Systems with high performance requirements often have a small number of critical components that operate close to mechanical design margins and that define the effective lifetime of the system. The concept of life extending control (LEC) proposes an active approach to simultaneously managing the damage accumulation of these components while maintaining dynamic performance to increase system effectiveness.

The concept depends on the prediction of fatigue life of the critical components. Currently, fatigue life prediction is based on local cyclic strain behavior such as:

\[ \delta e = \left( \frac{\sigma_t - \sigma_{am}}{E} \right) \left( \frac{2}{D_i} \right)^b + \varepsilon_f \left( \frac{2}{D_i} \right)^c \]  

Where \( D_i \) is the damage due to cycle \( i \), and \( \sigma_t \), \( E \), \( b \), \( \varepsilon_f \), and \( c \) are material constants and \( \sigma_{am} \) is an average tensile strength. Using the Palmgren-Minor approach the total damage is estimated as

\[ D = \sum_i D_i \]

The life usage predicted by this equation is directly related to strain magnitude (figure 1). Other more accurate and complex approaches are also possible. With these approaches, damage can only be estimated upon the completion of a stress-strain cycle. With current cyclic forms of damage modelling only indirect or implicit forms of LEC are possible.

**IMPLICIT LEC**

The implicit approach to Life Extending Control recognizes that current fracture/fatigue science can not predict the differential damage on less than a full cycle of strain. The implicit approach (see figure 2) selects a sequence of typical command transients (and disturbances) that are representative of those the system would experience in service. Two performance measures are defined: \( J_p \), an objective function that maximizes dynamic performance (possibly by minimizing quadratic state and control excursions) and \( J_D \) a damage measure which uses the best (current) fatigue/fracture theory available to calculate the damage accumulated over the sequence of command transients. An overall performance measure is defined where \( a \) represents the relative importance between

\[ J = J_p + aJ_D \]  

performance and life extension. The implicit approach then selects a "best" control algorithm which is applied for the full sequence of command transients.
The dynamic performance and damage accumulation over the sequence are optimised (relative to the selected measures) against the control algorithm parameters. The expectation is to find an algorithm such that the loss in dynamic performance is small (i.e., $J_{p,s,\text{min}} - J_{p,o,\text{min}}$ in figure 3), for a significant reduction in accumulated damage over the sequence of transients (i.e., $J_{p,o,\text{min}} - J_{p,s,\text{min}}$ is large and life is extended). Here the subscript o refers to optimizing for dynamic performance only. An actual operating gain set (point q in figure 3) is then chosen which satisfies the desired weighting between performance and damage (i.e., J). During the design process, two types of feedback variables are considered: 1) the performance variables normally used to manage dynamic performance and 2) nonlinear functions of the performance variable representative of the damage variables (stresses, strains, temperature and various rates). Various control algorithms are then examined within this feedback structure. That is, the sequence of selected performance and disturbance transients are applied to a simulated system with a trial control and performance J (or $J_p$ and $J_p$ separately) is calculated. Superior LEC algorithms can then be identified as those that minimize J (or $J_p$ and $J_p$ separately).

Algorithms for Implicit LEC may be formulated intuitively, i.e., minimizing the mean tensile stress, mean strain, and temperature levels and minimizing the cyclic amplitude of stress, strain, and temperature should minimize damage. Also, minimizing the number of cycles of stress and strain should contribute to extending critical component life.

**LIFE MANAGEMENT LEC**

A second indirect approach is called Life Management LEC and is shown in figure 4. Here the LEC would have a hierarchical structure similar to that found in other proposed intelligent control systems (ICS). At the coordination level, the task planner uses performance requirements and balances these against life usage and appropriate control commands or strategies are selected. This is accomplished by simulation of the system for a few pre-selected trajectories. From the results of this simulation and for a given performance definition, a commanded trajectory is selected that optimizes system performance and minimizes component damage over a sub-interval of the task. Within the task planner, information from a cyclic damage prediction model ranks various candidate trajectories of the successive interval in the planning and selection process. Outside of the task planner another cyclic prediction model assesses the actual damage accumulated during a sub-interval. The execution level implements the selected strategy in the interval by translating the commanded trajectory into control commands and applying these commands to the system.

**CONTINUOUS LIFE PREDICTION APPROACH**

The Implicit LEC approaches taken above do not directly control the damage rates of critical components. Direct control will require continuous forms of the damage laws instead of the current cyclic forms.

To achieve a continuous formulation of the life prediction process, an interdisciplinary approach is required. Here the knowledge of material properties and life prediction of fracture and fatigue scientists must be combined with the control engineers' knowledge of dynamics and modeling to develop these continuous forms. The objective is to functionally relate measurable performance information with a differential form of the damage laws. This would allow the direct use of the differential estimate of damage in the life extending control
law and, when integrated over complete cycles, would give equivalent or superior damage predictions to those associated with the cyclic theory. Two approaches are possible here: 1) derive such forms from basic theory (the current research thrust of the field) or 2) empirically select likely forms with a significant number of unspecified parameters and use optimization theory to best "fit" the parameters using available data sets. Adopting the second approach, several elementary forms, given in Table I, are proposed. These forms are determined either by intuition or by observing the important relationships embodied in the current theory of life prediction. Various weighted, linear and nonlinear combinations of these elementary forms would be linearly regressed against the available fatigue life data to obtain a continuous formulation.

MEASURED DAMAGE VARIABLES LEC

In this approach (figure 5) both the plant performance and the damage related variables (measured stresses, strains, temperatures, forces, etc.) associated with critical components are measured and used as feedback information for the control. Here the control attempts to directly regulate life as a resource. It is presumed that a "real-time" predictive damage model (described above) exists that would allow the prediction of the incremental damage as a continuous function of selected incremental control action. That is, the influence of changes in the performance variables (presumed to be controllable) on the behavior of the critical life variable is known. This is in the form of the local damage rates $DV_1, DV_2, ...$ in figure 5. Thus, in figure 6, at a time $A$, the damage associated with damage variable $DV_1$ can be predicted for any incremental control action (here actions $u_1, u_2, u_3$ are considered and result in damages $D_1, D_2, D_3$ respectively). (Note damage while shown as a continuous function of time will likely be modelled as a continuous function of local stresses, strains, etc.). The control problem then is to minimize damage of the critical life components while maximizing (dynamic) performance of the plant. The performance objective approach of equation (2) can be used to achieve this optimization.

One implementation of a measured damage variables LEC would achieve control performance by adaptively modifying the control feedback structure to permit damage to accumulate at a "setpoint" rate, a linear rate over time for example. The measured damage variables could be used directly in a feedback law or to modify the gains or even the structure of the existing control. The emphasis here is on obtaining desired system operation by an active, feedback control approach.

ESTIMATED DAMAGE VARIABLES LEC

Unlike the Measured Damage Variables LEC approach, this concept, shown in figure 7 uses a real time model to estimate the damage rates (and damage accumulation) of critical components. The models can be driven by performance variables or performance variables augmented by available damage measures. Conceptually the models can vary from simple, precomputed, linear, influence coefficients to detailed, non-linear, real time structural models which may require considerable computation. These models would be a direct consequence of the continuous life model described above and would result in a damage estimator that estimates real time damage rates. The controller design would follow in much the same manner as for the measured damage variables approach.
EXAMPLE SYSTEM

The example system of figure 8 is used to illustrate LEC. In figure 9 pulse sequence trajectory number 1 was applied to the system. Also shown in figure 9 are the system position and scaled force trajectory resulting from pulse trajectory 1. The performance endpoints were selected as \( X_1 = -X_2 = 1 \). In this case, \( N = 11 \) and \( T = 4.9 \) sec. and \( D = 0.0213 \) units of damage based upon a total component life of 1 unit were predicted.

A modified commanded pulse sequence trajectory, called trajectory number 2, was applied to the same system. The commanded trajectory, the system position, and the scaled force for case 2 is shown in figure 10. In case 2, \( N = 9 \) and \( T = 4.6 \) sec. and \( D = 0.0121 \). Because the commanded pulse trajectory has been slowed slightly, the resultant force trajectory has smaller peak magnitudes. Consequently, the stress-strain cycles have smaller magnitudes and the damage will be less. The example results are summarized in Table II.

CONCLUSIONS

The concept of Life Extending Control was introduced. Possible extensions to the cyclic damage prediction approach were presented based on the identification of a model from elementary forms. Several candidate elementary forms were presented. These extensions would result in a continuous or differential form of the damage prediction model.

Two possible approaches to Life Extending Control based on the existing cyclic damage prediction method, called implicit LEC and life management LEC approach were proposed. Two possible approaches to Life Extending Control based on the proposed continuous damage prediction method, called measured variables LEC and estimated variables LEC approach were defined. Here damage measurements or estimates would be used directly in the LEC. A simple hydraulic actuator driven, position control system example is used to illustrate the main ideas behind Life Extending Control. Results from a simple hydraulic actuator example demonstrate that overall system performance, that is, dynamic plus life, can be maximized by accounting for critical component damage in the control design.

REFERENCE

Figure 1. Life usage versus component strain parameterized by mean tensile stress.

Figure 2. Implicit life extending control approach
Figure 3. Effect of various life extending control algorithms on performance ($J_p$) and damage ($J_d$).

Figure 4. Life management life extending control approach.
Figure 5 Measured damage variables life extending control approach.

Figure 6. Incremental damage relationship

Figure 7 Estimated Damage Variables Life Extending Control Approach.
N = # of times $X = X_1$ or $X_2$
1 = time to achieve N
Damage(Rod) = $f(\text{FORCE}) = f(N,1)$

Figure 8 A hydraulic actuator position system.

Figure 9 Example results for Case 1 trajectory command.

Figure 10 Example results for Case 2 trajectory command.
Table I. Elementary damage prediction forms

\[
\begin{align*}
|\text{Mean Stress Level}| &= \frac{1}{2} \int_0^T \sigma(t) \, dt \\
|\text{Mean Strain Level}| &= \frac{1}{2} \int_0^T \varepsilon(t) \, dt \\
|\text{Cyclic Stress}| &= \int_0^T (\sigma^2 - \frac{1}{2} \omega^2 (t)) \, dt \\
|\text{Cyclic Strain}| &= \int_0^T (\varepsilon^2 - \frac{1}{2} \omega^2 (t)) \, dt \\
\int_0^T &\sigma \, dt = \int_0^T \sigma T \, dt \\
(a \sigma + b) &\int_0^T \sigma \, dt \\
\text{etc.}
\end{align*}
\]

Table II  EXAMPLE LIFE EXTENDING CONTROL RESULTS

<table>
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<tr>
<th>CASE</th>
<th>N</th>
<th>T, sec</th>
<th>N/T</th>
<th>F_{MAX}</th>
<th>CYCLES</th>
<th>D</th>
<th>T_f</th>
<th>N_{fail}</th>
<th>PERFORMANCE</th>
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<td>11</td>
<td>4.9</td>
<td>2.2448</td>
<td>1.6</td>
<td>5 MAJOR</td>
<td>0.0213</td>
<td>230</td>
<td>516</td>
<td>802</td>
</tr>
<tr>
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<td>9</td>
<td>4.6</td>
<td>1.9565</td>
<td>1.4</td>
<td>4 MAJOR</td>
<td>0.0121</td>
<td>380</td>
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