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TECHNICAL NOTES

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SIMPLE FORMULA FOR ESTIMATING AIRPLANE CEILINGS

By Walter S. Diehl,
Bureau of Aeronautics, U.S.N.

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Summary.

The absolute ceiling of an average airplane is given to a close approximation by

\[ H = 19000 \log_{10} \left( \frac{10000}{\left( \frac{W}{HP} \right)^2 \left( \frac{W}{S} \right)} \right) \]

where

- \( H \) = absolute ceiling, in feet
- \( \left( \frac{W}{HP} \right) \) = "power loading," based on full load and normal B.H.P.
- \( \left( \frac{W}{S} \right) \) = wing loading, based on full load.

The aeronautical engineer often has occasion to estimate the absolute ceiling of an airplane for which a detailed performance calculation is out of the question. In such cases it is customary to use either empirical performance charts or formulae. The performance charts which are given in several of the recent
works on aerodynamics and performance are satisfactory so long as
the airplane under consideration does not depart too far from the
average in its characteristics. The formulae, with one exception,
are no better. This exception is developed by Kann in Technische
Berichte 1-6 and is of the form

\[ H = C_1 \log_{10} \left( \frac{C_L^3}{C_D^2} \frac{\eta^2}{\left( \frac{W}{3000} \right)^3 \left( \frac{W}{S} \right)} \right) \]  

(1)

where

- \( H \) = absolute ceiling
- \( \eta \) = propeller efficiency
- \( \left( \frac{W}{3000} \right) \) = power loading (\text{gross load/normal BHP})
- \( \left( \frac{W}{s} \right) \) = wing loading (\text{gross load/wing area})
- \( C_L \) = absolute lift coefficient
- \( C_D \) = absolute drag coefficient
- \( C_1 \) & \( C_2 \) = constants.

The chief criticism of this formula is that it is too compli-
cated for extensive use.

Experience has shown that the terms \( \left( \frac{C_L^3}{C_D^2} \right) \) and \( \eta^2 \) may be
neglected without seriously affecting the results given by the
formula. That is, we may write

\[ H = K_1 \log_{10} \left( \frac{K_2}{\left( \frac{W}{3000} \right)^3 \left( \frac{W}{S} \right)} \right) \]  

(2)
and determine appropriate values of the constants $K_1$ and $K_2$ by calculation or from reliable performance data. Obviously $K$ may be arbitrarily assigned any value which will cause the term within the brackets to assume a reasonable value. The average value of \((\frac{W}{HP})^2 \cdot \frac{W}{S}\) is of the order of 1000 and it is desirable that the log term be about unity.

\[ K_2 = 10000 \]  \hspace{1cm} (3)

fulfills this requirement and the formula may be written:

\[ H = K \log_{10} \left( \frac{10000}{(\frac{W}{HP})^2 \cdot \frac{W}{S}} \right) \]  \hspace{1cm} (3a)

$K$ has been determined for a number of airplanes of all types and is found to have an average value of 19000 with individual values ranging from 17500 to 21000. In most cases, however, the value is quite close to 19000 as shown by Table 1, which contains a few representative determinations of $K$.

In any particular case for which the performance is known, the formula is quite accurate in predicting the effect of small changes in $W$, B.H.P., or $S$. To do this, Equation (3a) is solved for the exact value of $K$ by substitution of the original data and this value is used with the new values of \((\frac{W}{HP})\) or \((\frac{W}{S})\) to obtain the new ceiling.

The absolute ceiling for any common values of \((\frac{W}{HP})\) and \((\frac{W}{S})\) may be read directly from Fig. 1 which is based on Equation (3a) with $K = 19000$. 
Table 1.

Value of constant $K$ in

$$H = K \log_{10} \frac{10000}{\left(\frac{W}{HP}\right)^2 \left(\frac{W}{S}\right)}$$

<table>
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<tr>
<th>Airplanes</th>
<th>$\left(\frac{W}{HP}\right)$</th>
<th>$\left(\frac{W}{S}\right)$</th>
<th>$H$</th>
<th>$K$</th>
<th>Remarks</th>
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