The Probabilistic Structures Analysis Methods (PSAM) program integrates state-of-the-art probabilistic algorithms with structural analysis methods in order to quantify the behavior of SSME structures subject to uncertain loadings, boundary conditions, material parameters and geometric conditions. The initial 5-year program focused on advanced software to analyze probabilistic structural response. The second 5-year phase of PSAM is focusing on component and system structural reliability, material resistance, and risk, and on system qualification/certification, and health monitoring.

An advanced, efficient probabilistic structural analysis software program, NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) has been developed as a deliverable for this program. NESSUS contains a number of integrated software components to perform probabilistic analysis of complex structures, as seen in Figure 1. A nonlinear finite element module NESSUS/FEM is used to model the structure and obtain structural sensitivities. Some of the capabilities of NESSUS/FEM are shown in Figure 2. A Fast Probability Integration module NESSUS/FPI estimates the probability given the structural sensitivities. A driver module, PFEM, couples the FEM and FPI modules.

NESSUS, version 5.0, released in January 1991, addresses component reliability, resistance and risk. Recent enhancements include the development and implementation of advanced probabilistic algorithms to compute structural reliability, material resistance models including user defined resistance, and the risk of failure based on the consequences or severity of a failure.

The Advanced Mean Value with iterations (AMV+) algorithm has been shown to be very effective in obtaining accurate and efficient probabilistic solutions. The AMV+ algorithms are intended to minimize the number of finite element calculations needed, yet provide an accurate solution even at very high or low probabilities, i.e. in the tails of the distribution. Figure 3 shows the cumulative distribution function, CDF, of an elastic perfectly plastic cylinder under internal load. The nonlinearity caused by the transition from elastic to plastic material behavior is correctly determined by the AMV+ algorithm. Thus, highly nonlinear problems can be handled with NESSUS.

Material resistance governs the structure's ability to withstand a stress condition. NESSUS 5.0 can treat the traditional reliability problem, defined as \( g = R - S \) where \( R \) is the resistance and \( S \) is the stress, as well as more complicated reliability functions. For example, for fatigue crack growth the reliability function is defined as

\[
g = N_f - N_0 = \frac{2[a_f^{(1-\sqrt{n})} - a_i^{(1-\sqrt{n})}]}{C(2-n)(Y_{\text{max}}\sqrt{\pi})^n} - N_0, \quad n \neq 2
\]
where \( \sigma_{\text{max}} \) is obtained from a finite element model, \( a_f \) and \( a_i \) are the final and initial crack lengths, \( C \) is a crack growth material parameter, \( n \) is the Paris crack growth power and \( Y \) is a geometry factor. Failure occurs when \( N_f \) is less than the design life \( N_0 \). A very general user-defined reliability function option is provided in NESSUS. Figure 4 shows the results of a probabilistic fatigue analysis.

Structural risk with respect to cost is defined as the coupling of the probability of failure and the consequences of failure. The NESSUS/RISK module couples the probability of failure computed by the other NESSUS modules and the consequences of failure. In equation form this is defined as

\[
\text{Risk} = C_0(x) + P_f(x) C(x) + (1 - P_f(x)) NC(x)
\]

where \( x \) is the response quantity, \( C_0(x) \) is the initial cost, \( P_f(x) \) is the probability of failure, \( C(x) \) is the cost of failure and \( NC(x) \) is the cost of nonfailure or success. A schematic outline of the Risk module is shown in Figure 5. RISK can be run standalone or in conjunction with NESSUS/PFEM.

The capabilities for reliability, resistance and risk are all integrated within NESSUS. Figure 6 shows a schematic of the solution procedure in NESSUS 5.0. The user defines the structure, the resistance model, the random inputs, and the failure consequences. NESSUS computes the probabilistic response including sensitivity factors, and the risk of failure. These results can then be used for decision making by the user.

NESSUS is being extended to system reliability problems. One system reliability analysis method being developed is a probabilistic fault tree approach. In this method, the bottom events are analyzed probabilistically and the probability density functions are integrated upwards to determine the probability distribution of the top event.
NESSUS 5.0 CODE STRUCTURE

NESSUS PROBABILISTIC FINITE ELEMENT CAPABILITIES

ANALYSIS TYPES
- Static
- Natural Frequency
- Buckling
- Harmonic Excitation
- Random Vibration
- Transient Dynamics

NONLINEAR
- Material
- Geometry

ELEMENT LIBRARY
- Beam
- Plate
- Plane Stress
- Plane Strain
- Axisymmetric
- 3D solid
- Enhanced Elements

RANDOM VARIABLES
- Geometry
- Loads
- Forces
- Pressures
- Temperatures
- Material Properties
- Elastic Modulus
- Poisson's Ratio
- Shear Modulus
- Orientation Angle
- Yield Stress
- Hardening Parameters
- Damping
- Initial Conditions
CDF OF RADIAL STRESS FOR ELASTIC-PLASTIC CYLINDER

PROBABILISTIC FATIGUE ANALYSIS
**RISK/COST ALGORITHM**

\[
\text{RISK/COST} = \text{INITIAL COST} + \text{PROBABILITY OF FAILURE} \times \text{COST OF FAILURE}
\]

**PROBABILITY OF FAILURE**

\[
\begin{align*}
\text{TIME} & \quad \text{INITIAL COST} \\
\end{align*}
\]

**COST OF FAILURE**

\[
\begin{align*}
\text{TIME} & \quad \text{RISK} \\
\end{align*}
\]

**NESSUS 5.0 - COMPONENT RELIABILITY, RESISTANCE, AND RISK**