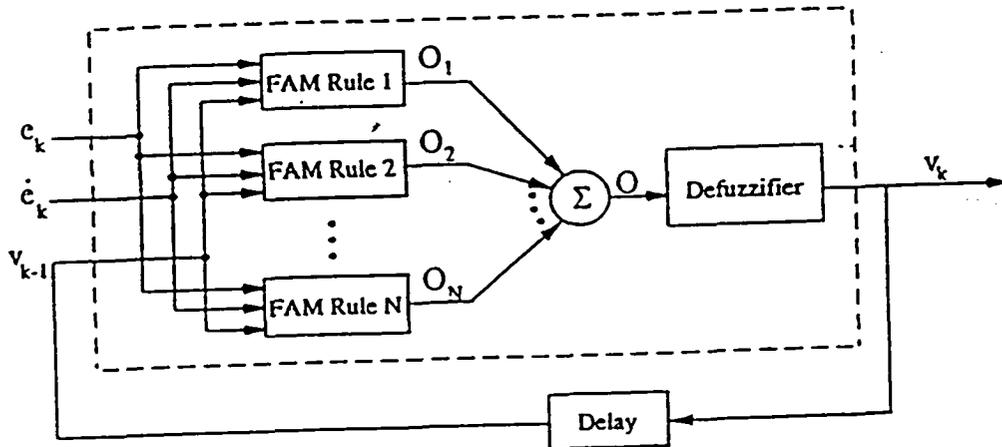


FIGURE 8 Fuzzy control system as a parallel FAM bank with centroidal output.

KALMAN FILTER CONTROLLER

We designed a one-dimensional Kalman filter to act as an alternative controller. The *state and measurement* equations take the general form

$$\begin{aligned} x_{k+1} &= \Phi_{k+1,k} x_k + \Gamma_{k+1,k} \dot{w}_k + \Psi_{k+1,k} u_k, \\ z_k &= H_k x_k + V_k, \end{aligned} \quad (24)$$

where V_k denotes Gaussian white noise with covariance matrix R_k . If V_k is colored noise or if $R_k = 0$, then the filtering-error covariance matrix $P_{k|k}$ becomes singular. The state x_k and the measurements z_k are jointly Gaussian. Mendel [1987] gives details of this model.

Assume the following one-dimensional model:

$$\begin{aligned} \Phi_{k+1,k} &= \Gamma_{k+1,k} = \Psi_{k+1,k} = H_k = 1 \text{ for all } k, \\ u_k &= e_k + \dot{e}_k . \end{aligned} \tag{25}$$

Let x_{k+1} denote the output velocity required at time k to exactly lock onto the target at time $k+1$. So the controller output at time k equals the "predictive" estimate $\hat{x}_{k+1|k} = v_k$. Note that

$$\begin{aligned} e_k &= x_k - \hat{x}_{k|k-1} \\ &= \tilde{x}_{k|k-1} , \\ \dot{e}_k &= e_k - e_{k-1} . \end{aligned}$$

Substituting (25) into (24), we get the new state equation

$$x_{k+1} = x_k + e_k + \dot{e}_k + w_k , \tag{26}$$

where w_k denotes white noise that models target acceleration or other unmodeled effects. The new measurement equation is

$$\begin{aligned} z_k &= x_k + V_k \\ &= \hat{x}_{k|k-1} + \tilde{x}_{k|k-1} + V_k \\ &= \hat{x}_{k|k-1} + V'_k . \end{aligned} \tag{27}$$

Since we assume $\tilde{x}_{k|k-1}$ and V_k are uncorrelated, the variance of V'_k is

$$\begin{aligned} R'_k &= E[V_k'^2] \\ &= E[\tilde{x}_{k|k-1}^2] + E[V_k^2] \end{aligned} \tag{28}$$

$$= P_{k|k-1} + R_k .$$

The general form of the recursive Kalman filter equations is

$$\begin{aligned} \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k [z_k - H_k \hat{x}_{k|k-1}] , \\ K_k &= P_{k|k-1} H_k^T [H_k P_{k|k-1} H_k^T + R_k]^{-1} , \\ \hat{x}_{k+1|k} &= \Phi_{k+1,k} \hat{x}_{k|k} + \Psi_{k+1,k} u_k , \\ P_{k|k-1} &= \Phi_{k,k-1} P_{k-1|k-1} \Phi_{k,k-1}^T + \Gamma_{k,k-1} Q_{k-1} \Gamma_{k,k-1}^T , \\ P_{k|k} &= [I - K_k H_k] P_{k|k-1} , \end{aligned} \tag{29}$$

where $Q_k = \text{Var}(w_k) = E[w_k w_k^T]$. Substituting (25), (27), (28) and the definition of v_k into (29), we get the following one-dimensional Kalman filter:

$$\begin{aligned} \hat{x}_{k|k} &= v_{k-1} + K_k V_k' , \\ K_k &= \frac{P_{k|k-1}}{R_k'} , \\ v_k &= \hat{x}_{k|k} + e_k + \dot{e}_k , \\ P_{k|k-1} &= P_{k-1|k-1} + Q_{k-1} , \\ P_{k|k} &= [1 - K_k] P_{k|k-1} . \end{aligned} \tag{30}$$

Unlike the fuzzy controller, this Kalman filter does not automatically restrict the output v_k to a usable range. We must apply a threshold immediately after the controller. To remain consistent with the fuzzy controller, we set the following thresholds:

$$|v_k| \leq 9 \text{ degrees azimuth} ,$$

$$|v_k| \leq 4.5 \text{ degrees elevation} .$$

Fuzzy and Kalman Filter Control Surfaces

Each control system maps inputs to outputs. Geometrically, these input-output transformations define *control surfaces*. The control surfaces are sheets in the input space (since the output velocity v_k is a scalar). Three inputs and one output give rise to a four-dimensional control surface, which we cannot plot. Instead, for each controller we can plot a family of three-dimensional control surfaces indexed by constant values of the fourth variable, the error e_k , say. Then each control surface corresponds to a different value of the error e_k .

The fuzzy control surface characterizes the fuzzy system's fuzzy-set value definitions and its bank of FAM rules. Different sets of FAM rules yield different fuzzy controllers, and hence different control surfaces. Figure 9 shows a cross section of the FAM bank when $e_k = ZE$. Each entry in this linguistic matrix represents one FAM rule with $e_k = ZE$ as the first antecedent term.

		v_{k-1}						
		LN	MN	SN	ZE	SP	MP	LP
\dot{e}_k	LN	LN	LN	LN	LN	MN	SN	ZE
	MN	LN	LN	LN	MN	SN	ZE	SP
	SN	LN	LN	MN	SN	ZE	SP	MP
	ZE	LN	MN	SN	ZE	SP	MP	LP
	SP	MN	SN	ZE	SP	MP	LP	LP
	MP	SN	ZE	SP	MP	LP	LP	LP
	LP	ZE	SP	MP	LP	LP	LP	LP

FIGURE 9 $e_k = ZE$ cross section of the fuzzy control system's FAM bank. Each entry represents one FAM rule with $e_k = ZE$ as the first antecedent term.

The shaded FAM rule is "IF $e_k = ZE$ AND $\dot{e}_k = SP$ AND $v_{k-1} = SN$, THEN $v_k = ZE$," abbreviated as $(ZE, SP, SN; ZE)$. Note the ordinal anti-symmetry of this FAM-bank matrix. The six other cross-section FAM-bank matrices are similar. We can eliminate many FAM rule entries without greatly perturbing the fuzzy controller's behavior.

The entire FAM bank—including cross sections for e_k equal to each of the seven fuzzy-set values LN , MN , SN , ZE , SP , MP , and LP —determines how the system maps input fuzzy sets to output fuzzy sets. The fuzzy set membership functions shown in Figure 2 determine the degree to which each crisp input value belongs to each fuzzy-set value. So both the fuzzy-set value definitions and the FAM bank determine the defuzzified output v_k for any set of crisp input values e_k , \dot{e}_k , and v_{k-1} .

Figure 10 shows the control surface of the fuzzy controller for $e_k = 0$. We plotted the control output v_k against \dot{e}_k and v_{k-1} . Since we use the same algorithm for tracking in azimuth and elevation, the control surfaces for the two dimensions differ in scale only by a factor of two.

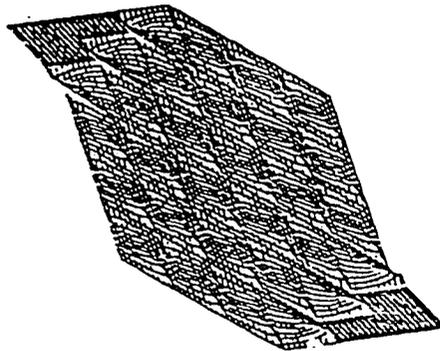


FIGURE 10 Control surface of the fuzzy controller for constant error $e_k = 0$. We plotted the control output v_k against \dot{e}_k and v_{k-1} along the respective west and south borders.

The Kalman filter has a random control surface that depends on a time-varying pa-

parameter. From (30) we see that

$$\begin{aligned}v_k &= \hat{x}_{k|k} + e_k + \dot{c}_k, \\ \hat{x}_{k|k} &= v_{k-1} + K_k V'_k,\end{aligned}$$

where V'_k denotes white noise with variance given by (28). Combining these two equations gives the equation for the random control surface:

$$v_k = v_{k-1} + e_k + \dot{c}_k + K_k V'_k. \quad (31)$$

At time k the noise term $K_k V'_k$ has variance

$$\begin{aligned}\sigma_k^2 &= K_k^2 R'_k \quad (32) \\ &= \frac{P_{k|k-1}^2}{R'_k} \text{ upon substituting from (30),} \\ &= \frac{P_{k|k-1}^2}{P_{k|k-1} + R_k},\end{aligned}$$

substituting from (28). Combining (31) and (32) gives a new control surface equation:

$$v_k = v_{k-1} + e_k + \dot{c}_k + \sigma_k V''_k, \quad (33)$$

where V''_k denotes unit-variance Gaussian noise. So the Kalman filter's control output equals the sum of the three input variables plus additive Gaussian noise with time-dependent variance σ_k^2 . For constant error e_k , we can interpret (33) as a smooth control surface in R^3 defined by

$$v_k = v_{k-1} + e_k + \dot{c}_k,$$

and perturbed at time k by Gaussian noise with variance σ_k^2 .

In our simulations the standard deviation σ_k converged after only a few iterations. We used unity initial conditions: $P_{o|o} = R_k = 1$ for all k .

Table 1 lists the convergence rates and steady-state values of σ_k for three different values of the variance $Var(w)$ of the white-noise, unmodeled-effects process w_k . For $Var(w) = 0$, σ_k decreases rapidly at first— $\sigma_8 = .10$, $\sigma_{17} = .05$ —but does not attain its steady-state value of zero within 100 iterations.

$Var(w)$	Steady-state value of σ_k	Number of iterations required for convergence
1.00	0.79	2
0.25	0.46	4
0.05	0.22	9

TABLE 1 Convergence rates and steady-state values of σ_k for different values of the variance $Var(w)$ of the white-noise, unmodeled-effects process w_k .

Figure 11 shows four realizations of the Kalman filter's random control surface for $e_k = 0$, each at a time k when σ_k has converged to its steady-state value. For each plot, we used output thresholds and initial variances for the azimuth case: $|v_k| \leq 9.0$, $R_k = P_{o|o} = 1.0$. As with the fuzzy controller, elevation control surfaces equal scaled versions of the corresponding azimuth control surfaces.

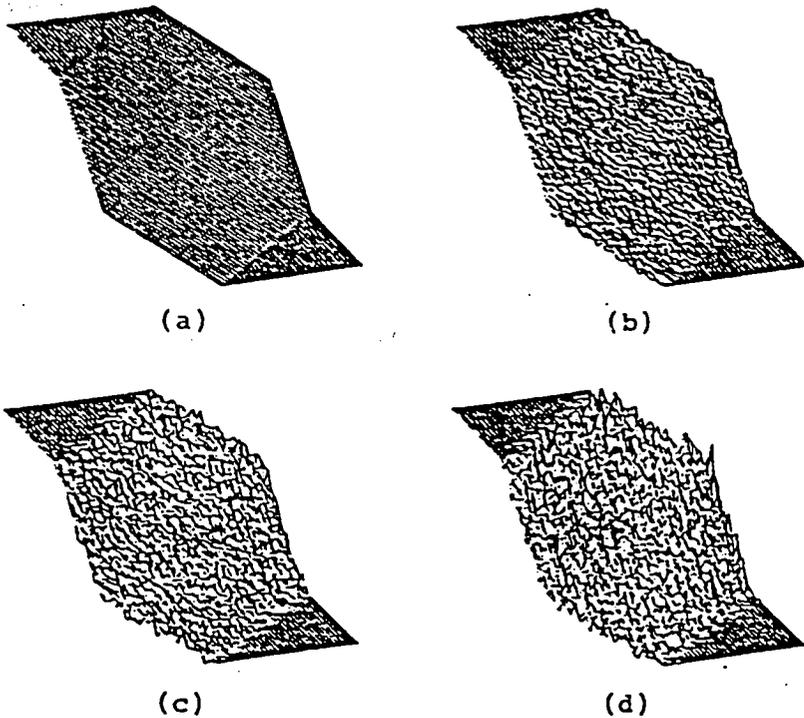


FIGURE 11 Realizations of the Kalman filter's random control surface with $e_k = 0$ for different values of the variance $Var(w)$ and steady-state values of the standard deviation σ_k : (a) $Var(w) = \sigma_k = 0$, (b) $Var(w) = .05$, $\sigma_k = .22$; (c) $Var(w) = .25$, $\sigma_k = .46$; (d) $Var(w) = 1.0$, $\sigma_k = .79$.

SIMULATION RESULTS

Our target-tracking simulations model several realworld scenarios. Suppose we have mounted the target tracking system on the side of a vehicle, aircraft, or ship. The system tracks a missile that cuts across the detection range on a straight flight path. The target maintains a constant speed of 1,870 miles-per-hour and comes within 3.5 miles of the

platform at closest approach. The platform can scan from 0 to 180 degrees in azimuth at a maximum rate of 36 degrees-per-second, and from 0 (vertical) to 90 degrees in elevation at a maximum rate of 18 degrees-per-second. The sampling interval is 1/4 of a second. The gain of the fuzzy controller equals 0.9. So the maximum error considered is 10 degrees azimuth and 5 degrees elevation. We threshold all error values above this level.

Figure 12 demonstrates the best performance of the fuzzy controller for a simulated scenario. The solid lines indicate target position. The dotted lines indicate platform position. To achieve this performance, we calibrated the three design parameters—upper and lower trapezoid bases and the gain. Figures 13 and 14 show examples of uncalibrated systems. Too much overlap causes excessive overshoot. Too little overlap causes lead or lag for several consecutive time intervals. A gain of 0.9 suffices for most scenarios. We can fine-tune the fuzzy control system by altering the percentage overlap between adjacent fuzzy sets.

Figure 15 demonstrates the best performance of the Kalman-filter controller for the same scenario used to test the fuzzy controller. For simplicity, $R_k = P_{0|0}$ for all values of k . For this study we chose the values 1.0 (unit variance) for azimuth and 0.25 for elevation. This 1/4 ratio reflects the difference in scanning range. We set Q_k to 0 for optimal performance. Figure 16 shows the Kalman-filter controller's performance when $Q_k = 1.0$ azimuth, 0.25 elevation.

Sensitivity Analysis

We compared the uncertainty sensitivity of the fuzzy and Kalman-filter control systems. Under normal operating conditions, when the FAM bank contains all fuzzy control rules, and when the unmodeled-effects noise variance $Var(w)$ is small, the controllers perform almost identically. Under more uncertain conditions their performance differs. The Kalman filter's state equation (26) contains the noise term w_k whose variance we must assume. When $Var(w)$ increases, the state equation becomes more uncertain. The fuzzy control

FAM rules depend implicitly on this same equation, but without the noise term. Instead, the fuzziness of the FAM rules accounts for the system uncertainty. This suggests that we can increase the uncertainty of the implicit state equation by omitting randomly selected FAM rules. Figures 17 and 18 show the effect on the *root-mean-squared error* (RMSE) in degrees when we omit FAM rules and increase $Var(w)$. Each data point averages ten runs.

The controllers behave differently as uncertainty increases. The RMSE of the fuzzy controller increases little until we omit nearly sixty percent of the FAM rules. The RMSE of the Kalman filter increases steeply for small values of $Var(w)$, then gradually levels off.

We also tested the fuzzy controller's robustness by "sabotaging" the most vulnerable FAM rule. This could reflect lack of accurate expertise, or a highly unstructured problem. Changing the consequent of the steady-state FAM rule ($ZE, ZE, ZE; ZE$) to LP gives the following nonsensical FAM rule:

IF the platform points directly at the target
AND both the target and the platform are stationary,
THEN turn in the positive direction with maximum velocity.

Figure 19 shows the fuzzy system's performance when this sabotage FAM rule replaces the steady-state FAM rule. When the sabotage FAM rule activates, the system quickly adjusts to decrease the error again. The fuzzy system is piecewise stable.

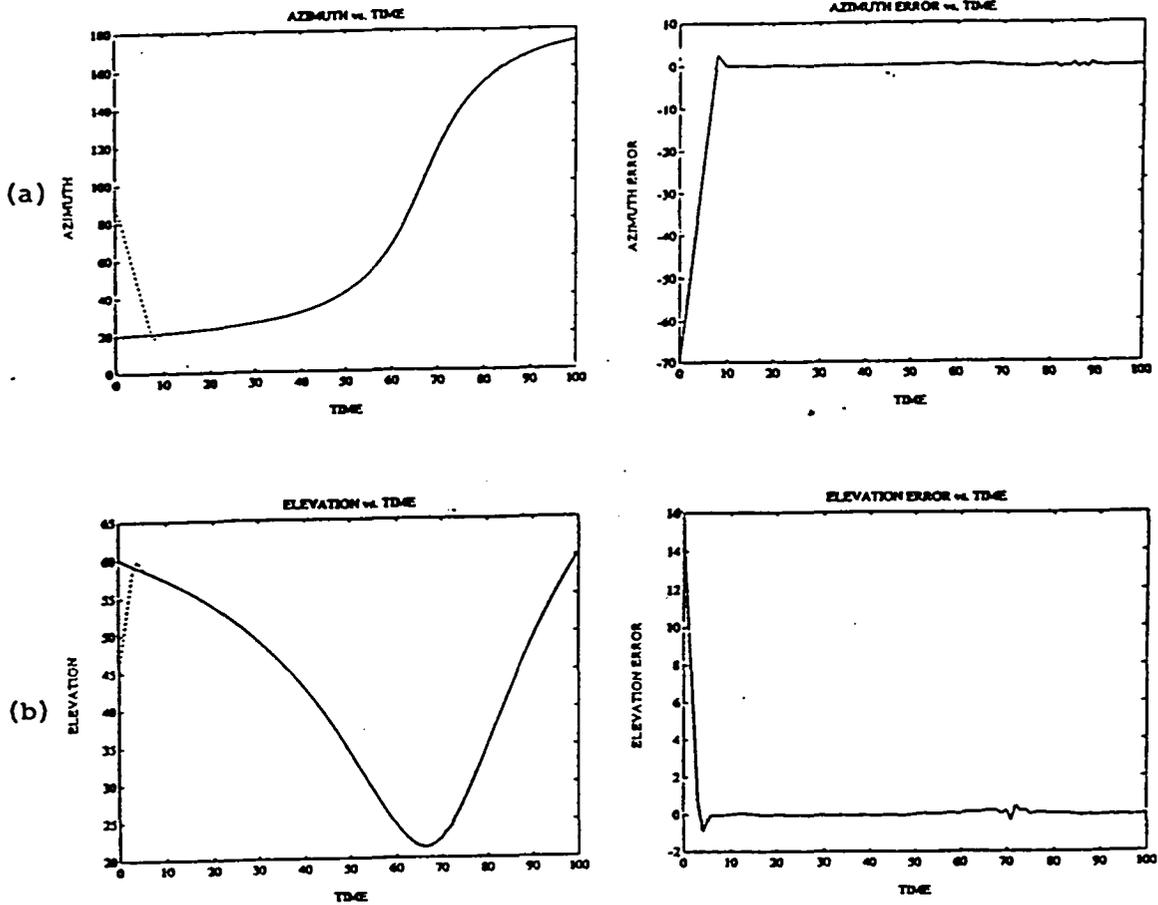


FIGURE 12 Best performance of the fuzzy controller: (a) azimuth position and error, (b) elevation position and error. Fuzzy set overlap is 26.2%.

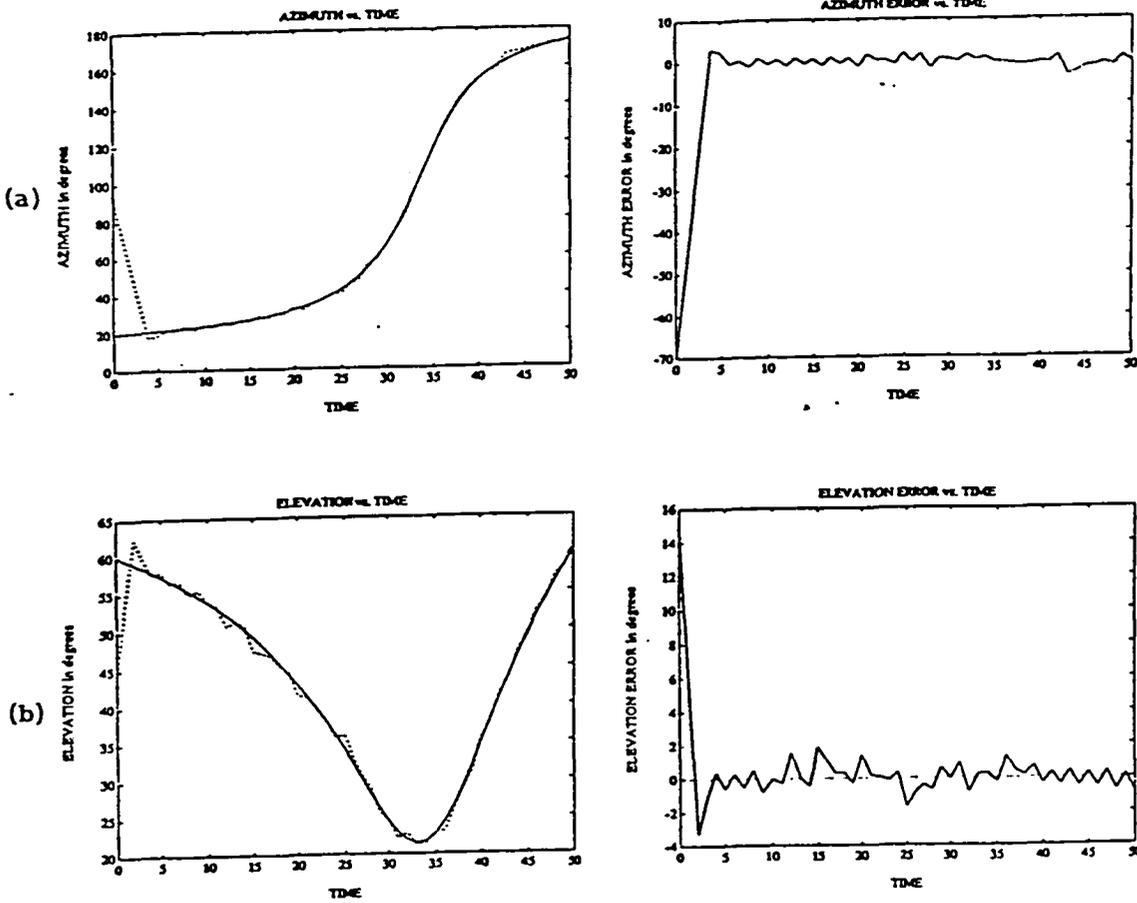


FIGURE 13 Uncalibrated fuzzy controller: (a) azimuth position and error, (b) elevation position and error. Fuzzy set overlap equals 33.3%. Too much overlap causes excessive overshoot.

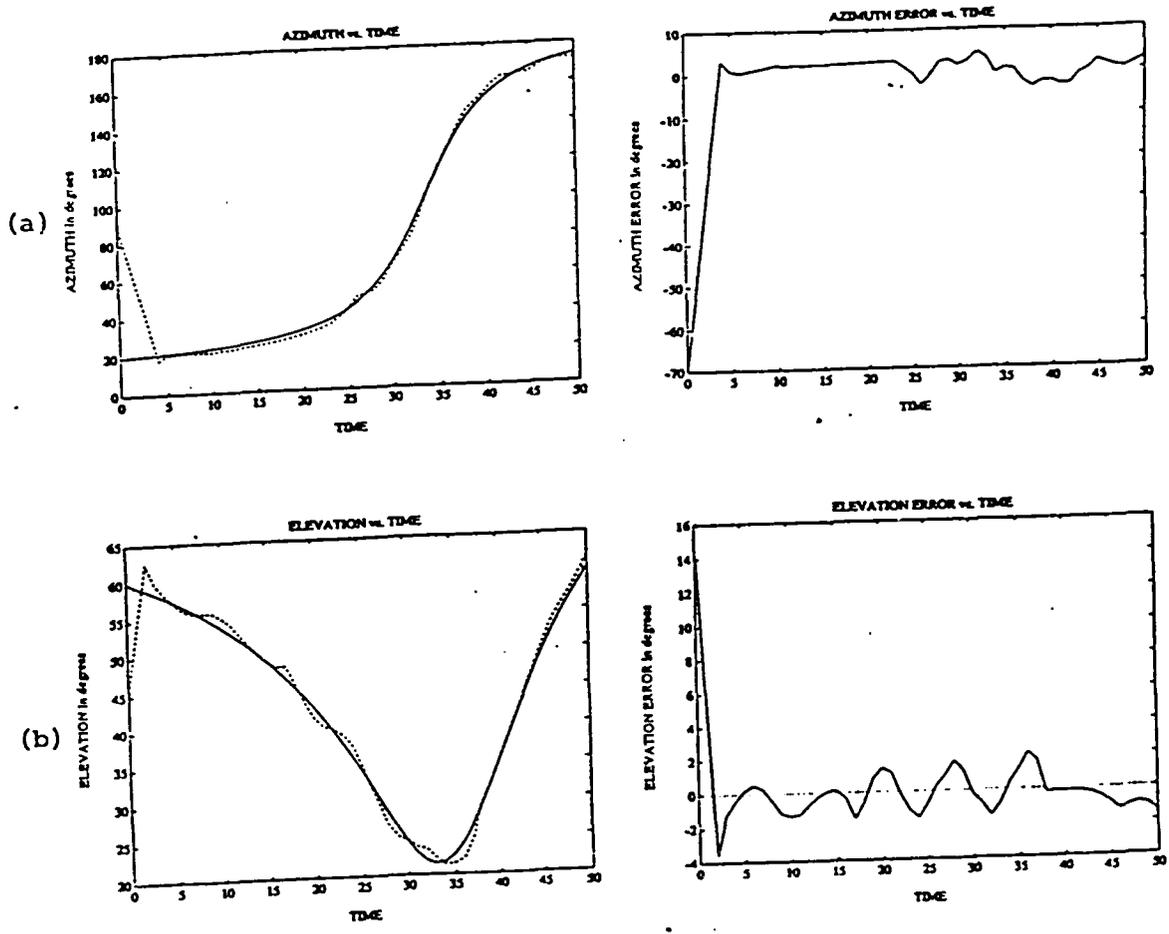


FIGURE 14 Uncalibrated fuzzy controller: (a) azimuth position and error, (b) elevation position and error. Fuzzy set overlap equals 12.5%. Too little overlap causes lead or lag for several consecutive time intervals.

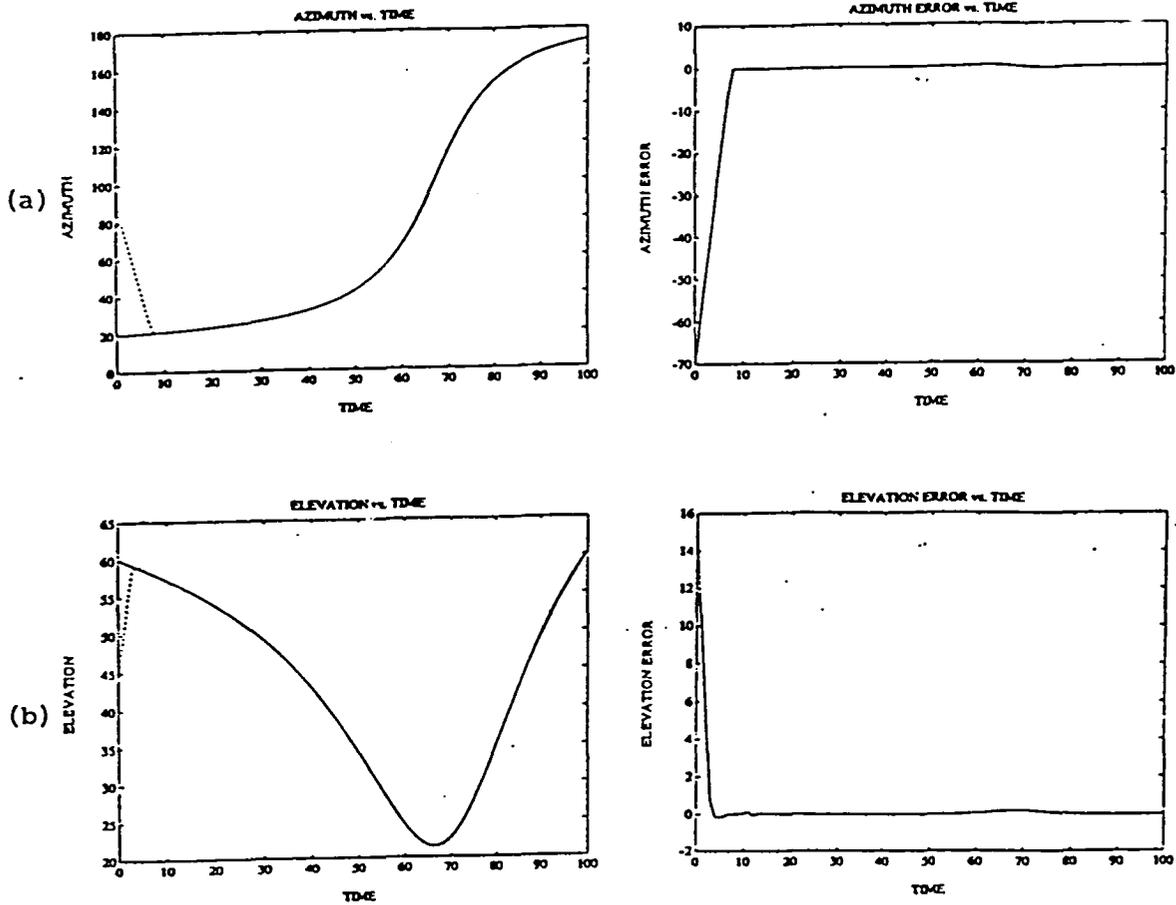


FIGURE 15 Kalman filter controller with unmodeled-effects noise variance $Var(w) = 0$: (a) azimuth position and error, (b) elevation position and error.

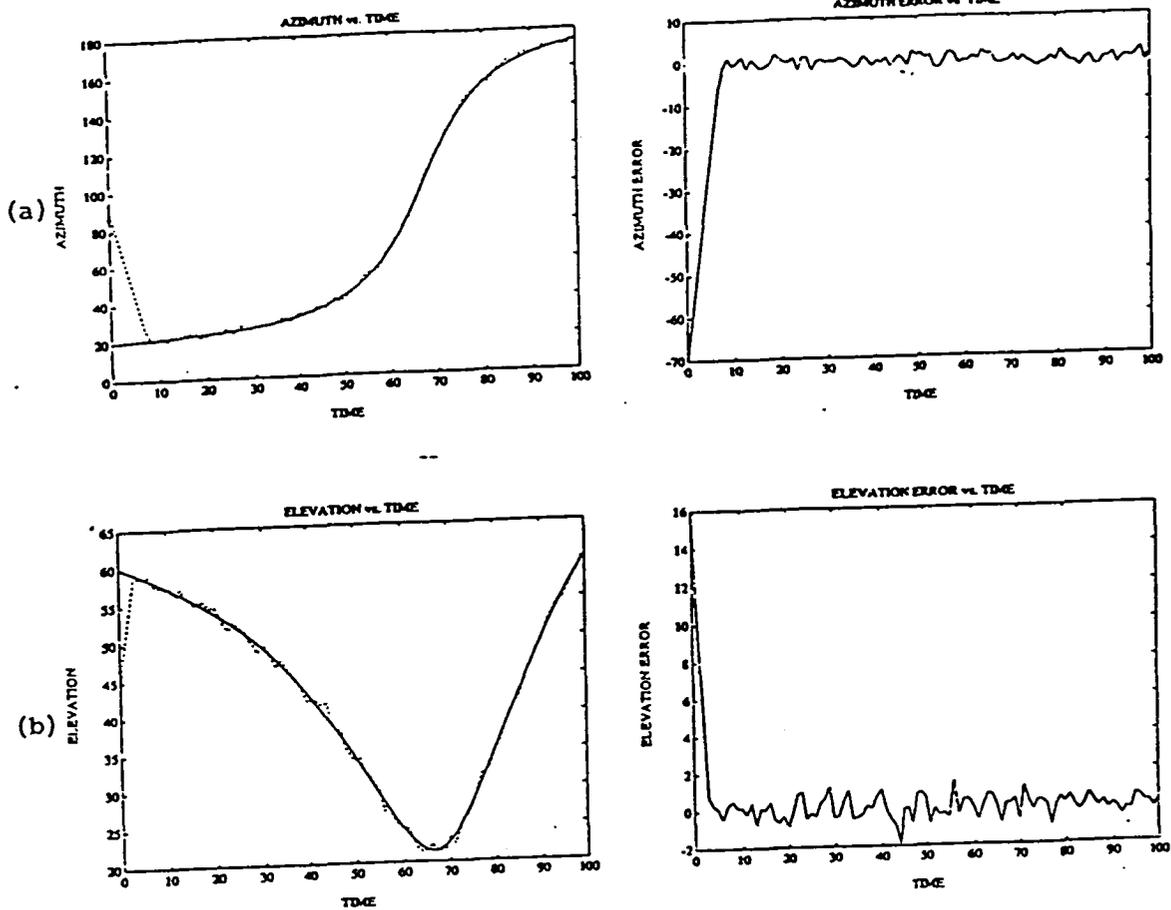


FIGURE 16 Kalman filter controller with $Var(w) = 1.0$ azimuth, 0.25 elevation: (a) azimuth position and error, (b) elevation position and error.

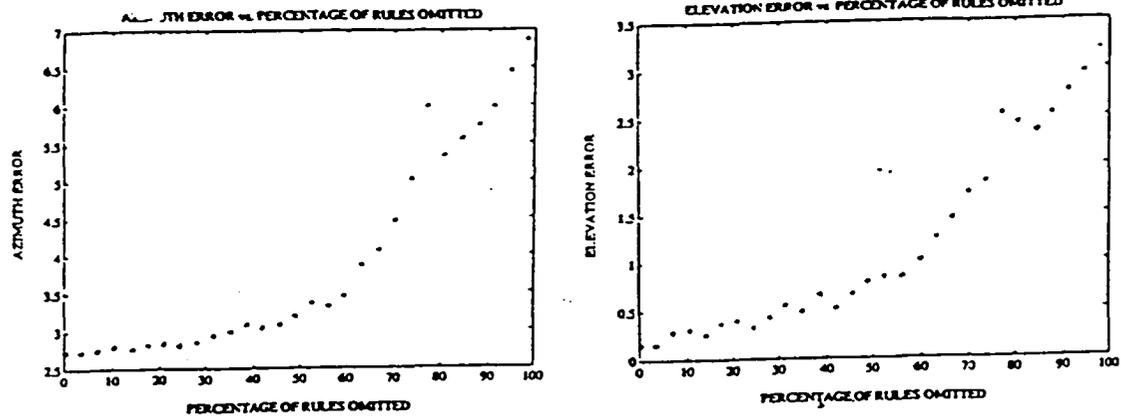


FIGURE 17 Root-mean-squared error of the fuzzy controller with randomly selected FAM rules omitted.

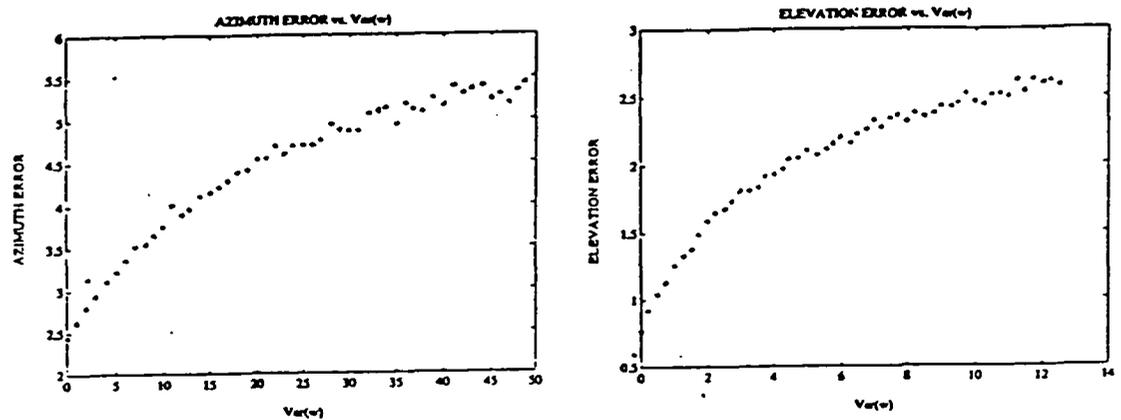


FIGURE 18 Root-mean-squared error of the Kalman filter controller as $Var(w)$ varies.

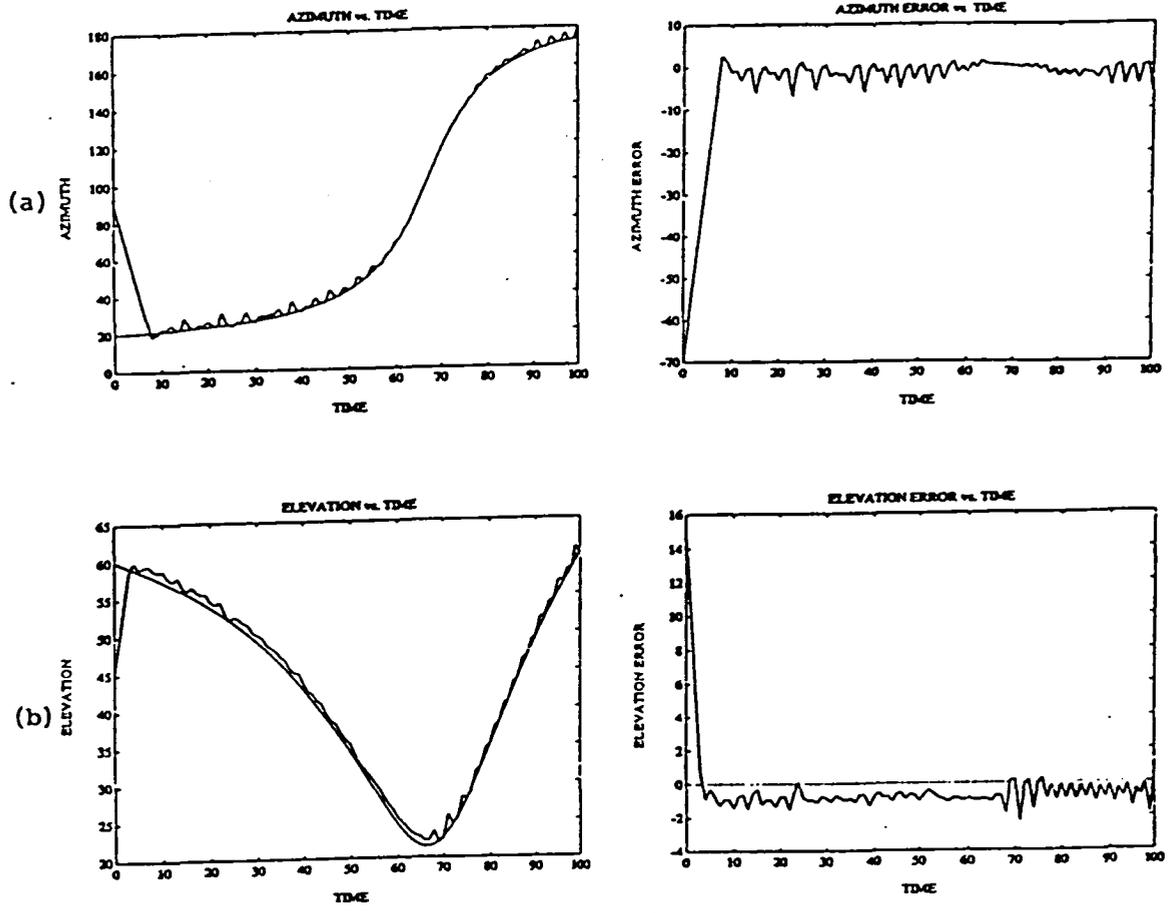


FIGURE 19 Fuzzy controller with a "sabotage" FAM rule: (a) azimuth position and error, (b) elevation position and error. The sabotage rule ($ZE, ZE, ZE; LP$) replaces the steady-state FAM rule ($ZE, ZE, ZE; ZE$). The system quickly adjusts each time the sabotage rule activates.

Adaptive FAM (AFAM)

We used unsupervised product-space clustering [Kosko, 1990a] to train an adaptive FAM (AFAM) fuzzy controller. Differential competitive learning (DCL) adaptively clustered input-output pairs. The Appendix describes product-space clustering with DCL. For this study, there were four input neurons in F_x . A manually-designed FAM bank and 80 random target trajectories generated 19,236 training vectors. Each product-space training vector $(e_k, \dot{e}_k, v_{k-1}, v_k)$ defined a point in R^4 .

Symmetry allowed us to reflect about the origin all sample vectors with negative errors e_k . We then trained 3,000 synaptic quantization vectors ($p = 3,000$) in the positive error half-space. For each sample vector, we defined the 10 closest synaptic vectors as "winners" ($N = 10$). The matrix W of F_y within-field synaptic connection strengths had diagonal elements $w_{ii} = 2.9$, off-diagonal elements $w_{ij} = -0.1$. After training, we reflected the 3,000 synaptic quantization vectors about the origin to give 6,000 trained synaptic vectors.

The product-space FAM cells uniformly partitioned the four-dimensional product space. Each FAM cell represented a single FAM rule. The four fuzzy variables could assume only the 7 fuzzy-set values $LN, MN, SN, ZE, SP, MP,$ and LP . So the product space contained $7^4 = 2401$ FAM cells.

At the end of the DCL training period, we defined a FAM cell as occupied only if it contained at least one synaptic vector. For some combinations of antecedent fuzzy sets, synaptic vectors occupied more than one FAM cell with different consequent fuzzy sets. In these cases we computed the centroid of the consequent fuzzy sets weighted by the number of synaptic vectors in their FAM cells. We chose the consequent fuzzy set as that output fuzzy-set value with centroid nearest the weighted centroid value. We ignored other FAM rules with the same antecedents but different consequent fuzzy sets.

Figure 20(a) shows the $e_k = ZE$ cross section of the original FAM bank used to generate the training samples. Figure 20(b) shows the same cross section of the DCL-estimated FAM bank. Figure 21 shows the original and DCL-estimated control surfaces for constant error $e_k = 0$.

for constant error $e_k = 0$.

The regions where the two control surfaces differ correspond to infrequent high-velocity situations. So the original and DCL-estimated control surfaces yield similar results. Table 2 compares the controllers' root-mean-squared errors for 10 randomly-selected target trajectories.

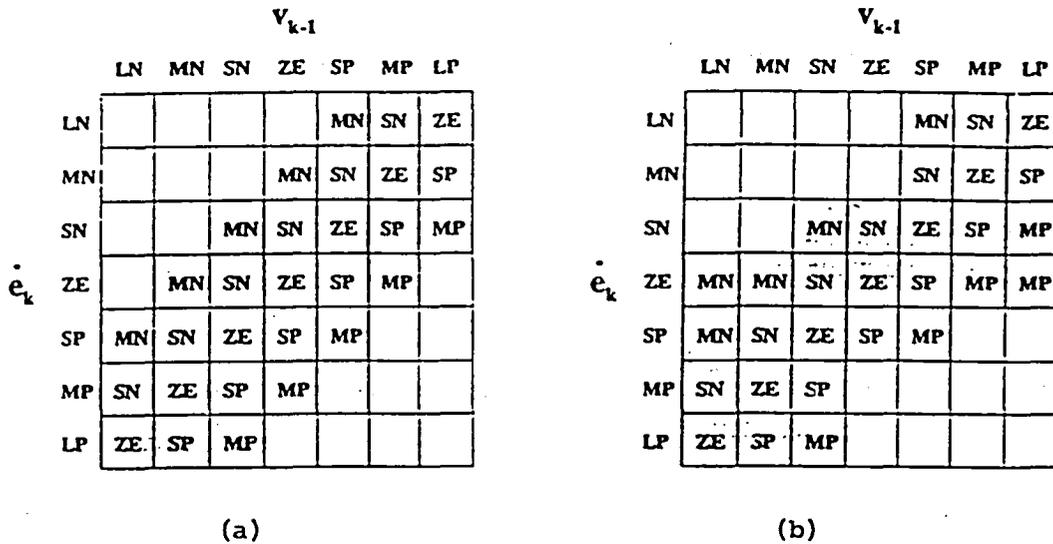


FIGURE 20 Cross sections of the original and DCL-estimated FAM banks when $e_k = ZE$: (a) original, (b) DCL-estimated.

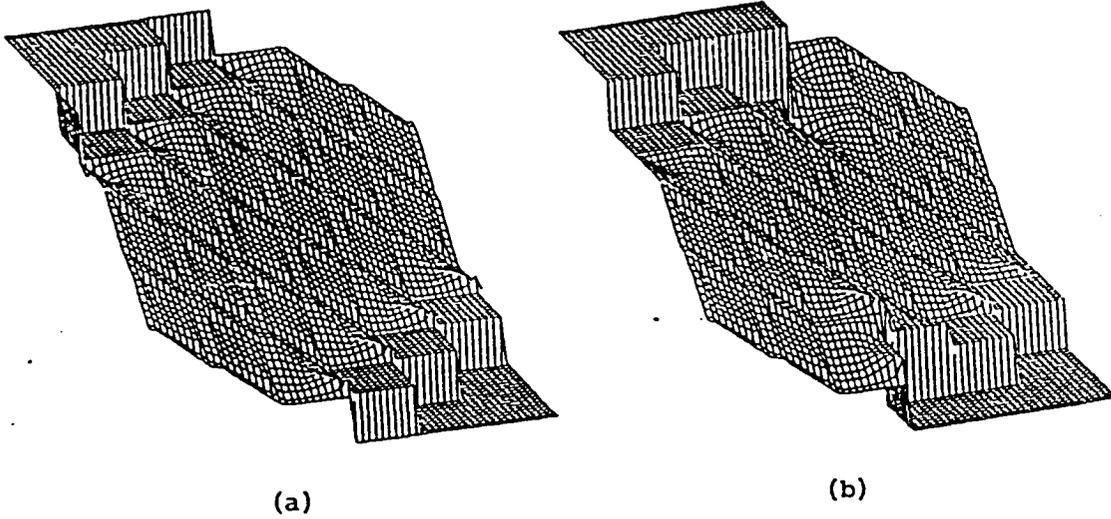


FIGURE 21 Control surfaces for constant error $e_k = 0$: (a) original, (b) DCL-estimated.

Trajectory Number	<i>Azimuth</i>		<i>Elevation</i>	
	Original	Estimated	Original	Estimated
1	2.33	2.33	3.31	3.37
2	4.14	4.14	3.03	2.89
3	6.11	6.11	3.69	3.68
4	3.83	3.83	3.32	3.30
5	4.02	4.02	3.11	3.10
6	2.84	2.84	1.20	1.21
7	3.22	3.22	3.04	2.98
8	0.75	0.74	2.00	2.00
9	9.28	9.27	5.50	5.41
10	1.81	1.81	2.29	2.29
Average	3.83	3.83	3.05	3.02

TABLE 2 Root-mean-squared errors for 10 randomly-selected target trajectories. The original and DCL-estimated FAM banks yielded similar results since they differed only in regions corresponding to infrequent high-velocity situations.

Conclusion

We developed and compared a fuzzy control system and a Kalman-filter control system for realtime target tracking. The fuzzy system represented uncertainty with continuous or fuzzy sets, with the partial occurrence of multiple alternatives. The Kalman-filter system represented uncertainty with the random occurrence of an exact alternative. Accordingly, our simulations tested each system's response to a different family of uncertainty environments, one fuzzy and the other random. In general representative training data can "blindly" generate the governing FAM rules.

These simulations suggest that in many cases fuzzy controllers may be a robust, computationally effective alternative to linear Kalman filter, indeed to nonlinear extended Kalman filter, approaches to realtime system control—even when we can accurately articulate an input-output math model.

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Appendix: Product-space Clustering with Differential Competitive Learning

Adaptive Vector Quantization

Product-space clustering [Kosko, 1990a] is a form of stochastic adaptive vector quantization. Adaptive vector quantization (AVQ) systems adaptively quantize pattern clusters in R^n . Stochastic competitive-learning systems are neural AVQ systems. Neurons compete for the activation induced by randomly sampled patterns. The corresponding fan-in vectors adaptively quantize the pattern space R^n . The p synaptic vectors m_j define the p columns of the synaptic connection matrix M . M interconnects the n input or linear neurons in the input neuronal field F_X to the p competing nonlinear neurons in the output field F_Y . Figure 22 below illustrates the neural network topology.

Learning algorithms estimate the unknown probability density function $p(x)$, which describes the distribution of patterns in R^n . More synaptic vectors arrive at more probable regions. Where sample vectors x are dense or sparse, synaptic vectors m_j should be dense or sparse. The local count of synaptic vectors then gives a nonparametric estimate of the volume density $P(V)$ for volume $V \subset R^n$:

$$P(V) = \int_V p(x) dx \quad (34)$$

$$\approx \frac{\text{Number of } m_j \in V}{p} \quad (35)$$

In the extreme case that $V = R^n$, this approximation gives $P(V) = p/p = 1$. For improbable subsets V , $P(V) = 0/p = 0$.

Stochastic Competitive Learning Algorithms

The metaphor of competing neurons reduces to nearest-neighbor classification. The AVQ system compares the current vector random sample $\mathbf{x}(t)$ in Euclidean distance to the p columns of the synaptic connection matrix M , to the p synaptic vectors $\mathbf{m}_1(t), \dots, \mathbf{m}_p(t)$. If the j th synaptic vector $\mathbf{m}_j(t)$ is closest to $\mathbf{x}(t)$, then the j th output neuron "wins" the competition for activation at time t . In practice we sometimes define the nearest N synaptic vectors as winners. Some scaled form of $\mathbf{x}(t) - \mathbf{m}_j(t)$ updates the nearest or "winning" synaptic vectors. "Losers" remain unchanged: $\mathbf{m}_i(t+1) = \mathbf{m}_i(t)$. Competitive synaptic vectors converge to pattern-class centroids exponentially fast [Kosko, 1990b].

The following three-step process describes the competitive AVQ algorithm, where the third step depends on which learning algorithm updates the winning synaptic vectors.

Competitive AVQ Algorithm

1. Initialize synaptic vectors: $\mathbf{m}_i(0) = \mathbf{x}(i), i = 1, \dots, p$. Sample-dependent initialization avoids many pathologies that can distort nearest-neighbor learning.
2. For random sample $\mathbf{x}(t)$, find the closest or "winning" synaptic vector $\mathbf{m}_j(t)$:

$$\|\mathbf{m}_j(t) - \mathbf{x}(t)\| = \min_i \|\mathbf{m}_i(t) - \mathbf{x}(t)\| , \quad (36)$$

where $\|\mathbf{x}\|^2 = x_1^2 + \dots + x_n^2$ defines the squared Euclidean vector norm of \mathbf{x} . We can define the N synaptic vectors closest to \mathbf{x} as "winners."

3. Update the winning synaptic vector(s) $\mathbf{m}_j(t)$ with an appropriate learning algorithm.

Differential Competitive Learning (DCL)

Differential competitive "synapses" learn only if the competing "neuron" changes its competitive status [Kosko, 1990c]:

$$\dot{m}_{ij} = \dot{S}_j(y_j)[S_i(x_i) - m_{ij}] , \quad (37)$$

or in vector notation,

$$\dot{\mathbf{m}}_j = \dot{S}_j(y_j)[\mathbf{S}(\mathbf{x}) - \mathbf{m}_j] , \quad (38)$$

where $\mathbf{S}(\mathbf{x}) = (S_1(x_1), \dots, S_n(x_n))$ and $\mathbf{m}_j = (m_{1j}, \dots, m_{nj})$. m_{ij} denotes the synaptic value between the i th neuron in input field F_X and the j th neuron in competitive field F_Y . Nonnegative signal functions S_i and S_j transduce the real-valued activations x_i and y_j into bounded monotone nondecreasing signals $S_i(x_i)$ and $S_j(y_j)$. \dot{m}_{ij} and $\dot{S}_j(y_j)$ denote the time derivatives of m_{ij} and $S_j(y_j)$, synaptic and signal velocities. $S_j(y_j)$ measures the competitive status of the j th competing neuron in F_Y . Usually S_j approximates a binary threshold function. For example, S_j may equal a steep binary logistic sigmoid,

$$S_j(y_j) = \frac{1}{1 + e^{-cy_j}} , \quad (39)$$

for some constant $c > 0$. The j th neuron wins the laterally inhibitive competition if $S_j = 1$, loses if $S_j = 0$.

For discrete implementation, we use the DCL algorithm as a stochastic difference equation [Kong, 1991]:

$$m_j(t+1) = m_j(t) + c_t \Delta S_j(y_j(t))[S(\mathbf{x}(t)) - m_j(t)] \text{ if the } j\text{th neuron wins,} \quad (40)$$

$$m_i(t+1) = m_i(t) \text{ if the } i\text{th neuron loses.} \quad (41)$$

$\Delta S_j(y_j(t))$ denotes the time change of the j th neuron's competition signal $S_j(y_j)$ in the

competition layer F_Y :

$$\Delta S_j(y_j(t)) = \text{sgn}[S_j(y_j(t+1)) - S_j(y_j(t))] \quad (42)$$

We define the signum operator $\text{sgn}(x)$ as

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \quad (43)$$

$\{c_t\}$ denotes a slowly decreasing sequence of learning coefficients, such as $c_t = .1(1 - t/2000)$ for 2000 training samples. Stochastic approximation [Huber, 1981] requires a decreasing gain sequence $\{c_t\}$ to suppress random disturbances and to guarantee convergence to local minima of mean-squared performance measures. The learning coefficients should decrease slowly,

$$\sum_{t=1}^{\infty} c_t = \infty, \quad (44)$$

but not too slowly,

$$\sum_{t=1}^{\infty} c_t^2 < \infty. \quad (45)$$

Harmonic-series coefficients, $c_t = 1/t$, satisfy these constraints.

We approximate the competitive signal difference ΔS_j as the activation difference Δy_j :

$$\Delta S_j(y_j(t)) = \text{sgn}[y_j(t+1) - y_j(t)] \quad (46)$$

$$= \Delta y_j(t) \quad (47)$$

Input neurons in feedforward networks usually behave linearly: $S_i(x_i) = x_i$, or $S(x(t)) = x(t)$.

Then we update the winning synaptic vector $m_j(t)$ with

$$m_j(t+1) = m_j(t) + c_t \Delta y_j(t) [x(t) - m_j(t)] \quad (48)$$

We update the F_Y neuronal activations y_j with the additive model

$$y_j(t+1) = y_j(t) + \sum_i^n S_i(x_i(t)) m_{ij}(t) + \sum_k^p S_k(y_k(t)) w_{kj} \quad (49)$$

For linear signal functions S_i , the first sum in (49) reduces to an inner product of sample and synaptic vectors:

$$\sum_i^n x_i(t) m_{ij}(t) = \mathbf{x}^T(t) \mathbf{m}_j(t) \quad (50)$$

Then positive learning tends to occur— $\Delta m_{ij} > 0$ —when \mathbf{x} is close to the j th synaptic vector \mathbf{m}_j .

Since a binary threshold function approximates the output signal function $S_k(y_k)$, the second sum in (49) sums over just the winning neurons: $\sum_k w_{kj}$ for all winning neurons y_k .

The $p \times p$ matrix W contains the F_Y within-field synaptic connection strengths. Diagonal elements w_{ii} are positive, off-diagonal elements negative. Winning neurons excite themselves and inhibit all other neurons. Figure 22 shows the connection topology of the laterally inhibitive DCL network.

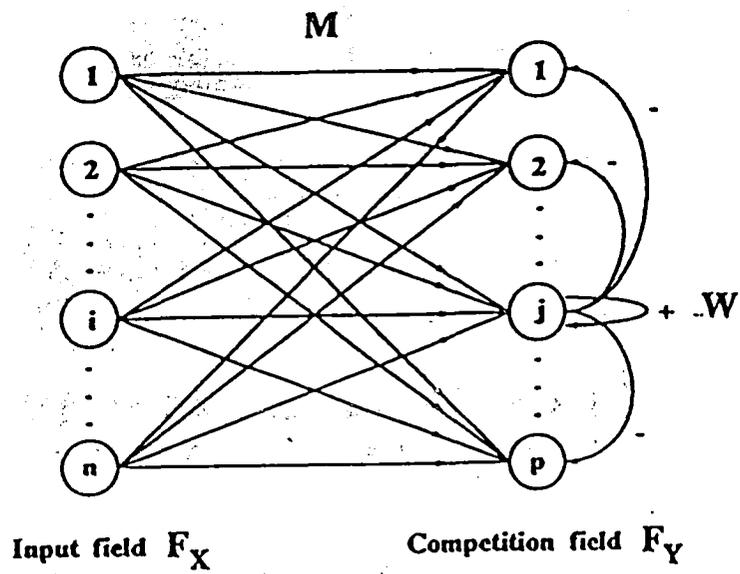
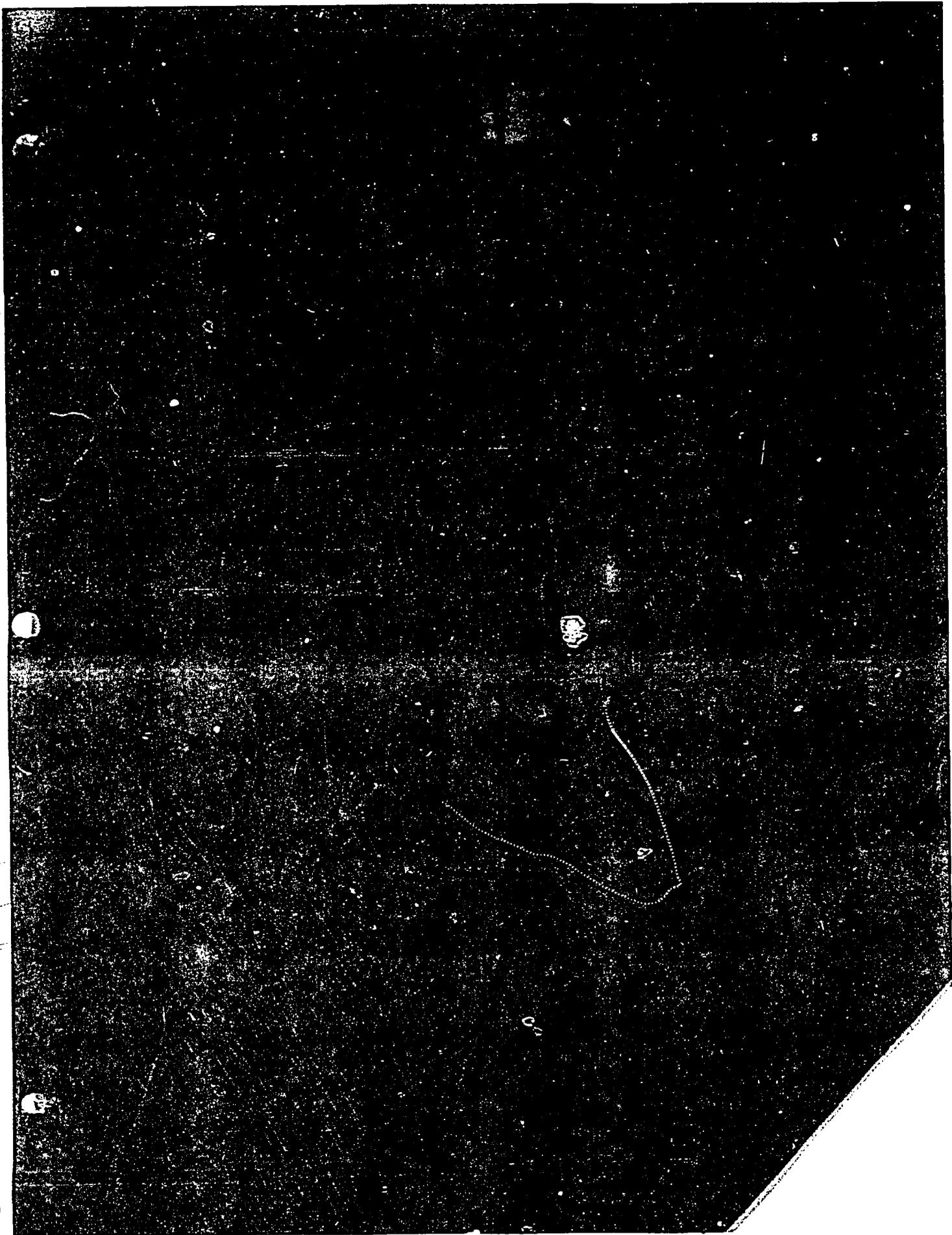


FIGURE 22 Topology of the laterally inhibitive DCL network.



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31 47

LAS

**SPACE STATION FREEDOM
ATTITUDE DETERMINATION
AND CONTROL SYSTEM
OVERVIEW**

**For Presentation at the NASA/MDSSC
Workshop on Fuzzy Control Systems
and
Space Station Applications**

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pg 5
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14, November 1990
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INTRODUCTION

- SS Freedom is a highly variable system which requires a general, robust control system
 - Mass properties vary orders of magnitude during assembly
 - Vehicle orientation for normal operations changes significantly during assembly

- Operational environment
 - Disturbances typically 0-2 times orbit frequency
 - Aerodynamic, gravity gradient, gyroscopic torques
 - Sensors sampled at 5 Hz, effectors commanded at 2.5 Hz

- Vehicle is controlled using two different systems
 - CMG's (Control Moment Gyroscopes)
 - RCS (Reaction Control System)

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HIGHLY DYNAMIC PLANT

■ Vehicle changes significantly after each assembly stage

● Mass properties variations

- Mass varies from 60,000 - 800,000 lb
- Center of Mass (cm) location moves over 100 ft.
- MOI's (Moments of Inertia) vary by 2 orders of magnitude
- Delta MOI's (I_{xx} - I_{zz} etc) can change sign during planned operations resulting in the gravity gradient torque derivatives changing sign

● RCS system variations

- Number and location of thrusters changes depending on assembly stage
- Blowdown ACS thrust varies from 25 - 9 lbf.
- Reboost varies from 50 - 20 lbf.

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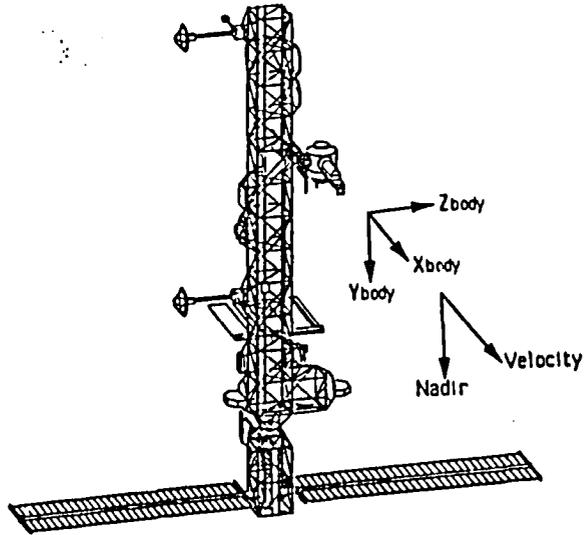
HIGHLY DYNAMIC PLANT (cont.)

- Mass properties change significantly during each assembly phase
 - Orbiter docking adds approximately 220,000 lb and causes significant changes in cm and MOI's
 - Arm motion with heavy payload attached causes continually varying mass properties during operations

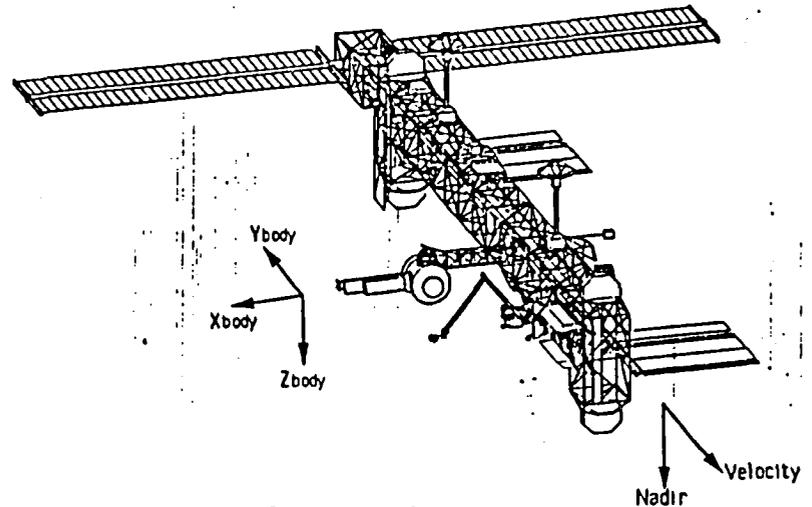
- Vehicle orientation during normal operations changes during assembly flights
 - Assembly flights 1-6
 - Nominal attitude is gravity gradient stable with the truss aligned with the local vertical
 - During reboost 'Arrow' orientation is maintained with the truss aligned with the velocity vector
 - After assembly flight 6 vehicle maintains TEA (Torque Equilibrium Attitude) during normal operations, LVLH (Local Vertical Local Horizontal) during reboost

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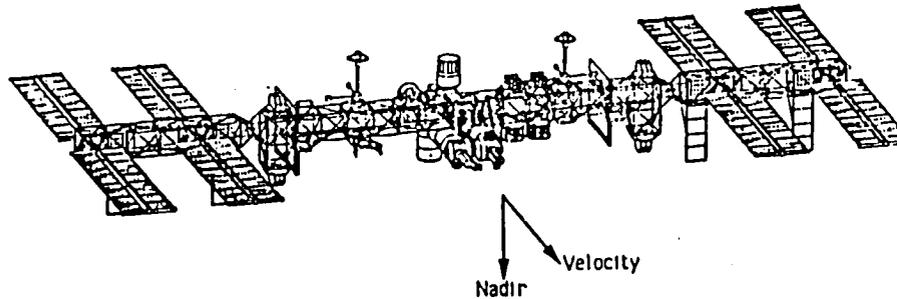
SS FREEDOM FLIGHT ATTITUDES



Gravity Gradient Orientation



Arrow Orientation



Normal Orientation

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EFFECTORS

■ CMG (Control Moment Gyroscopes)

- Dual Gimbal, constant speed rotors store angular momentum (3500 ft-lbf-sec each)
- Provide torque to the vehicle by changing the net angular momentum vector of the CMG(s)
- Linear actuator
- Relatively small angular momentum storage capacity (4 CMG's in current design)

■ RCS (Reaction Control System)

- Uses small, rocket thrusters to provide vehicle attitude control & reboost
- Requires consumable propellant
- Non-linear actuator

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SENSORS

■ Inertial Attitude Sensors

● ISA (Inertial Sensor Assembly)

- Three axis ring laser gyroscope
- Provides body rate information
- 3 ISA's on the Attitude Reference Assembly

● Star Tracker

- Provides inertial attitude based on star catalog
- Software provides 'deadstart' capability
- 2 star trackers on Attitude Reference Assembly

■ CMG angular momentum measured via sensed gimbal angles

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CONTROL SYSTEM PERFORMANCE REQUIREMENTS

- Attitude control
 - TEA must be within $\pm 5^\circ$ of LVLH (currently not doable)
 - Attitude variations from TEA of less than $2.5^\circ/\text{orbit}$

- Minimize propellant consumption during program lifetime
 - Requires a CMG momentum management system
 - Manages CMG momentum using gravity gradient torques
 - Can fly TEA and consume no propellant
 - RCS must use minimum propellant
 - TEA seeker minimizes propellant consumption
 - Use of fuel optimal jets for every firing

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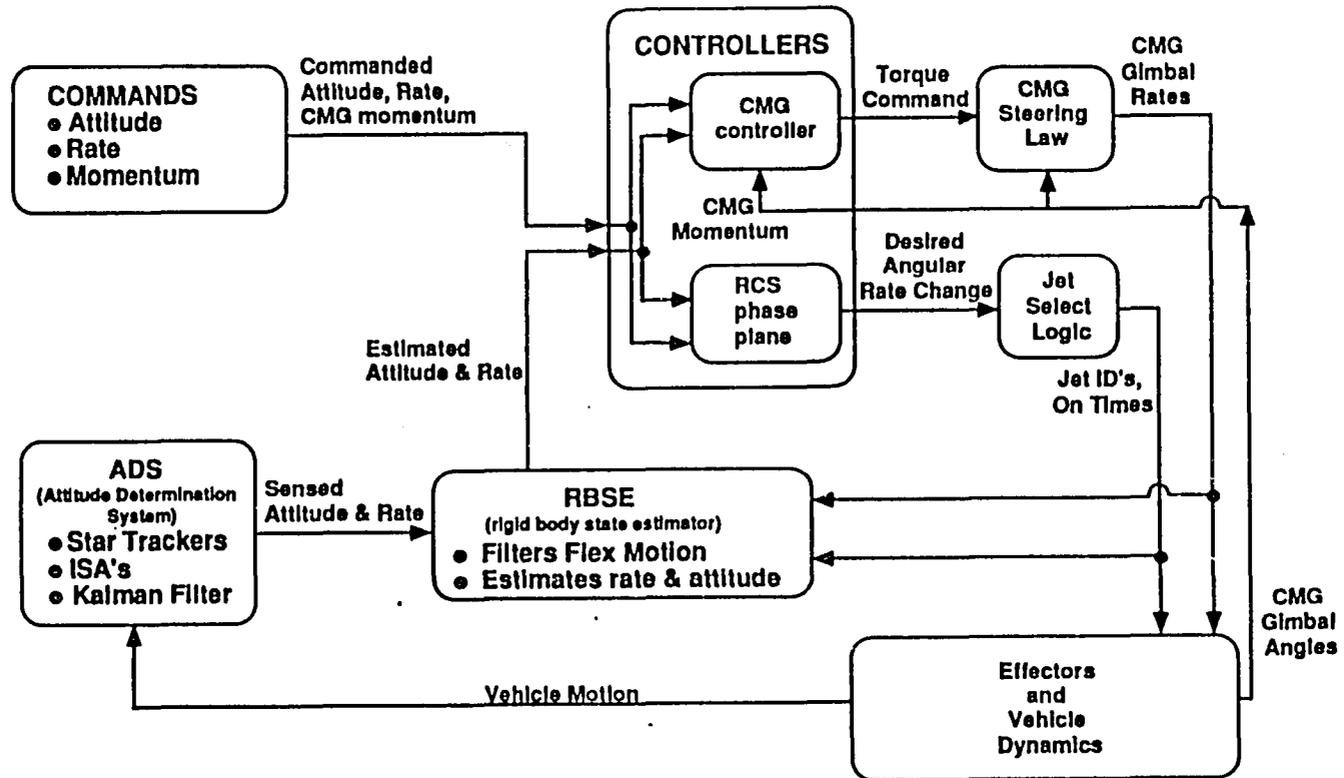
CONTROL SYSTEM FUNCTIONAL REQUIREMENTS

FUNCTION	CMG	RCS	COMMENTS
Primary Attitude Control (Normal operations)	X		Orbit at TEA using momentum manager
Backup Attitude Control		X	Orbit in TEA seeker mode
Attitude Maneuvers		X	CMG's in reset mode
Reboost attitude control	X	X	CMG's in reset
Large Transient Disturbances	X	X	Cooperative CMG/RCS control - RCS assists CMG's to prevent saturation
Adequate Flex Stability Margin	X	X	No active damping of structural vibrations. Controllers must not excite significant flex-body modes.
LVLH Attitude Hold	X	X	
Inertial Attitude Hold	X	X	
Failure Accomodation	X	X	CMG controller must handle failed CMG(s) RCS must handle failed thruster(s)

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CONTROLLER ARCHITECTURE

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CONTROLLER OVERVIEW

■ Momentum Manager/CMG Controller

- Maneuvers the vehicle to generate time-varying gravity gradient torques to manage the CMG momentum and thus use no consumables
- Gravity gradient torques about roll and pitch axes only. Yaw momentum must be dissipated via roll maneuvers 1/4 orbit after momentum buildup occurs
- Low Bandwidth (poles are .3-10 times orbit frequency), continuous controller

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■ RCS Controller

- Generates control torques using pre-processed fuel optimal sets of jets
- 50 millisecond minimum on-time
- Bang-off-bang controller with variable phase plane parameters

■ Structural flex-body mode filters used

SUMMARY

- Robust, multi-variable stable control system is required for SS Freedom attitude control due to dramatic variations of the vehicle mass and aerodynamic properties
- CMG controller requires a guidance momentum management algorithm so no consumable is required during normal operations
- RCS system must minimize propellant consumption by using sets of fuel optimal jets for maneuvers
- Both systems must be fault tolerant and operate with failed components (thrusters and individual CMG's)
- Multiple opportunities for fuzzy logic controllers, particularly the RCS if fuel use can be reduced below current levels

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LAS

BLOWDOWN THRUST PROFILE PREDICTION

by

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■ Outline

- Introduction
- Thrust Prediction Problem
- Solution Approach
- Limitations of Current Approach
- Summary
- Panel Discussion

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

■ INTRODUCTION

- Space Station Freedom propulsion system consists of a blowdown (non-constant thrust) hydrazine system. The thrust depends on the tank pressure which "blows down" as the thrusters are fired.
- The station is periodically reboosted to maintain orbital lifetime and reduce orbital eccentricity. Due to the low thrust to mass ratio available, typical reboost maneuvers can require hours of continuous thrusting.
- Computation of the burn on/off times for a reboost require accurate prediction of the thrust profile for the length of the burn.
- Computation of the burn on/off times onboard require a means of generating a predicted thrust profile without resorting to a 6-dof simulation.
- Prediction of the thrust profile is complicated by the following:
 - Thruster cycling to maintain attitude control.
 - Both reboost and attitude control thrusters are cycled for attitude control depending on the relative thrust miss-match on the modules.

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

- Introduction (continued)
 - Thruster cycling is frequency limited due to structural constraints.

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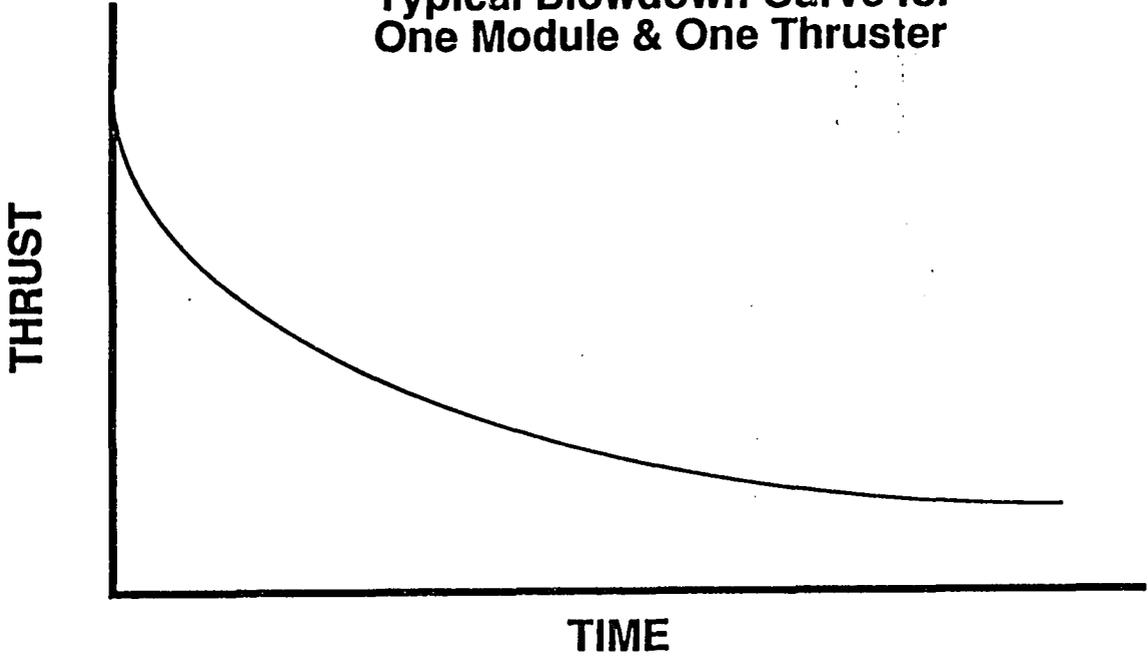
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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

Typical Blowdown Curve for
One Module & One Thruster



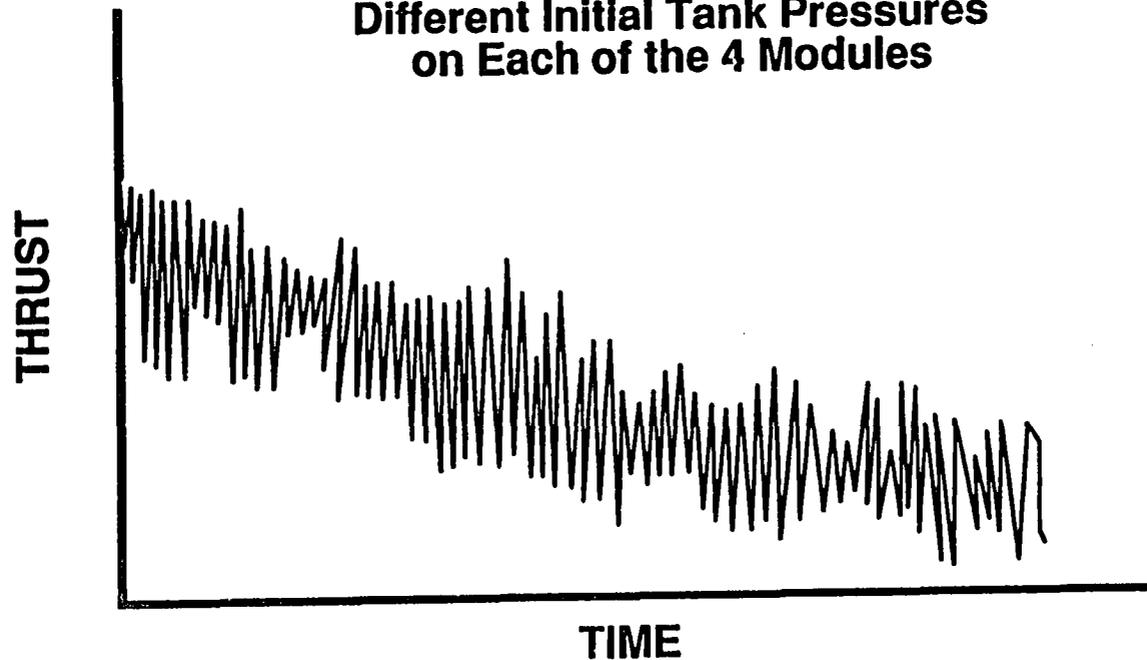
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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

Typical Total Thrust Profile With
Thruster Cycling &
Different Initial Tank Pressures
on Each of the 4 Modules



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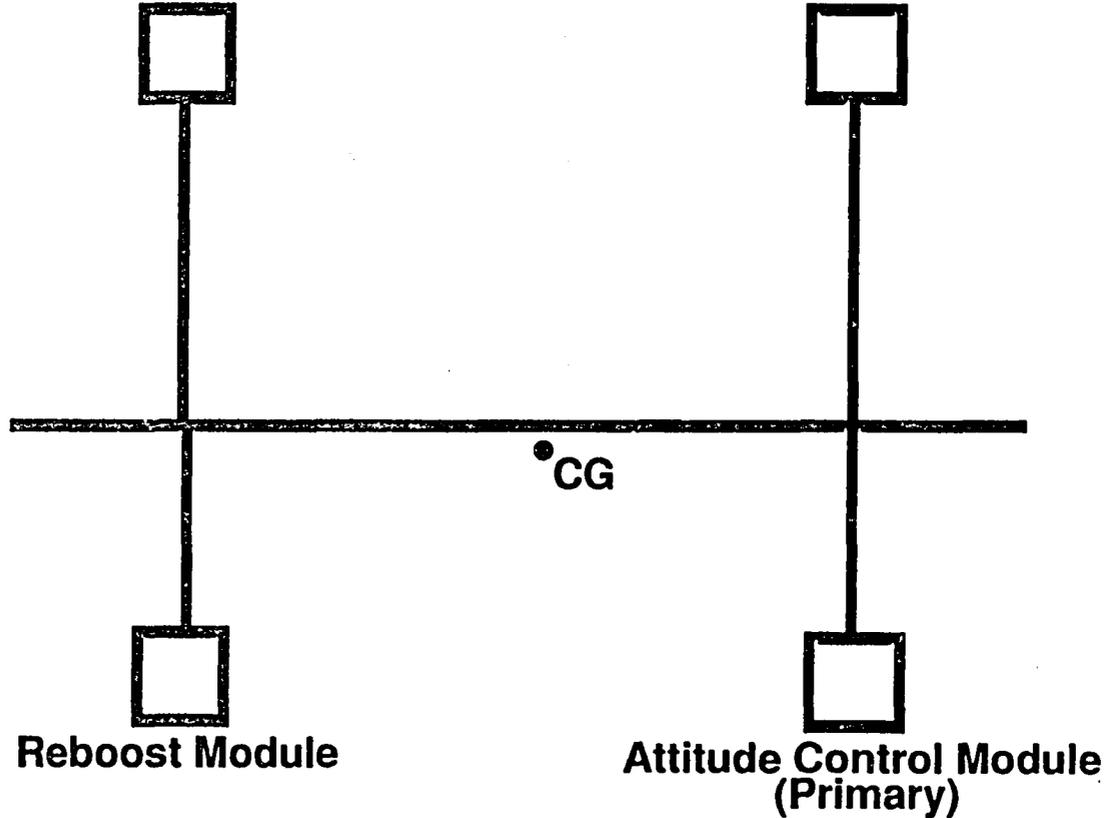
BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

Propulsion Module Geometry

Attitude Control Module

Reboost Module



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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

■ Solution Approach

- The current solution approach seeks to determine an approximation to the average thrust curve representing the blowdown thrust profile. The curve is smooth and does not contain the spikes associated with thruster cycling.
- The algorithm for determining the thrust curve runs as follows:
 - Determine whether attitude can be maintained by pulsing attitude control jets only with both reboost jets on full or whether one reboost jet must be cycled with the other reboost jet on full. This can be determined by considering the geometry.
 - Select a burn duration for the full-on reboost jet.

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

■ Solution Approach (Continued)

- Form the equation for the static moment of the two reboost and single attitude control module. This provides two equations (one each in pitch and yaw) in two unknowns (scale factors on the burn times of the attitude control module and the reboost module that has the pulsing jet). Mathematically this is expressed as:

$$\int_0^T \{ \bar{r}_1 \times \bar{f}_1(t) + \bar{r}_2 \times \bar{f}_2(\alpha t) + \bar{r}_3 \times \bar{f}_3(\beta t) \} dt = 0$$

where the \bar{r}_i are the position vectors (relative to the truss center) of the full on reboost module, the pulsing reboost module and the attitude control module respectively. The \bar{f}_i are the thrust vectors of the respective modules (all in the +X body direction) and the α and β are time scale factors.

- Solve the above system of equations for the time-scale constants.

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

- Iteration on the burn time for the full-on module provides time-scale factors for the other two modules over the length of the reboost.
- The total thrust profile as a function of time is then given by:

$$\text{Thrust}(t) = \bar{f}_1(t) + \bar{f}_2(\alpha t) + \bar{f}_3(\beta t) \quad 0 \leq t \leq T$$

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

■ Limitations of Current Approach

- ⊕ Although full numerical evaluation of the algorithm has not yet been completed the following points can be made:
 - The predicted thrust curve is more accurate towards the end of the reboost time T than it is at the beginning. This is a drawback since the predicted curve is used for maneuver performance monitoring as well as targeting.
 - The thrust profile is also more accurate in predicting the integrated thrust the longer the time span chosen. Once again though, the accuracy of the curve near the start of the burn is reduced.

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BLOWDOWN THRUST PROFILE PREDICTION

by
Kelly D. Murdock
(Continued)

■ Summary

- A means of generating an estimate of the total thrust curve encountered during a reboost using a blowdown propulsion system has been developed.
- The algorithm is straight forward and does not involve a large amount of code.

■ Question for Panel Discussion:

- Can a fuzzy logic algorithm be developed that would provide a thrust profile curve that would predict the integrated thrust of the propulsion system over time spans of a few hundred seconds rather than over 1000's of seconds with equal accuracy over all regions of the curve?

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- Topic: Blowdown Thrust Prediction
 Presenter: Butch Stegall
- Comment: (Stegall) Environment on the ground is worse than in space. They currently have an analytical math model (running in Fortran and Ada). GN&C experts are available at JSC, Marshall, JPL, etc.
 - Highly dynamic system during assembly & operation – opinion is that this is a good area for uncertainty management
 - The largest criteria for deciding which method to use is the minimization of the code size.
- Question: (Stegall) What can be done? Current algorithms are simple, but they are approximations – large amount of uncertainty and many combinations
- Q (Berenji): Why is source reduction a goal?
 A: \$450. per line of code for development for manned flight software systems; Large life cycle costs; 30 year program (SSFP); software maintenance
- Q (Lawler): Over suite of combinations, is it still fuel optimal?
 A: Yes, 98% optimal.
- Q (Lawler): If fuzzy logic came up with an algorithm to match curve (see figure from presentation of "typical" thrust profile with thruster cycle), how much precision will they gain?
 A: Don't know; full blown Monte Carlo simulations have not been run. This is not a "large" software application – contains less than 40 lines of code. The tradeoff is in performance and MIPS.
- Q (Lawler): What is the value added?
 A: in MIPS – any reduction in MIPS is a value. Right before reboost, they are predicting crunch (reboost --> every 90 days).
- C (Stegall): Reboost case is a long-term high fuel utilization.
- Q (Lawler): Why can't this be done on the ground?
 A: Zone of Exclusion, tank management, attitude control – need to be onboard in order to run closed loop and to deal with errors.
- Q (Kosko): On control system, what kind of sensitivity studies have been done and what will the benchmarking be done against?
 A: 5 out of 21 Shuttle flights are being looked at in detail for standardized set of disturbances; The main concentration is using analytical verification. Example: Verified shuttle aero-variables have some range. In test and verification, GN&C does Monte Carlo simulations; destructive testing, trying to break it.
- C (Lea): There is a good deal of work on phase plane control attitude using forty for attitude hold. Also, we have used fuzzy logic on pre-editing of filtering rate attitude data from sensors (sensor data preprocessing).
 On the attitude control we are still testing but have seen a significant decrease in fuel usage (over 1/3) for an attitude hold. Propellant consumption is propulsion metric.
- C (Sugeno): If the flight control system is controlled on the helicopter, you have also succeeded in controlling Space Station (less complicated control sequence).

Topic: Rendezvous Guidance
Presenter: Andy McGuire

Q (Berenji): The LQT assumptions – which one can you make with confidence?

Comment: (Lea) What fuzzy gain functions don't require reinitialization.
A candidate is to attack the drawbacks.

Comment: (McGuire) The software is still in preliminary design phase.

Comment: (Berenji) observation - no restriction combining fuzzy logic with detailed domain model; use the fuzzy logic application as a high level controller.
Possibility → use fuzzy logic for doing reinitialization.
Gain → Adaptability, flexibility

Q (Lawler): How "bad" off are they? Computational? i.e. can traffic control use it's own SDP; what's throughput; sizing problem? What are the metrics? [All systems need to answer this].

A: Fuel Performance. Run against performance using existing information.

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LAS

PROXIMITY OPERATIONS IN SPACE STATION ENVIRONMENT

by

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■ Outline

- Introduction
- Requirements & Constraints
- Problem Formulation
- Proposed Approach
- Summary
- Panel Discussion

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PROXIMITY OPERATIONS INTRODUCTION

- Proximity operations encompasses all free-flying vehicle relative motion within 1 km radius sphere of the Station, the proximity operations zone (POZ)
- U.S. efforts traditionally include manual piloting of an active chase vehicle about a passive target
- Primary active vehicle is the Shuttle
 - Greatest effects on Station components and operations
 - Highest frequency of interaction
- Station era may involve unmanned cooperative vehicles
 - Potential candidates for automated flight
 - Man-Tended Free-Flyer (MTFF) - currently plans Station visits
 - Assured Crew Return Vehicle (ACRV)
 - Flight Telerobotic Servicer (FTS)
 - Remote manual piloting from Station cupola or ground
 - Shuttle- and Station-based OMVs / OTVs
 - Crew Equipment Retrieval System (CERS)
 - Flight Telerobotic Servicer (FTS)
- Direct influences on system design, SE&I and operations

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PROXIMITY OPERATIONS TASK REQUIREMENTS & CONSTRAINTS

- **Flight techniques / procedures development accounts for**
 - **Lighting / visibility constraints**
 - **Trajectory and attitude control**
 - **Plume impingement & environment effects**
 - **Docking / berthing contact conditions**
 - **Structural clearances**
 - **Contingency planning**
 - **Crew timelines**
 - **Ground rules**
 - **Fuel usage**

- **Flight performance directly dependant on**
 - **Flight techniques development and piloting skill**
 - **Control system authority, capacity and performance**
 - **Propulsion system functionality and effectiveness**
 - **Relative navigation sensor accuracy and precision**
 - **Orbital environmental effects**

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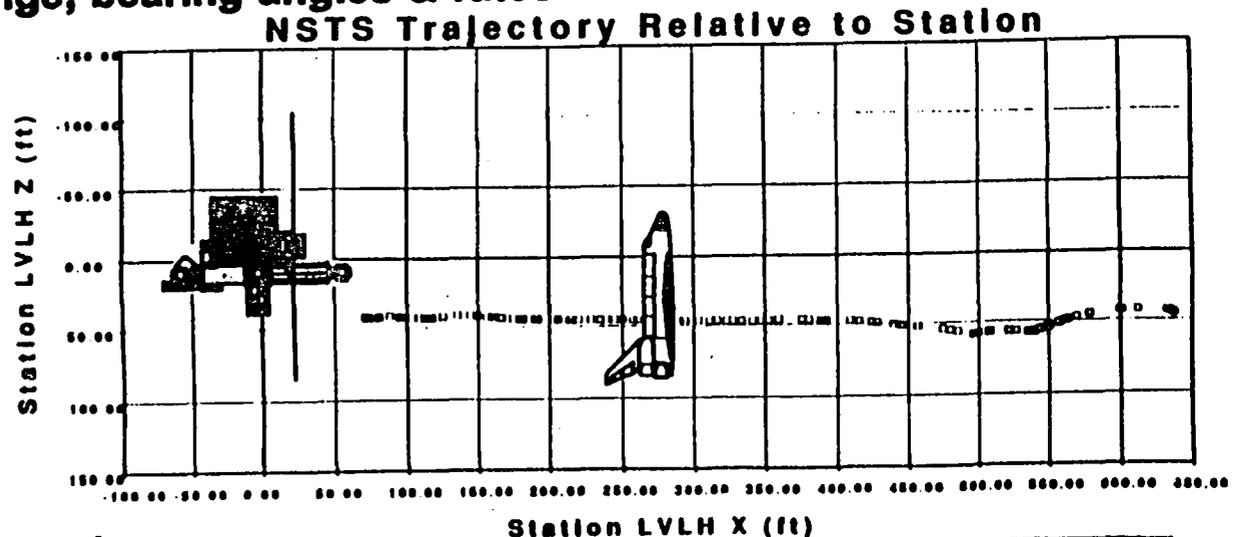
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PROBLEM FORMULATION

- Control task is regulating the six degree-of-freedom (DOF) relative motion between vehicles to achieve the desired trajectory
 - Chaser performs translational and rotational maneuvers
 - Target drifts or actively maintains an attitude profile
 - Critical flight parameters wrt relnav sensor line of sight (LOS)
- Traditionally, flight profile manually flown by refined procedures -
 - Relative state control is split into three distinct aspects:
 - Rotational attitude and attitude rates
 - Range, closing or opening velocity
 - Cross range, bearing angles & rates

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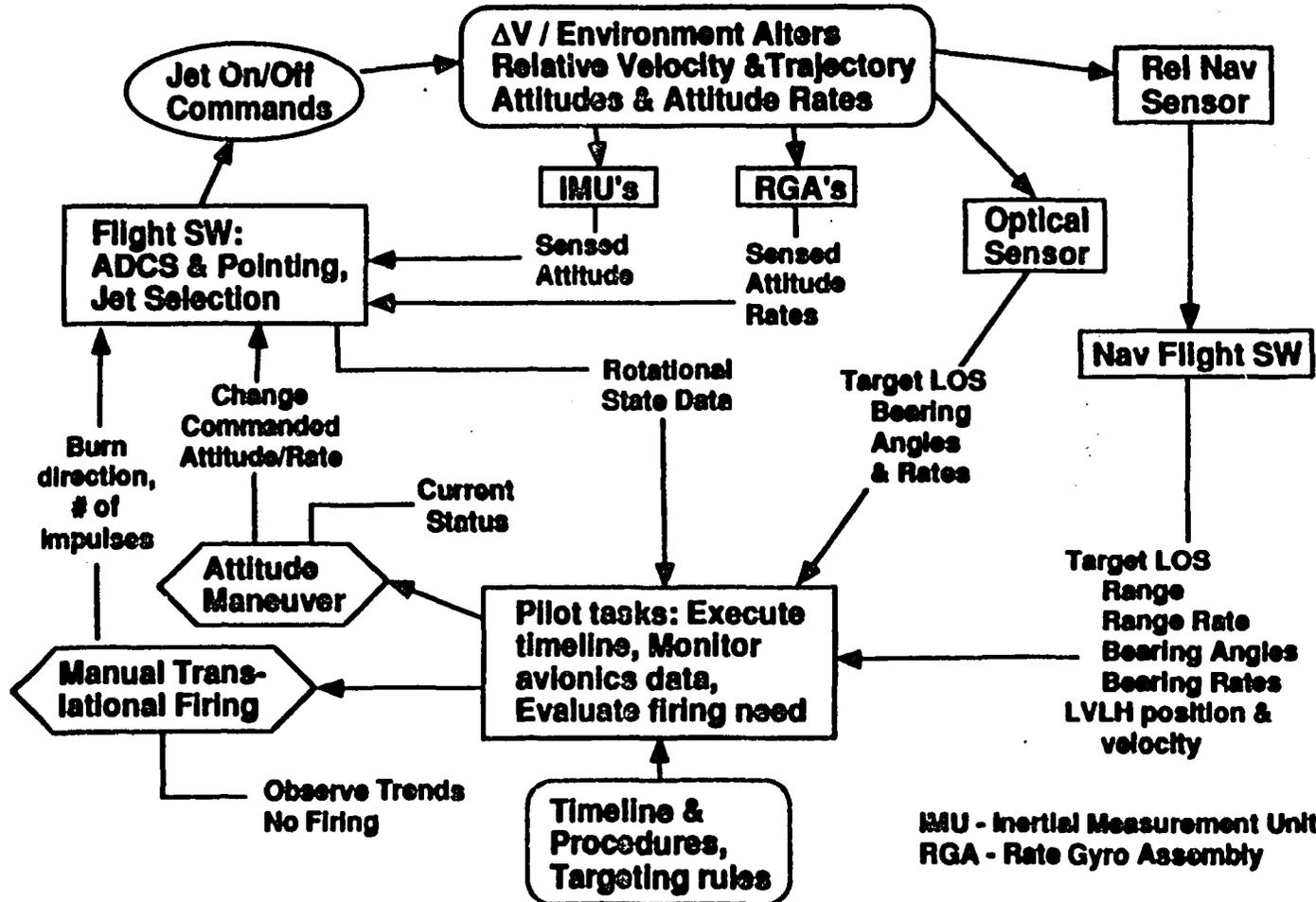
- Vbar approach as example trajectory -



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PROBLEM FORMULATION (CONTINUED)

■ Flight parameter flow / pilot interaction for proximity operations task



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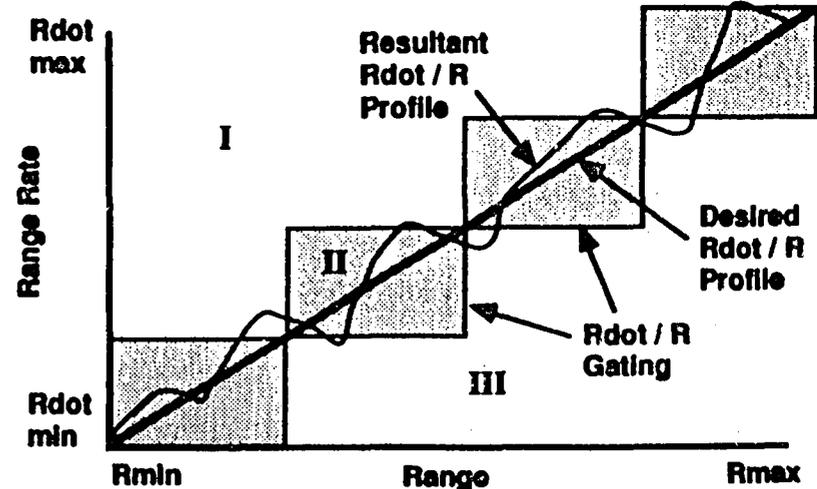
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PROBLEM FORMULATION (CONTINUED)

- Rotational state (3 DOF) automatically maintained by attitude determination / control and pointing systems
 - Command activation as a function of time and / or position
- Piloting task centers on execution of RCS translational firings (3 DOF)

- Range rate flown as function of range / time, the relation may be
 - Simply linear (see plot)
 - Based on non-linear empirical data



- Gate boundaries sized to subject vehicle and desired profile tightness

● Firing Decision Logic

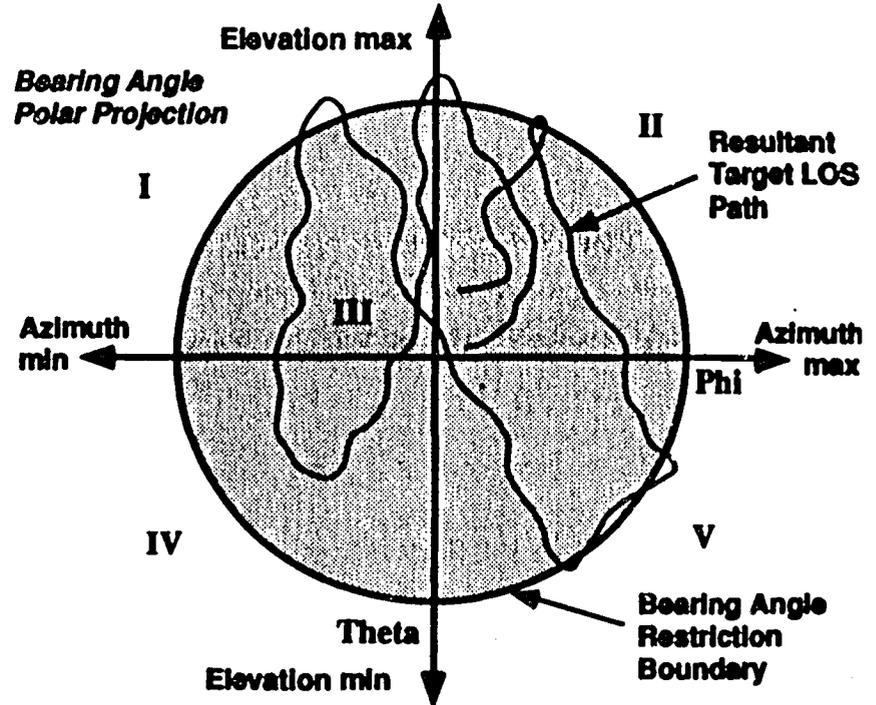
REGION	Rdot State	Thrust Reaction
I	Constant or Increasing	Decrease Rdot
	Close to Box & Decreasing	N/A, follow trend
II	Moderate or no Δ In Rdot	N/A, follow trend
	Rapid Δ In Rdot	Decrease Δ rate
III	Close to Box & Increasing	N/A, follow trend
	Constant or Decreasing	Increase Rdot

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PROBLEM FORMULATION

- Cross range flown as function of LOS bearing
 - Translational burns \approx normal to LOS
 - Data from CCTV, COAS, relative nav avionics
- Angle boundaries sized to subject vehicle and desired profile tightness
- Firing Decision Logic independent for azimuthal and elevation parameters in each zone



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REGION	Phi dot State	Thrust Reaction	Theta dot State	REGION
II, V	Constant or Increasing	Induce neg rate	Constant or Increasing	I, II
	Decreasing	N/A, follow trend	Decreasing	
I, IV	Constant or decreasing	Induce pos rate	Constant or decreasing	IV, V
	Increasing	N/A, follow trend	Increasing	
III	$ \text{Phi dot} > \text{Phi dot max}$	Decrease rate to below max	$ \text{Theta dot} > \text{Theta dot max}$	III

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SUMMARY

- Techniques exist that model, augment, automate prox ops piloting
- Overall task requires flexibility for broad application and poses multiple requirements constraints
- Automation may be applied to specific aspects of the prox ops task
- Traditional simulation paper pilots are tuned for their environment
 - Contributing vehicle factors:
 - Digital autopilot (DAP) / RCS configurations
 - Propulsion system characteristics
 - Sensor data quality
 - Tuning parameters:
 - Frequency of response
 - Number of input impulses per response
 - Filtering and / or trend evaluation of the sensor data
- Could fuzzy logic provide a way to develop generic automatic flight control applicable to various active vehicles for prox ops?

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— **Space Station Freedom**

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Topic: Proximity Operations
Presenter: Andy McGuire

Comment: Work has been done during the last two years on automatic flight control with a demonstration in 1988. --> Rotational Controller containing 31 rules.

N93-

3150

LAS

RENDEZVOUS GUIDANCE

BY
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ENGINEER

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N 93-31450

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■ Outline

- Introduction
- Problem Formulation
- Proposed Approaches
- Summary
- Panel Discussion

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— Space Station Freedom

McDonnell Douglas • GE • Honeywell • IBM • Lockheed

A. T. McGuire

RENDEZVOUS GUIDANCE

BY

ANDREW T. MCGUIRE

■ Introduction

- Space Station onboard software provides maneuver commands to cooperative unmanned vehicles attempting to rendezvous.
- Constraints affecting rendezvous include, station safety, fuel consumption, time limitations, etc.
- Several targeting algorithms may be employed to obtain the relative guidance maneuver commands.
- Two of these targeting algorithms will be addressed.
 - Lambert Targeting (point to point guidance).
 - Linear Quadratic Targeting (LQT) (closed loop guidance).

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— Space Station Freedom

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A. T. McGuire

11/13/90-page-2

file

RENDEZVOUS GUIDANCE

BY
ANDREW T. MCGUIRE

■ Problem Formulation

- Onboard software responsible for unmanned vehicles in the Command and Control Zone (CCZ).
- CCZ dimensions: 12 km radially, thickness of 8 km
- The rendezvous maneuver brings the vehicle to a holding point at the edge of the proximity operations zone (POZ).
- POZ dimensions: 1km sphere centered at the Station.

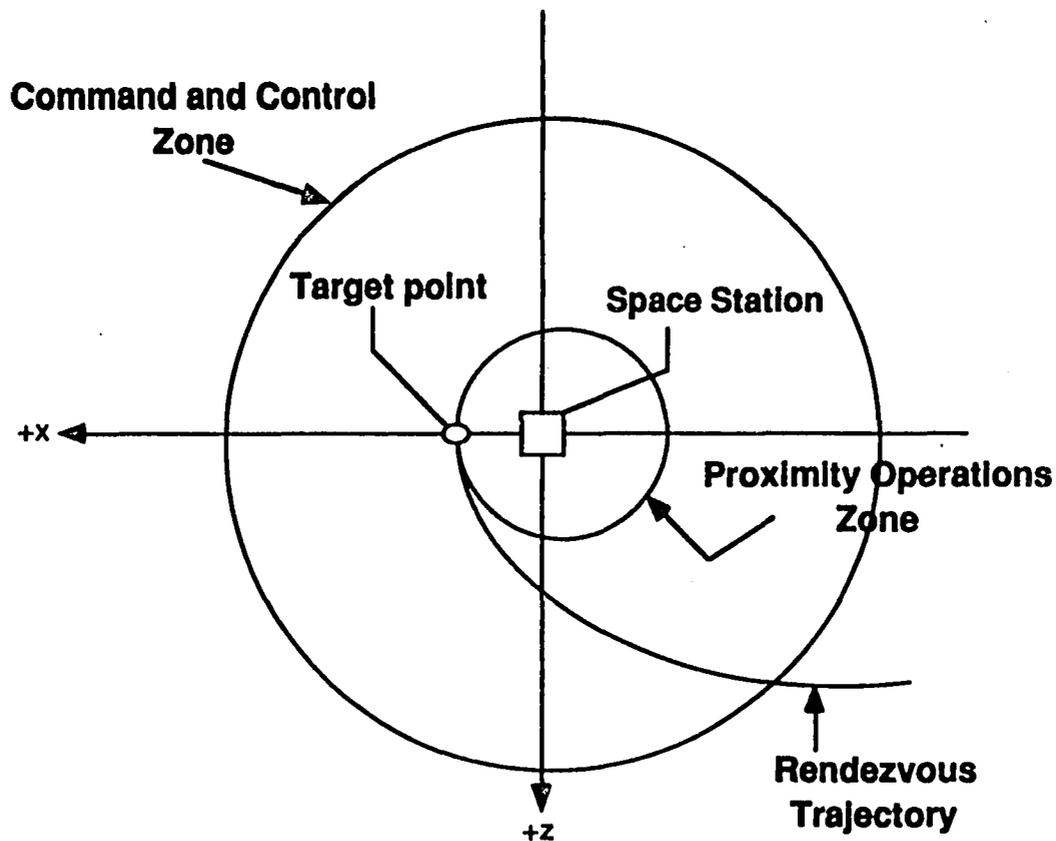
359

RENDEZVOUS GUIDANCE

BY

ANDREW T. MCGUIRE

- A generic rendezvous scenario in LVLH coordinates.



360

— Space Station Freedom

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11/13/90-page-4

A. T. McGuire

file

RENDEZVOUS GUIDANCE

BY

ANDREW T. MCGUIRE

■ Proposed Approaches

● Lambert Targeting Solution

- Relative guidance solves the rendezvous targeting error function for zero. $\delta(\bar{x}) = f_c(\bar{x};t) - f_t(t) = 0$

— $f_t(t)$ is the target state at the rendezvous time t , $f_c(\bar{x};t)$ is the rendezvous vehicle state.

— \bar{x} is the control vector

$\bar{x} = \begin{bmatrix} \Delta \bar{V}_1 \\ \Delta \bar{V}_2 \end{bmatrix}$ where $\Delta \bar{V}_1$ and $\Delta \bar{V}_2$ are the impulsive maneuver burns

— The vector prediction function is defined as, $f(\bar{x};t) = \begin{bmatrix} \bar{R} \\ \bar{V} \end{bmatrix}$ where

as \bar{R} and \bar{V} are the inertial position and velocity vectors of the vehicle performing the rendezvous at time t .

— f evaluated with a predictor and $\delta(\bar{x})$ with Newton's method.

- Lambert Targeting only satisfies the end conditions.

— Space Station Freedom

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A. T. McGuire

RENDEZVOUS GUIDANCE

BY
ANDREW T. MCGUIRE

■ Proposed Approaches (cont.)

- Linear Quadratic Targeting (LQT) Solution
- The LQT scheme is a closed loop optimal control problem.
- Forces the system to track a desired trajectory over a given time interval.
- Using the Clohessy-Wiltshire equations our system is modelled as: $x_{k+1} = Ax_k + Bu_k$, $k > i$, with system output being $y_k = Cx_k$.
- It is desired to make the output state follow a desired reference state r_k over a time interval $[0, N]$ so the cost function can be minimized.

$$J_i = \frac{1}{2}(y_n - r_n)^T P(y_n - r_n) + \frac{1}{2} \sum_{k=i}^{N-1} \left[(y_k - r_k)^T Q(y_k - r_k) + u_k^T R u_k \right].$$

- x is the system state, u is the control, and the weighting matrices are $P \geq 0$, $Q \geq 0$, $R > 0$, with all three being symmetric.

RENDEZVOUS GUIDANCE

BY
ANDREW T. MCGUIRE

■ **Proposed Approaches cont.**

- **The reference trajectory is the desired rendezvous trajectory.**
- **The weighting matrices, are user specified.**
 - P weights the terminal state values, Q weights the state trajectory values, and R weights the control values.
- **By manipulating the weighting matrices you can sculpt the resulting trajectory to fit endpoint dispersion constraints, fuel consumption constraints, etc.**
- **Advantage over Lambert targeting**
 - **Desired trajectory can be arbitrary. (Can be contrary to orbital dynamics)**

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RENDEZVOUS GUIDANCE

BY

ANDREW T. MCGUIRE

■ Summary

- Lambert Targeting results in fairly high end-point dispersions .
- Can reduce dispersions by incorporating mid-course correction burns.
- Lambert Targeting is bound to orbital dynamics during the coast phase of the maneuver.
- LQT can significantly reduce endpoint dispersions with little or no additional fuel consumption.
- LQT can follow any reference trajectory desired regardless of orbital dynamics involved.
- The LQT weighting matrices must be reinitialized with each new reference trajectory in order to perform optimally.
- Fuzzy Logic Control has been discussed as being potentially applicable to the rendezvous guidance control.
- (Opening question for Panel discussion)

Can Fuzzy Logic control offer any advantages over LQT in computational simplicity, or ability to eliminate weighting matrix reinitialization.

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Topic: Path Planning Control
Presenter: Malcolm McRoberts

No notes were taken during this presentation.

N93-

31 51

CLAS

PATH PLANNING CONTROL

by

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- **Problem Statement**

- Motion planning for redundant robots in a constrained environment

- **Current Approaches**

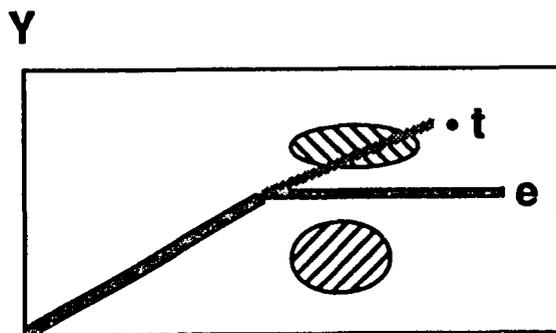
- Model Based
- Sensor Based
- Hybrid

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INTERCOMPARISON
PAGE

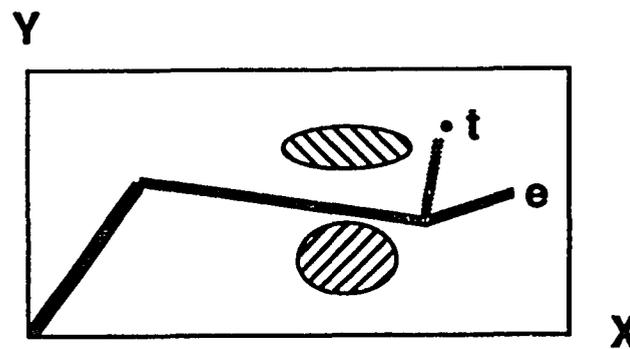
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PLANAR MODEL

Task Space

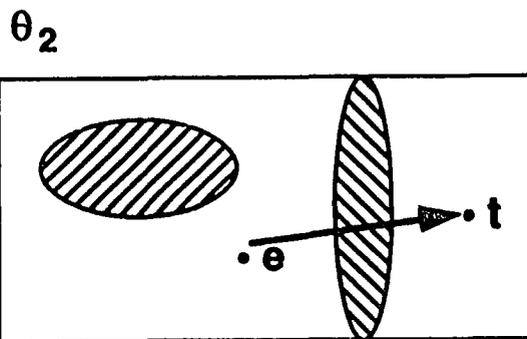


2 Degree of Freedom

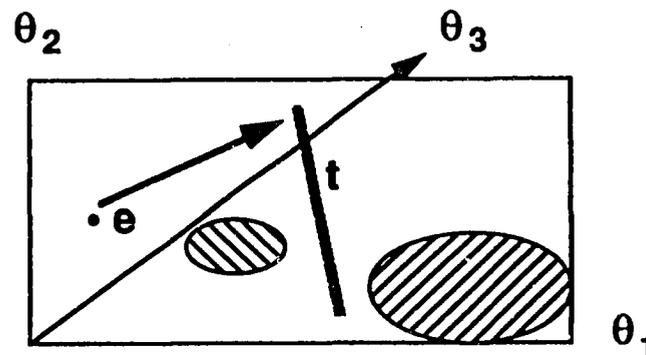


3 Degree of Freedom
(redundant)

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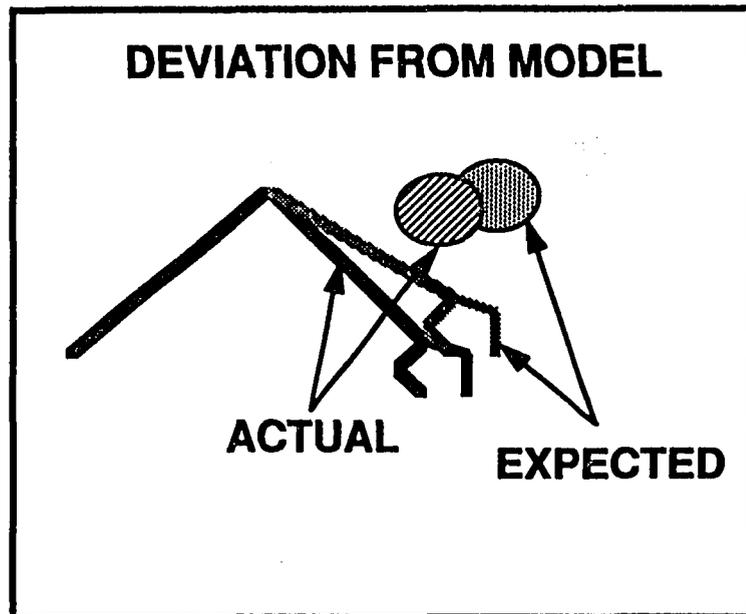
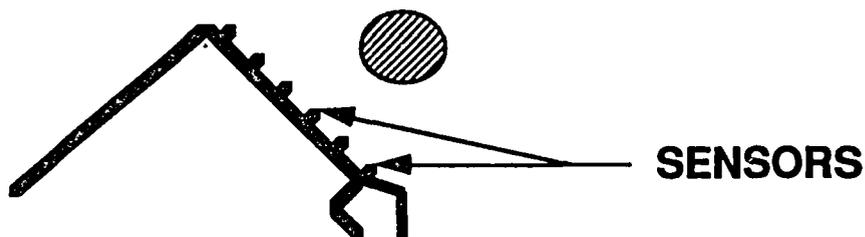
Joint Angle Space



APPROACHES

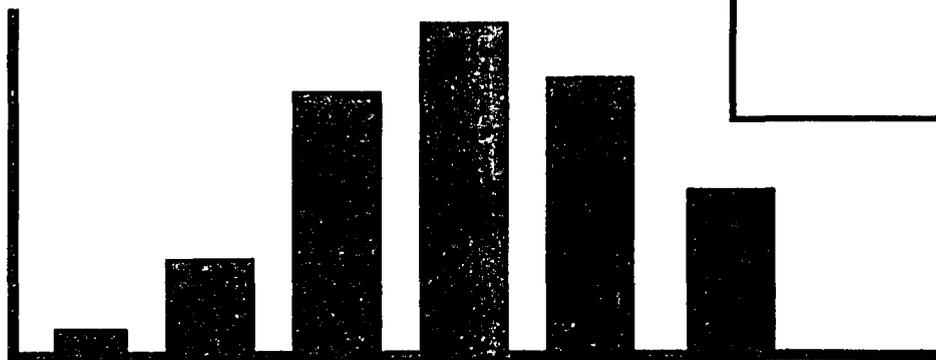
- **Model Based Path Planning**
 - **Uses CAD model (a priori knowledge)**
 - **Transformations done offline**
 - **Uses large granularity for efficiency**
 - **Large search space (exponential in DOF)**
 - **Environment must be static (i.e. no humans)**
- **Sensor Based Path Planning**
 - **Detect obstacles before collision**
 - **Non-optimal path (may wander)**
 - **May have very high degree of sensor redundancy**
 - **Sensors usually located on robot (work in robot space)**
- **Hybrid Path Planning**
 - **Best of both worlds**
 - **Requires fusion of model and sensor information**

PROXIMITY SENSOR ARRAY



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SENSOR RESPONSE



SENSOR LOCATION

APPLICATIONS FOR FUZZY LOGIC

- **Model based**
 - **No closed form solutions for redundant manipulators**
 - **Large search space**
 - **Interpolations between tessellations**
- **Sensor based**
 - **Combine redundant information**
 - **Can calculate approximate range and size of obstacles**
 - **Optimal path between 2 obstacles**
- **Hybrid**
 - **Same as model and sensor**
 - **Resolving conflicts between model and sensor data**

Topic: Path Planning Control
Presenter: Malcolm McRoberts

No notes were taken during this presentation.

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31 52

PLAS

400

N 9 3 - 3 1 4 5 2

ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by

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■ Outline

- Introduction
- Problem Formulation
- Proposed Approaches
- Simulation Demonstration
- Summary
- Panel Discussion

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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ INTRODUCTION

- The Shuttle Remote Manipulator System (SRMS) is a highly flexible dynamical structure which results in vibration in its links, most commonly seen at the end-effector.
- The vibration is caused by several factors: deformable material used for construction of the links, non-rigidity of the brakes and gears at the joints, characteristics of the controllers of the joint servos.
- In this report, we will address the following:
 - (i) Modeling of a flexible manipulator dynamical structure
 - (ii) Designing control law criterion that minimizes vibration
 - (iii) Candidate Application of Fuzzy Logic Control Law to the Problem

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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

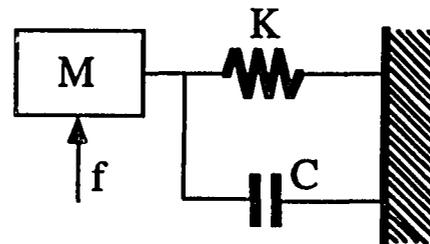
by
Trung T. Pham
(Continued)

■ PROBLEM FORMULATION

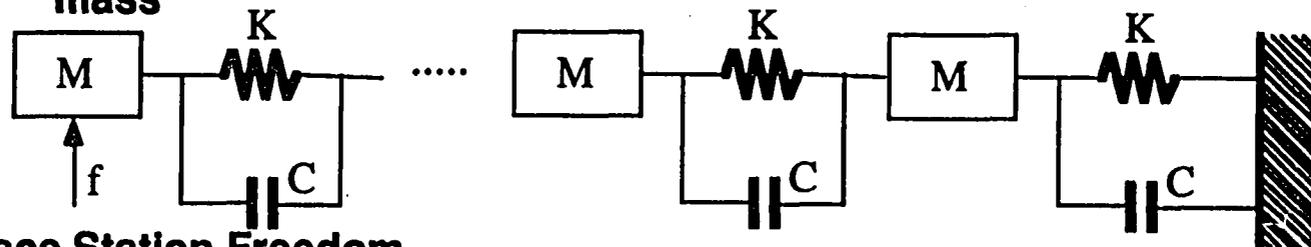
- A point mass in vibration is governed by the equation of motion:

$$M a + C v + K r = f$$

where M is the mass, K the stiffness, C the damping, f the external force, r the displacement, v the rate of displacement, and a the rate of v



- A classical approach in formulating a long flexible rod is to treat the rod as a set of small segments, each representing a point mass



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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ PROBLEM FORMULATION (cont.)

● Control Laws:

$$\min J(q,u) \quad (\text{J is the cost function})$$

s.t.

$$\dot{q} = f(q) + g(q) T \quad (\text{the equation of motion})$$

$$\frac{E_i}{\rho_i} \frac{\partial^2 u_i}{\partial x_i^2} - \frac{\partial^2 u_i}{\partial t^2} = \xi(T,q) \quad (\text{the equation of vibration})$$

where q is the state of the manipulator, u the vibration, T the joint torque vector, E_i the Young's modulus, ρ_i the mass density, and x_i the position on link i with respect to joint i .

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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ PROPOSED APPROACHES

- Definition of sensor data; type, rate: The followings are assumed provided by external sensors: state (joint angles and joint angular velocities), joint positions, mass of payload; and by analysis: equation of motion of the manipulator, equation of vibration
- Definition of positive control authority: The manipulator is put into motion by the joint torques. There are limitation in the amount of joint torques, joint angles, and joint angular velocities
- Formulate the Control Criteria: minimizing vibration, minimizing response time, minimizing error & error rate, minimizing energy

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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ PROPOSED APPROACHES (cont.)

- It can be shown that the bang-bang control is one of the most efficient treatment of the previously defined optimal control problem.

if $v(q,u) < D$ then $T = T_{max}$
else $T = T_{min}$

- Since u is an estimation of the vibration, one can update this model by camera observation data, with some relatively confident factor p .

ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ SIMULATION DEMONSTRATION

- This simulation illustrates the following concepts:
 - (i) Flexible Model of a Long Rod
 - (ii) Non-Rigid Model of a Joint Brake
 - (iii) PID Controller of a 1-Link, 1-DOF Flexible Manipulator

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ACTIVE DAMPING IN A FLEXIBLE MANIPULATOR

by
Trung T. Pham
(Continued)

■ SUMMARY

- Techniques to model the flexible dynamics of a manipulator have been identified
- Control Criteria (specifications) for motion control of the flexible manipulator have been formulated
- Fuzzy Logic Control Law has been discussed with application to the Shuttle Remote Manipulator System
- (Opening question for Panel discussion)
Given the sensor information and the control criterion, can Fuzzy Logic offer an advantage in computational simplicity, and what approach would you recommend, eg. additional sensory information, sensor fusion, and feedback control.

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Topic: Active Damping in a Flexible Manipulator
Presenter: Trung Pham

Comment: (Sugeno) To apply fuzzy logic, should not use feed forward because this system should include time data. Feedback is not so good if there is a time delay.

Comment: (Pham) Would be interested in comparing these two approaches.

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5

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LAS

TETHERED SYSTEMS CONTROL

MDSSC-SSD

OUTLINE

- Introduction
- Problem Formulation
- Proposed Approaches
 - Controller Structure
 - Proposed Controllers
- Simulation Examples
 - Deployment
 - Retrieval
- Summary

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P.9

N 93-81453

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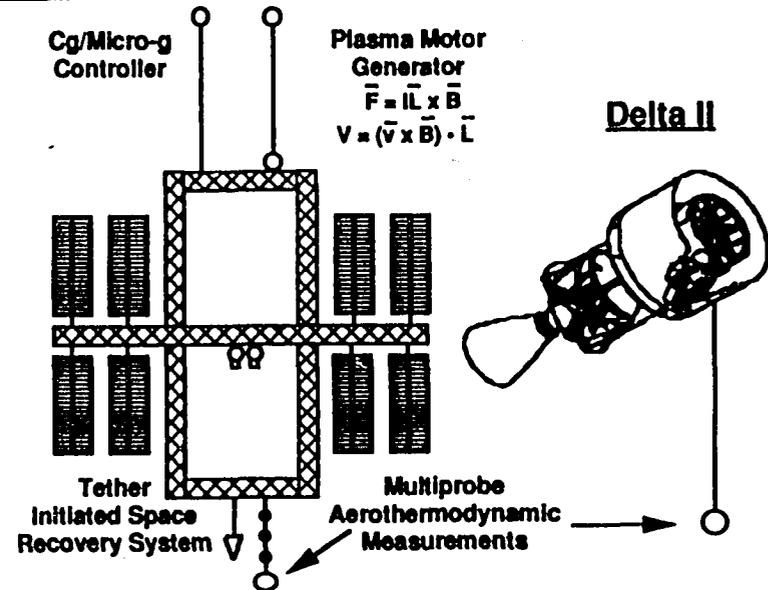
TETHERED SYSTEMS CONTROL

MDSSC-SSD

Introduction

- Tethers can provide
 - Momentum exchange
 - Mechanical/electrical conversion
 - Electrical/mechanical conversion

- Technology applications
 - Acceleration environment control
 - Payload transportation
 - Electrical power generation
 - Attitude and orbital control



Relevance

- Project applications
 - Atmospheric science research
 - Payload boost/deboost
 - Spacecraft power generation
- Advantages
 - Data gathering
 - Application alternatives (Safety)
- NASA near term activities
 - Delta II/SEDS flight experiment
 - STS/Tethered Satellite System

Problem Overview

- Operational phases
 - Deployment, retrieval
 - Stationkeeping
- Dynamics
 - Nonlinear, time-varying, coupled
 - Unstable, elastic, uncertain
- Performance criteria
 - Libration magnitude
 - Deployment/retrieval time
 - Payload disturbances

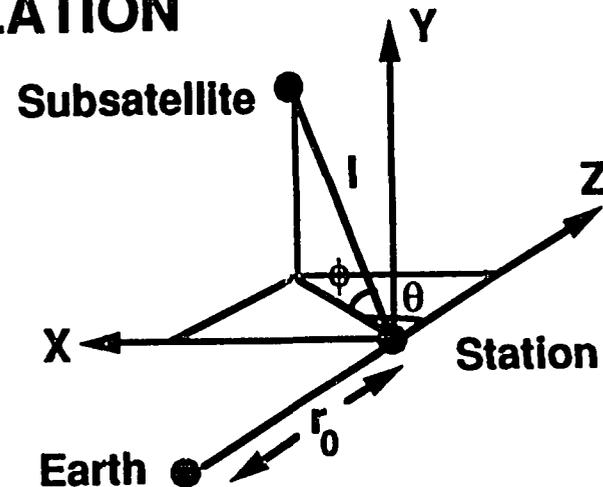
TETHERED SYSTEMS CONTROL

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PROBLEM FORMULATION

■ Rigid body simplifications

- Point masses (Station and subsatellite)
- Station mass \gg Subsateellite mass
- Massless, rigid tether
- Spherical earth
- Circular orbit



$$\ddot{l} - [\dot{\phi}^2 + \cos^2 \phi (\omega_0 + \dot{\theta})^2 - \omega_0^2 + 3\omega_0^2 \cos^2 \phi \cos^2 \theta] = \frac{Q_l}{m_p}$$

$$\ddot{\theta} + \frac{2\dot{k}(\omega_0 + \dot{\theta})}{l} - 2 \tan \phi (\omega_0 + \dot{\theta}) \dot{\phi} + 3\omega_0^2 \sin \theta \cos \theta = \frac{Q_\theta}{m_p l^2 \cos^2 \phi}$$

$$\ddot{\phi} + \frac{2l\dot{\phi}}{l} + \cos \phi \sin \phi [(\omega_0 + \dot{\theta})^2 + 3\omega_0^2 \cos^2 \theta] = \frac{Q_\phi}{m_p l^2}$$

■ Observations

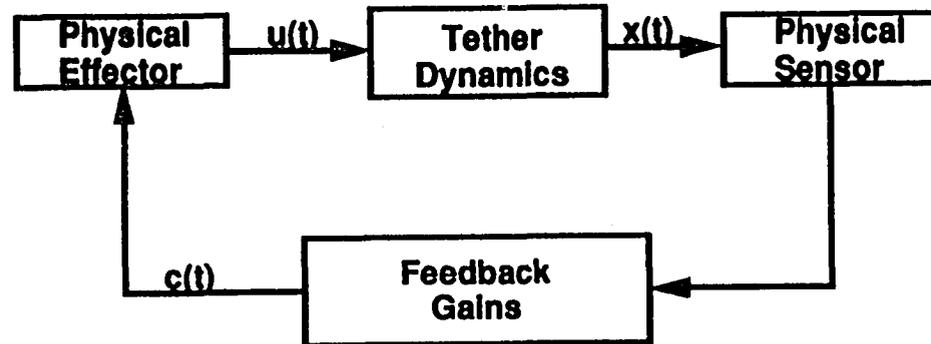
- Dynamics are nonlinear (trig, products of states)
- Retrieval dynamics involve negative libration damping

TETHERED SYSTEMS CONTROL

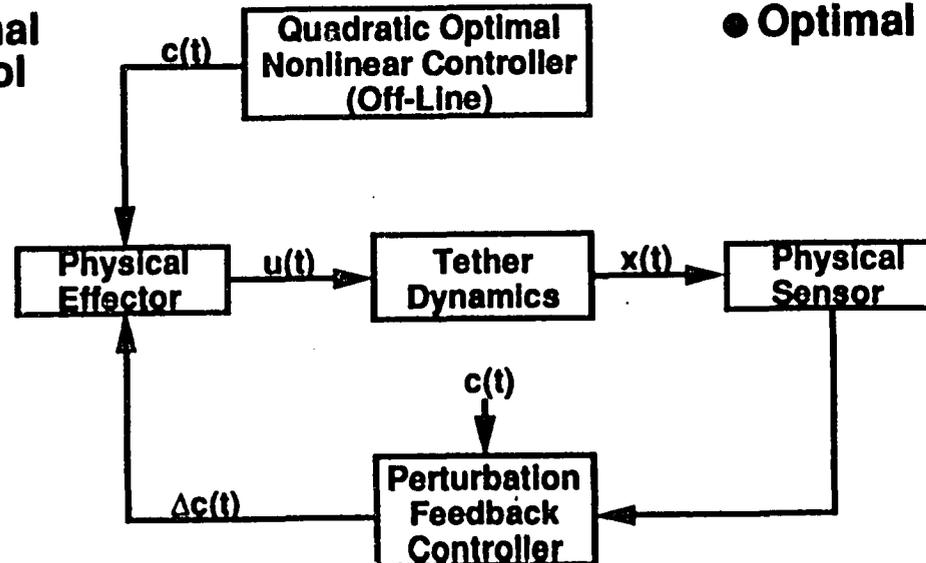
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PROPOSED APPROACHES (CONTROLLER STRUCTURE)

■ Feedback control



■ Quadratic optimal nonlinear control



Recent approaches

- Nonlinear feedback
- Optimal gains

TETHERED SYSTEMS CONTROL

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PROPOSED APPROACHES (PROPOSED CONTROLLERS)

- Quadratic optimal nonlinear controllers
 - Tension Optimal Nonlinear Controller
 - Newton-Raphson, steepest descent
 - Divergence (explicit feedback, tension delay)
 - Length Rate Optimal Nonlinear Controller (LRONC)
 - Steepest Descent
 - Direct damping control
 - Suboptimal Nonlinear Controller (SONC)
 - Feedback structure (gain optimization)
 - Powell's Method

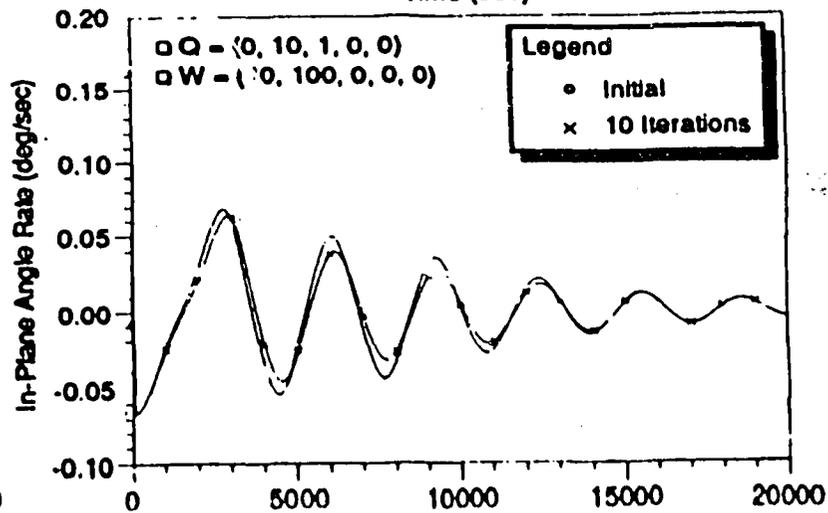
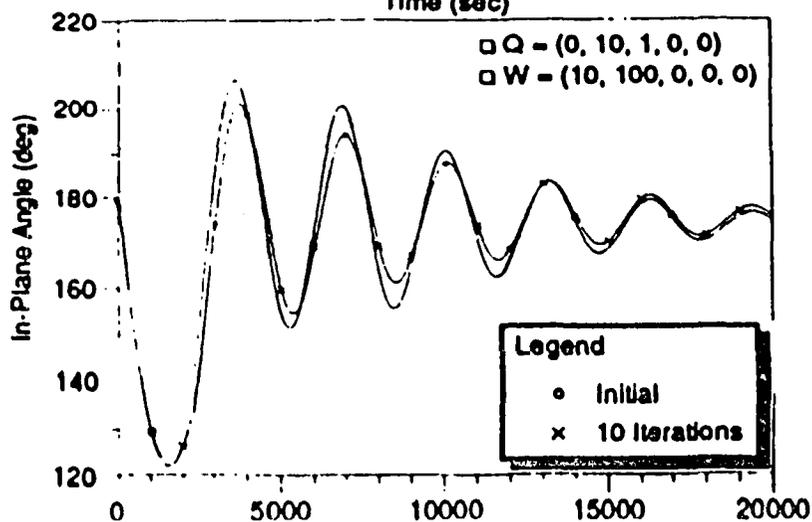
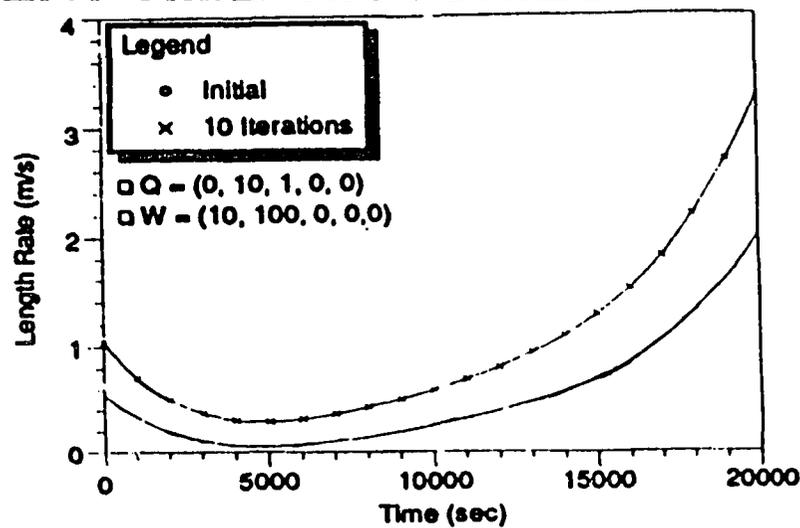
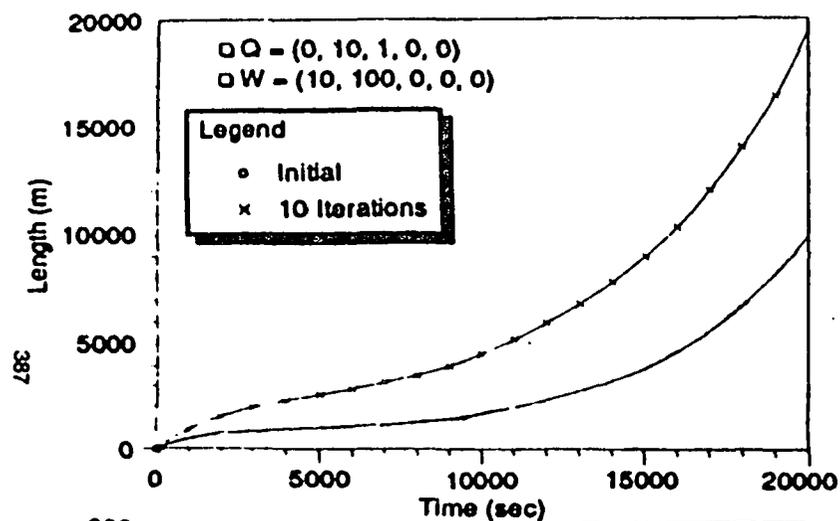
- Lyapunov-based nonlinear controllers
 - Mission/distance function concept
 - Mission Function Control (MFC)
 - Lyapunov Optimal Feedback Controller (LOFC)

- Sensor/actuator options
 - Measurements - length, tension, deployment attitude, P/L accels
 - Actuators - spring ejection mechanism, reel, thrusters

TETHERED SYSTEMS CONTROL

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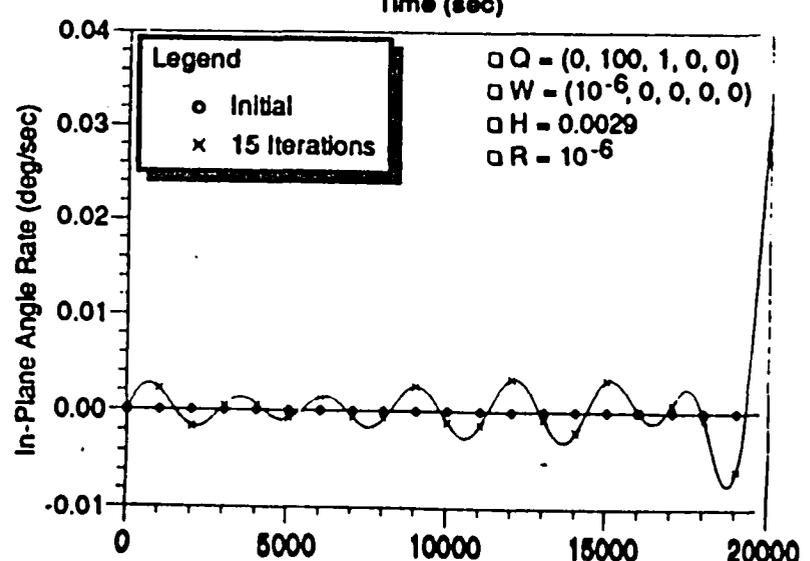
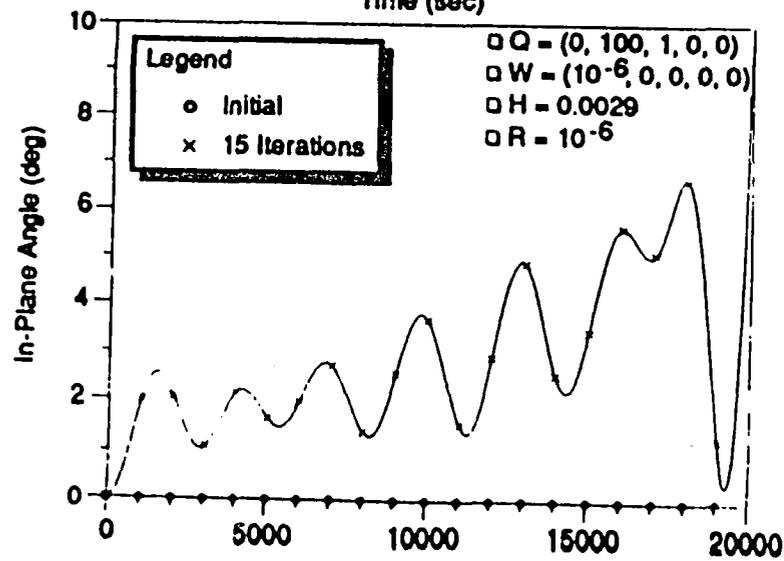
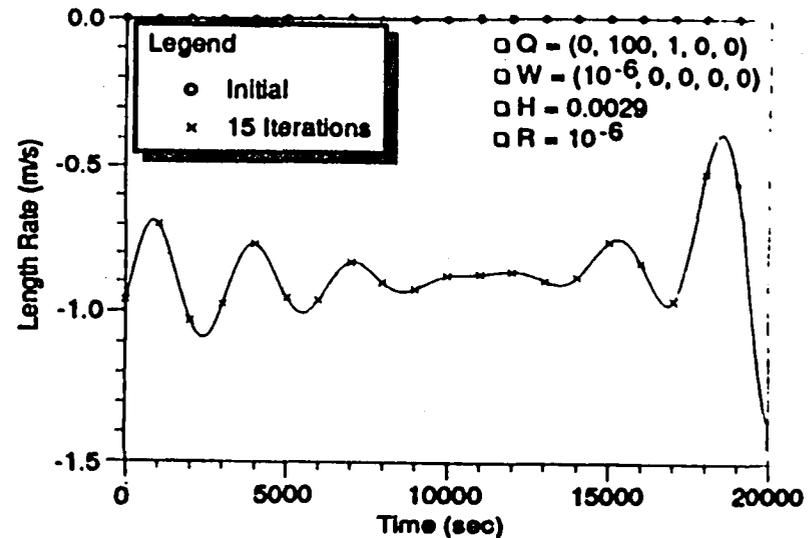
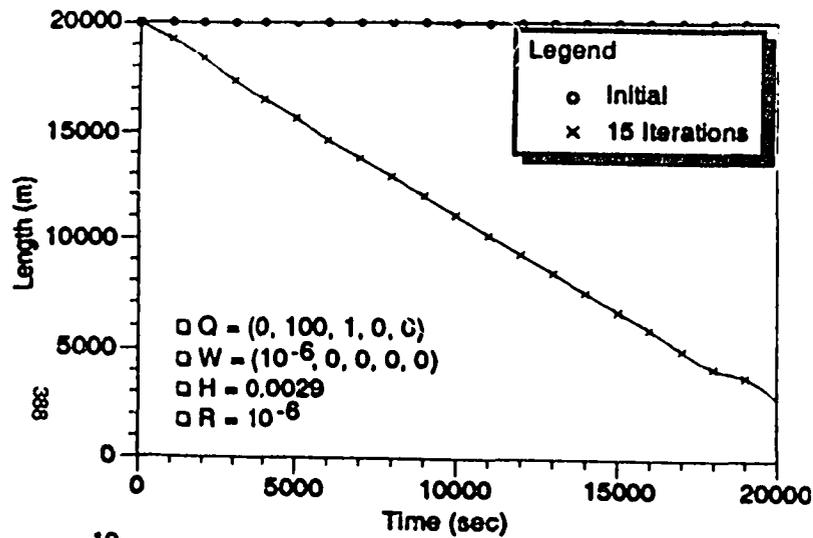
LRONC- DEPLOYMENT TIME HISTORIES



TETHERED SYSTEMS CONTROL

MDSSC-SSD

LRONC - RETRIEVAL TIME HISTORIES



TETHERED SYSTEMS CONTROL

MDSSC-SSD

SUMMARY

- Tethered systems have many potential applications
- Tether dynamics encompass typical spacecraft control issues (stability, nonlinearities, coupled dynamics)
- Tether control improvements could provide benefits in such areas as retrieval stability and mission timelines
- Question - Can fuzzy logic control techniques help with the tethered systems control problem ?

Topic: Tethered Systems Control
Presenter: Dan Nowlan

No notes were taken during this presentation.

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CLAS

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HYPERBARIC TREATMENT

by
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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ OUTLINE

- Introduction
- Problem Formulation
- Proposed Approaches
- Summary
- Panel Discussion

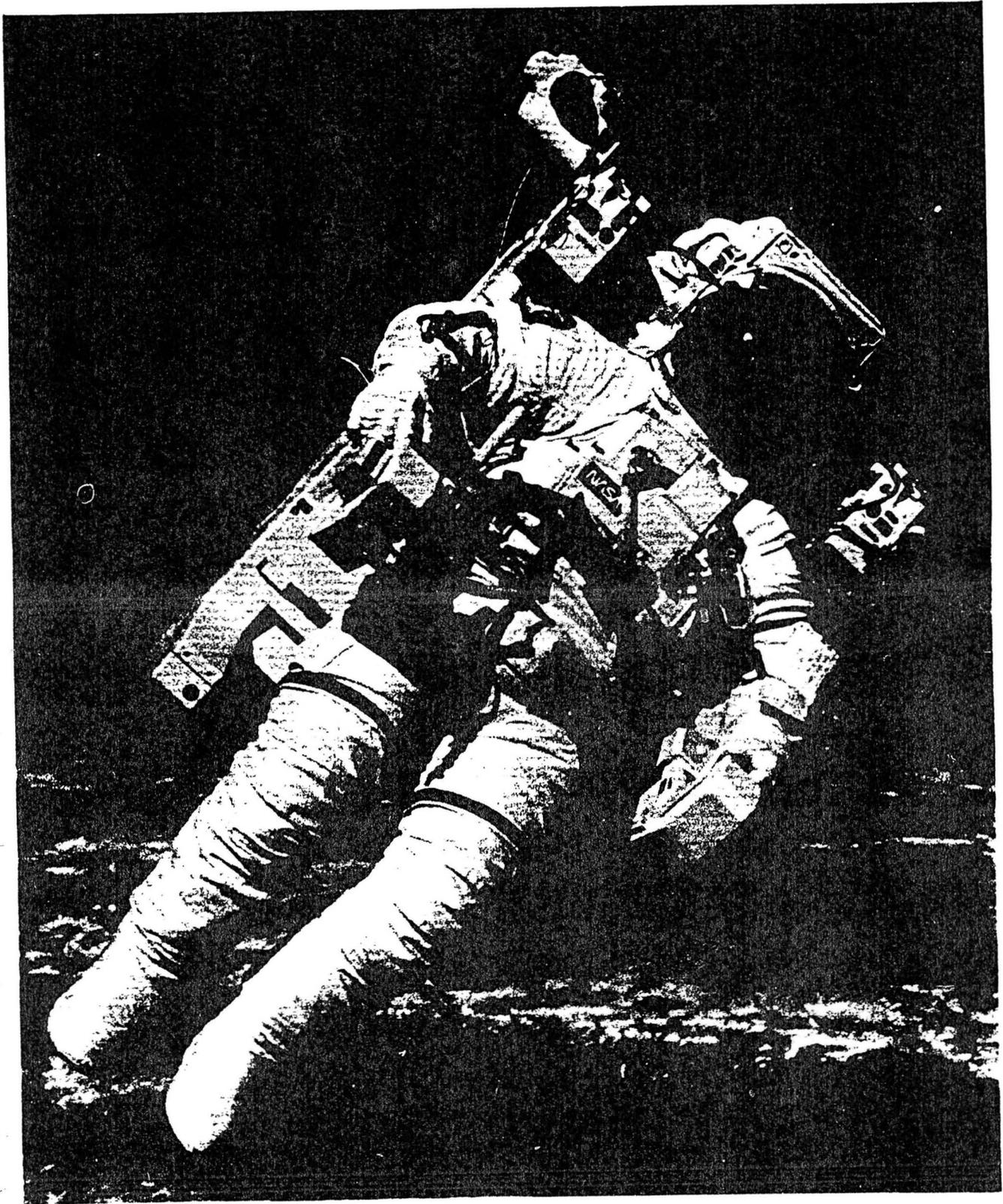
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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ INTRODUCTION

- **Hyperbaric Treatment - Purpose**
- **Decompression sickness - Description**
- **Sources of decompression sickness**
- **Physical description**
- **Forms of decompression sickness**
- **Hyperbaric treatment of decompression sickness**
- **Duration of treatment**



HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ PROBLEM FORMULATION

- Determination of duration of hyperbaric treatment
- Decision points in determining the duration of treatment
- Relationship of exhaled breath gas composition to decompression sickness
- Required equipment to monitor inert gases in exhaled breath
- Space Station unique circumstances

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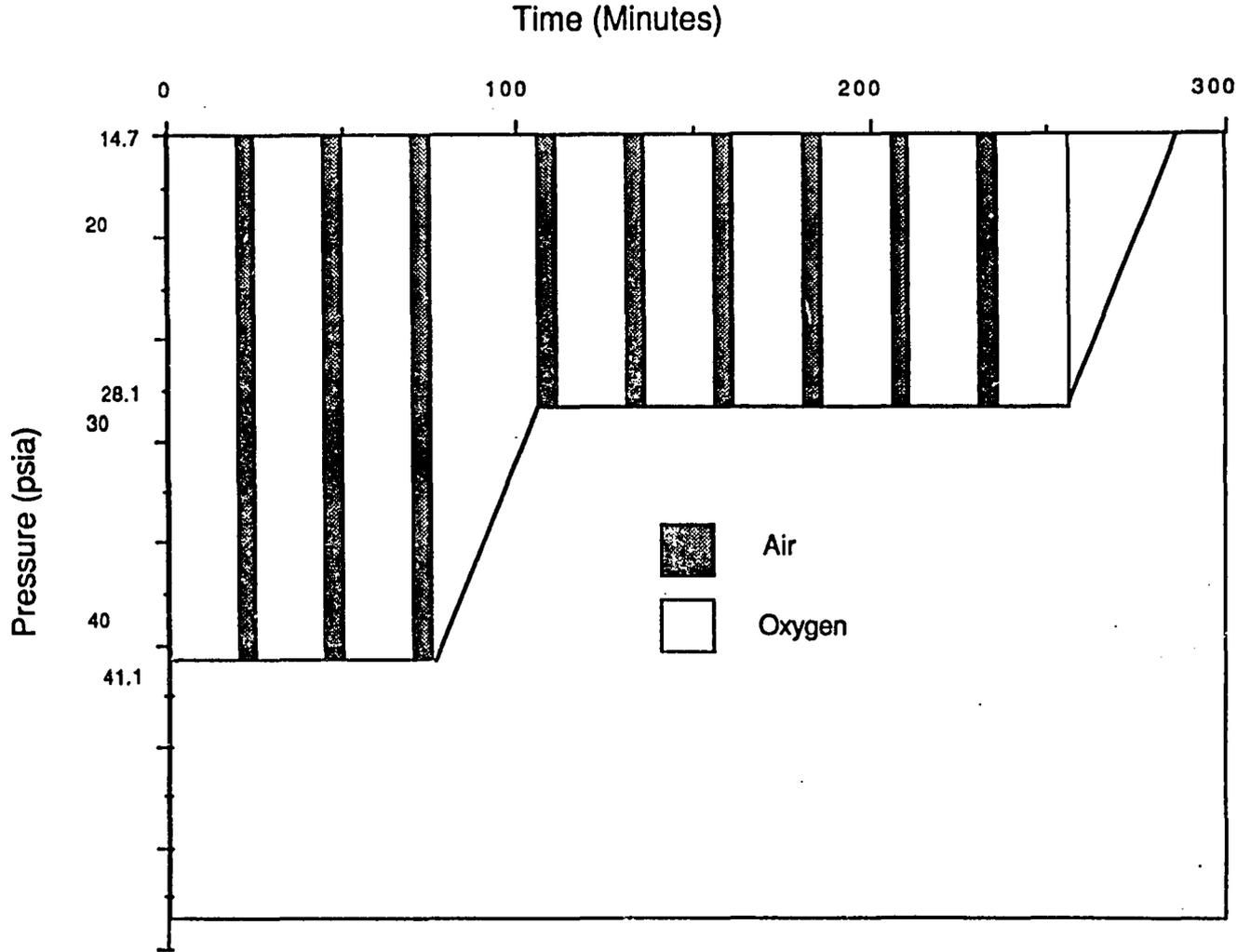


Figure 1 Hyperbaric Treatment Table 6.

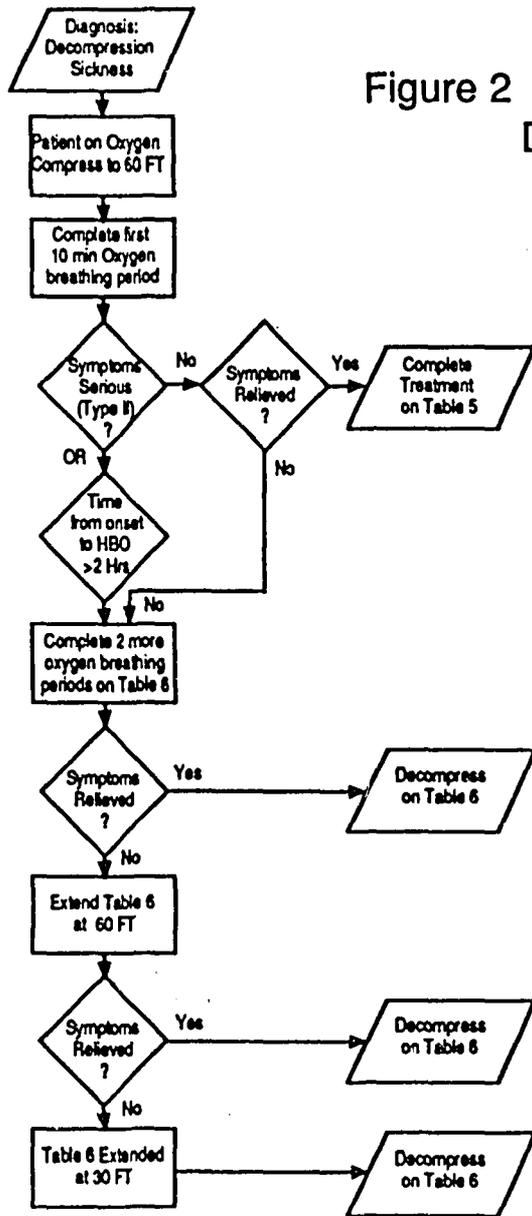
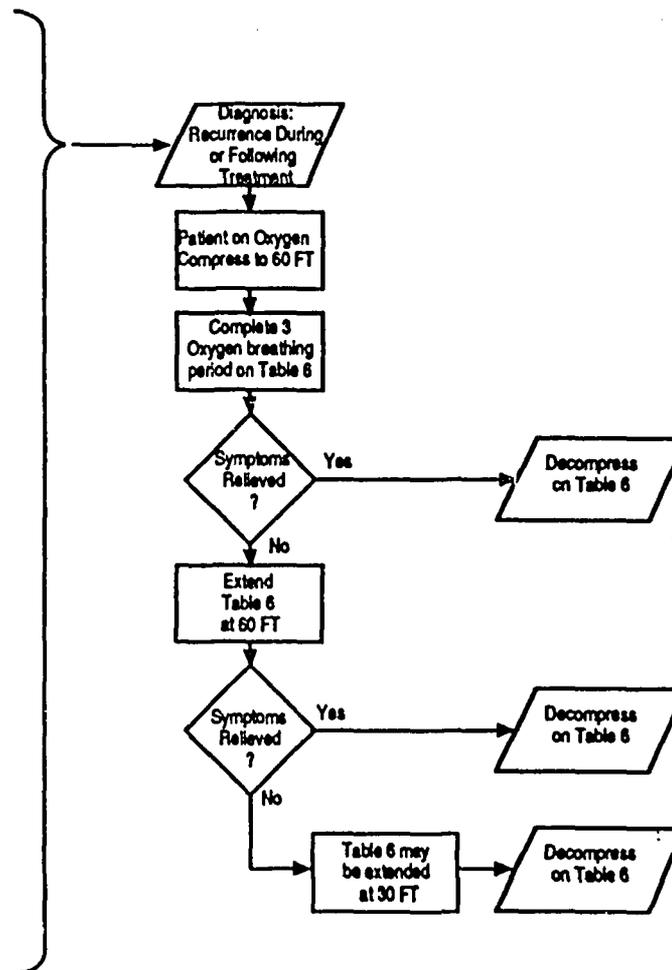


Figure 2 Decision Points for the Treatment of Decompression Sickness



HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ PROPOSED APPROACHES

- Optimize the duration of hyperbaric treatment**
 - Reasons**
- Utilize all information available**
 - Types of data available**

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ SUMMARY

- **Derived design goals of Hyperbaric Treatment**
- **Traditional treatment protocols**
- **Instrumentation already available**
- **Hundreds of hyperbaric chambers could benefit**

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ **SUMMARY (Continued)**

- (Opening question for Panel discussion)

Is it possible to use Fuzzy Logic and the additional information available to accurately determine the required duration of hyperbaric treatment?

– The additional information is :

- 1). Instrument Data: hyperbaric chamber pressures, pressure rate of change, and the exhaled breath gas compositions
- 2). The objective data from the examining attendant: patient's vital signs
- 3). The subjective data: from the patient's responses to the questions of the attendant
- 4). The exposure data: duration and pressure of exposure

HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ SUPPLEMENTAL INFORMATION

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HYPERBARIC TREATMENT

by

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Space Station Freedom

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ OUTLINE

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- Introduction
- Problem Formulation
- Proposed Approaches
- Summary
- Panel Discussion

HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ INTRODUCTION

- **Hyperbaric Treatment is the medical protocol used to treat Decompression Sickness.**
- **Decompression Sickness can result from exposure to a significant reduction in ambient pressure.**
- **Space Station crewmembers risk decompression sickness when they perform an extra-vehicular activity (working outside the Space Station). Another risk is the unexpected decompression of a Space Station module.**
- **Decompression sickness is characterized by inert gases in the body coming out of solution and forming gas bubbles in the body tissues.**
- **The most common form of decompression sickness is the bends. Severe cases can be fatal.**

HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ INTRODUCTION (Continued)

- The only definitive treatment for decompression sickness is hyperbaric treatment - treating the patient with pressure and oxygen for sustained periods of time.
 - A common hyperbaric therapy protocol is shown in Figure 1. This table and others were developed by the Navy, essentially by trial and error.
- Hyperbaric treatments for Space Station will range from 2 Hours to a maximum of 67 Hours at pressures as high as 2.8 atmospheres.
- Hyperbaric treatment requires 3 persons, in addition to the person being treated. This involves the entire Space Station crew.

HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ PROBLEM FORMULATION

- There are no quantitative measurements which can be monitored to accurately determine the required duration of hyperbaric treatment.
- Figure 2 shows decision points that terrestrial observers use to determine the duration of hyperbaric treatment.
 - The decision points are boolean. No provisions are made for partial relief of symptoms.
- Examination of the relationship between the amount of inert gases in the exhaled breath and the amount of inert gases remaining in the body.
 - Not a deterministic relationship. Other data needed.
- A form of mass spectrometer can determine the inert gasses in the exhaled breath.
 - This equipment will be available on Space Station.

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ **PROBLEM FORMULATION (Continued)**

- **Unique circumstances onboard Space Station**
 - **Minimal crew of 4**
 - **Far away from help**
 - **No Medical Doctor**
 - **Limited amount of resources**

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ PROPOSED APPROACHES - DESIGN GOALS

- **Optimize duration of Hyperbaric Treatment to:**
 - **Get medical treatment to a crewmember in hyperbaric treatment.**
 - **Return injured crewmember to earth as soon as possible.**
 - **Alleviate other crewmembers from hyperbaric duties to perform critical activities.**
 - **Conservation of Resources.**
- **Utilize all available information in determination of ideal duration of treatment.**
 - **Objective instrument data**
 - **Objective medical attendant data**
 - **Subjective data from patient's responses**
 - **Exposure data**

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ SUMMARY

- **Derived design goals of Hyperbaric Treatment of decompression sickness**
 - Reduce delays in treating other injuries
 - Return patient to earth as soon as possible
 - Free up other crewmembers for critical activities
 - Conserve vital resources
- **Traditional treatment protocols do not optimize the design goals**
 - Boolean decisions based on primarily subjective inputs
- **The instrumentation to provide objective inputs will be available on Space Station.**
- **Hundreds of hyperbaric chambers in terrestrial use could benefit from an accurate determination of required hyperbaric treatment.**

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HYPERBARIC TREATMENT

by
Michael T. Amoroso
(Continued)

■ SUMMARY (Continued)

● (Opening question for Panel discussion)

Is it possible to use Fuzzy Logic and the additional information available to accurately determine the required duration of Hyperbaric Treatment?

– The additional information is :

- 1). Instrument data: hyperbaric chamber pressures, pressure rate of change, and the exhaled breath gas compositions.
- 2). The objective data from the examining attendant: patient's vital signs.
- 3). The subjective data: from the patient's responses to the questions of the attendant.
- 4). The exposure data: duration and pressure of exposure.

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Topic: Hyperbaric Treatment Control
Presenter: Mike Amoroso

Comment: (Amoroso) Every time astronauts do EVA, they risk decompression sickness. With decompression sickness, pressure and oxygen treatment must be in precise control. Three people are required to perform hyperbaric treatment – timekeeper, chamber controller and an astronaut is needed to control the pressure. They monitor three variables – oxygen, pressure, and time.

Q (Brown): Is there a correlation between past treatments and additional data? (refer to the summary on page 7 of presentation)

A: - Exhaled breath can be used, but more information is needed.
- Duration of exposure and time of overall treatment is not currently being looked at by anyone.
- No data to clarify change to treatment tables; they were developed through trial and error. SCUBA diving incidents make up the most cases of the bends.
- Active modelling is not being performed.
- No bends on Space Activity. It is estimated that there is a 2-3% treatable bends risk per EVA.

Q (Lawler): What about data from the Russians?

A (Spoor): The Russians say they have no problems. There has been one instance of joint pains which may or may not be attributable to the bends (both the U.S. and Russia have reported this).

Comment: (Spoor) When dealing with human sickness, you often get "fuzzy" answers (subjective information). For example, trying to detect the extent of numbness, how "weak" is an arm, "are you 'ok'?". The inputs to the decision maker are not distinct values.

Q (Aldridge): Has there been treatment based on anything other than the hyperbaric chamber?

A (Spoor): It's possible to bring the Shuttle cabin down as low as is feasible; pump up space suit as high as possible (1.3 - 1.5 atmosphere).

Q (Lawler): Will Life Sciences "buy off" on this application?

A (Spoor): They see the potential.

Q (Lawler): Have you talked with the CHeCS people (on the NASA side)

A (Spoor): Yes, we have been talking to Dr. Norfleet (Crew Hyperbaric).

Comment: Value added of using fuzzy logic application --> elimination of using an extra crew member for treatment.

Q (Stegall): Is there some way to implement the application as an advisory position?

A: Absolutely. The ultimate (medical) decision is with the doctor (on the ground).

Comment: (McRoberts) There are three tests:

- (1) How does the patient feel?
- (2) Schedule of decompression
- (3) Executing - sequential

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CLAS

N93-31455

DAILY EXERCISE ROUTINES

by

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■ OUTLINE

- Introduction
- Problem Description
- Proposed Approaches
- Summary
- Panel Discussion

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PAGE

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INTERNATIONAL

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DAILY EXERCISE ROUTINES

by

Patrick L. Anderson and Michael T. Amoroso
(Continued)

■ INTRODUCTION

- **Daily exercise and periodic stress testings are required on Space Station.**
 - **Exercise Equipment:** Treadmill, Dual Bike, Muscular Output Trainer, and portable exercise equipment.
 - **Physiological Monitors: Daily -** Heart Rate Monitor. **Periodic -** Blood Pressure Monitor, Electrocardiograph, Metabolic Gas Monitor, and Body Mass Measurement Device.
- **The equipment is configured either automatically or manually.**
- **Data is obtained from the physiological monitors and logged by the system.**
- **Exercise physiologist monitors data and updates the protocols.**

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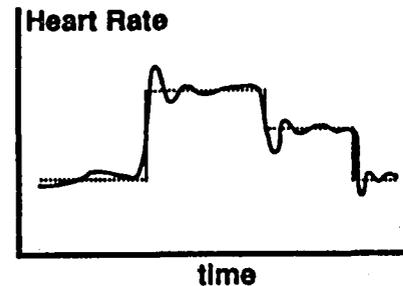
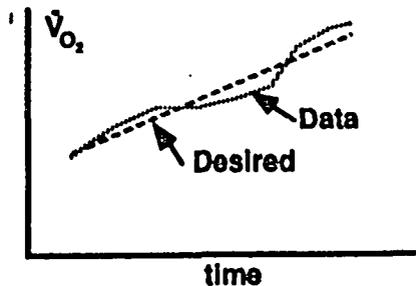
DAILY EXERCISE ROUTINES

by

Patrick L. Anderson and Michael T. Amoroso
(Continued)

■ PROBLEM FORMULATION

- Exercise protocols strive to maintain cardio-vascular, and musculo-skeletal systems at levels established by doctors and/or exercise physiologists.
- To provide consistent data during stress testing, it is desirable to maintain physiological levels without exceeding targets.



- Metabolic Gas Monitor data, exercise workload data, and perceived exertion levels from interviews (subjective data) are used to ascertain the physical condition of crew members to determine modifications to protocols.

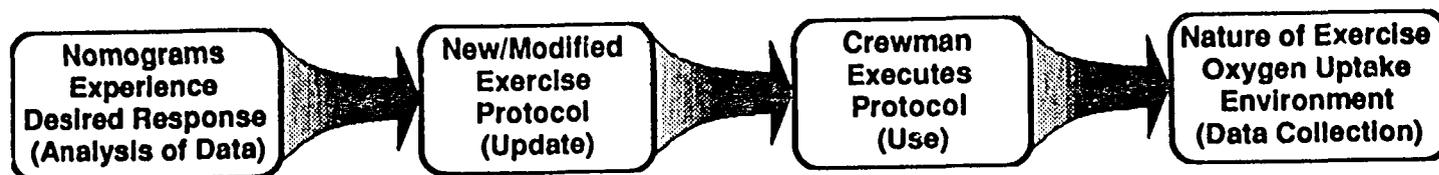
DAILY EXERCISE ROUTINES

by

Patrick L. Anderson and Michael T. Amoroso
(Continued)

■ PROPOSED APPROACHES

- Daily exercise data and periodic stress test data is collected on-orbit and sent to the ground.
- Exercise protocols are modified based on the analysis of data by an exercise physiologist.



- New exercise protocols are uploaded.
- Crew member executes new protocol.
- During stress testing, step function feedback to exercise equipment is provided to reduce overshoot of target physiological parameters.

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DAILY EXERCISE ROUTINES

by
Patrick L. Anderson and Michael T. Amoroso
(Continued)

■ SUMMARY

- **Analysis of exercise data is performed by an expert and utilizes subjective input.**
- **Equipment control is provided by step functions using time and magnitude as input parameters.**
- **(Opening question for Panel discussion)**
Is Fuzzy Logic a viable alternative to the current approach?

417

Topic: Daily Exercise Protocol
Presenter: Pat Anderson

Comment: (Berenji) Wherever there is a control problem that experts can define heuristics for the fuzzy rules, it makes sense to model the process using rules and fuzzy logic.

Comment: (Weiss) There is lots of literature available. If you treat system where output is heartrate, you can control heartrate by workload (i.e. adjust bungi cords so that the optimum heart-rate is achieved).

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31 56

PLAS

N93-31456

ATMOSPHERIC CONTROL SYSTEMS

by

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■ Outline

- Introduction
- Problem Formulation
- Proposed Approaches
- Summary
- Panel Discussion

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ATMOSPHERIC CONTROL SYSTEMS

by
Melanie Mankamyer
(Continued)

■ INTRODUCTION

- The Space Station (and future manned space missions) require atmospheric control systems
- Spacecraft atmosphere control systems consist of:
 - Temperature control
 - Composition control
 - Pressure control
- Composition control includes controlling the major constituents, humidity, and trace contaminants
- Temperature, pressure, and composition control are all interrelated

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ATMOSPHERIC CONTROL SYSTEMS

by Melanie Mankamyer (Continued)

■ PROBLEM FORMULATION

- **Cabin temperature is selectable between 64.4 °F - 80.6 °F and must be maintained within 1 °F**
- **Cabin atmospheric pressure is set at 14.7 psia and maintained within 0.2 psi**
- **Oxygen partial pressure is set at 2 psia**
- **Relative humidity is maintained between 25% and 70% (dew point temperature is always maintained above 59 °F)**

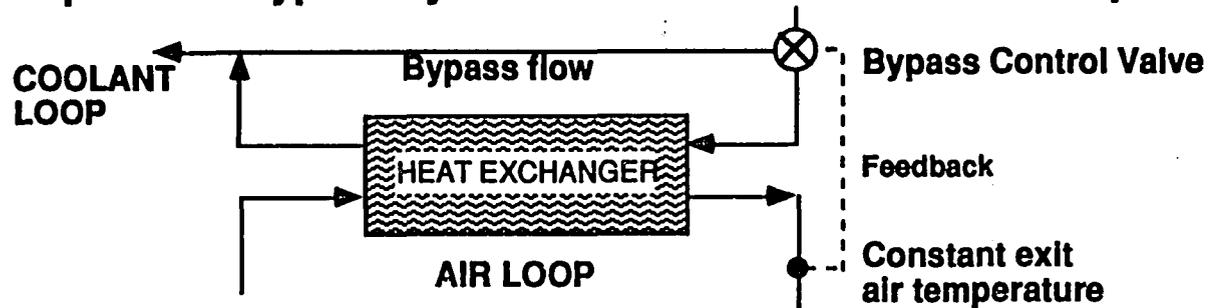
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ATMOSPHERIC CONTROL SYSTEMS

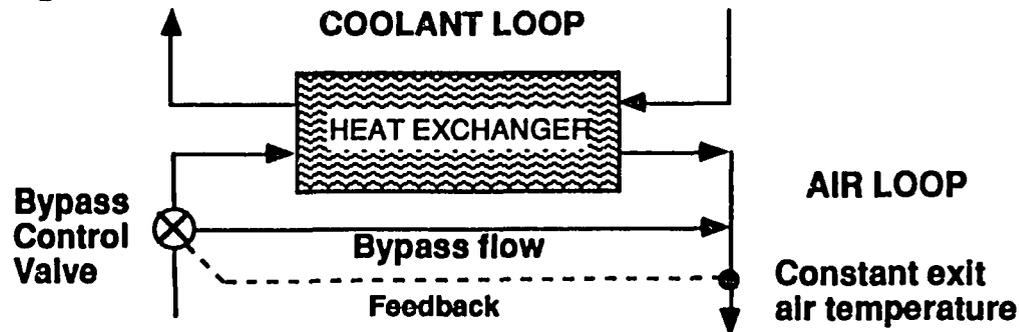
by Melanie Mankamy (Continued)

■ PROPOSED APPROACHES

- The classical approach to temperature control is to use a heat exchanger with flow bypass.
- A liquid side bypass system is smaller but slower to respond



- The air side bypass system provides a quicker response and can also maintain humidity with the use of a condensing heat exchanger

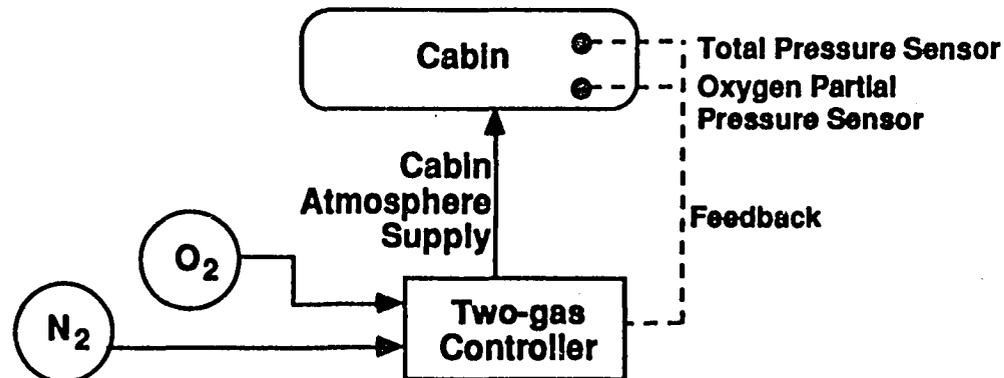


ATMOSPHERIC CONTROL SYSTEMS

by Melanie Mankamyer (Continued)

■ PROPOSED APPROACHES (cont.)

- Major atmospheric constituent control is coupled with pressure control and is accomplished with a 2-gas controller
 - Sensor registers a drop in total cabin pressure of 0.2 psi
 - Controller checks the partial pressure of oxygen (sensor)
 - If oxygen level low, a valve is opened and oxygen added until the partial pressure is met, then nitrogen is added until total pressure is met
 - If oxygen level is good, then a valve is opened and nitrogen is added until total pressure is met



Space Station Freedom

11/9/90

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ATMOSPHERIC CONTROL SYSTEMS

by Melanie Mankamyer (Continued)

■ SUMMARY

- ⊗ Techniques to maintain atmospheric control parameters have been identified
- Fuzzy Logic Control Law has been mentioned for application to atmospheric control
- (Opening question for Panel discussion)

What advantages does fuzzy control logic offer to the rather simplistic controls of the atmospheric control systems?

424

- Topic:** Atmospheric Control
Presenter: Melanie Mankamyer
- Comment:** (Mankamyer) Space Station is not using automatic oxygen control due to scrub activities. Astronaut must manually check gauges and manually adjust the levels as needed. No active monitoring in the contamination system.
- Comment:** (Lea) Better cost and temperature control with fuzzy logic usage. Such a system keeps responding to the various changes (temperature, etc).
- Comment:** (Mankamyer) Systems are mainly worked in WP-01. Temperature control is in nodes/airlock.
- Q (Lawler):** How much oxygen can you go without before you can detect the loss of oxygen (referring to faulty gas control).
A (Spoor): You can detect subtle symptoms; a person can possibly go to 14,000 - 14,500 feet, after that you need oxygen compensation.
- Q (Brown):** What if there is a valve problem?
A (Spoor): Carbon Dioxide sensors are part of ECLSS (caution & warning system)
- Comment:** (Berenji) Cooperating expert systems is a strong possibility (Thermal, power, etc).
- Comment:** (Brown) Due to scrub activities, the type of work Dr. Berenji mentioned would not be possible (no automated power).

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LAS

ROTATIONAL SPEED CONTROL

by

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N93-81457

■ Outline

- Background
- Requirements
- Application and Typical Solution
- Summary
- Panel Discussion

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163091
P. 8

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FIG 426
INTERIOR PANEL

ROTATIONAL SPEED CONTROL

■ BACKGROUND

- A life sciences centrifuge is scheduled to fly aboard Space Station
- Live animal and plant specimens will be carried on the rotor and compared with microgravity specimens in racks
- The centrifuge provides both a one-g control environment and variable gravity capability
- Experimenters indicate that specimens are sensitive at the milli g level under microgravity conditions
- Centrifuge gravity level (speed) variation must be minimized
- Acute studies involving constant angular acceleration spin-up and spin-down profiles are also desired for neurovestibular research

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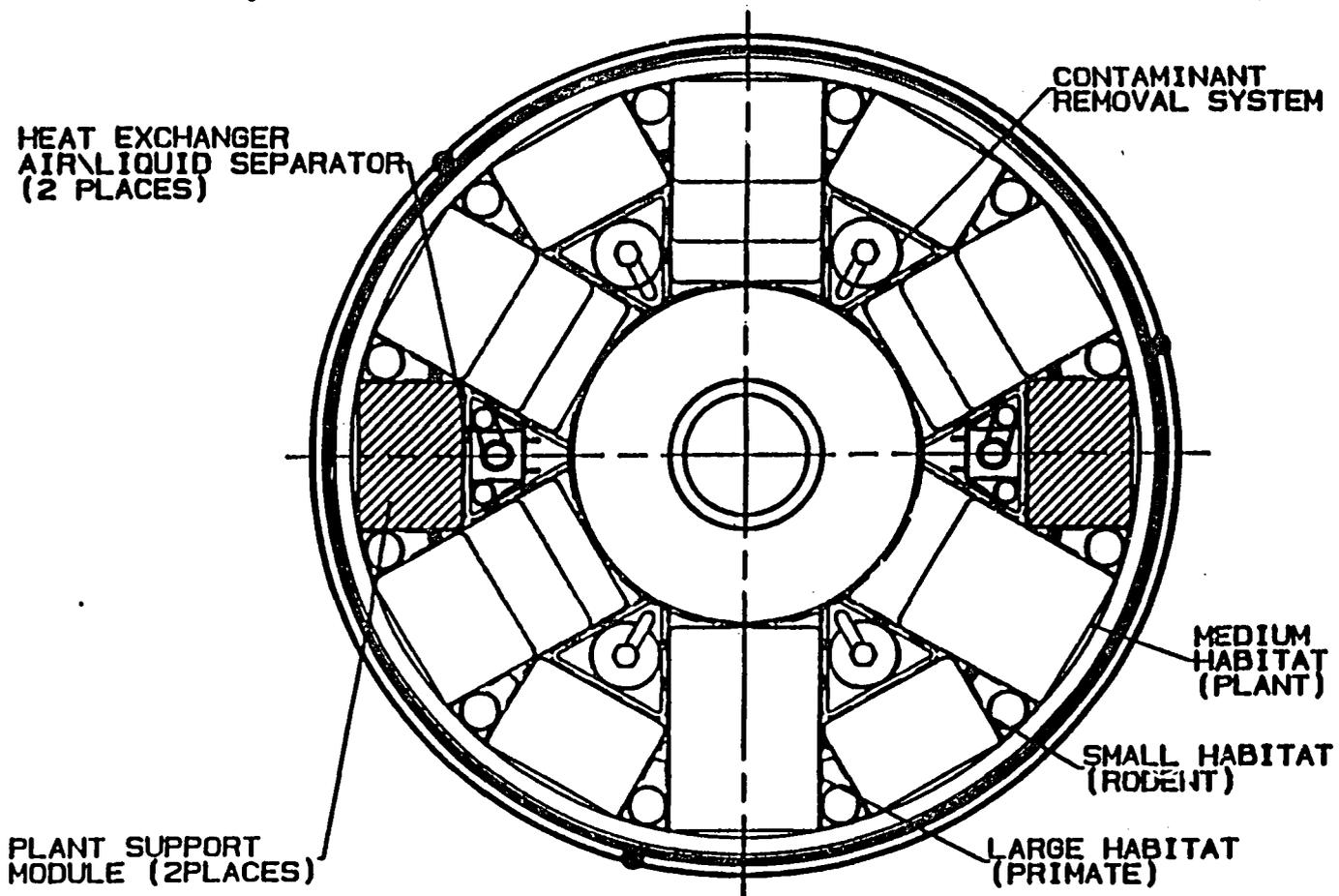
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LAYOUT, SECTION B-B

Habitat and major subsystem accommodations on rotor:



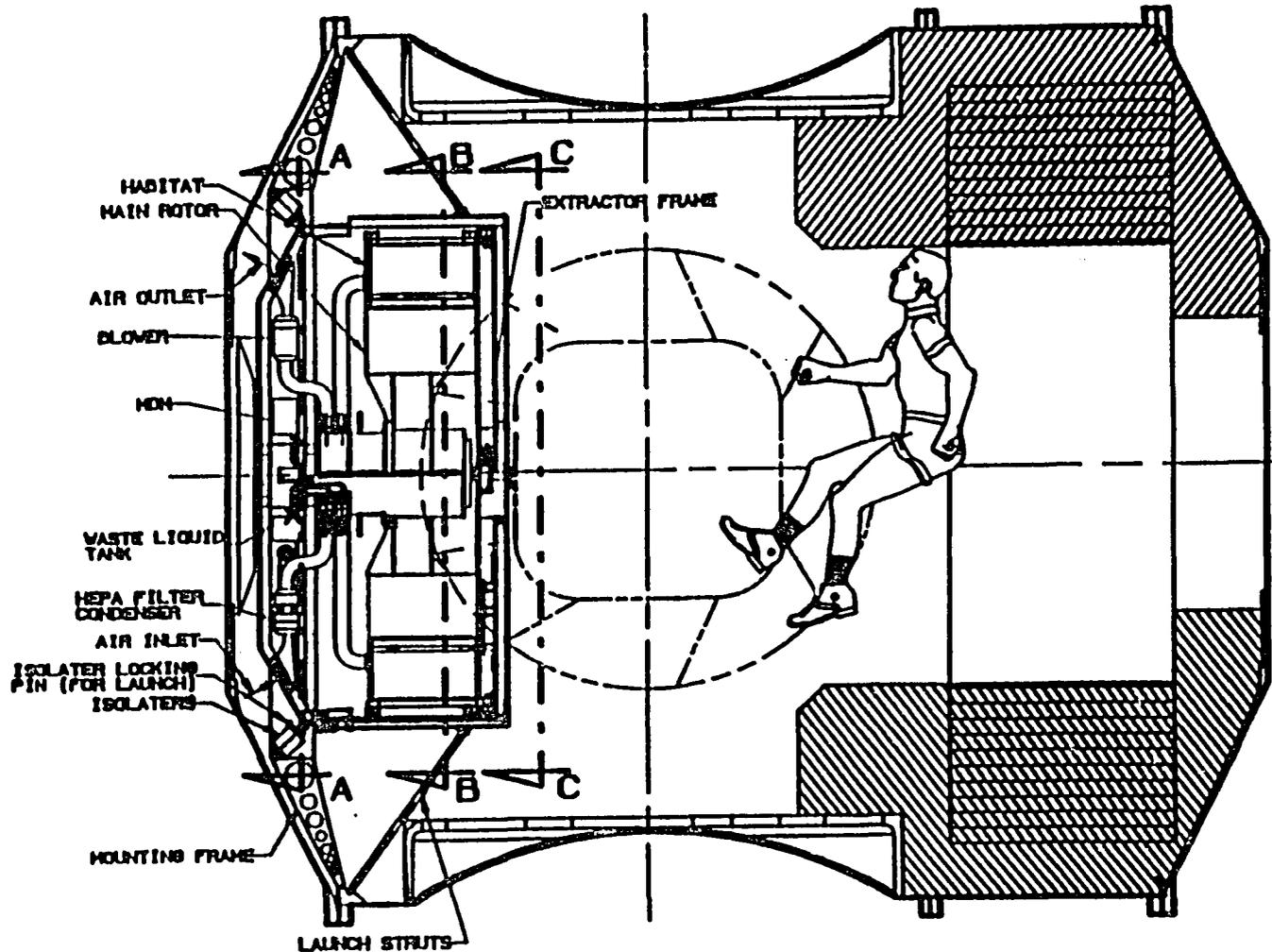
SECTION B-B



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LAYOUT, PLAN VIEW

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ROTATIONAL SPEED CONTROL

■ REQUIREMENTS

- The life sciences centrifuge holds specimens at variable gravity levels between 0.01 g and 2.0 g (approximately 2.8 to 40.0 RPM)
- Steady-state gravity level to be maintained constant to within 7.07×10^{-4} g RMS or 2%, whichever is greater
- Maximum specimen radial jerk ("g-dot") of 0.01 g/sec during spin-up, spin-down, and changes in acceleration
- Repeatability of gravity levels to within 1%
- Specimen extractor rotor to provide angular rates up to 30 deg/sec² with rates selectable in increments of 1 deg/sec² or less

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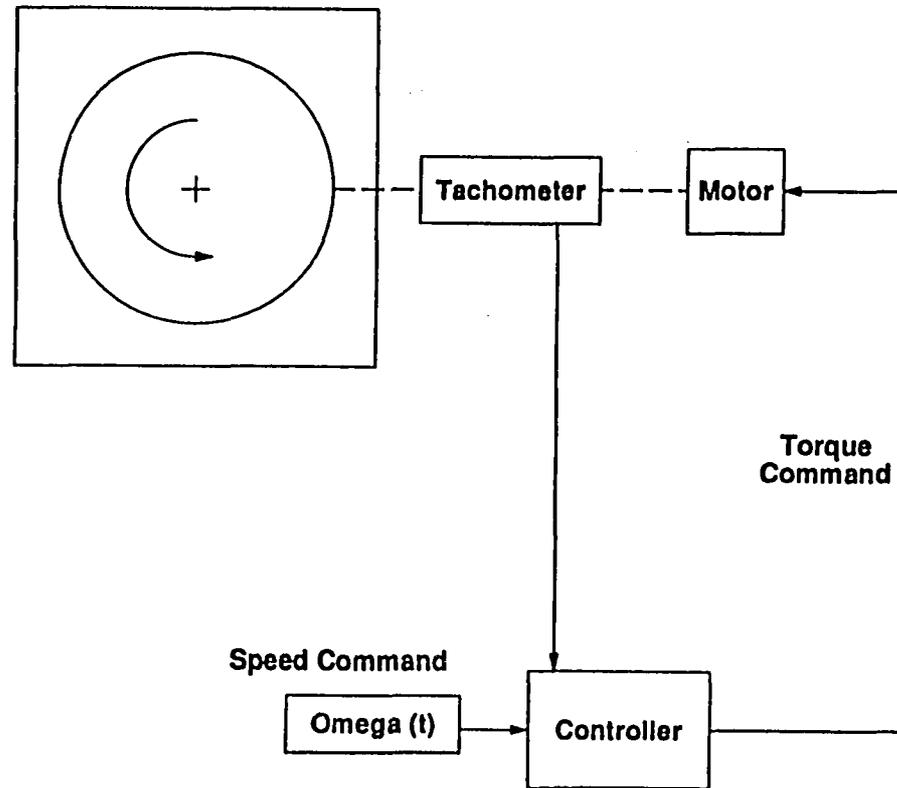
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ROTATIONAL SPEED CONTROL

■ APPLICATION AND TYPICAL SOLUTION

- Centrifuge rotor speed control



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Author

Filename

ROTATIONAL SPEED CONTROL

■ SUMMARY

- Key requirements for a case example involving rotor speed control have been identified
- (Opening questions for Panel discussion)

How can fuzzy logic control be applied to rotational speed control systems to meet desired performance requirements at a lower cost or complexity than conventional methods?

What are typical tolerances that can be held using fuzzy logic? Does the particular example discussed have high suitability for the fuzzy logic control approach?

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Topic: Rotational Speed Control
Presenter: Paul Bastin

Comment: (Bastin) System is continuously running - requirement for up to 90 days operation.

Comment: (Berenji) Fuzzy logic is able to handle imprecisions and preconditions in the rules appear to have more slack. Overall, fuzzy logic system is more robust and has better adaptation with weight changes.

Comment: (Brown) The Japanese use Fuzzy logic control in applications such as in the subway trains in order to smooth the starting and stopping of the trains. The goal of this application is similar in terms of the goal of smoothness of operation; fuzzy logic can handle the large variations.

Q (Lawler): How much "change" in the system?
A: Not alot - 5% to 10%

Q (Lawler): How much evolution in terms of the size of experiments are being allowed for?

A: The system is able to accomodate new experiments and the current design allows for the size of the envelopes to be modified and heavier loads.

Comment: (Jani) The acceleration should be measured by accelerometers.

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LAS

N 93-31458

VIBRATION ISOLATION

by

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■ Outline

- Introduction
- Space Station Vibration Requirements
- Proposed Application
- Typical Solutions
- Summary
- Panel Discussion

SR-54
163092
P, 9

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VIBRATION ISOLATION

■ INTRODUCTION

- Many subsystems, payloads, and events on Space Station are potential disturbance sources in the microgravity environment. Examples include
 - Exercise equipment - treadmill
 - Rotating equipment - centrifuge
 - Planned events - berthing and docking
 - Unplanned events - crew motion
- Isolation may be required both to attenuate vibration from "noisy" subsystems as well as to protect sensitive payloads
- Low frequency disturbance sources are an important concern due to sensitivity of Space Station flexible structure modes

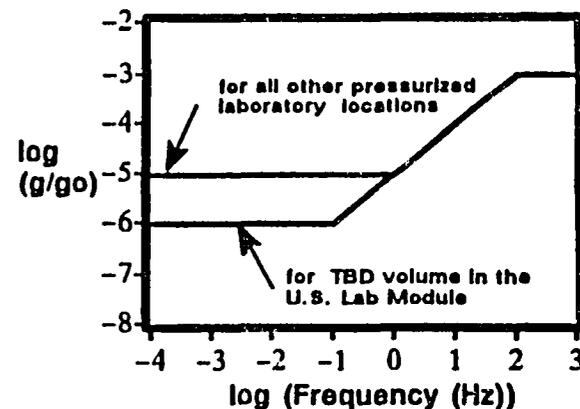
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MICROGRAVITY ASSESSMENT CRITERIA

- For continuous periods of at least 30 days and 50% of the operational year, the following criteria applies:
 - No greater than 1 micro-g steady-state acceleration levels in the rack volume of the U.S. Laboratory
 - Torque Equilibrium Attitude +/- 5 degrees of Local Vertical Local Horizontal (LVLH)
 - Induced Vibration acceleration levels:
 - $\leq 1 \times 10^{-6}$ g for $f < 0.1$ Hz
 - $\leq (1 \times 10^{-5} \times f)$ g for $0.1 \leq f \leq 100$ Hz
 - $\leq 1 \times 10^{-3}$ g for $f > 100$ Hz
 - Integrated acceleration response for a 10 second moving window less than 1×10^{-6} g-seconds



VIBRATION ISOLATION

■ PROPOSED APPLICATION

● Life Sciences Centrifuge

- Provide variable gravity centrifugation from 0.01 to 2 g for specimens (maximum 40 RPM [0.7 Hz] rotation rate)
- Rotating mass approximately 2200 lb
- 2.5 meter diameter
- Mounted in Space Station Node endcone
- Independent extraction rotor spins up and down to remove a pair of specimen habitats from rotating main rotor
- Disturbance to Space Station limited to 25% of microgravity requirement
- Disturbance to specimens limited to 1×10^{-3} g in frequency range 0.1 to 100 Hz

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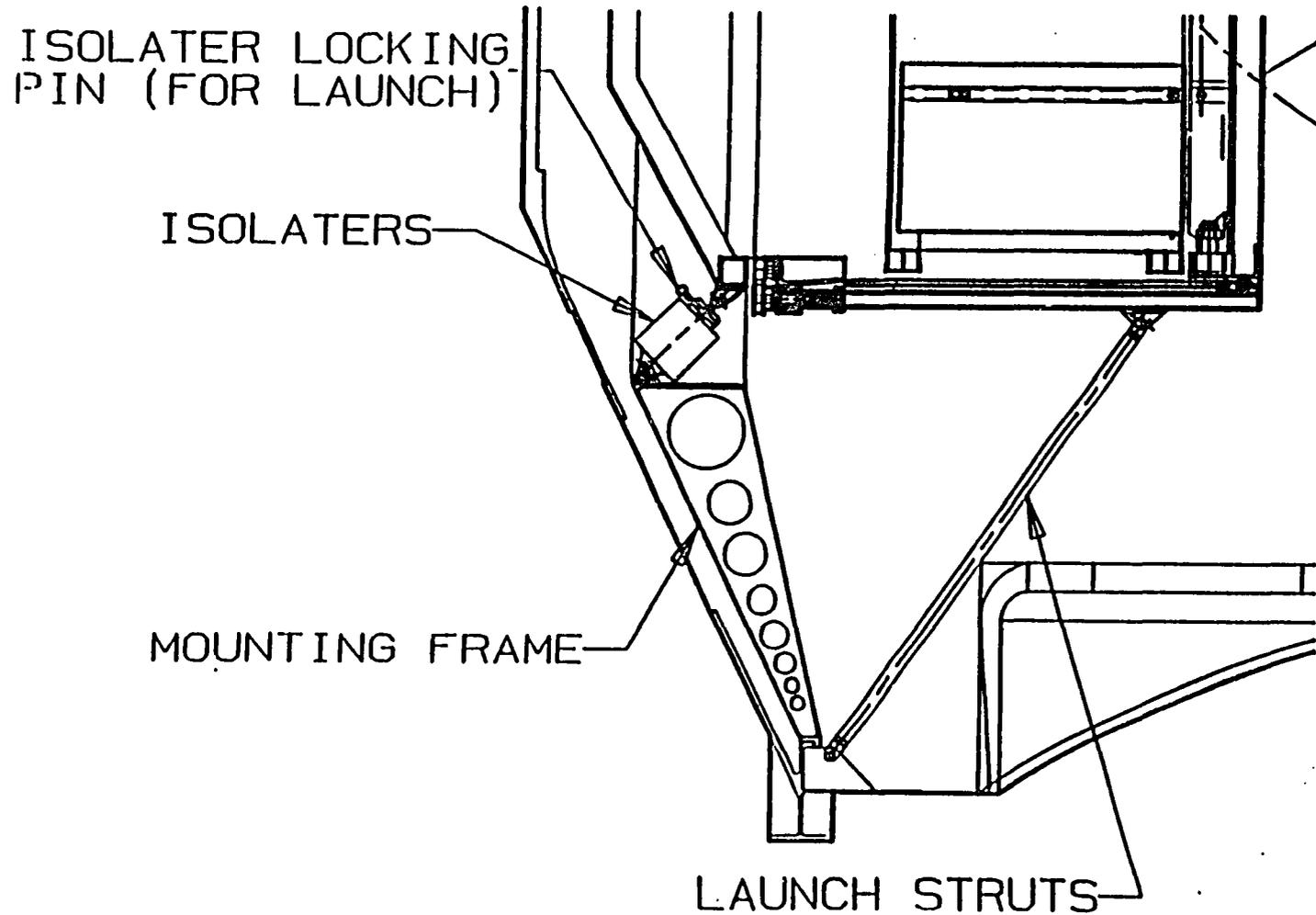
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ADDITIONAL DESIGN DETAILS

NASA / AMES RESEARCH CENTER

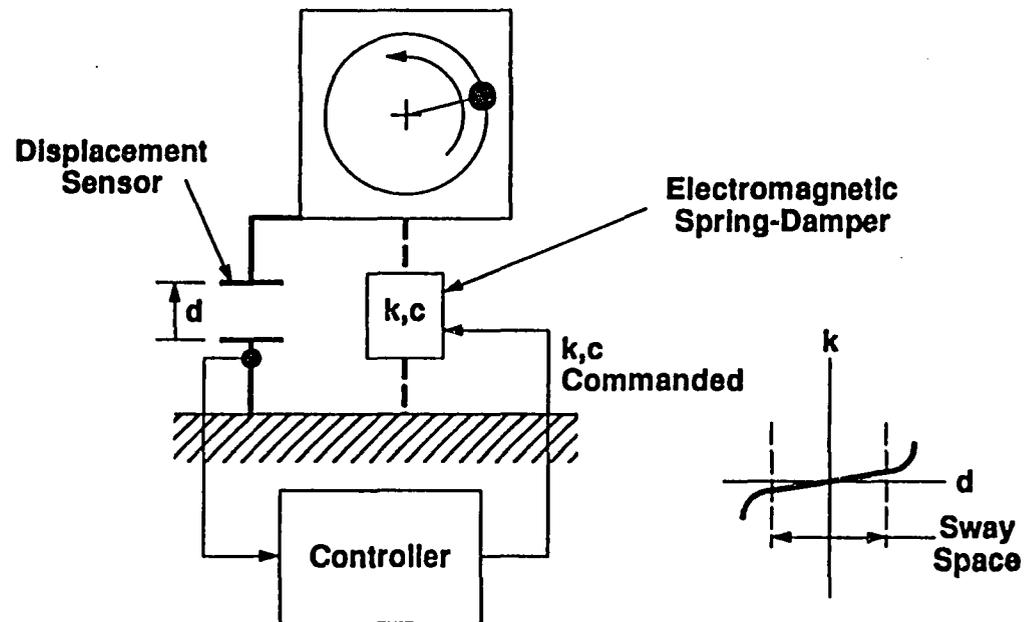
FLIGHT SUPPORT EQUIPMENT FOR LAUNCH



VIBRATION ISOLATION

■ TYPICAL SOLUTIONS

● Active vibration isolation system



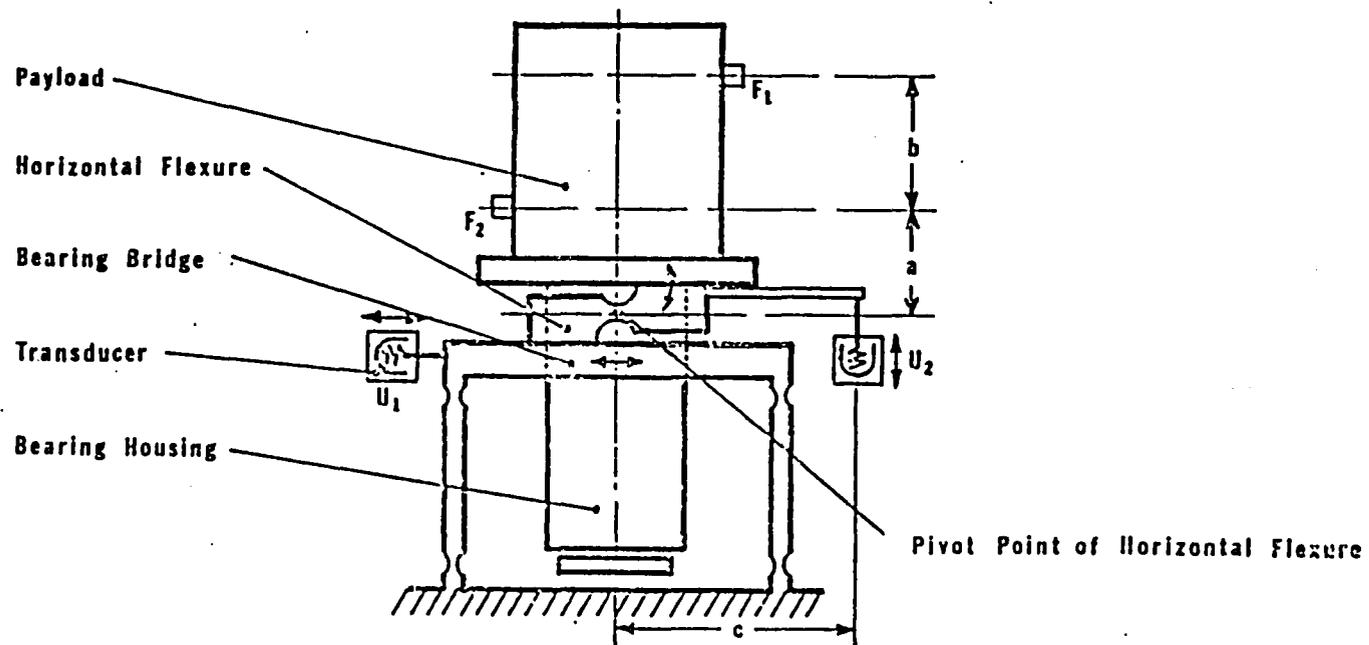
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VIBRATION ISOLATION

■ TYPICAL SOLUTIONS (continued)

● Active balancing system



$$U_1 = F_1 + F_2$$

$$U_2 = F_1 \cdot \frac{a+b}{c} + F_2 \cdot \frac{a}{c}$$

Data courtesy of Schenk Trebel

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VIBRATION ISOLATION

■ SUMMARY

- ④ Techniques to control and isolate centrifuge disturbances have been identified
- ④ (Opening questions for Panel discussion)

How can fuzzy logic be applied to Space Station vibration isolation systems to improve performance over passive approaches or reduce the cost and complexity of active approaches?

Could fuzzy logic control be used to facilitate an inexpensive active low frequency isolator? Could a simple active controller be implemented to augment a conventional passive isolator?

Can fuzzy logic control be used to improve signal to noise ratio or response time?

Topic: Vibration Isolation
Presenter: Paul Bastin

Comment: (Knackstedt) The problem of vibration isolation may have more stringent requirements than those of Rotational Speed Control.

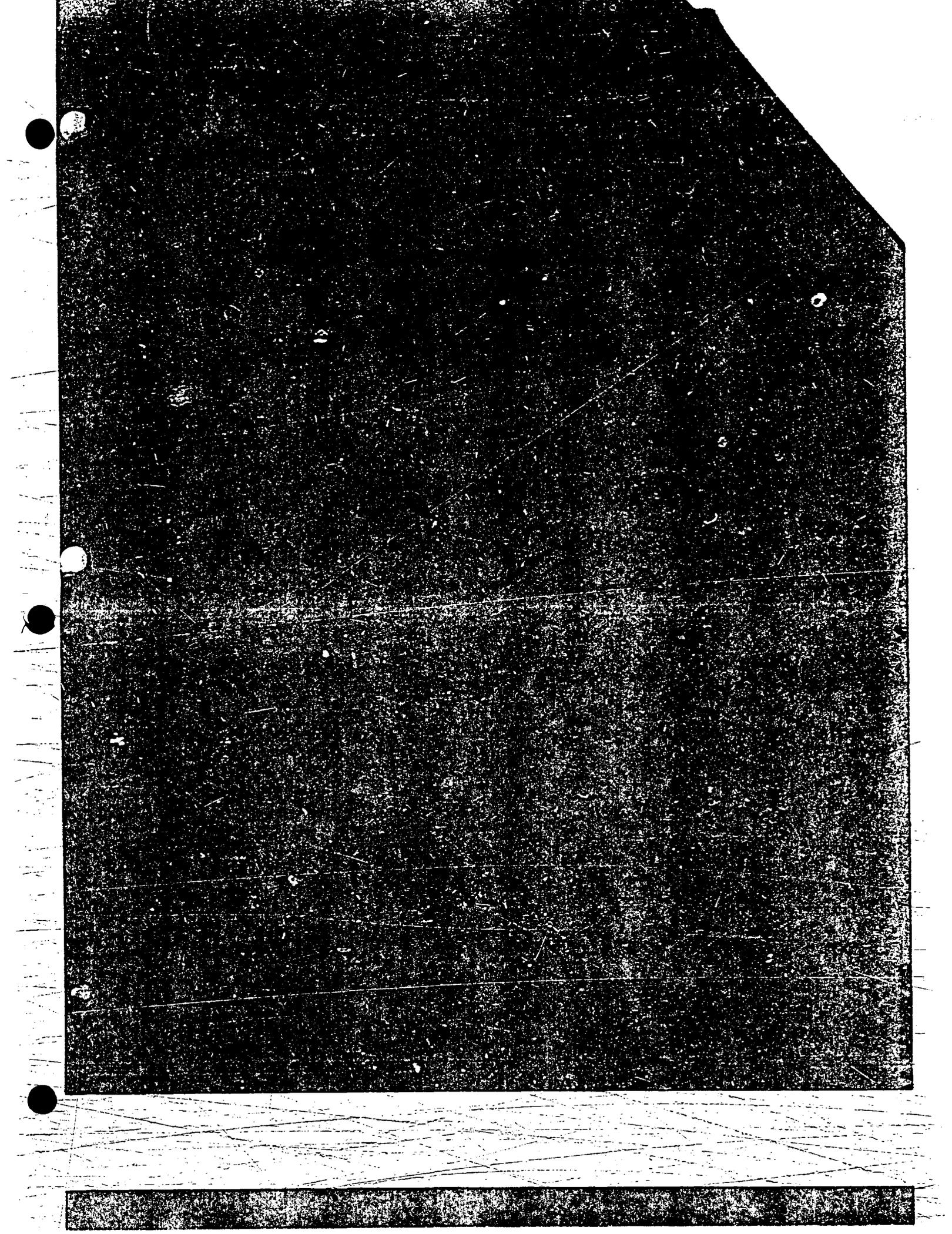
Comment: (Brown) The conflicting requirements mentioned have no effect on the fact that there is a serious problem of isolation of vibrations. The question of whether Fuzzy logic can be used to build an active isolator or to design a passive isolator to improve performance over cost should be considered.

Comment: (Berenji) The experiments on board need a steady, vibration-free platform. These systems must be stable, fuzzy logic is applicable. Is this problem to different from those of which fuzzy logic can be used?

Comment: (Jani) Fuzzy logic should not be too difficult in implementing a solution to this problem.

Comment: (?) The "fuzzy" control surfaces shown in Dr. Kosko's examples were "pipe organ-ish" showing discontinuity. With Vibration Isolation problem, there is a great concern for smoothing the operation.

Comment: (Berenji) Agreed with comment that a non-smooth transition can create problems. However, the examples shown were not necessarily optimal (which is problem dependent); multiple fuzzy rules can create a smoothing operation



WORKSHOP SUMMARY

BOB BROWN
MDSSC-SSD

SUMMARY - Bob Brown

The objectives of this workshop were to demonstrate that (1) Fuzzy Logic is a mature technology, (2) to prove that it was well founded in basic mathematical principles, (3) to show that the experts in the field were knowledgeable control systems engineers and scientists, (4) to demonstrate that fuzzy logic could solve non-linear problems and (4) that this technology was target to complex applications. In addition, we wanted to show-case potential applications to our customers (NASA). In so doing, we sought to present a wide spectrum of applications that could potentially result in two funded projects.

Dave Wensley set the tone of the workshop and McDonnell Douglas's commitment to advanced automation.

Mark Gersh, NASA Headquarters, stated NASA's position and plans in his area of responsibility (Space Station Advanced Automation) and what NASA was looking for in the way of future projects.

Dr. Togai gave an excellent presentation on the wide ranging commercial applications which demonstrated the maturity of fuzzy controls.

Professor Sugeno discussed his projects in the development of autonomous flight control for helicopters. Professor Sugeno, a control systems theorist, is funded by the Japanese government to develop and demonstrate this capability. During the workshop Prof. Sugeno suggested that he would like to join McDonnell Douglas in a complex application of fuzzy logic on the Space Station Program. Prof. Sugeno's project caught the attention of the audience because of the complexity of the problem and his extreme confidence that he would succeed.

Dr. Aldridge did an excellent job in preparing the workshop for the transition from traditional control theory to fuzzy control. His example of the inverted pendulum gave the audience a good demonstration of the basic concepts and how to apply them to a control problem.

Dr. Berenji offered several alternatives for providing training to McDonnell Douglas personnel. Once the projects are selected we will discuss the specifics with the Ames Research Center as part of our Technical Exchange Agreement.

Dr. Bart Kosko did an excellent job in the mathematical derivation of Fuzzy Estimation Theory. Many of the skeptics have had to rethink their objections to Fuzzy Logic as a result of Dr. Kosko's work. Dr. Kosko gave the workshop the equivalent of a 6 hour lecture in 90 minutes.

The panel was impressed with the wide ranging applications presented by the speakers. Their conclusion was that all presentations were good candidates for Fuzzy Control.

Mark Gersh, in his summary, said he plans to return to Washington to tell the Space Station Program management that this technology is mature and that they should fund an effort.

The selection of the candidates will be based on how well they meet NASA and McDonnell Douglas' criteria.

All the comments I have received indicate that this was a very successful workshop and accomplished all of our objectives. NASA liked the format and plans to use it in future workshops.

CONCLUDING REMARKS

Mark Gersh, NASA HQ

Hamid Berenji, PhD., NASA AMES

CONCLUDING REMARKS - Mark Gersh

The NASA Headquarters Autonomous Systems Manager for Space Station Engineering, Mark Gersh, made some concluding remarks which expressed his impressions of the workshop. Overall, Mr. Gersh noted that he was "very impressed with the knowledge and expertise" attending the workshop. He stated that there were scores of potential applications for Fuzzy Logic control on the Space Station Freedom Program (SSFP). In addition, Mr Gersh said he would take back the message that this was a mature technology ready for application and would recommend that funds be set aside to fund two projects.

Mr. Gersh gave some guidelines and comments for pursuing these projects. They included:

1. Due to the high degree of maturity of Fuzzy Logic Control technology, it should be considered for use in SSFP.
2. This technology is grounded in theory and science.
3. We need to find candidate programs to showcase the benefits of this technology.
4. Considering the needs of SSFP, we should investigate how Fuzzy Logic Control technology can help us reduce power usage and weight constraints.
5. We need to utilize this technology to help the SSFP meet it's requirements of limited computational environment.
6. The technology should be used to help maximize the SSFP's usage of crew time.
7. In logistics, such as fuel re-supply, Fuzzy Logic should help reduce the need due to it's optimization qualities.
8. In development of a project we need to consider how well the baseline issue is handled.
9. Any Fuzzy Logic project in SSFP should entail user participation.
10. The project should influence the baseline program.
11. The early demonstration of Fuzzy Logic technology should be provided for in any funded project.
12. The application should be simplistic enough to solve yet complex enough to show the importance of the problem.
13. There should be measurable success in terms of solving a baseline problem and benchmarking against conventional control techniques.

In conclusion, Mark Gersh thought that this technology should be showcased and that he would propose the support of 2 projects on SSFP to be selected based on NASA's criteria.

Concluding Remarks - Hamid Berenji

This workshop brought together a number of distinguished researchers in the field of fuzzy logic and also a number of fuzzy logic control application specialists. Although NASA has taken the lead in the U.S. by sponsoring two previous conferences on Fuzzy Logic and Neural Network applications at the Johnson Space Center (1988 and 1990), the current workshop held at McDonnell Douglas was clearly a major step forward in further exposing this technology to a number of engineers and scientists involved in control of major subsystems of the Space Station. The systems presented at this workshop were unique in many respects and pose serious challenges to us in the design and development of controllers for their actual deployment in space. The panel members and the other workshop attendants developed significant understandings of the requirements which have to be met by these control systems. In almost all the systems presented, panel identified and foresaw areas for application of the fuzzy logic control technology. These observations were encouraged in part by discussing the similarities between the subsystems presented and those already very successful commercial applications of the fuzzy logic control.

I believe that this workshop was very successful in many respects. Although the field of fuzzy logic control requires further research in some areas, it has already matured enough to be applied in many real life control problems. A follow up workshop, including reports of actual experiences in applying this technology to some of the subsystems of the Space Station, is recommended.

Finally, I would like to acknowledge the excellent works of Mr. Bob Brown and others in organizing this workshop. Also, many thanks to the members of my panel who had to travel long distances (e.g., Professor Sugeno who came from Japan) to be at this workshop.

Hamid R. Berenji
November 1990

END

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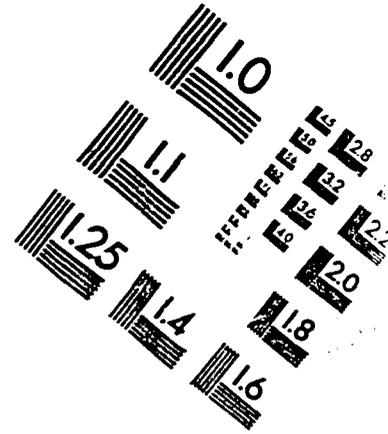
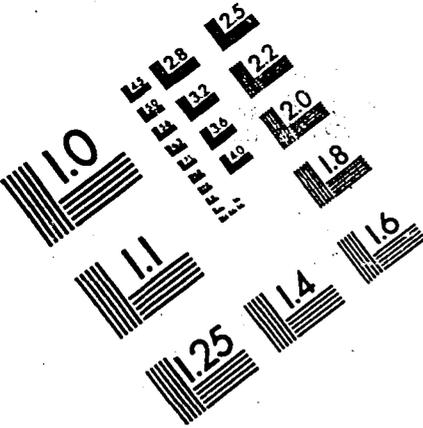
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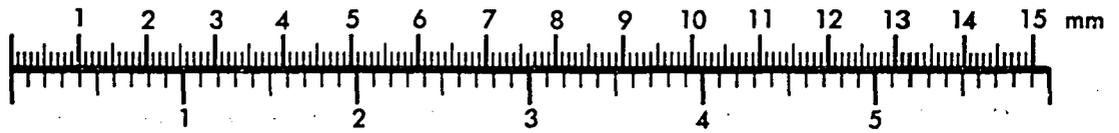


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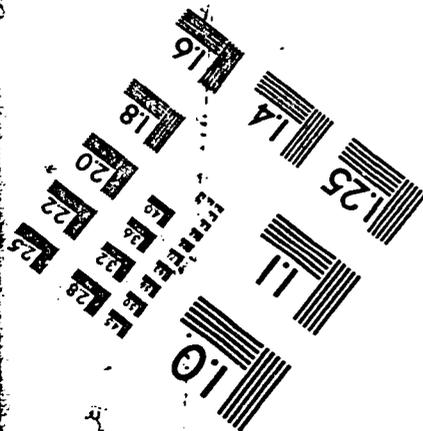
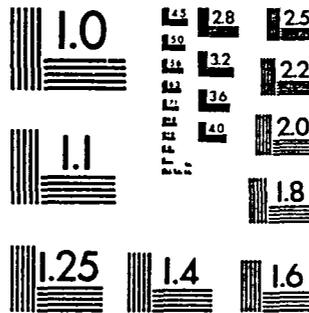
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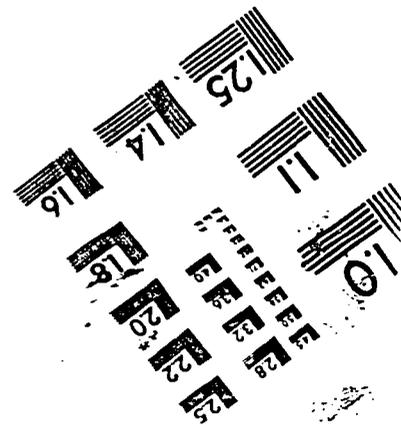
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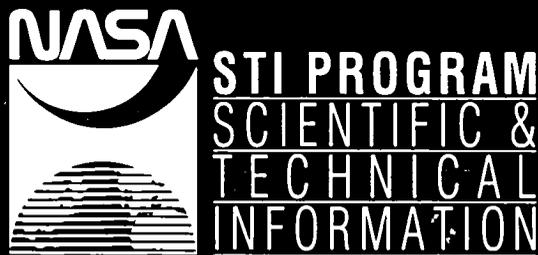


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