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TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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No. 58

ABSOLUTE COEFFICIENTS AND THE GRAPHICAL REPRESENTATION OF AEROFOIL CHARACTERISTICS.

By
Max Munk,
National Advisory Committee for Aeronautics.

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Introduction.

It is customary to examine the aerodynamic qualities of an aerofoil by considering the coefficients of the forces, rather than the actual forces, corresponding to any particular set of conditions. Such a coefficient, being always non-dimensional (absolute), is the ratio of the actual force to some standard force corresponding to the given area, relative velocity, and air density. It is only by the use of such coefficients that the designer is able to judge the qualities of a profile and to compare, on the same basis, one profile with another. The reports of tests whether on models or in free-flight usually include both the observed forces and the calculated coefficients, although it is not uncommon to find in published reports only the coefficients. The angles of attack are always given to make these data complete.

The use of absolute coefficients is almost universal in all sciences in all countries, which shows their advantage. Unfortunately in the science of aeronautics there is some

lack of agreement as to conventions and there are not only several kinds of coefficients in use in various countries but there are also differences in the methods used in plotting them. In some countries there have been changes even in the standard both for the coefficients and for the methods of plotting. As a result, the older publications using obsolete methods are confusing to the average reader who is, perhaps, familiar with the current methods only. The fact that such changes have been made is sufficient proof that there are certain advantages or disadvantages connected with each scheme in use, since it is hardly likely that an entire country would change from one system to another if all were of equal merit. Indeed, it may be shown that the lift and drag of an aerofoil supply an example of those quantities which require the use of a certain absolute coefficient and a particular method of graphical representation, in order that the results may be interpreted fully.

Coefficients.

In aeronautics there are two types of absolute coefficients which demand particular attention. The first kind is in common use in the United States and England, the second kind in Germany. The essential difference lies in the "standard force" which is only half as great in the second kind as in the first. That is, the absolute coefficients of lift and drag are determined in the United States by the expression

$$F = C \frac{\rho}{g} S V^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

where F is the force

$\frac{\rho}{g}$ the mass-density of the air

S the area of the aerofoil

V the relative velocity

C the absolute coefficient of the force F.

While the absolute coefficients of lift and drag are determined in Germany by the expression

$$F = C \frac{\rho}{2g} S V^2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

It appears upon casual examination that the system in use in the United States is the more natural and therefore the better on account of the omission of the coefficient 1/2. Upon a careful study, however, it is evident that the second system is superior to the first in two respects. Since both sides of equations (1) and (2) represent a force, the expressions $\frac{\rho}{g} V^2$ and $\frac{\rho}{2g} V^2$ must represent a force per unit area, that is, a pressure, and it is of especial importance to understand clearly the exact pressure to which each refers. Otherwise it is not likely that the exact significance of the absolute coefficients will be understood. The second expression $\frac{1}{2} \frac{\rho}{g} V^2$, is the difference between the maximum pressure on the surface of a body due to air having a velocity V and the pressure in air at rest. This pressure difference is that given by the common Pitot-tube and may be called the "Dynamical Pressure."

In the German publications it is denoted by the symbol q. On the other hand the first expression $\frac{\rho}{g} V^2$ has no physical meaning and can only be understood and felt as "Twice the Dynamical Pressure." The two expressions are closely related to the expression for the kinetic energy of a moving body, $1/2\ MV^2$. The coefficient 1/2 resulting from the integration of VdV cannot be avoided here. The expression MV^2 is never considered.

In forming absolute coefficients the choice of the natural expression $\frac{1}{2} \frac{\rho}{g} V^2$ instead of the meaningless expression $\frac{\rho}{g} V^2$ as the standard pressure, not only gives the coefficient a definite physical meaning, but also renders the quantities easily understood by enabling the density and the square of the velocity to be always grouped together and considered as the dynamical pressure.

There are additional advantages connected with the use of the natural absolute coefficients based on the expression $\frac{1}{2} \frac{\rho}{g} V^2$. The advantages are apparent when it is necessary to make use of certain theorems connecting lift, drag and angle of attack (see Technische Berichte II-2). These theorems and the formulae resulting from them are not only interesting from a physical point of view but are also of great practical value to the designer of aircraft. Furthermore, these formulae are quite simple and it requires no more mathematical knowledge and calculation to understand and to apply them than it requires to apply the simplest formulae for the stresses in bent beams.

These formulae demand the use of the natural absolute coefficients based on the actual dynamical pressure if the simple form is to be retained. The use of the absolute coefficients which are now standard in the United States and England, introduces additional factors confusing and likely to lead to error in substitution.

Graphical Representation.

There are two principal methods of representing graphically the characteristics of an aerofoil. In the United States and England it is customary to plot lift and drag coefficients as ordinates against angles of attack as abscissae, thus obtaining two curves. The continental method, sometimes called the "polar diagram"* employs but a single curve in which the lift coefficients are plotted as ordinates and the drag coefficients as abscissae. The angles of attack are commonly indicated on this diagram by figures along side of the curve.

The results of a test on a model wing are plotted in Fig. 1, according to the usual American and British practice, and in Fig. 2, the same data are plotted according to Continental usage. These methods differ greatly and the true points of difference are not always well understood. In the first place the angle of attack has no definite significance aerodynamically,

* If lift and drag are plotted in the same scale, the line of connection between any point of the curve and the origin of the system of coordinates is the vector of the force on the wing as to direction and size. Therefore, this diagram can be considered to be a polar diagram, the radii representing the absolute magnitude of the forces and the angle representing the angle between the force and the direction of motion.

for it is merely an agreement or convention which considers the chord as the direction of the section. Further, the definition of "chord" is not clear in all cases. It fails entirely when a wing is twisted (wash-in or wash-out) or when the chords of a system of two or more wings are not parallel. Consequently, in plotting coefficients against angle of attack there is obtained no natural comparison between the characteristics of various aerofoils. The position of the Y-axis has no special physical meaning and is unimportant for the qualities of the aerofoil. Hence by using this method the designer renounces one of the advantages - and the simplest too - which are connected with plotting at all.

The designer usually desires a large lift and a small drag. These two quantities and their relation to each other are most important in making an estimate of the value of an aerofoil. The angle of attack is merely a structural consideration. In order to obtain a connection between the lift and drag when separate curves are plotted against angle of attack, it is necessary to carry through tedious mental processes and the final result can not compare in vividness with the mental picture given by a glance at a polar diagram.

There are also reasons why the polar diagram is the "natural" method of representing aerofoil characteristics graphically. Aerodynamical theorems and actual tests prove that the lift depends not upon the angle of attack but upon the flow about the

wing. That is to say, the air flow around wings of the same sections but of different plan form is the same for equal lift coefficients and not necessarily for equal angles of attack. Furthermore, the drag may be divided into two parts, one of which depends upon the lift but neither of which depend upon the angle of attack. One of the formulae previously mentioned (Technische Berichte II-2) provides a very simple method by which one may calculate that part of the drag which is due only to the particular arrangement and proportions of the lifting surfaces. This part of the drag is independent of the aerofoil section and is called the "induced drag." The induced drag may be considered as the minimum limit of drag consistent with the aspect ratio used and is an ideal which may be approached through the reduction of "section drag" but which can never be equaled. This "section drag" is conditioned by the aerofoil section and must be obtained from tests either on models or in free flight. This part of the drag is determined for example, by the change in the performance of an airplane when only the total load is changed. The first and sometimes the more important part of the drag may be calculated very quickly with a slide rule, and without the necessity of tests, may be plotted as a parabola, dependent upon the lift. The formula for this "induced" drag is:

$$D = \frac{1}{\pi} \frac{L^2}{V^2 \frac{\rho}{2}} \cdot \frac{1}{B^2} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where L is the lift; B the span; and $V^2 \rho/2$ the dynamical

pressure. Written in absolute coefficients, defined by

$$D = D_C \frac{\rho}{2} V^2 S, \text{ etc.}$$

where S is the area, the same formula becomes

$$D_C = \frac{1}{\pi} L_C^2 \frac{S}{B^2} \quad \ldots\ldots\ldots\ldots\ldots (1a)$$

This formula holds for single aerofoils; and for systems of aerofoils, B is to be replaced by kB where k is a coefficient somewhat different from 1. This formula represents a parabola which however, cannot be plotted dependent upon the angle of attack, without tests, because there is no definite relation between the lift and the angle of attack. The designer who uses the plots of lift and drag against angle of attack instead of the polar diagram gives up half of the advantages to be obtained from the use of the formulae.

Regardless of the attitude of the designer towards the method which he uses to plot aerofoil characteristics, it is certain that his conclusions are influenced by these diagrams, and that an unfavorable or obscure diagram may lead to a wrong conclusion. The curves of lift plotted against angle certainly do this very often. For instance, a designer may be led to compare two different wings, or even two different sections, at the same angle of attack instead of at the same lift or drag coefficient. An ingenious man usually draws correct conclusions; but it is an advantage to use diagrams which may be also interpreted by men who are not specialists in aerodynamics.

Other Possibilities of Graphical Representation.

The principal difference between the two kinds of plotting mentioned is the change of the variables. There are special advantages connected with the plotting of the lift and drag coefficients directly against each other. Now these advantages would not vanish if, instead of plotting the coefficients themselves, functions of them were taken. It is worth while to compare the advantages of several such diagrams.

Any two such diagrams are mathematically connected with each other. Any construction in the one diagram can be repeated in the other, and to each curve drawn in the one belongs a corresponding curve in the other. In general, the corresponding curve is not a straight line if the original curve is a straight line. The chief difference between different diagrams lies in the type of curve by which the most important relations are represented.

In the diagram generally used, L_c against D_c the curves of constant L_c and D_c of constant L/D, and of constant velocity are straight, and the "induced" coefficient curve and the important curves for constant power $\frac{D_c}{L_c^{3/2}}$ = const. are curved lines. If one coordinate is the drag coefficient itself, the addition of a constant drag coefficient, for instance, when proceeding from the wings to the entire airplane, can be represented by merely transferring the origin of the system of coordinates. The original curve remains unaltered. This quality of the diagram is so useful that a diagram without it would be inferior. Whence it

follows that D_c should always be plotted directly in one direction. To so choose coordinates that the curves for constant L/D are straight is in a smaller degree advantageous. It is true that L/D is frequently considered in present practice, but this is done, not because it means very much, but because in the present diagrams this quantity is the only one giving a direct relation between L_c and D_c. It would be better if the curves for constant power are straight, for the power is more important than the angle of gliding. This can be obtained by plotting $L_c^{3/2}$ instead of L_c against D_c. The induced coefficient remains a curved line, and all advantages of the first diagram remain too. It is not even necessary to calculate and to put in the values of the 1.5 power of L_c; for, as in logarithmic diagrams, it is quite sufficient to use a proper variable graduation of the corresponding axis of coordinates.

Another possibility would be the plotting of $D_c^{3/2}/L_c$ against D_c. The power would be plotted as it were against the drag, whereas in the preceding diagram it can be considered as being plotted against the lift. This seems to be more natural.

L/D against L_c sometimes used in England, gives straight lines for constant D_c; but the addition of a constant D_c requires a new curve; nor are the curves for the induced coefficient or for constant power nor for L_c straight. The drawing of a new curve when adding a constant D_c is still more complicated than before.

There remains therefore only the diagram $L_c^{3/2}$ against D_c

as a competitor to the diagram L_c against D_c. The differences between the two diagrams are not considerable. It is convenient to have straight lines for constant power but the odd power of L_c is sometimes confusing. In any case the advantages are not sufficient to compensate for the disadvantage of using diagrams differing from those used in most other countries.

Conclusions.

In addition to the important features connected with the use of natural absolute coefficients in polar diagrams there are several minor advantages. A few of the special applications are given in the above references. On the whole it appears that the use of natural absolute coefficients in a polar diagram is the logical method for presentation of aerofoil characteristics. Serious consideration should be given to the advisability of adopting this method in all countries. The actual adoption would be a great advancement of uniformity and accuracy in the science of aeronautics.

%	Up'r	Lw'r
0	4.40	4.40
1¼	6.90	2.15
2½	8.10	1.48
5	10.00	.90
7½	11.35	.70
10	12.40	.55
15	13.97	.28
20	15.08	.10
30	16.02	0
40	15.80	0
50	14.82	0
60	13.25	0
70	10.87	0
80	8.10	0
90	4.83	.38
95	3.17	.79
100	1.40	1.40

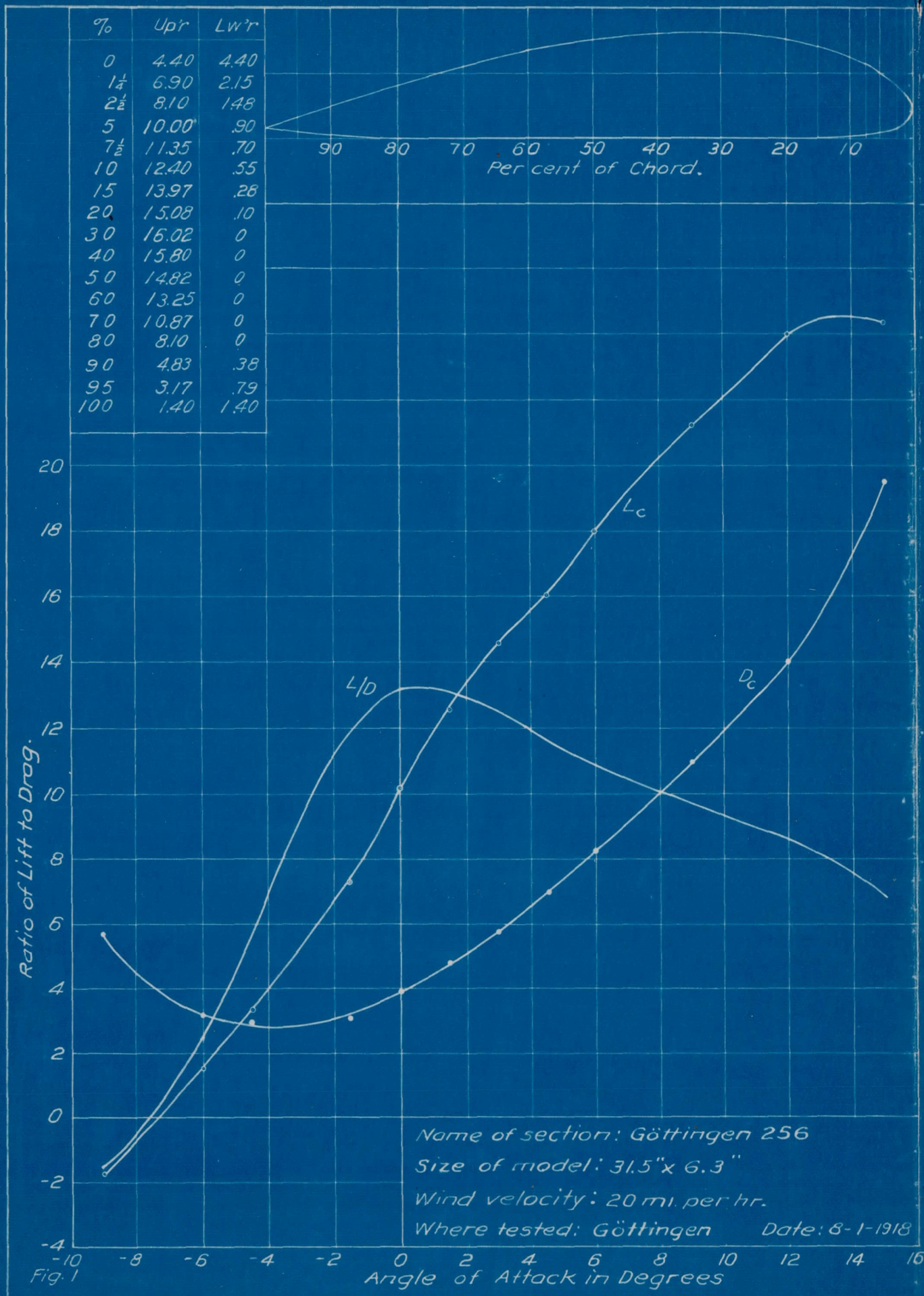

Fig. 1

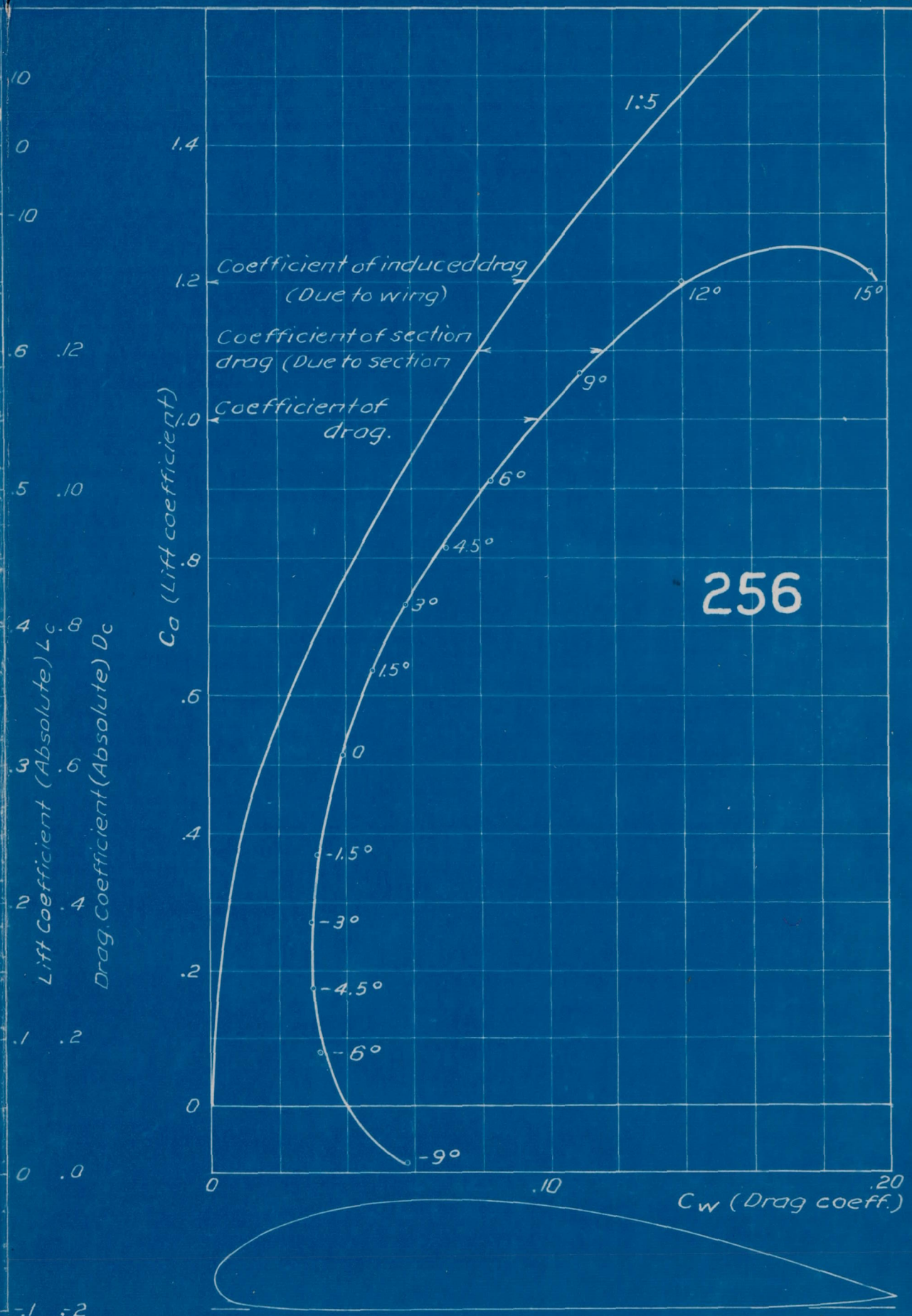

Fig. 2

Figs. 1-2.