THE FACTORS THAT DETERMINE THE MINIMUM SPEED OF AN AIRPLANE.

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March, 1931.
A large range between the maximum and minimum speeds of an airplane is of undisputed value, either to permit safe landings in small fields with the medium or slow speed machine, or to permit landing at all with very high speed machines. The factors which limit the maximum speed are well understood, but rather strangely the limiting factors of the minimum speed have seldom been recognized. The whole question of minimum speed has usually been settled by the statement that the wings have reached the point of maximum lift, whereas there are very few airplanes that can be flown at, or beyond, this point, and a great many that can not reach within 50 of it. Because of this general misunderstanding of the principles of flight at low speed there are a large number of machines that could be made to fly several miles slower than at present by slight modifications. In the following paragraphs, therefore, the factors that affect the minimum speed will be discussed with the hope that some of the present uncertainty will be cleared up.
The wing section has a large effect on the minimum speed of an airplane because this determines the maximum lift coefficient of the supporting surface. This lift coefficient is usually found from model tests in the wind tunnel, and in order to show the range of values obtained and the approximate sections for each, the following table is given:

<table>
<thead>
<tr>
<th>Section</th>
<th>Shape</th>
<th>$L_0$ - M.P.H.</th>
<th>$L_0$ - Lbs/Sq.ft</th>
<th>Absolute</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.A.F. 15</td>
<td></td>
<td>.00274</td>
<td>.538</td>
<td></td>
</tr>
<tr>
<td>R.A.F. 6</td>
<td></td>
<td>.00308</td>
<td>.604</td>
<td></td>
</tr>
<tr>
<td>U.S.A.T.S. 16</td>
<td></td>
<td>.00323</td>
<td>.770</td>
<td></td>
</tr>
<tr>
<td>Martin with flap</td>
<td></td>
<td>.00516</td>
<td>1.005</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1

Of course if a low speed were the only consideration, the highest lift wing would be chosen; but usually it is speed range that is the object, and the selection of a wing for this pur-
pose comes beyond the scope of the discussion. In order to show how greatly the wing section affects the minimum speed, curves are plotted in Fig. 2 against various loadings.

If a number of lift curves from model tests are examined it will be found that the lift falls off beyond the maximum in some cases slowly as in curve (1) in Fig. 3, some rapidly as in curve (2) and some, especially the high lift sections, drop off suddenly (curve 3). Now it certainly would be most awkward when pulling up the nose of an airplane in making a landing to have the lift fall off suddenly 25 or 50 per cent, and for this reason it was formerly thought unwise to use wings that showed a discontinuous lift curve in the model. However, more recent tests have proved that if these sections are run at a high enough speed the discontinuity disappears. Also the fuselage in combination with the wings has the property of flattening the burble point. These facts are shown very strikingly in Fig. 4, where the lift curves are plotted for a model wing, the same wing in a model airplane, and the full-sized airplane.* The lift values show a close agreement up to 16° where they begin to diverge, the full-sized machine continuing straight on, the model wing falling off rapidly, and the model airplane taking an intermediate path.

The disposition of the wings on the airplane slightly affects the lift coefficient and a few cases will be discussed. The aspect ratio has a slight effect on the maximum lift as shown in Fig. 5.** A biplane will have a maximum lift of about

* N.A.C.A. Report #96.

** Bairstow - Applied Aerodynamics, p.137.
96% of that of a monoplane, while a triplane will give only 92%. In some cases a monoplane seems to give an abnormally high lift due to the cushioning effect of the air between the ground and the wing, but no really accurate tests have been made of this. However, a model of the JN biplane has been tested in the tunnel at varying distances from a flat surface representing the ground and it was found that the lift and drag at the three point landing angle with the wheels just free of the ground were each increased 5%.* It would be expected that a monoplane with a wing close to the ground would show an even greater effect than this. Stagger also has a slight effect on the lift as shown in Fig. 6**, and gap chord ratio has still less (Fig. 7).**

It has been found that the lift coefficients from models can not be applied directly to full-sized machines, and this is especially true in regard to the high values in which we are interested. It is difficult to obtain values of the lift coefficient in full flight at the burble point due to the great skill required to fly a machine steadily at this angle. The burble point, however, was reached with a JN4h airplane (Fig. 4) in one case. It has been the practice to compute the landing speed of a machine from the maximum lift coefficient obtained on the model wing, and by a coincidence this procedure is very nearly correct as the full-sized machine lands at an angle of attack much lower than the burble point. It is necessary therefore to make a distinction between landing speed and minimum

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* Variation in Resultant Pressure upon Landing Due to Proximity of the Earth. A. A. Memil - The Ace, December, 1930.
** Bairstow - Applied Aerodynamics, p.146.
speed, the former occurring between 10° and 14° and the latter between 18° and 20°. As will be shown later this difference is due mainly to the fact that the controls are not powerful enough to safely hold the nose of the machine up in a glide.

There is one other factor associated with the wings that has a definite, although usually slight, effect on the minimum speed, and that is the extra lift exerted by the slip stream on the wings. If the weight of the machine \( W \) is assumed to be supported only by the wings:

\[
W = L_c A V^2 - m L_c \cdot n A \cdot p^2 V^2
\]

where

\( L_c \) is the maximum lift coefficient of the wings,
\( A \) is the area of the wings,
\( V \) is the minimum speed,
\( m \) is the ratio of the lift coefficient at the angle between the wings and the thrust line to the maximum lift coefficient,
\( n \) is the ratio of the effective area in the slip stream to the total area,
\( p \) is the ratio of the velocity in the slip stream to the air speed.

then

\[
V = \sqrt{\frac{W}{A}} \cdot \frac{1}{L_c} \cdot \sqrt{\frac{1}{1 - mnp}}
\]

The last radical contains only those terms affected by the slip stream. In Fig. 8 are plotted a few curves with various values of the constants \( m \) and \( n \). On the usual tractor machine the percentage of effective wing area is very small so that the reduction in speed from this cause is at most only a few per cent.
When an airplane is flying slowly with the throttle open (climbing) the thrust axis is inclined upward several degrees so that there will be a vertical component of the thrust given by:

\[ Z = T \sin \theta \]

where \( T \) is the thrust and \( \theta \) the angle of the thrust axis to the horizontal. It is possible to fly a powerfully controlled airplane at a very steep angle even when a constant altitude is held. In Fig. 9 is plotted a curve showing the decrease in speed due to the direct lift of the airscrew on a 2000 pound machine with a 400 pound thrust. It is noticed that with a 30° inclination - the largest that is likely to occur - the decrease in speed is only 4¾.

It may happen that on low powered airplanes the minimum speed in level flight is determined by the engine power, that is, as the power increases with a decrease in speed for low speeds, the power may not be sufficient to allow reaching the minimum speed. This is shown in Fig. 10 for a JN4 with a 150 and a 90 horsepower motor; the latter power giving a minimum speed 3 m.p.h. greater than the former. In gliding flight this factor would not, of course, enter in.

Every pilot knows that it is necessary to hold the stick well back when flying at the minimum speed, and this is especially true in a glide when the elevators are not in the slip stream. In a great many machines the controls are pulled back to their greatest extent when flying slowly, and in such cases the longitudinal control is the limiting factor of the minimum
speed. In the majority of the flying range - from 10 m.p.h. above the minimum to the maximum - the movement of the controls is very slight, but below a certain critical velocity the stick must be pulled back rapidly. This is shown clearly by a few control position curves from free flight tests plotted in Fig. 11.* It is also evident that as far as the longitudinal control is concerned a lower air speed can be obtained by an open throttle.

The reason for this break in the control position curves is due mainly to the fact that the center of pressure travel on the wing changes from an unstable to a stable direction at this speed; that is, at the lower air speeds the machine becomes very stable and attempts to nose down strongly, so that only a powerful tail force can hold it in slow speed equilibrium. There seems to be no way in which this break in the control position curve can be prevented, so that this factor imposes a serious obstacle to the safe and comfortable attainment of the lower speeds. All that can be done, and this is in other ways detrimental, is to use a powerful elevator, or a tail heavy and an unstable machine.

That the longitudinal control can have an important effect on the minimum speed was recently demonstrated on an experimental JN4h with a special tail to provide great stability. With this tail the minimum speed that could be reached was 50 m.p.h. while with the regular tail the minimum speed was 40 m.p.h., a very considerable difference.

We now come to the last and most important factor affecting the minimum speed, the lateral control. The lateral control

*N.A.C.A. Report #93.
is seldom associated with the ability to fly at very low speeds, but nearly every pilot will say that the reason he can not fly more slowly is that the machine stalls, and a stall is falling into a side slip or spin because of the ineffectiveness of the ailerons and rudder. As the speed of an airplane approaches its minimum the action of the ailerons is seen to be very sluggish; in fact, if the stick is pushed sharply over the machine does not roll, but yaws sharply toward downward aileron. The ineffectiveness of the ailerons is shown very strikingly by a few curves taken from a model test* (Fig. 13). As the angle of incidence is increased the rolling moment grows smaller, becoming zero for no yaw at about 17° angle of attack, and at higher angles becoming negative. This means that at 17° the ailerons could not produce any rolling moment for this particular test, and the conditions would be nearly the same for any type of machine.

The other member of the lateral control, the rudder, is more effective at high angles of incidence than the ailerons and has the additional advantage of being in the slip stream, but it can not directly produce a rolling moment. It is used, however, almost entirely to produce lateral balance by causing an angle of yaw which in turn produces a rolling moment as shown by the curves in Fig. 13. A pilot uses the rudder almost entirely when flying at very low speeds to keep his lateral balance, and the more skillful he is the slower can he fly without pulling into a spin.

* R. & M. No.152, British Advisory Report.
The stalling speed of an airplane is usually not any definite figure for a certain machine but is a function also of the pilot. As an instance of this a pilot was able to fly a certain machine no lower than 43 m.p.h. even after repeated trials; another pilot on the same machine and with the same weight, after considerable practice was able to reach a steady speed of 40 m.p.h. because of his greater skill in using the rudder to prevent the machine from falling into a spin.

When designing a machine the preceding conditions for low speed should be considered, as they do not in general conflict with the other desirable properties. In particular, care should be taken to provide a powerful lateral control as most pilots quite properly refuse to make full use of the low speed properties of their machine because of the chance of pulling into a spin or sideslip. A great many crashes or landings can be traced to a lack of lateral control. Excessively large ailerons cannot be used on a high speed machine because they are too stiff, that is, it is necessary to slow down before it is possible to go into a turn with any considerable bank, but it would seem possible to increase their efficiency without making them larger.

In conclusion, the following list of factors affecting the minimum speed of an airplane is given with the approximate magnitude of their influence in per cent based on the maximum variation of the factors that is likely to occur. The percentages given are of necessity quite arbitrary and are only intended in a general way to show the relative unimportance of all the fac-
tors except Nos. 1, 10, and 11. The wing loading is assumed to be specified and so does not come into the discussion.

1. Wing Section - 26%.
2. Wing Loading - a given condition.
3. Aspect Ratio - 4%.
4. Gap Chord Ratio - 4% (Compared with Monoplane).
5. Stagger - 2%.
6. Scale - 3%.
7. Slipstream on wings - 1%.
8. Vertical component of Air Screw Thrust - 3%.
10. Longitudinal Control - 20%.
11. Lateral Control - 15%. 
Effect of gap/cord ratio on minimum speed.
No stagger.

Fig. 7

Gap/cord = 1

Fig. 6

Figs. 6 and 7
Effect of slip stream on the minimum speed.

$m$ is the ratio of the lift coefficient at the angle between the wings and thrust line to the maximum lift coefficient.

![Graph](image)

**Fig. 8.**

Percentage of normal landing speed vs. fraction of wing in slip stream.

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The lowering of the air speed by the vertical component of the thrust.

![Graph](image)

**Fig. 9.**

Percent of normal landing speed vs. angle of incidence.

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Power available vs. power required.

![Graph](image)

**Fig. 10.**

Engine HP vs. air speed (m.p.h.).
Figure 11: Variation of elevator angles.

Figure 12: The effect of angle of incidence on rolling moment.

- Right aileron down 10°
- Left aileron up 10°

For angles of incidence:

- +10° yaw
- 0°
- -10° yaw

Figures 11, 12.