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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 214

NOTE ON THE KATZMAYR EFFECT ON AIRFOIL DRAG.

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The reduction in drag of an airfoil when the air stream is oscillating is called the Katzmayr effect. This effect was first described by Dr. Katzmayr, director of the Vienna Aerodynamical Laboratory, in "Zeitschrift für Flugtechnik und Motorluftschiffahrt," March 31 and April 15, 1922 (Reference 1). Confirming experiments have been made by Toussaint, Kerneir and Girault, and are given in a report translated by the National Advisory Committee for Aeronautics in Technical Note No. 202 (Reference 2). That report confirms Katzmayr's results, yet contains no explanation or reason why an oscillating wind should reduce the airfoil drag. The purpose of this note is to offer a simple explanation of the cause of the Katzmayr effect.

Consider first the condition in a perfectly steady air stream. The drag, of course, is measured by a balance the linkages of which are so aligned that the component of force along the wind stream can be read directly. As no artificial air stream is absolutely steady in velocity or direction, it is necessary that the balance be heavily damped by dashpots or equivalent arrangements. Furthermore, as the forces to be measured are not inconsiderable, the balance will inevitably have a large moment of inertia and a long natural period of oscillation. The drag balance therefore will not follow a rapid oscillation in the true magnitude of the drag component, even though the amplitude of the variation be large, but will give a mean reading representing the integral of the instantaneous drag with respect to time divided by the time of a complete oscillation.

If the air stream is oscillating in direction a correctly aligned drag balance will read the mean of the components of force parallel to the normal steady wind. As the wind direction changes from normal the drag balance is no longer in correct alignment with the instantaneous direction, so a component of lift is added to or subtracted from the true drag. In the case of an airfoil within the ordinary working range of angles of attack the components to be subtracted are larger than those to be added, as the component of lift measured on the drag arm is negative when the change of wind direction is such as to increase the true angle of attack and therefore when the lift is largest (β positive, as shown in Fig. 1). The mean component parallel to the wind direction, which is what the balance reads, will therefore be reduced by the existence of the oscillation (Reference 3). When the frequency of the oscillation is such that the wing chord is only a small fraction of a wave length, the angle of attack may be considered constant along the chord at any instant during an oscillation, and the lift and drag components parallel to the normal wind direction can be calculated from values made in a normal steady wind. If the wave is so short that it approaches the wing chord, the above assumption cannot be made and the calculation becomes invalid. The changing direction of the stream, like the induced drag on an airfoil of low aspect ratio, then acts to change the effective curvature of the section. However, even at the highest frequency and the slowest speed of test mentioned in N.A.C.A. Technical Note No. 202 (Reference 2), the wave length in the Toussaint experiments was five times the wing chord.

On the assumption that the angle of attack is constant along the chord the process of calculation becomes comparatively simple. In Fig. 1

α = hypothetical angle of attack, based on the mean wind direction

L = true lift at α°

D = true drag at α°

β = angle between mean wind direction and wind direction at any instant, + when angle of attack is increased.

L_1 and D_1 = true lift and drag at $(\alpha + \beta)^\circ$

L_2 and D_2 = true lift and drag at $(\alpha - \beta)^\circ$

D' = reading of balance when $\beta = +\beta_1^\circ$

D'' = reading of balance when $\beta = -\beta_1^\circ$

$$D' = D_1 \cos\beta_1 - L_1 \sin\beta_1$$

$$D'' = D_2 \cos\beta_1 + L_2 \sin\beta_1$$

From the method of generation the wave form of the oscillation is approximately harmonic, or $\beta \pm K \sin t$, where "K" is the amplitude and "t" the time. The mean effect may be found by integrating the mean force when the oscillation is $\pm\beta^\circ$, one-half of the curve for a complete cycle being folded back on the other part, that is:

$$F = \text{mean drag reading} = \frac{\int \frac{D' + D''}{2}\,dt}{\int dt}$$

If K is small, the slope of the lift and the change of slope of the drag may be considered constant. The form of the mathematical solution was originally due to Prof. Edward P. Warner. The introduction of curvature, giving a somewhat closer approximation, was suggested by Dr. H. L. Dryden.

$$\frac{D' + D''}{2} = \frac{D_1 + D_2}{2} \cos\beta - \frac{L_1 - L_2}{2} \sin\beta$$

Replacing $\cos\beta$ by 1 and $\sin\beta$ by β

$$\frac{D_1 + D_2}{2} = D + \frac{1}{2}\,\frac{d^2D}{d\alpha^2}\,\beta^2$$ since change of slope is assumed constant.

$$\frac{D' + D''}{2} = D + \frac{1}{2}\,\frac{d^2D}{d\alpha^2}\,\beta^2 - \frac{dL}{d\alpha}\,\beta^2$$

Integrating over a half cycle (the combination of the positive and negative angles making this equivalent to integration for a complete cycle).

$$F = \frac{\int_0^{\pi} \frac{D' + D''}{2}\,dt}{\int_0^{\pi} dt} = \frac{D\int_0^{\pi/\omega} dt + \left(\frac{1}{2}\frac{d^2D}{d\alpha^2} - \frac{dL}{d\alpha}\right)\int_0^{\pi/\omega} \beta^2\,dt}{\int_0^{\pi/\omega} dt}.$$

$$\int_0^{\pi/\omega} \beta^2\,dt = K^2\int_0^{\pi/\omega} \sin^2 \omega t\,dt = \frac{K^2}{2}\left(t - \frac{\sin 2\omega t}{2\omega}\right)\Bigg|_0^{\pi/\omega} = K^2 \times \frac{\pi}{2\omega}$$

$$F = D + \frac{K^2}{4}\frac{d^2D}{d\alpha^2} - \frac{K^2}{2}\frac{dL}{d\alpha} \quad K \text{ and } \alpha \text{ in radians}$$

$$F = D + \frac{K^2}{4}\frac{d^2D}{d\alpha^2} - \frac{K^2}{114.6}\frac{dL}{d\alpha} \quad K \text{ and } \alpha \text{ in degrees.}$$

In the use of this formula it is more convenient to replace forces by coefficients

$$C_F = C_D + \frac{K^2}{4}\frac{d^2C_D}{d\alpha^2} - \frac{K^2}{114.6}\frac{dC_L}{d\alpha}$$

Fig. 6 of Technical Note No. 202, reproduced as Fig. 2, in this paper, at 0° C_D is .041, $\frac{dC_L}{d\alpha}$ is .070, $\frac{d^2C_D}{d\alpha^2}$ is about .0007. With an amplitude of oscillation of $\pm 10^\circ$

$$C_F = .041 + \frac{100}{4}\,.0007 - \frac{100}{114.6}\,.070 = .041 + .018 - .061 = -.002.$$

The experimental value is +.003. If, instead of the values of $\frac{dC_L}{d\alpha}$ and $\frac{d^2C_D}{d\alpha^2}$ at 0° the mean values from -10° to $+10^\circ$ are used, the agreement with the experimental result is even better. It then seems clear that the instantaneous lift and drag of an airfoil are functions only of the altitude at that instant, and quite independent of the fact that the angle of attack may be in process of change.

The mean drag coefficient C_F may also be found by an arithmetical integration which makes due allowance for all variations in C_D and C_L. The work is given in Table I for a mean angle of 0°. C_D' is calculated for values of β spaced at 2° intervals from $+10^\circ$ to -10°. Assuming that each C_D' affects the mean C_F

in proportion to the time it persists, and using 0° as the mean from -1° to 1°, 2° from $+1^\circ$ to $+3^\circ$ and so on ($9\frac{1}{2}^\circ$ is used in place of 10° as the last point, for obvious reasons), the mean C_F is found. C_F works out by this method, in the case just given, at -.003.

Similar computations have been made by the mathematical and arithmetical methods for $\alpha = \pm 4^\circ$, each with 10° amplitude.

α	Math.	Arith.	Exper.
-4°	-.026	-.022	-.015
0	-.002	-.003	+.003
$+4^\circ$	+.027	+.028	+.031

The agreement between the calculated and experimental results is fair, certainly good enough to indicate that the explanation offered is correct.

By the mathematical analysis it appears that the reduction in drag coefficient should vary as the square of the amplitude of oscillation. In Fig. 9 of Technical Note No. 202 (Reference 2), the reduction of coefficient due to a 10.5° amplitude is .038, while that due to 16.5° is only .058, instead of .094, if the square law held. But in Fig. 12, the reductions are:

Amplitude	8°	13°	17°
Reduction	.036	.079	.110
Varied as square based on 13°	.030	.079	.135

These variations are nearer the square. Such aberrations as exist for the large amplitudes are accounted for by the increase of the angle of attack in the course of the oscillation to far beyond the point of maximum, invalidating the assumptions on which the analysis was based.

A further application of the Katzmayr effect, suggested by Mr. Walter F. Eade, is that it may be the cause of part of the variation of drags measured in various tunnels. A small natural oscillation of direction of the wind stream would have exactly the same effect as an artificially produced oscillation. Using the mathematical analysis and value of C_D, $\frac{dC_L}{d\alpha}$, and $\frac{dC_D^2}{d\alpha^2}$ for a good wing near minimum drag, the reduction in drag, due to an oscillation of 1° amplitude, is 5.2%, and that due to $1/2^\circ$ is 1.3% ($C_D = .0120$, $\frac{dC_L}{d\alpha} = .100$, $\frac{dC_D^2}{d\alpha^2} = .001$). It is certainly not inconceivable that there may be an oscillation in wind direction of $1/2^\circ$ at some wind speed in almost any type of tunnel. Furthermore, the fact that this oscillation may change with wind speed would offer an explanation of the strikingly different characteristics shown by "scale effect curves" taken in different tunnels.

TABLE I.

Effective Drag Reading at 0° Mean Angle

$\alpha+\beta$	C_L	C_D	$C_D \cos\beta$	$-C_L \sin\beta$	D'	t	C_D't
$-9\frac{1}{2}°$	-.02	.016	.016	-.003	.013	.451	.0059
-8	.11	.016	.016	.015	.031	.343	.0107
-6	.27	.018	.018	.028	.046	.253	.0117
-4	.42	.023	.023	.029	.052	.219	.0114
-2	.58	.031	.031	.020	.051	.205	.0104
0	.72	.041	.041	.000	.041	.200	.0082
2	.86	.054	.054	-.030	.024	.205	.0049
4	.98	.067	.067	-.069	-.002	.219	-.0004
6	1.10	.084	.084	-.115	-.031	.253	-.0079
8	1.21	.104	.103	-.168	-.065	.343	-.0223
$9\frac{1}{2}$	1.27	.121	.119	-.210	-.091	.451	-.0410
						3.142	-.0084

$\alpha = 0°$

Values of C_L and C_D from Fig. 6, N.A.C.A. Technical Note No. 202 (Reference 2).

TABLE II.

Effective Drag Reading at -4° Mean Angle.

$\alpha+\beta$	β	C_L	C_D	C_D'	$C_D't$
$-13\frac{1}{2}^{\circ}$	$-9\frac{1}{2}^{\circ}$	-.31	.027	-.018	-.0081
-12	-8	-.21	.021	-.008	-.0027
-10	-6	-.05	.016	+.011	.0028
- 8	-4	.11	.016	.024	.0053
- 6	-2	.27	.018	.029	.0059
- 4	0	.42	.023	.023	.0046
- 2	2	.58	.031	.011	.0023
0	4	.72	.041	-.009	-.0020
2	6	.86	.054	-.036	-.0091
4	8	.98	.067	-.070	-.0240
$5\frac{1}{2}$	$9\frac{1}{2}$	1.07	.078	-.100	-.0451
					-.0701

TABLE III.

Effective Drag Reading at +4° Mean Angle.

$\alpha+\beta$	β	C_L	C_D	C_D'	$C_D't$
$-5\frac{1}{2}°$	$-9\frac{1}{2}°$	.30	.019	.068	.0306
-4	-8	.42	.023	.081	.0278
-2	-6	.58	.031	.092	.0233
0	-4	.72	.041	.091	.0199
2	-2	.86	.054	.084	.0172
4	0	.98	.067	.067	.0013
6	2	1.10	.084	.046	.0094
8	4	1.21	.104	.019	.0042
10	6	1.30	.127	-.010	-.0050
12	8	1.37	.156	-.037	-.0127
$13\frac{1}{2}$	$9\frac{1}{2}$	1.40	.175	-.059	-.0266
					.0894

References.

1. R. Katzmayr: Effect of Periodic Changes of Angle of Attack on Behavior of Airfoils. Translation from "Zeitschrift für Flugtechnik und Motorluftschiffahrt," March 31 and April 13, 1922. N.A.C.A. Technical Memorandum No. 147. 1922.

2. Toussaint, Kerneis and Girault: Experimental Investigation of the Effect of an Oscillating Air Stream (Katzmayr Effect) on the Characteristics of Airfoils. Translation from the French. N.A.C.A. Technical Note No. 202. 1924.

3. A. F. Zahm: Influence of Model Surface and Airflow Texture on Resistance of Aerodynamic Bodies. N.A.C.A. Technical Report No. 139. 1922.

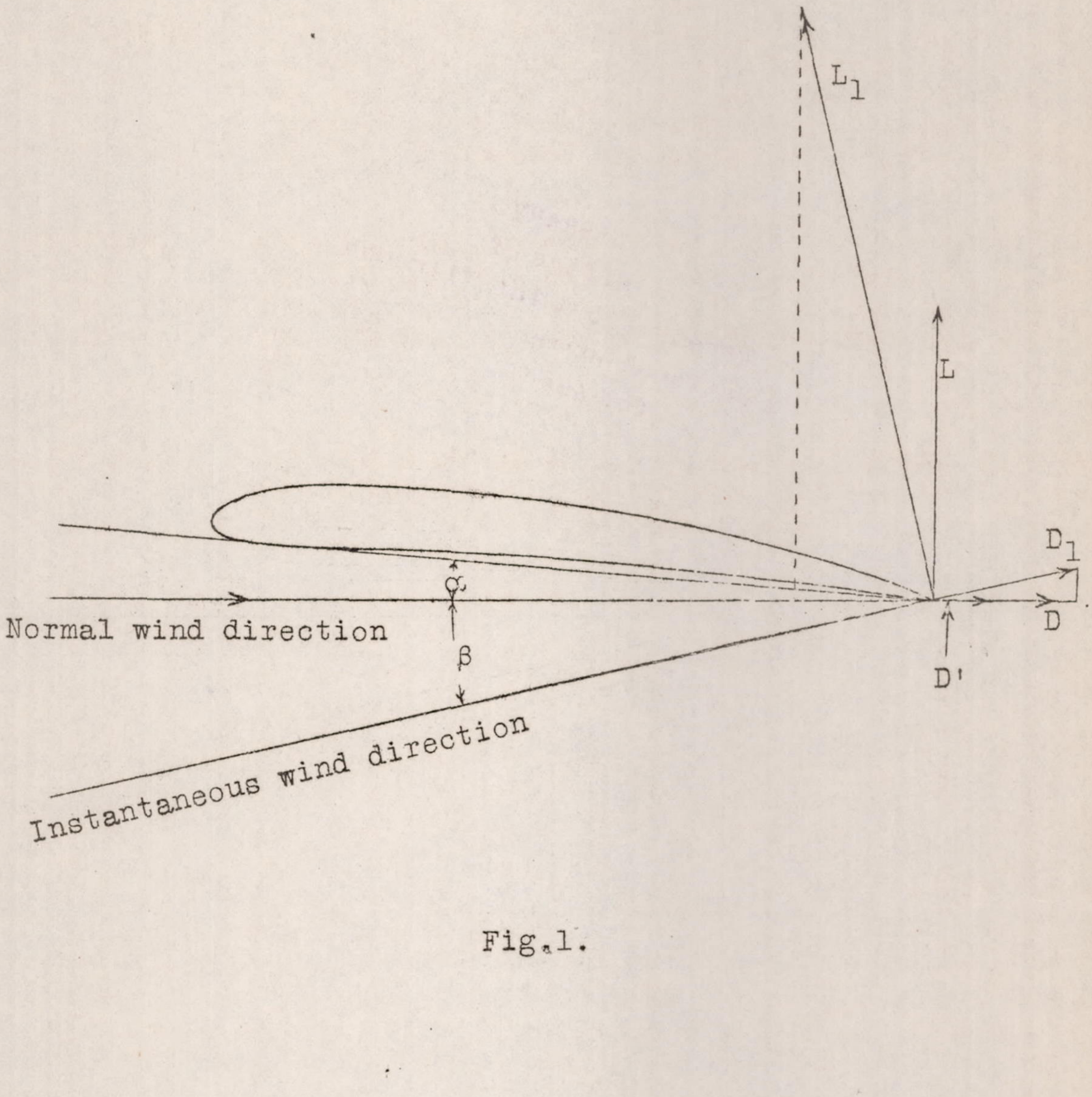
L_1
L
D_1
α
Normal wind direction
D
β
D'
Instantaneous wind direction

Fig.1.

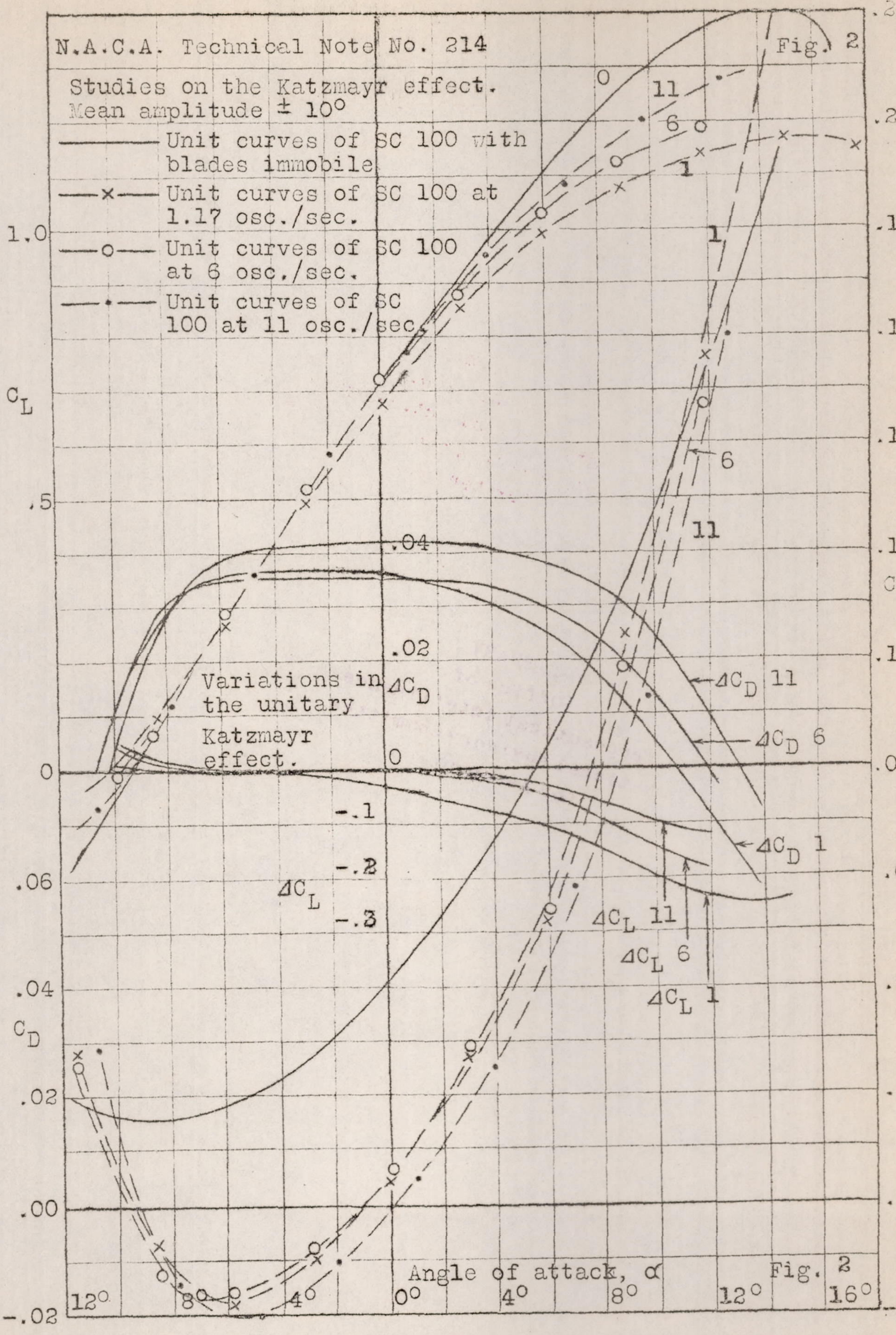

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Fig. 2
Studies on the Katzmayr effect.
Mean amplitude ± 10°
Unit curves of SC 100 with blades immobile
Unit curves of SC 100 at 1.17 osc./sec.
Unit curves of SC 100 at 6 osc./sec.
Unit curves of SC 100 at 11 osc./sec.
Variations in the unitary Katzmayr effect.
ΔC_D 11
ΔC_D 6
ΔC_D 1
ΔC_L 11
ΔC_L 6
ΔC_L 1
ΔC_D
ΔC_L
C_L
C_D
Angle of attack, α
12°
8°
4°
0°
4°
8°
12°
16°
1.0
.5
0
.06
.04
.02
.00
-.02
.22
.20
.18
.16
.14
.12
.10
.08
.04
.02
-.1
-.2
-.3
Fig. 2